MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Harmonische Analyse
und Darstellungstheorie topologischer Gruppen

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Die Tagung fand unter der Leitung von Herrn R. Howe (New Haven) und Herrn D. Poguntke (Bielefeld) statt. Es wurden vor allem Fragen der kommutativen und nichtkommutativen harmonischen Analyse, der Darstellungstheorie lokal kompakter, insbesondere Lie'scher Gruppen und der Theorie invarianter Differentialoperatoren behandelt.

Vortragsauszüge

M. Bozejko: Positive definite functions on the free group and the noncommutative Riesz product

Let $\mathbb{F}_2$ be the free group generated by the two free generators a and b, and $A = gp\{a\}$, $B = gp\{b\}$. For functions $u,v : A \to \mathbb{C}$ we define the free product $\varphi = u \circ v$ in the following way:

If $x = a_1 b_1 a_2 b_2 \dots a_k b_k$, where $a_j \in A$, $b_j \in B$, then

$$\varphi(x) = u(a_1)v(b_1)u(a_2)v(b_2) \dots u(a_k)v(b_k).$$

We show the following

Theorem: If u and v are positive definite functions on A and B, respectively, then the free product $\varphi = u \circ v$ is a positive definite function on the free group $\mathbb{F}_2$.

We can also extend this result to the free group with an infinite number of generators and then we have the following

Corollary 1: If $u_k \in P(A_k)$, $u_k(e) = 1$, then the free product

$\varphi = \bigcirc_{k=1}^{\infty} u_k$ is a positive definite function on the free group $\mathbb{F}_\infty$.

Corollary 2: The function

$$R = \bigcirc_{k=1}^{\infty} (\delta_1 + \alpha_k \delta_{n_k} + \bar{\alpha}_k \delta_{n_k}), \quad n_k \in A_k = \mathbb{Z}, \quad |\alpha_k| \leq \frac{1}{2},$$

(we called R the noncommutative Riesz product) is positive definite on $\mathbb{F}_\infty$. We can also obtain the following result:

Proposition: Let $\delta = 4 \sum_{k \neq 1} |\alpha_k||\alpha_1|$, then

(1) $R \in P_\lambda(\mathbb{F}_\infty) \Leftrightarrow \delta \leq 1.$

(2) $R \in l^2(\mathbb{F}_\infty) \Leftrightarrow \delta < 1.$

(3) If $\sum_{k=1}^{\infty} |R(n_k)|^2 = \infty$, then $R \in P_\lambda^\perp$.

Those results extend the theorems of De Michele and Figa-Talamanca and Alisene, De Michele.

J. Faraut: Spherical Laplace transform on a hyperboloid

Let $G = SO_0(1,n)$, and $H = SO_0(1,n-1)$. There exists a commutative convolution algebra of functions on G, biinvariant under H. These functions are not integrable, but it is possible to convolve two of them because they are supported by a semigroup contained in G. The spherical Laplace transform $L(f)$ of such a function f is an analytic function on a half-plane in the complex plane, and we have $L(f_1*f_2) = L(f_1)\cdot L(f_2)$. Using horospheric coordinates, the spherical Laplace transform appears as the composition of a Radon transform on G/H and the ordinary Laplace transform. From this decomposition follows the inversion formula of the spherical Laplace transform.

R. Felix: Solvability of certain differential equations and harmonic analysis on nilpotent Lie groups

Sei X ein nilpotenter Endomorphismus von $\mathbb{R}^n$ und L das durch $L\varphi(x) := d\varphi(x)(Xx)$, $\varphi \in \mathcal{D}(\mathbb{R}^n)$ definierte Vektorfeld. Der Lösungsraum in $\mathcal{S}'(\mathbb{R}^n)$ der homogenen Gleichung $Lu = 0$ besteht aus den unter $\operatorname{Exp} \mathbb{R}X$ invarianten temperierten Distributionen. Wir geben eine Fundamentalmenge M_X für diesen Lösungsraum an und zeigen, daß genau für diejenigen Funktionen $f \in \mathcal{S}$, die von M_X annulliert werden, die Gleichung $Lu = f$ eine Lösung $u \in \mathcal{S}$ hat, d.h. es ist $L\mathcal{S} = M_X^{\perp}$. Die Lösung ist eindeutig und kann durch die Formel $u(x) = \int_0^\infty f(\operatorname{Exp} tXx)dt$ explizit angegeben werden. Folglich ist ein Differentialoperator D, der sich als Produkt solcher Vektorfelder L schreiben läßt, ein topologischer Isomorphismus von $\mathcal{S}$ auf einen abgeschlossenen Unterraum von $\mathcal{S}$, so daß die Gleichung $Du = f$ für jede temperierte Distribution

$f \in S'$ eine Lösung $u \in S'$ hat. (Die entsprechende Aussage für $\mathcal{D}$ bzw. $\mathcal{D}'$ ist im allgemeinen falsch.)

