MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 31/1981

DISCRETE GEOMETRY

12.7. bis 18.7.1981

Die Tagung fand unter der Leitung der Herren H.S.M. Coxeter (Toronto), L. Fejes Tóth (Budapest) und A. Florian (Salzburg) statt. Sie wurde von 60 Teilnehmern besucht, von denen 48 Vorträge hielten.

Thema der Tagung waren die diskreten Mannigfaltigkeiten geometrischer Gebilde. Dies umfaßte Packungen und Überdeckungen von Räumen, Theorie der Polytope, Zellkomplexe und Raumzerlegungen, Theorie der Punktgitter und kombinatorische Geometrie. Hinzu kamen Beiträge, die die Codetheorie, Konfigurationen von Punkten und Kreisen u.a. betrafen. Probleme dieser Art wurden vom metrischen oder vom kombinatorischen Gesichtspunkt aus untersucht. Nahestehende Gebiete sind etwa die Geometrie der Zahlen, Gruppentheorie und hyperbolische Geometrie. In neuerer Zeit werden auch Methoden der Codierungstheorie, etwa bei Packungsproblemen, mit Erfolg angewendet. Insgesamt wurde ein weites Spektrum geometrischer Fragestellungen behandelt. Die derzeit sechste Ausgabe einer Sammlung aktueller Probleme der diskreten Geometrie umfaßt 70 Titel mit (soweit bekannten) Resultaten und Literaturangaben und ist aus einer viel bescheideneren Liste offener Probleme hervorgegangen, die 1977 in Oberwolfach vorgestellt wurde.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Organisatoren und Teilnehmer haben die -wie stets-ausgezeichneten Voraussetzungen des Forschungsinstitutes für die Arbeit bei der Tagung und den Aufenthalt sehr zu schätzen gewußt.

Vortragsauszüge

R.P. BAMBAH:

On the Remak-Dyson-Skubenko-Lemma

Problem A: Given a lattice Λ in $\mathbb{R}^n$, do there exist the reals $\lambda_1,\dots,\lambda_n$ such that the ellipsoid $\sum \lambda_i x_i^2 \leq 1$ has no point other than O of Λ inside, but n linearly independent ones on the boundary? $\Longleftrightarrow$ Does there exist a transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ of the type $Tx_i = \mu_i x_i$, $\mu_i > 0$, $\Pi \mu_i = 1$, such that the lattice $T\Lambda$ has n linearly independent points on the boundary of the smallest sphere that contains non-zero points of $T\Lambda$?

Problem B: If Λ is a lattice like $T\Lambda$ above, i.e. among the points of Λ, other than O and nearest to O, exists a full set of n linearly independent points, does Λ provide a covering by $\sum x_i^2 \leq n/4$ and hence $|\Pi x_i| \leq 2^{-n}$ if det $\Lambda = 1$?

If for any n the answer to both A and B is yes, then a classical conjecture of Minkowski (i.e. every lattice of det = 1 provides a covering by $|\Pi x_i| \leq 2^{-n}$) follows.

The answer to B is known to be yes for n ≤ 6 (n = 5,6 A.C. Woods).

The answer to A is known to be yes for n = 2,3 (Remak), 4 (Dyson), 5 (Ryskov). The last one under mild restrictions on Λ allowed for proving Minkowski's conjecture (because of a result of Birch and Swinnerton-Dyer).

Simpler(!) proofs for n = 4,5 under some restriction on Λ, allowed for proving Minkowski's conjecture, for problem A are given in R.P. Bambah and A.C. Woods, J. Number Theory 1974, 1980.

U. BETKE:

Nichtkonvexe Ecken polyedrischer Mannigfaltigkeiten

Der Vortrag behandelt eine gemeinsame Arbeit mit P. Gritzmann. Für eine polyedrische Realisierung im R^3 einer geschlossenen, orientierbaren Mannigfaltigkeit (M,g) vom Geschlecht g sind Innengebiet I(M) und Außengebiet A(M) eindeutig bestimmt. Ein Punkt $x \in M$ heiße dann konvex, wenn es eine Kugel $U_\varepsilon(x)$ mit Mittelpunkt x und Radius ε gibt, so daß $U_\varepsilon(x) \cap I(M)$ oder $U_\varepsilon(x) \cap A(M)$ konvex ist. Andernfalls heißt x nicht konvex.
Es wird folgender Satz bewiesen:
Satz: Jede Realisierung von (M,g) hat mindestens 5 nichtkonvexe Ecken.

G. BLIND:

Packungen mit Reuleaux-Dreiecken

Um eine Schranke für die Packungsdichte von Reuleaux-Dreiecken zu gewinnen, wird zunächst das einem Reuleaux-Dreieck umbeschriebene Sechseck vom kleinstmöglichen Flächeninhalt bestimmt. Es hat dieselben Symmetrieeigenschaften wie das Reuleaux-Dreieck selbst. Daraus ergibt sich die Abschätzung $D \leq 0{,}9473$. Die vermutete beste Schranke ist 0,9229.

R. BLIND:

Die Anzahl der primitiven Polytope im $\mathbb{R}^d$ mit 2d-2 Facetten

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Zu einer polyedrischen Menge P des $\mathbb{R}^d$ sei N(P) die Menge der äußeren Normalen ihrer Facetten. Ein konvexes d-Polytop P heißt "primitiv", wenn es kein konvexes d-Polytop Q mit $N(Q) \subsetneq N(P)$ gibt.