Durch Verschärfung der Aussage $LS = M_X^{\perp}$ für den Spezialfall, daß die Jordansche Normalform von X aus nur einem einzigen Jordankästchen besteht, können wir für eine bestimmte Folge (N_n) nilpotenter einfach zusammenhängender Liegruppen, in der Gruppen beliebig hohen Nilpotenzgrades vorkommen, das von C. Herz gestellte Syntheseproblem, ob die zentralen temperierten Distributionen auf N_n approximiert werden können durch Linearkombinationen von Charakteren irreduzibler Darstellungen, für $n \neq 2$ positiv beantworten. Für $n = 2$ ist die Antwort negativ nach einem Beispiel von Dixmier.

R. Gangolli: Some analogies between Number Theory and Riemannian Geometry

Let G be a connected semisimple Lie group with finite center, K a maximal compact subgroup of G. We assume $\mathrm{rank}(G/K) = 1$. Let $\Gamma' \subset \Gamma$ be discrete torsion free uniform subgroups of G such that Γ' is normal and of finite index in Γ. Let $M' = \Gamma' \backslash G/K$, and $M = \Gamma \backslash G/K$. Then the natural map $\pi : M' \to M$ is a Riemannian covering (with the obvious Riemannian structures on M', M) with group $H = \Gamma/\Gamma'$. For any character χ of H viewed as a character of Γ, one has the zeta function $Z_\Gamma(s,\chi)$ attached to (G,K,Γ,χ) as in Gangolli (Ill. J. Math. 1977). The set up $(M',M,H,Z_\Gamma,Z_{\Gamma'})$ is analogous to $(Q',Q,\mathrm{Gal}(Q'/Q), \zeta_{Q'},\zeta_Q)$ where Q' is a finite normal extension of Q, and $\zeta_{Q'},\zeta_Q$ are the Dedekind and Riemann zeta functions of Q',Q. A rather precise analogy can be formulated, in which the set of closed primitive geodesics in M,M' play the rôle of primes, and all the density theorems of analytic number theory (Dirichlet, Chebotarev) as well as the basic theorem of class field theory (Artin reciprocity) can be formulated and proved for $(M,M',H,Z_\Gamma,Z_{\Gamma'})$.

R. Goodman: Iwasawa Decompositions of Loop Groups

Let $SL_n(\mathcal{W})$ be the group of $n \times n$ matrices of determinant one, with entries in the Wiener algebra $\mathcal{W}$ of absolutely convergent Fourier series on the circle. This is a Banach-Lie group, whose Lie algebra is the tensor product $\underline{s\ell}_n(\mathbb{C}) \otimes \mathcal{W}$. Let $SU_n(\mathcal{W})$ be the Lie subgroup of unitary matrices over $\mathcal{W}$, and V_n the Lie subgroup of elements whose negative Fourier coefficients vanish and constant term is I_n. Let A_n and N_n be the usual subgroups of $SL_n(\mathbb{C})$ of positive diagonal matrices and uppertriangular unipotent matrices, resp.

Theorem: There is a unique factorization $SL_n(\mathcal{W}) = V_n \cdot N_n \cdot A_n \cdot SU_n(\mathcal{W})$, which is an isomorphism as a product of real-analytic manifolds.

An analogous decomposition is obtained for certain Banach Lie groups $\hat{G}$ over $\mathcal{W}$ associated with a real form G of a complex reductive algebraic group. (Joint work with N.R. Wallach).

R. Gustafson: Non cuspidal discrete series representations of Chevalley groups over local fields

The construction of non-cuspidal discrete series representations of the split semi-simple group G over the p-adic field h which occur in the unramified principal series of G is reduced to examining the components of certain degenerate principal series representations $\mathrm{ind}_P^G \chi$ where χ is an unramified character of a maximal parabolic subgroup P and ind is unnormalized induction. An involution $\wedge$ on the unramified principal series is constructed, which maps the trivial representation of G to the Steinberg representation. The degenerate principal series is decomposed in the case $G = Sp(2n,h)$ by the explicit computation of the action of certain Hecke operations in $H(G,B)$ i.e. the Iwahori-subgroup B-bi-invariant functions with compact support, acting on the space of B-fixed vectors in $\text{ind}_p^G \chi$.

S. Helgason: Integral representations of eigenfunctions

In this lecture we give an expository survey of results obtained within the last to years on integral representations of joint eigenfunctions of invariant differential operators on homogeneous spaces G/H.
A detailed discussion will be given of the following cases:

(i) G/H = non-Euclidean disk
(ii) G/H = symmetric space of the noncompact type.
(iii) G/H = symmetric space of zero curvature
(iv) G/H = semisimple symmetric space.

References will be given to work of Kashiwara, Konata, Minemura, Okamoto, Oshima, Tanaka, Sekiguchi, Hashizume, Hiraoka, Matsumoto, Morimoto, J. Lewis and the speaker.

R.W. Henrichs: Onesided harmonic analysed and uniform continuity

H. Leptin has introduced the left Wiener property.
(L) Every closed, proper left ideal I in $L^1(G)$ is annihilated by a non-zero continuous positive definite function φ $(\varphi \in P(G))$.
It is a fact that every closed, proper left ideal I in $L^1(G)$ is annihilated by a right uniformly continuous function. Let $C_u^b(G)$ be the set of right and left uniformly continuous bounded functions; clearly $P(G) \subset C_u^b(G)$.

Theorem 1. Let $F(G)$ be $P(G)$ or $C_u^b(G)$. If every closed, proper left ideal I in $L^1(G)$ is annihilated by some $0 \neq \varphi \in F(G)$, the (multiplicative) characters of closed subgroups always have extension $\varphi \in F(G)$.

Theorem 2. Let G be almost connected. Then the following conditions are equivalent.

(i) G is a semidirect product $G = \mathbb{R}^h \times_\mu K$ where K is compact and $\mu(K)$ finite.

(ii) G has the extension property as in Theorem 1.

(iii) G is a (SIN)-group, i.e. the left uniform structure and the right uniform structure are equivalent.

For connected groups we obtain a short proof of a result of Leptin.

C. Herz: Boundaries and Poisson kernels

I $\underline{g}$ is a semi-simple Lie algebra of non-compact type the symmetric space Σ of $\underline{g}$ is the set of Cartan involutions viewed as a submanifold of $Aut(\underline{g})$ with a Riemannian structure from the Killing form. The boundary B of $\underline{g}$ is the set of maximal unipotent subalgebras of $\underline{g}$ given the manifold structure of a homogeneous space of $Aut(\underline{g})$. If V is a vector space and G an analytic subgroup of $Aut(V)$ with Lie algebra $\underline{g}$ each $b \in B$ corresponds to a flag $V_1(b) \subset V_2(b) \ldots \subset V_r(b)$ of subspaces giving a 'triangular' form to b. The spaces $V_1(b)$ as b ranges over B form a smooth subvariety of a Grassmann manifold B_1 on which $INT(\underline{g})$, the adjoint group, acts. The same is true for the bundle of spaces $V_2(b)$ over B_1. One gets a sequence $B_1 \leftarrow B_2 \leftarrow \ldots B_r = B$ of bundles whose fibres are subvarieties of Grassmann manifolds. Given $\sigma \in \Sigma$, put $K(\sigma)$ for the centralizer of σ in $INT(\underline{g})$, the adjoint group. It is easy to construct inductively $K(\sigma)$-invariant measures on B_r. One gets the invariant measure $\mu(\sigma,b)$ on B and $\mu(\sigma,b) = P(\theta,\sigma,b)\mu(\theta,b)$ gives the Poisson kernel.

Andrzej Hulanicki: A multiplier theorem for the heat semi-group on on statified nilpotent Lie groups

This is a joint work with E.M. Stein.

Let G be a stratified nilpotent Lie group, L the homogenous sublaplacian on G. Let E_λ the spectral resolution of L on $L^2(G)$.

Theorem Let $m(\lambda)$ be a bounded function on $\mathbb{R}^+$ such that for a positive integer K $(K = K(G))$

$$\left| \frac{d^\ell}{d\lambda^\ell} m(\lambda) \right| \leq \frac{C}{\lambda^\ell} \qquad \ell = 0,1,\ldots,K$$

Then the operator

$$T_m f = \int_0^\infty m(\lambda)\, d\, E_\lambda f \,, \quad f \in L^2(G),$$

is of weak type $1 - 1$ and bounded on all $L^p(G)$, $1 < p < \infty$.

Palle E.T. Jørgensen: Harmonic analysis of infinitesimal generators for ∞-dimensional group representations

Let $\pi : G \to L(B)$ be a strongly continuous representation of a Lie group G in a Banach space B, and let $d\pi$ be the corresponding infinitesimal representation of the Lie algebra $\mathcal{G}$. Then $L = d\pi(\mathcal{G})$ is a Lie algebra of, generally unbounded, operators on B. Any pair of operators $X,Y \in L$ satisfies certain commutation relations. We shall reverse the point of view, considering a given system of operators defined on a common dense domain in B. The results are applied to the representation theory of $s\ell_2(\mathbb{R})$, and to the automorphism groups of the C^*-algebra of the canonical anti-commutation relations. The following questions are addressed:

(i) Exponentiability (alias integrability) of infinitesimal systems,

(ii) structure theory and classification,

(iii) perturbations and spectra.