Bekanntlich hat ein primitves Polytop mindestens d+1 Facetten und ist dann ein Simplex und höchstens 2d Facetten und ist dann ein Parallelotop. Ebenso sind die primitiven Polytope mit d+2 und die mit 2d-1 Facetten bekannt.

Die Verhältnisse bei den primitiven Polytopen mit 2d-2 Facetten sind wesentlich unübersichtlicher. So wird zunächst nur die Anzahl ihrer kombinatorischen Typen bestimmt.

A. BLOKHUIS:

Systems of lines with few angles in indefinite spaces

A system of lines in $\mathbb{R}^{d,1}$ with all mutual angles equal to arccos α and $\alpha^2(d+1) \geq 1$ has cardinality at most $\binom{d+1}{2}$.

A system of lines in $\mathbb{R}^{d,1}$ with all mutual angles equal to $\frac{\pi}{2}$ or arccos α has cardinality at most $\binom{d+2}{3}$.

U. BOLLE:

Mehrfache Kugelpackungen und -überdeckungen

Sei $k \in \mathbb{N}$, $K(\underline{x})$ die offene Einheitskugel mit Mittelpunkt $\underline{x}$ im $\mathbb{R}^m$ und G ein Gitter des $\mathbb{R}^m$.

$\{K(\underline{g})\}_{\underline{g} \in G}$ heißt eine k-fache gitterförmige Einheitskugel-packung bzw. -überdeckung im $\mathbb{R}^m$ genau dann, wenn jeder Punkt des $\mathbb{R}^m$ in höchstens bzw. mindestens k Kugeln aus $\{K(\underline{g})\}$ liegt. Die Dichte d(G) einer solchen gitterförmigen Kugelanordnung im $\mathbb{R}^m$ sei wie üblich definiert.

Schließlich sei

$$d_k^{(m)} := \sup \{d(G) \mid G \text{ liefert eine k-fache Packung}\}$$

$$D_k^{(m)} := \inf \{d(G) \mid G \text{ liefert eine k-fache Überdeckung}\}.$$

Dann gilt folgender

Satz: Sei $m \geq 2$ und seien $c_m, C_m > o$ geeignete nur von m abhängige Konstanten.

Dann gilt: a) für $m \not\equiv 1,5(8)$

$$\frac{d_k^{(m)}}{k} \leq 1 - c_m k^{-\frac{m+1}{2m}}, \quad \frac{D_k^{(m)}}{k} \geq 1 + C_m k^{-\frac{m+1}{2m}}$$

b) für $m \equiv 1,5(8)$

$$\frac{d_k^{(m)}}{k} \leq 1 - c_m k^{-\frac{m+3}{2m}}, \quad \frac{D_k^{(m)}}{k} \geq 1 + C_m k^{-\frac{m+3}{2m}}$$

K. BÖRÖCZKY:

The planes connecting the points of a finite set

The well-known Sylvester problem is the following: given n non-collinear points in the plane, thenthey determine a line, containing exactly two of the points.

In 1951 Motzkin formulated the following conjecture: Any n points in d-space that are not on one hyperplane determine at least one ordinary hyperplane, that is, a

connecting hyperplane on which all but one of the given points are on one linear (d-2)-space.
Motzkin proved his conjecture for d=3, and Hansen proved it for all d.

We prove the following theorem:

Any n points in 4-space, that are not on a hyperplane determine at least 5 ordinary hyperplanes.

Fan R.K. CHUNG:

Tilings of Rectangles with Rectangles

We consider the following geometrical packing problem: How can a rectangle be partitioned into subrectangles so that no proper subset of more than one subrectangle itself forms a rectangle? We will discuss some results on this and related problems.

H. S. M. COXETER:

The 600-cell {3, 3, 5} as a shadow of 4_{21}, the E_8 polytope

Anyone looking at the most symmetrical plane projections of the regular 600-cell in 4 dimensions, and the uniform polytope 4_{21} in 8 dimensions (Coxeter, Regular Complex polytopes, p. 42 and frontispiece) will notice that the 120 vertices of the former appear as 4 concentric 30-gons while the 240 vertices of the latter appear as 8 concentric 30-gons, 4 of these 8 being the same as for the 600-cell. This apparent coincidence is now explained by the observation that the 600-cell actually occurs as an orthogonal projection of half the vertices of 4_{21}. The remaining half form a smaller 600-cell, reduced in the golden ratio 1 : τ.

L. DANZER:

Locally finite tilings of $\mathbb{E}^2$ with topological discs, each of which has a rotational symmetry of $\frac{2\pi}{n}$ (n = 1,2,3,4,5,6,...).

This talk is closely related to Shephards lecture.

H. E. DEBRUNNER:

Tiling d-space wiht congruent simplices

Constructions are presented which yield monohedral tilings of euclidean d-space with prototype a simplex. Basically there are two ways: on the one hand "Hadwiger - Hill tilings" which arise by subdividing lattice tilings by parallelotopes, and which yield one parameter families of simplicial space fillers; on the other hand "Coxeter tilings" for some special simplex types which are fundamental regions of Coxeter groups. Cutting and pasting in these tilings give additional types and further modifications allow tilings with pairwise directly congruent tiles. No method is known to show completeness of the enumeration, but specialization to d=3 yields all known simplicial space fillers of E^3.