Adam Koranyi: Geometric properties of Heisenberg type groups

On the Heisenberg group and on some of its generalizations there is a natural left-invariant distance function defined with the aid of a 'gauge' (or 'homogeneous norm'). It is shown here that the associated notion of acr length coincides with the arc length obtained from a certain generalized Riemannian metric M_0 which was introduced by B. Gaveau. M_0 is in a natural way the limit of a family of ordinary Riemannian metrics M_c as c tends to zero. This remark is exploited here for the purpose of studying geodesics in M_0 and for the construction of some examples in Riemannian geometry.

H. Leptin: Verschränkte Faltungen

Sei F eine beliebige zusammenhängende Liesche Gruppe und $c : (x,y) \to c(x,y) \in \Pi$ ein glatter 2-Cozykel auf F mit Werten in $\Pi = \{z \in \mathbb{C};\ |z| = 1\}$. Für eine Distribution $T \in \mathcal{E}'(F)$ und eine Testfunktion $\varphi \in \mathcal{D}(F)$ sei $T_c(\varphi)(x) = \int_F \varphi(y^{-1}x)c(y,y^{-1}x)d\,T(y)$. Für $c = 1$, d.h. $1(x,y) \equiv 1 \in \Pi$ ist $T_1(\varphi) = T * \varphi$ die gewöhnliche Faltung. Allgemein ist $T_c : \varphi \to T_c(\varphi)$ ein linearer Operator auf $\mathcal{D}(F)$.
Mit Hilfe funktional-analytisch-gruppentheoretischen Oberlegungen und des Faktorisierungssatzes von Dixmier und Malliavin wird der folgende Satz bewiesen: Sei $1 < p < \infty$. Der Operator T_c ist dann und nur dann beschränkt in $L^p(F)$, wenn T_1 in $L^p(F)$ beschränkt ist. Für $F = \mathbb{C}^n$ und den Weyl'schen Cozykel c erhält man ein von M. Cowling mit anderen Methoden bewiesenes Ergebnis.

Wen-Ch'ing Winnie Li: On Converse Theorems of GL_1 and GL_2

Let F be a global field and A_F the adele ring of F. Let G be either GL_1 or GL_2, and let π be an admissible irreducible representation of the adelic group $G(A_F)$. We give a criterion for π to be automorphic.

This criterion, similar to that of Jacquet-Langlands in the cuspidal case, involves the analytic continuations and functional equations of the L-series attached to the twists of π by the suitable idele class characters which are unramified outside of a prescribed finite set of places of F. In particular, if F is a number field of class number one, we need only consider characters whose conductor divides the conductor of the representation. This is an exact generalization of a result of Hecke on classical modular forms.

Jean Ludwig: Irreducible representations of exponential solvable Liegroups and operators with smooth kernels

Let G be an exponential, solvable Lie group, π an element of $\hat{G}$. There exists a realization of π on $L^2(\mathbb{R}^k)$ which has the following property: for every element F in a certain subspace of the Schwartz-space of rapidly decreasing smooth functions of $\mathbb{R}^k \times \mathbb{R}^k$, there exists an element f_F in $L^1(G)$ such that $\pi(f)$ is a kernel operator with kernel F. This implies for instance that there exists a dense subspace W of the representation space $L^2(\mathbb{R}^k)$ of π so that for every $w \in W$ the orthogonal projector P_w of $L^2(\mathbb{R}^k)$ on $\mathbb{C}w$ is contained in $\pi(L^1(G))$. Hence the linear subspace V generated by the set $V' = \{\pi(g)\zeta \mid \zeta \in L^2(\mathbb{R}^k),\ g \in L^1(G)$ with rank $\pi(g) < \infty\}$ is an algebraically irreducible $L^1(G)$-module. Another implication is the fact that a symmetric solvable exponential Liegroup is *-regular in the sense of J. Boidol. Furthermore, if in the dual $\mathfrak{g}^*$ of the Lie algebra $\mathfrak{g}$ of such a group there exists a non regular element ℓ such that $G\cdot(\ell_{|[\mathfrak{g},\mathfrak{g}]})$ is a closed subset of $([\mathfrak{g},\mathfrak{g}])^*$ then G does not have the Wiener property.