M. DEZA:

On some extremal (0, ±1)-vector problems

We shall discuss some extremal problems of sets of vectors whose entries can be only 0, +1, and -1, and which have prescribed scalar products. We shall also give analogs of some extremal results for binary vectors. This work was done jointly with Peter Frankl.

J. DOYEN:

Dissections of polygons

Let $\mu(P_m, Q_n)$ denote the minimum number of pieces in a dissection of P_m into Q_n (using the full group of isometries of the Euclidean plane to rearrange the pieces), where P_m and Q_n are convex regular m- and n-gons having the same area ($3 \leq m < n$). Theorem: $3 \leq \mu(P_m, Q_n) \leq (2m + 4)(n + 1)$.
A polygon P is called rep-k if it can be dissected into $k \geq 2$ pairwise congruent polygons, all similar to P. The convex rep-k polygons with $2 \leq k \leq 4$ have already been classified by Valette and Zamfirescu. Theorem: The only convex rep-5 polygons are (up to a similarity) the right triangle with sides of lengths 1, 2, $\sqrt{5}$ and the parallelograms with sides of lengths 1, $\sqrt{5}$.

I. FARY:

Electrical circuits and two complexes

It is usual to represent an electrical circuit with a graph whose edges correspond to the two terminal elements of the circuit. We cannot represent then a circuit containing transistors, etc. We describe a different method of representing electrical circuits. With our method, we can describe circuits containing transistors, etc.

G. FEJES TÓTH:

Packing of incongruent circles

Three different results concerning packing of incongruent circles are mentioned:

If a packing of circles is obtained as a plane section of a packing of equal balls in a finite-dimensional Euclidean space then the density of the circles in the plane is at most $\pi/\sqrt{12}$.

A packing of circles is said to be totally separable if each pair of the circles can be separated by a straight line avoiding all circles of the packing. In a joint paper with L. Fejes Tóth we phrased the conjecture that the density of a totally separable packing of circles is at most $11\pi/(24\sqrt{3}) = 0.831...$ and equality is attained for the face-incircles of the quasi-regular tiling (3,6,3,6). Here it is proved that the density of a totally separable packing of circles is at most 0.99.

In the third part of the talk the following result is proved: If the circles $C_1,...,C_n$ are packed into a convex polygon P with at most six sides and α is a real number ≤ 0.912 then the density d of the circles in P satisfies the inequality

(*) $$d \leq \frac{\pi}{\sqrt{12}} \frac{M_1(C_1,\ldots,C_n)}{M_\alpha(C_1,\ldots,C_n)} .$$

Here $M_x(C_1,\ldots,C_n)$ denotes the power mean of exponent x of the areas of the circles $C_1,\ldots,C_n$. Previously (*) has been proved for $\alpha \leq 0.77$ by F. Fejes Tôth.

L. FEJES TÓTH:

Packing and covering with convex discs

We denote a domain and its area with the same symbol and prove the following theorem: If the circles $c_1,\ldots,c_n$ are packed into a convex polygon P with at most six sides, then the packing-density $d = (c_1+\ldots+c_n)/P$ satisfies the inequality

$$\frac{1}{d} \geq 1 + \frac{\sqrt{12}-\pi}{\pi} \frac{M_{1/3}(c_1,\ldots,c_n)}{M_1(c_1,\ldots,c_n)} .$$

Here M_x denotes the power-mean of exponent x. Various applications are discussed. Similar theorems hold for general convex discs instead of circles, as well as for the covering instead of the packing.

A. FLORIAN:

Some remarks to an inequality on convex functions

Let $P = \{p_0, p_1, p_2, \ldots\}$ be a strictly increasing sequence of real numbers with $p_0=0$, $p_1=1$. Let δ_j $(j\geq 1)$ be defined by $\delta_j = \min\{p_{j+1}-p_j, p_j-p_{j-1}\}$. W. FLEISCHER [Monatsh. Math., to appear] proved the following theorem: Let f be a positive convex function, defined on $[1,+\infty)$, such that $\sum_{k=1}^{\infty} f(k) < +\infty$. Then

$$\sum_{j=1}^{\infty} f(p_j)\delta_j \leq \sum_{k=1}^{\infty} f(k). \qquad (1)$$

If f is strictly convex, equality occurs only if $p_j = j$ $(j = 1,2,\ldots)$.

Question: Does there exist a convex function f and a sequence $P \neq \{0,1,2,\ldots\}$ such that equality occurs in (1)?

The answer is in the negative.

R. L. GRAHAM:

Recent developments on the Steiner tree problem

In this talk I will discuss some recent developments which have occured in the so-called Steiner tree problem, i.e., the problem of finding the shortest connected network joining together a given set of points (usually in the plane).