Giancarlo Mauceri: Riesz means for the eigenfunction expansions for a a class of hypoelliptic differential operators

We investigate Riesz summability for the eigenfunction expansions of a class $\mathbb{D}$ of the left invariant, hypoelliptic differential operators on the Heisenberg group H_n. The operators in $\mathbb{D}$ are homogeneous with respect to group dilations and invariant under the action of the unitary group. We prove the following results. Let P be an operator in the class $\mathbb{D}$, homogeneous of degree $2d$ and define the Riesz mean of order α of P by $S_R^\alpha = \int_0^R (1-\lambda/R)^\alpha \, dE_\lambda$, where E_λ is the spectral resolution of P. Then if $Q = 2n + 2$ is the homogeneous dimension of H_n and $1 \leq p \leq 2$, $S_R^\alpha f \to f$ as $R \to \infty$:

i) in L^p if $\operatorname{Re}\alpha > (Q-1)(1/p - 1/2)$,

ii) at Lebesgue points of f if $\operatorname{Re}\alpha > (Q-1)/p$,

iii) almost everywhere if $\operatorname{Re}\alpha > (Q-1)[(2/p)-1]$.

If f vanishes in a neighborhood of $g \in H_n$, then $S_R^\alpha f(g) = o(R^{[(Q-1)/p-\operatorname{Re}\alpha]/2d})$ provided $\operatorname{Re}\alpha > (Q-1)/p$. A similar result holds for $2 < p < \infty$.

O. Carruth McGehee: Questions of Littlewood and Lusin on the behavior of Fourier transforms

The Littlewood conjecture about the norms of exponential sums now has a simple proof. The Lusin rearrangements problem has been solved (by Kahane and Katznelson, and independently by Olevskii). The talk will cover these works, and the general subject area to which they belong.

Calvin C. Moore: Homogeneous Flows and Representation Theory

G will denote a connected Lie group, Γ a closed subgroup such that G/Γ has a finite G-invariant measure and satisfying a condition analogous to one considered by Dani that $(\overline{R\cdot\Gamma})_0$ is a compact extension of R where R is the radical. By a theorem of Auslander any discrete Γ satisfies this.

For $X \in \mathcal{G}$, the Lie algebra of G, $x_t = \exp(tx)$ defines a one parameter group of measure preserving diffeomorphisms of G/Γ. We want to study the ergodic properties of such flows and the principal tool will be representation theory.

Among all such G/Γ one isolates the special class of Euclidean solvmanifolds which are first of all submanifolds (G solvable) and which can be characterized among solvmanifolds in several ways - that G is locally isomorphic to $\mathbb{R}^n \cdot T^k$ with the torus T^k acting on $\mathbb{R}^n$ linearly - or equivalently that $\pi_1(G/\Gamma)$ has an abelian group of finite index. Flows on Euclidean solvmanifolds by a reformulation of a result of Auslander look just like Kronecker flows on a torus and one has a simple necessary and sufficient criterion for ergodicity in terms of rational linear independence of some of the coordinates of X in $\mathcal{G}$.

Another class of spaces G/Γ of special interest are the semi-simple manifolds, i.e., those arising from semi-simple G. Refining earlier results of the author, one obtains again very simple necessary and sufficient conditions for ergodicity - namely H be the product of the non-compact factors of G and $H = G_1 \times \ldots \times G_n$ be the decomposition of H determined by Γ where the projection of Γ into $H = G/C$ (C = compact factors) is commensurable with a product $\Gamma_1 \times \ldots \times \Gamma_n$ $\Gamma_i \subset G_2$, irreducible lattices. Then x_t is ergodic on G/Γ iff (1) $\overline{\Gamma \cdot H} = G$ and (2) if X_i is the component of X in $\mathcal{G}_i$, ad $(\exp(tx_i))$ does not have compact closure in $\mathrm{Aut}(\mathcal{G}_i)$. These conditions are easy to check.

For a general group G we can give a criterion for ergodicity using these two special cases. Any $G/\Gamma = M$ has unique maximal quotients E and S which are respectively a Euclidean solvmanifold and a semi-simple manifold.

<u>Theorem</u> X_t is ergodic on M iff it is ergodic on both E and S. Moreover, the infinitesimal generator X_M of $L^2(M)$ of the one parameter group has pure discrete spectrum in $L^2(E) \subset L^2(M)$ and on $L^2(E)^{\perp}$ is has Lebesque spectrum of uniformly ∞ multiplicity.

The key fact from representation theory used is the Mautner phenomenon - namely, let $X \in G$; the ad-compact subgroup of X is the unique smallest normal subgroup H_X with $ad(\exp tX)$ contained in a compact subgroup of $Aut(G/H_X)$. Then the result is as follows.