R. J. HANS-GILL:

Some results on covering by star bodies defined by indefinite quadratic forms

Let $C_{r,n-r}$ be the infimum of all constants C such that for any indefinite quadratic form of type (r, n-r), $0 < r < n$, and real numbers $c_1,...,c_n$, there exist $(x_1,...,x_n) \equiv (c_1,...,c_n)$ (mod 1) such that

$$|Q\ (x_1,...,x_n)| \leq (C|D|)^{1/n}.$$

Hans-Gill and M. Raka (Monatsh. für Math. 88 (1979), 305-320 and Indian J. Pure and Appl. Math. 11 (1980), 75-91) have proved that $C_{2,3} = C_{3,2} = 1/4$ and $C_{1,4} = C_{4,1} = 1/2$. Recently, Raka has obtained $C_{r,n-r}$ for signatures $s = \pm 1, \pm 2, \pm 3, \pm 4$. This problem is related to the problem of covering by star bodies of the type

$$|x_1^2 + .. + x_r^2 - x_{r+1}^2 ... - x_n^2| \leq 1.$$

Several asymmetric results have also been obtained by the speaker jointly with Professors R. P. Bambah and V. C. Dumir.

H. HARBORTH:

1) Simple arrangements of straight lines in the plane
2) Point sets with equal numbers of unit-distant neighbors

1) In the plane n straight lines with all $\binom{n}{2}$ points of intersection being different is called a simple arrangement of these n lines. For infinitely many n we construct simple arrangements where (i) in the projective plane the maximum number of triangles is attained, (ii) in the projective plane no quadrilaterals occur, (iii) in the Euclidean plane the maximum number of colored regions of one color is attained for two-colorings where regions

with common sides have different colors. - So far extremal arrangements were known only for some $n \leq 16$.

2) Let n_k denote the smallest number of points in the plane such that (I) each point has distance 1 from exactly k other points, (II) all these unit distances do not intersect one another. - Results: $n_1 = 2$, $n_2 = 3$, $n_3 = 8$, $n_4 \leq 52$, and n_k does not exist for $k \geq 6$. It is an open problem whether n_5 exists.

A. HEPPES:

On two and three-saturated circle packings

In the Euclidean plane we shall consider a set of closed unit circles / circular discs /. If no two circles have inner points in common then the circles are said to form a packing. If any closed unit circle intersects at least k circles of the set then the set is said to be k-saturated.

The problem of finding the thinnest k-saturated set of unit circles is equivalent with the problem of the thinnest k-fold covering of the plane with equal circles. It is easy to see that in the Euclidean plane k-saturated packings of unit circles do not exist for $k > 3$.

The thinnest simple covering with equal circles is known its density is equal to $2\pi/\sqrt{27}$. Since the associated set is automatically a packing we can state that the density of the thinnest 1-saturated packing of unit circles is equal to $\pi/\sqrt{108}$. But the thinnest 2- and 3-saturated packings of unit circles cannot be determined in a similar way. On the one hand, the thinnest 2- and 3-fold coverings with equal circles are not known, on the other hand we do not know wether the associated sets are packings or not.

The problem itself, conjectures for the extremal configurations and a close lower bound for the density of the thinnest 2-saturated packing are due to L. Fejes Tôth. Recently A. Bezdek improved this result finding the extremal value of the density. The presentation is devoted mainly to the author's solution of the problem for k=3.

F. HERING:

A Steinitz Theorem

Let M denote a finite set of order $m = \# M$, let $\mu = \{M_1, \ldots, M_r\}$ be a partition of M such that $\# M_i \geq 2$, $i = 1, \ldots, r$, (i.e. $2r \leq m$), let $D \subseteq \{I \subset M : \# (I \cap M_i) = 1,\ i = 1, \ldots, r\}$ be nested and $Q = \{J \subset M : \# J = r+1$, there exists exactly one $I \in D, I \subset J\}$ and $q = \# Q$. Then there exists a convex $(m-r-1)$-polytope P_μ with m vertices and q facets, whose combinatorial structure is described as follows: There exists a bijective map α of the set of vertices of P_μ onto M and a bijective map β of the set of facets of P_μ onto Q such that for every vertex V and every facet F the relations $V \in F$ and $\beta(F) \not\ni \alpha(v)$ are equivalent. In the case r=2 this construction gives all simple m-3 polytopes with m vertices.

J. HORVÁTH:

Über die Enge der k-fachen Kugelpackungen und die Lockerheit der k-fachen Kugelüberdeckungen

Eine Menge von offenen (geschlossenen) Kugeln bildet eine k-fache Packung (Überdeckung) im E^n, wenn jeder Punkt des Raumes zu höchstens (mindestens) k-Kugeln gehört. Die Kugelpackung (Überdeckung) ist gitterförmig, wenn die Kugelmittelpunkte ein Punktgitter bilden.

Unter der Enge (Lockerheit) einer k-fachen Packung (Überdeckung) von Kugeln mit dem Radius R verstehen wir das Supremum der Radien (r) von Kugeln, die wir in den höchstens (k-1)-mal (mindestens (k+1)-mal) überdeckten Teilen einfügen können. Also $e_k^n(R)=\sup r$, wo $e_k^n(R)$ die Enge der Kugelpackung ist, und $\ell_k^n(R)=\sup r$, wo $\ell_k^n(R)$ die Lockerheit der Überdeckung ist. Wenn die Packung (Überdeckung) gitterförmig ist, dann verwenden wir die Bezeichnung $e_{k\Gamma}^n(R)$ $(\ell_{k\Gamma}^n(R))$.