Theorem If π is any unitary representation G, $X \in G$ H_X its ad-compact subgroup, then the set of vectors $v \in H(\pi)$- the Hilbert space of π, fixed by H_X, say $H(\pi)^{H_X}$ is G-invariant and the spectrum of $d\pi(X)$ on the orthogonal complement of $H(\pi)^{H_X}$ is Lebesque - in particular any eigenvector of $d\pi(X)$ is fixed by H_X.

The theorem on ergodicity results from this theorem plus some geometric arguments. The result contains as special case all previous work of Gelfand-Fomin, Parasyk, Mautner, Green, Auslander-Green, Auslander, Dani, Stepin and the author. The result on ergodicity is joint work with J. Brezin.

Delef Müller: On the sythesis problem for smooth submanifolds of $\mathbb{R}^n$

Let $F^1(\mathbb{R}^n)$ be the Fourieralgebra on $\mathbb{R}^n$. Associate to a closed subset E in $\mathbb{R}^n$ the following closed ideals:

$k(E) = \{f \in F^1(\mathbb{R}^n) : f(E) = \{o\}\}$ and

$j(E) = \{\varphi \in \mathcal{D}(\mathbb{R}^n) : \varphi \text{ vanishes near } E\}^-$.

Then E is a set of synthesis iff $k(E) = j(E)$. E is called to be of weak synthesis, if $k(E) \cap \mathcal{D}(\mathbb{R}^n)$ is dense in $k(E)$.

Let M be a smooth, k-dimensional submanifold of $\mathbb{R}^n$, and for $m \in M$ let $\nu(m)$ be the relative index of nullity at m (which for $k = n-1$ is just the dimension of the radical of the second fundamental form). Define for a closed subset E in $\mathbb{R}^n$.

$d(E) := \min\{\ell \in \mathbb{N} \cdot \overline{(k(E) \cap \mathcal{D}(\mathbb{R}^n))^\ell} = j(E)\}$.

Then we prove the following results:

Theorem 1: If $\nu(m) = \nu$ for all $m \in M$, then for any 'nice' compact subset E in M one has $d(E) \leq \left[\frac{k-\nu}{2} + 1\right]$.

Theorem 2: If $k = n-1$ and $\nu(m) = \nu$ for all $m \in M$, then any 'nice' compact subset of E in M is of weak synthesis, and $d(E) = \left[\frac{n-1-\nu}{2} + 1\right]$.

Furthermore, we study the behavior at infinity of the fouriertransforms of smooth surfacemeasures carried by smooth curves in $\mathbb{R}^n$ and give estimates from above and below.

Niels Vigand Pedersen: On the semicharacters on connected Lie groups

We discuss various results on semicharacters on locally compact groups. The following results will be mentioned: 1. For a separable locally compact group G, and a continuous homomorphism $\chi : G \to \mathbb{R}_+$ the space $\hat{G}_\chi$ of representations associated with a χ-semicharacter is a Borel subset of the quasi-dual $\hat{G}$ (with the Mackey Borel-structure), and $\hat{G}_\chi$ is standard in the induced Borel structure. 2. When G is a connected Lie group, any semicharacter on G gives rise to a normal representation of G. 3. We give a complete characterization of those connected Lie groups for which any normal representation has a distribution semicharacter. 4. We give a very explicit characterformula for completely solvable Lie groups, hereby extending earlier results of Charbonnel and myself. We conjecture that an analogous result is true in general.

Richard Penney: Non-elliptic Laplace operators on nilpotent Lie groups

Let N be a connected, simply connected nilpotent Lie group which admits a left invariant complex structure. Let $\mathcal{N}$ be the Lie algebra. A non-degenerate, alternating, J-invariant bi-linear form ϕ on $\mathcal{N}$ is called p-k if

$$\phi([X,Y],W) + \phi([X,W],X) + \phi([W,X],Y) = 0.$$

If ϕ is considered as a two form on N, then N becomes a complex, pseudo-Kählerian manifold. In this talk we analyze the non-elliptic "Laplace" type operator defined from the pseudo-Kählerian structure on N, acting either on $L^2(N)$, $L^2(\Gamma\backslash N)$ or certain homogeneous line bundles. Under some additional hypotheses, we are able to completely describe the discrete and continuous spectrum of the operator. We describe a technique, which, in principle yields the inverse and exponential of the operator. As applications, we obtain new proofs and generalizations of results of Folland-Greiner-Stein on the $\overline{\partial_b}$ operator on the Heisenberg group and some counterexamples to a non-elliptic Hodge theorem.

Tadeus Pytlik: Poles of resolvent functions in Banach algebras

It was shown that if an invertible element x in a Banach algebra with unit satisfies

$$||x^n|| = O(|n|^r) \quad \text{as} \quad |n| \to \infty$$

for some r, than any isolated point of the spectrum x is a pole and the order of the pole is not greater than r+1.

The proof is based on some investigations concerning analytic semigroups in radical Banach algebras.

Some applications were also given.

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Fulvio Ricci: Singular Integrals on Nilpontent Lie Groups

Let $\underline{n}$ be a nilpotent Lie algebra over $\mathbb{R}$ endowed with a one-parameter group of dilating automorphisms. We let $|x|$ be a homogeneous pseudo-norm on $\underline{n}$ and consider singular kernels K on $\underline{n}$ satisfying either one of the following conditions:

(1) $$\int_{|x| > C|y|} |K(x-y) - K(x)|\, dx \leq A \quad \text{for every} \quad y \neq 0$$

(2) $$\int_{|x| > C|y|} |K(xy^{-1}) - K(x)|\, dx < A \quad \text{for every} \quad y \neq 0.$$

Here C is a fixed large constant and xy denotes the product in the connected and simply connected Lie group N associated to $\underline{n}$.

Theorem. Suppose that the Fourier transform $K(\zeta) = \int_{\underline{n}} K(x) e^{-i(\zeta,x)} dx$ is a bounded function on $\underline{n}^*$. If K satisfies (1), the operator of left convolution (on N) by K is bounded on $L^2(N)$. If K satisfies (2), it is bounded on every space $L^p(N)$, $1 < p \leq 2$.

This is an improvement of previous results, the first of which due to Knapp and Stein (Annals of Math. 93-1971). The proof does not make use of Cotlar's Lemma, but of an induction argument on the dimension of $\underline{n}$.

W. Rossmann: The tempered dual of a reductive group

We give another approach to the Knapp-Zuckermann classification of tempered representations.

There is a natural finite-to-one map from the set of equivalence classes of tempered irreducible representations of a reductive group G to the set of G-orbits in the dual of the Lie algebra of G. The image of this map consists of all closed, twisted-integral orbits with quasi-split isotropy groups. The fibre over such an orbit Ω consists of the constituents of representations π parabolically induced from discrete series. The decomposition $\pi = \Sigma\, \pi_i$ of π

into irreducibles π_i is described in terms of a decomposition $\Lambda = \cup \Lambda_i$ of the family $\Lambda = \Lambda(\Omega)$ of orbits of maximal dimension with Ω in their closures into subfamilies Λ_i. These Λ_i are themselves the orbits Λ of a subgroup $P(\pi)$ of the 'rho-group' $P(\Omega)$ which operates simply transitively on Λ.

G. Schlichting: The type structure of certain multiplier representations of discrete groups

Let Γ be a discret group and σ a multiplier on Γ; then the following is proved

Theorem 1 Equivalent assertions are

(1) irreducible σ-representations are finite dimensional

(2) σ-factor representations are type I

(3) the 'left regular σ-representation' λ_σ^Γ is of type I

(4) σ' is trivial on a cofinite abelian subgroup.

Theorem 2 Let $\Gamma_f = \{x \in \Gamma \; ; \; [x]^\Gamma$ finite conjugacy class$\}$

$\Gamma_o = \Gamma \{\ker \pi \; ; \; \pi$ ordinary representation of Γ, $\dim \pi < \infty\}$, then equivalent assertions are

(1) λ_σ^Γ has non-zero typ I part $\lambda_{\sigma,I}^\Gamma$

(2) $|\Gamma_o| < \infty$, Γ/Γ_o almost abelian, σ is similar to multiplier $\dot{\sigma}$ lifted from Γ/Γ_o and σ almost trivial.

(3) $[\Gamma:\Gamma_f] < \infty$, $|\Gamma_f'| < \infty$, σ similar to $\dot{\sigma}$ and almost trivial.

(4) $\exists \; \Gamma_1 \leq \Gamma_f$, $[\Gamma':\Gamma_1] < \infty$, $|\Gamma_1'| < \infty$ and

$$\Gamma_1 < \{x \in \Gamma_f \; ; \; \sigma(x,y) = \sigma(y,x) \; \forall \; y \in \Gamma_f\} .$$

If this is the case, then $\lambda_{\sigma,I}^\Gamma$ is equivalent (in the obvious way) to the left regular $\dot{\sigma}$-representation of Γ/Γ_o on $\ell^2(\Gamma/\Gamma_o)$.