Von L. Fejes Tóth wurde die folgende Frage aufgeworfen: Wie klein kann der Wert $e_k^n(R)$ $(\ell_k^n(R))$ sein? Bei welchen Packungen (Überdeckungen) tritt dieser Wert auf?

Im Vortrag bestimmen wir das Minimum von $e_{k\Gamma}^n(R)$ $(\ell_{k\Gamma}^n(R))$ und die zu diesem Minimum gehörigen Packungen (Überdeckungen), wenn (n,k) = (2,2); (2,3); (2,4); (3,2) sind. Weiterhin wird das Minimum von $e_2^2(R)$ $(\ell_2^2(R))$ bestimmt, wenn die Packung (Überdeckung) doppelgitterförmig ist.

E. JUCOVIČ:

A notion of combinatorial regularity of cell-complexes

A new concept of dimensional regularity of complexes, maps etc. is presented.

Results on 1-regular cell-decompositions of orientable surfaces and on 2-regular 3-dimensional polytopes are stated.

The connection between 1-regular planar graphs and a type of packings of congruent discs in the plane, is mentioned.

V. KLEE:

Some frightening tilings of infinite-dimensional Banach spaces

While the theory of 2-dimensional tilings is thriving (forthcoming book by Grünbaum & Shephard), and there was a recent "breakthrough" concerning n-dimensional tilings (1980 paper by McMullen in Mathematika), the theory of infinite-dimensional tilings seems to be nonexistent. However, recent results suggest that

(a) it may be possible to develop an interesting theory of infinite-dimensional tilings, and

(b) if so, it will probably be very different from the finite-dimensional theory. As evidence, we offer the following result, for which the 3-dimensional analogue would be a translational tiling of E^3 by regular octahedra:

If n is a regular infinite cardinal such that $n^{\aleph_0} = n$, then the Banach space $l^1(n)$ can be covered by a family of pairwise disjoint translates of the closed unit ball .

(There are arbitrarily large cardinals with the indicated property and under the generalized continuum hypothesis it is possessed by every infinite successor cardinal.)

P. KLEINSCHMIDT:

Combinatorial and affine automorphisms of convex polytopes

In 1969 the following problem was discussed by Grünbaum and Shepard:

Let ϕ be a combinatorial automorphism of the boundary-complex of a (convex) polytope P. Is there a polytope P', combinatorially equivalent to P, such that ϕ is induced by a symmetry of P' (i.e. an orthogonal transformation of P' onto itself)?

We give a negative answer to this problem by presenting a 4-polytope with 10 vertices, whose group of combinatorial automorphisms contains several elements which are not "affinely realizable". Several problems of the same kind will be discussed.

J. LINHART:

Oberfläche und Umkugelradius konvexer Polyeder

Sei P ein konvexes Polyeder im $\mathbb{R}^3$ mit e Ecken, k Kanten und f Flächen, welches in einer Kugel mit Radius R enthalten ist.

Dann gilt für die Oberfläche F von P vermutlich folgende Ungleichung

$$F/R^2 \le k \sin \frac{\pi f}{k} \left(1 - \cot^2 \frac{\pi f}{2k} \cot^2 \frac{\pi e}{2k}\right),$$

wobei im Falle der regulären Polyeder das Gleichheitszeichen gilt. Diese Ungleichung konnte bisher nur unter der Voraussetzung ("Fußpunktbedingung") bewiesen werden, daß die Fußpunkte der vom Kugelmittelpunkt auf die Flächenebenen und Kantengeraden gefällten Lote auf den entsprechenden Flächen bzw. Kanten liegen. Im Vortrag wird ein Beweis diskutiert, der nur die Fußpunktbedingung bezüglich der Kanten benützt.

E. MAKAI, Jr.:

Analogues of a theorem of Erdös-Pach

L. Fejes Tóth posed a question dual to Tarski's plank problem: how thin can be a system of points $\{P_i\}$ in the plane, for which for any straight line l for some P_i distance $(P_i, l) \le 1$. Erdös-Pach proved that in terms of $r_i = OP_i$ (O = origin) a necessary and sufficient condition is $\sum r_i^{-1} = \infty$ (for $r_i \to \infty$). Joining to a question of L. Fejes Tóth and Strauss we say that a system of points $\{(x_i, y_i)\}$ in the plane controls some family F of functions, if for any $f \in F$ for some i $|f(x_i)-y_i| \le 1$. We ask how thin $\{x_i\}$ can be, such that there exists $\{y_i\}$, $\{(x_i,y_i)\}$ controlling F. For F = {linear functions} a necessary and sufficient condition is $\sum(|x_i|+1)^{-1} = \infty$. We treat the case of linear functions

$R^n \to R^m$ too. For $F = \{\text{Lipschitz-functions: } [0,\infty) \to R^m\}$, supposing $x_1 \leq x_2 \leq \ldots$ and $x^m_{i+1} - x^m_i$ has a limit, a necessary and sufficient condition is that this limit is 0. For $F = \{\text{polynomials of degree} \leq n\}$ we conjecture the condition $\sum(|x_i|^n + 1)^{-1} = \infty$. The proofs are geometrical.

P. McMULLEN:

Regular polyhedra and their near relations

This talk is centred around the general area of regularity, but also concerns itself with more or less closely related ideas. For example, a closed polyhedral manifold is equivelar if each (convex) face has the same number p of sides, and each vertex has the same valence q. It is natural to ask which pairs p, q give rise to equivelar polyhedra in E^3. The results here are perhaps somewhat surprising, because their consequences go rather against intuition. Also considered are related regular polyhedra with convex faces in higher dimensions, and yet further polyhedra, which are regular in Grünbaum's more extended sense. Again, an obvious problem is whether such polyhedra can be realized geometrically in E^3.

J. MOLNÁR:

On the packing of congruent spheres in certain convex cylinders

The main result is the following THEOREM: If A resp. P denotes the area resp. the perimeter of the convex base of an orthogonal cylinder of altitude a $(2 \leq a \leq 2+\sqrt{2})$, in which are packed n unit spheres, then

$$n \leq \frac{1}{2\sqrt{4a-a^2-1}} A - \frac{1}{2}\left(\frac{1}{\sqrt{4a-a^2-1}} - \frac{1}{\sqrt{4a-a^2}}\right) P$$

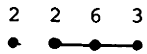
$$- \pi\left(\frac{1}{\sqrt{4a-a^2}} - \frac{1}{2\sqrt{4a-a^2-1}}\right) + 1 .$$

Denoting by $\{C_i\}$ the circlesystem obtained by the orthogonal projection of the unit spheres onto the base of the cylinder, equality holds if and only if the base of the cylinder is the convex hull of $\{C_i\}$ and the convex hull h of centers of $\{C_i\}$ has one of the following properties: a) h is a set of triangles of sides $2,\lambda,\lambda$ $(\lambda = \sqrt{4a-a^2})$ having the vertices the centers of the circlesystem and forming a poligon of sides with length some multiple of λ; b) h is a segment of straight line of length λk, containing $k+1$ centers of circles; c) h is a point.

B. MONSON:

<u>Variants of Mennicke's "Torte" in Hyperbolic Space</u>

In "Theorie der Automorphen Functionen", Fricke and Klein determined the unit group G for various integral quadratic forms $f = px^2 - qy^2 - rz^2$. A somewhat easier and more general approach is to use-f to define the metric in the hyperbolic plane H^2. Then the subgroup F generated by reflections in G has a fundamental region P, which may be determined by Vinberg's geometric algorithm. Another instance is Mennicke's form f, where $-f = -x^2 + 2y^2 + 3z^2 + 3u^2$ is equivalent to the simplicial form



Then 12 copies of P fit around one edge to form a "Torte" T, which tiles H^3 and has only right dihedral angles.

(T is formed by interlocking two hexagonal "flowers", each with 6 pentagonal "petals").

Other simplicial forms yield a variety of interesting Torten.

W. MOSER:

<u>Problems in Discrete Geometry</u>

At the July 1977 Discrete Geometry week I presented 14 problems, for research, proposed by my brother Leo Moser (1921-1970). I now present the preliminary version of the sixth edition of this collection: Research Problems in Discrete Geometry. This edition contains 70 problems, each with known results and relevant bibliography. All participants will receive a copy late this year.

A. NEUMAIER:

Discrete hyperbolic geometry

1. Classification of reflection groups in hyperbolic space: Techniques an partial results.
2. Combinatorial configurations in hyperbolic space; in particular an attempt to find strongly regular graphs with 75,76,95 and 96 vertices.
3. The hyperbolic Leech lattice and related lattices.

J. PACH:

Some Helly Type Theorems

The aim of this paper is to establish some quantitative versions of Helly's Theorem. One of our results, proved jointly with I. Bárány and M. Katchalski, is the following. Let C be an arbitrary finite family of convex sets in the d-dimensional euclidean space. Then there exists a constant c(d), depending only on d, such that if the intersection of every 2d members of C is of volume at least c(d), then the intersection of all members belonging to C has volume ≥ 1. A similar theorem is true for diameters, instead of volumes. A common root of these two results is the following lemma: Suppose there is a set V of vectors in the d-dimensional space, whose convex hull (conv V) contains a ball of radius r, around the origin. Then there exists a subset V' of V, which has at most 2d elements, and conv $\underline{V}'$ contains a ball of radius $r\varepsilon(d)$, around O. Here $0 < \varepsilon(d) \leq 1$ is an absolute constant.

J. F. RIGBY:

Configurations of circles and points

In the real inversive plane, every Money-Coutts configuration of nine cycles each anti-touching four others can be derived from a (9_4) configuration of circles and points. Certain (9_4) configurations can be embedded in a $(15_4 10_6)$ configuration.

The circles of a $(15_4 10_6)$ configuration (called the auxiliary circles) meet again in 60 P-points. The P-points lie (i) by fours on 45 M-circles which touch by threes at the P-points, (ii) by fours on 15 O-circles orthogonal to the M-circles, (iii) by threes on 20 S-circles orthogonal to M-circles, and (iv) by fours on 60 K-circles. The (60_4) configuration of K-circles and P-points splits up into six (10_4) configurations, each of which can be embedded (using five of the auxiliary circles) in a $(15_4 10_6)$ configuration. It is conjectured that these six $(15_4 10_6)$ configurations are inversively isomorphic to each other but not to the original auxiliary configuration.