Jorge Soto Andrade: Some remarks on generalized Weil representations and related problems

We point out the close relationship between the existence of Weil models for the irreducible unitary representations of $GL(2,\mathbb{R})$ and the classical Barnes' Lemma for the Gamma function. Actually for $GL(2,\mathbb{F}_q)$ the equivalence between the existence of those models and two forms of the finite Barnes' Lemma is already proved and the idea of the proof seems to carry over to local fields.

We give an example of a generalized Weil representation in the space $L^2(k^3)$ for the group $SL_3(k)$ (k a finite field), which is associated in a natural way to the cubic form $x_1x_2x_3$. We finally present a 'Weil representation' for the group $GL_3(k)$ in the space $L^2(\mathbb{K}^\times \times (k^2 \setminus \{o\}) \times k^\times)$ (where $\mathbb{K}^\times$ denotes the unique cubic extension of k), associated to the cubic norm of $\mathbb{K}$ over k.

Janusz Szmidt: The Selberg trace formula for the Picard group $SL(2,\mathbb{Z}[i])$

In his paper of 1979 (Proceedings of the Tata Institute) Don Zagier has given a new proof of the Selberg trace formula in the case of the group $G = SL(2,\mathbb{R})$ and its discrete subgroup $\Gamma = SL(2,\mathbb{Z})$. It rests on the consideration of the integral

$$I(S) = \int_{\Gamma\setminus G} K_0(g,g)E(g;s)dg$$

where K_0 "the discrete part" of the kernel of the integral operator which occurs in the trace formula and $E(g;s)$ is the Eisenstein series corresponding to the cusp. The residue of this integral at the critical point gives the trace formula. This method can be also applied to the case $G = SL(2,\mathbb{C})$ and $\Gamma = SL(2,Z[i])$, $Z[i]$ is the ring of Gaussian integers. It leads to some connections with the Dedekind zeta functions of number fields.

Aleksander Strasburger: Differential equation for conical vectors and its applications

We write $G = KAN$ for the Iwasawa decompostion of a connected semi-simple Lie group and we denote by M the centralizer of A in K, K the maximal compact subgroup . We shall consider representations of the principal spherical series of G, acting on smooth functions over K/M according to the formula $\tau_\lambda(g)f(kM) = e^{(i\lambda-\rho)H(g^{-1}k)}f(g^{-1}kM)$. A distribution ψ on K/M is called λ-conical if it is fixed unter $\tau_\lambda(MN)$. In this work we introduce an M-invariant differential operator X^λ such that λ-conical distributions are precisely the M-invariant solutions of the equation $X^\lambda\psi = 0$. Using a theorem of C. Herz characterizing distributions invariant under a compact Lie group we are able to solve the equation completely for $SO_0(n,1)$, describing solutions in terms of periodic distributions in one variable. The results are compared with those previously obtained by Helgason, Harzallah and Hu. Applications to determination of differential intertwining operators are given.

Yitzhak Weit: Spectral Analysis in spaces of vector-values functions

Let $L_1(G;H)$ denote the L_1 space of functions from a L C A group G of a separable Hilbert space H. We say that spectral analysis holds for $L_1(G;H)$ if every closed invariant subgroup of $L_1(G;H)$ is included in a closed co-dimension one invariant subspace. We prove that spectral analysis holds for $L_1(G;H)$ if and only if H is finite dimensional which provides a short and direct proof to a theorem of Beurling for general motion groups. The corresponding result for the space of radial vector-valued functions on $\mathbb{R}^2$ provides sets which admit spectral synthesis and sets which are not of spectral synthesis in the space of one-sided maximal ideals in the group algebra of the Euclidean motion group.

Daoxing Xia: Unitary representation of the group of diffeomorphisms

Let Diff(X) be the group of all such diffeomorphisms f in the n-dimensional manifold X, which are identical mappings beyond some compact sets K_f, m be a measure on the Schwartz's space $K^*(X)$ of distributions on X, which is quasi-invariant with respect to all f in Diff(X), and U be the unitary representation

$$(U(f)F) = F(f*g)(dm(f\ g)/dm(g))^{\frac{1}{2}},$$

defined by the measure m. Under certain conditions we give the general form of the Radon-Nikodym derivative

$$dm(f*g)/dm(g) = \exp(c(\int gdx)(g, \ln D(x,f)) + P(f*g) - P(G))$$

where D(x,f) is the Jacobian of f at x, C(·) is a function and P(·) is a functional. Zang Yinnan gives a necessary and sufficient condition of the unitary equivalence of two unitary representations defined by two smooth measures m_1 and m_2 on $K^*(X)$. By means of tangent bundles, we also give another elementary series of irreducible representations of Diff(X) which are different from that given by Versik, Gelfand and Graev.

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