Also connected with these configurations are various (8_4) Clifford configurations and symmetric (32_6) Clifford configurations.

D. SCHATTSCHNEIDER:

Tiling the plane with congruent pentagons--the non-convex case

Pentagons which tile the plane are classified into types according to minimal conditions on their sides and angles which ensure the existence of a tiling. Thirteen types of convex pentagons are known to tile the plane; just five of these types can tile isohedrally. Convexity ensures all thirteen types tile properly, i.e., all vertices of the tiling have valence ≥ 3. Nine of the thirteen types also have non-convex pentagons which satisfy the defining relations. If one distinguishes the position of non-convex angle, there are fifteen distinct classes of non-convex pentagons which tile properly; ten of these classes have isohedral tilings.

For non-convex pentagons, the sum of two angles can be 360°; if these two angles are fitted together in a tiling, we say the tiling is improper. Nine types of non-convex pentagons which can only tile improperly have been found; five of these types have isohedral tilings.

Five distinct types of equilateral pentagons tile. Two improper types each have an infinite range of tiles, two proper types each have an infinite range of convex and non-convex tiles, and one proper type has a unique convex tile.

E. SCHULTE:

Reguläre Inzidenzkomplexe

Das Konzept der "regulären (Inzidenz-) Komplexe" verallgemeinert den klassischen Begriff des regulären Polyeders im kombinatorischen und gruppentheoretischen Sinne. Ein regulärer Komplex ist eine spezielle teilweise geordnete Struktur, deren Regularität über die Fahnentransivität ihrer Automorphismengruppe definiert wird.

Ausgehend von einer gruppentheoretischen Charakterisierung der kombinatorischen Struktur regulärer Komplexe kann zu geeigneten Gruppen U ein Komplex konstruiert werden, auf dem U fahnentransitiv operiert. Das führt u.a. zu vielen interessanten 2-dimensionalen regulären Komplexen. Ferner entstehen auf diese Weise eine Reihe von 4-dimensionalen Komplexen, deren Facetten und Eckfiguren zu euklidisch regulären 3-Polytopen oder zu einer der regulären Torus-Pflasterungen $\{4,4\}_{b,c}$, $\{6,3\}_{b,c}$, $\{3,6\}_{b,c}$ (b=c oder c=0) isomorph sind.

J.J. SEIDEL:

Discrete hyperbolic geometry

Some examples of hyperbolic geometry as a tool for combinatorial investigations: permanents, root systems, lattices, codes, graphs, 2-distance sets.

G.C. SHEPHARD:

Tiles with 5-fold symmetry

The crystallographic restriction asserts that no plane figure with discrete symmetry group can have more than one centre of 5-fold rotational symmetry. However this does not imply that it is impossible to find a tiling in which each tile has 5-fold rotational symmetry.

In this talk, a number of attempts to tile the plane in this manner will be described. In particular, if we measure the size of a "patch" of tiles (each with 5-fold symmetry) by

ρ = (diameter of largest circular disk covered by the tiles in the patch) / (maximum of the diameters of the tiles in the patch),

then the largest known patch has $\rho = 4.786$. It is an unsolved problem whether this is the largest possible value of ρ, or if, indeed a tiling of the whole plane by bounded tiles is possible.

F.A. SHERK:

Configurations in PG (1,q)

Two sets of n points in PG (1,q) ($q=p^k$, p prime) belong to the same equivalence class if and only if there is a projectivity or anitprojectivity mapping one onto the other. For given small values of q, these n-configurations are classified, and the symmetry group of each n-configuration is found. Special attention is paid to the case q=16, where for example it is shown that these are four classes of 6-configurations and seven of 7-configurations.

It is shown how configurations in PG (1,q) can be used to classify translation planes of order q. The classification is illustrated in detail in the case q=16.

N.J.A. SLOANE:

Voronoi regions of lattices and quantization

This talk, which is based on joint work with J.H. Conway, studies the Voronoi regions of various lattices and the second moments of these regions about their centers.
The vertices of the Voronoi region are the "holes" in the lattice, and the most distant vertices from the origin determine the covering radius of the lattice. The lattices considered are the root lattices A_n, D_n, E_6, E_7, E_8 and the polar lattices A_n^*, D_n^*. For the Leech lattice in joint work with Richard Parker we have determined that the covering radius is $\sqrt{2}$ times the packing radius, but have not yet found the Voronoi region. The results have applications to the design of quantizers (analog-to-digital converters) and to the selection of signals for transmission over a band-limited channel.

B.A. TROESCH:

On the tiling of simply - and multiply - connected patios.

A patio has to be covered by rectangular non-overlapping tiles of arbitrary size. What is the minimum number of tiles needed? By scaling, the problem is identical to: On a large checkerboard, squares are marked; cover them with the minimum number of tiles. Upper and lower bounds are easily obtained. It is global question in that the configuration far away influences the local tiling. The best tiling is not unique. Although there are similarities with the geometrical approach to switching theory, those results (Quince-McCluskey algorithm) are not applicable. By considering significant elements of the configuration (corners, connecting lines between corners, and isthmuses which separate different parts of the configuration) an empirical formula is given for the minimum number of tiles.

H. TVERBERG:

A Generalization of Radon's Theorem

In 1964, the following theorem was proved (JLMS 41 (1966), 123-128): Let A be a set of rd+r-d points in real d-space. Then A can be partitioned into r sets $A_1,\ldots,A_r$ in such a way that $(\operatorname{conv} A_1) \cap (\operatorname{conv} A_2) \ldots \cap (\operatorname{conv} A_r) \neq \emptyset$. The case r=2 is Radon's theorem. The proof was quite complicated, and in this talk a simpler proof will be presented. The main idea of the proof is to consider partitions of A for which $\operatorname{conv} A_1 \neq \emptyset$, $(\operatorname{conv} A_2) \cap \ldots \cap (\operatorname{conv} A_r) \neq \emptyset$ (the existence of them is assured by the case of the theorem when $(r-1)d+(r-1)-d$ points are given), and choose such a partition for which the distance between these two sets is minimal. The assumption that this minimum is positive is shown to give a contradiction.

G. VALETTE:

Polyhedral mitosis

If P is polyhedron of E^3, we call mitosis of P a dissection {P', P"} of P such that P' and P" are combinatorially equivalent to P.
We describe the pairs (P,π) where π is the plane which separates the elements P' and P" of a mitosis of P.

G. WEGNER:

Extremale Groemerpackungen

L. Fejes Tóth stellte 1975 die Frage, wie man n Einheitskreise in der Ebene packen muß, damit die Fläche der konvexen Hülle dieser Packung minimal wird. Mit $n = 1 + 6\binom{a}{2} + ab + c$, wobei $a > 0$, $0 \leqq b \leqq 5$ und $0 \leqq c \leqq a-1$, gilt für jede Packung G aus n Einheitskreisen

$$F(\text{conv } G) \geqq F_o(n) := (n-1)2\sqrt{3}+p_o(n)(2-\sqrt{3})+\pi \quad \text{mit}$$

$$p_o(n) := 6(a-1)+b-\delta_{o,b+c}+1.$$

Dabei kann Gleichheit nur eintreten für geeignete sechseckförmige Ausschnitte der dichtesten gitterförmigen Packung (Groemerpackungen). Eine Groemerpackung heiße extremal, wenn das Gleichheitszeichen eintritt, und n heiße Ausnahmezahl, wenn es keine extremale Groemerpackung aus n Einheitskreisen gibt. Ich vermute, daß die Ausnahmezahlen genau gegeben werden durch $b = 2 \wedge a - c \equiv -6m \bmod 9^{m+1}$ und $b = 5 \wedge a - c \equiv 6.9^{m} \bmod 9^{m+1}$ $(m = 0, 1, 2, \ldots)$. Im Falle m = 0 sind dies tatsächlich Ausnahmezahlen. Außerdem können Serien von Zahlen angegeben werden, die keine Ausnahmezahlen sind.

J.B. WILKER:

Topologically Equivalent N-Dimensional Isometries

Let G be the group of isometries of the N-sphere, Euclidean N-space or hyperbolic N-space, the group of similarities of Euclidean N-space or the group of Möbius transformations of the N-sphere. Two elements g and g' of G are topologically equivalent if there is a homeomorphism h of the space on which G acts such that $g' = h^{-1}gh$. In each case, we determine the equivalence classes when N=2 and make progress with determining them when N>2. Our results for N=2 help to explain the intuitive appeal of the homeomeric classification of patterns on the 2-sphere and in the Euclidean plane.

J.M. WILLS:

Minimal width of lattice-point-free convex bodies

(joint paper of P. McMullen and J.M. Wills)

Let K be a convex body in d-dimensional euclidean space E^d, let $\Delta(K)$ denote its minimal width, $D(K)$ its diameter, and $G^o(K) = \text{card}((\text{int } K) \cap \mathbb{Z}^d)$ the number of lattice points in the interior of K. If $G^o(K) = 0$, we call K lattice-point-free. Further let

$\Delta_d = \max\{\Delta(K)/K \subset E^d,\ G^o(K) = 0\}$. Then we prove:

Theorem 1 $\Delta_d > (\sqrt{2}+1)(\sqrt{d+1}-\beta)$, where $\beta \sim 1{,}0181$

Theorem 2 For a lattice-point-free $K \subset E^d$ holds:

a) $$\left(\frac{\delta_{i(K)}}{\Delta_{d-1}} - 1\right)(D_i(K)-1) \leq 1 \qquad \text{for } i=1,\dots,d$$

b) there are constants λ_d, μ_d (independent of K) with

$$\frac{1}{\sqrt{2}} \leq \lambda_d \leq 1,\ \frac{1}{\sqrt{d}} \leq \mu_d \leq 1 \quad \text{such that}$$

$$\left(\lambda_d \frac{\Delta(K)}{\Delta_{d-1}} - 1\right)(\mu_d D(K)-1) \leq 1.$$

The case d=2 was solved by P.R. Scott in 1973 resp. 1979.

T. ZAMFIRESCU:

Magic mirrors

Theorem 1. For most convex planar curves, most points of the plane lie on infinitely many normals.

Suppose now the curves reflect light like mirrors.

<u>Theorem 2.</u> For most convex planar curves and most pairs of points in the plane, the one point sees infinitely many images of the other.

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