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This conference on Banach spaces was organized by Professors B. Fuchssteiner (Paderborn), J. Lindenstrauss (Jerusalem) and A. Pelczynski (Warsaw).
Unfortunately, due to sickness, Prof. B. Fuchssteiner and Prof. J. Lindenstrauss had to cancel their participipation in the meeting.

49 mathematicians from 17 countries attended the conference and contributed in lectures and many discussions to the success of this meeting.

The scientific program included 33 lectures on various topics in Banach space theory such as isomorphisms on L_p and $C(K)$, operators on L_1, operators on finite dimensional Hilbert spaces, extreme operators, norm attaining operators, operator ideals, invariant subspaces, fixpoints of non expansive maps, Darboux integrability, invariant functionals, interpolation , isometric theory of duality, transfinite duals, uniform approximation property, K-convexity, tensor products, uniqueness (up to a permutation) of unconditional bases, weak compactness, subbushes, martingales, H^1, H^∞, L^1/H^1, finite dimensional subspaces of L_p-spaces, embedding of l_1^k into finite dimensional spaces, Banach spaces containing $l_1(\tau)$, Lipschitz and uniform classification of Banach spaces, measures, smoothpoints and smooth norms.

Furthermore, a problem session was held where open questions from various areas of Banach space theory were posed and discussed.

D. E. ALSPACH: Small into isomorphisms on L_p spaces

If T is an isomorphism of $L_p(\mu)$ into $L_p(\nu)$, $||T||\cdot||T^{-1}||$ is a measure of the size of T. In particular if $||T||\cdot||T^{-1}||$ is near one then T is said to be small and at least in one sense like an isometry. We show that smallness in fact guarantees that T is a perturbation of an isometry. Precisely: For each p, $1 \leq p < \infty$, $p \neq 2$, there is a function $\tau: [0, \varepsilon) \to [0, \delta)$ such that if T is an isomorphism of $L_p(\mu)$ into $L_p(\nu)$ with $||T||\cdot||T^{-1}|| < 1 + s$, $0 < s < \varepsilon$, then there is an isometry S of $L_p(\mu)$ into $L_p(\nu)$ such that $||T - S|| < \tau(s)$. Moreover $\lim_{s\to 0^+}\tau(s) = 0$.

J. BATT: Weak compactness in the space of Bochner integrable functions.

The aim of the talk is to give necessary and sufficient conditions for the relative weak compactness of a subset K in the space $L_1(\mu,X)$ of Bochner integrable functions on a finite positive measure space with values in a Banach space X. A unified approach leads to (partial) extensions of results of BOURGAIN'S and DIESTEL'S. Other new results on weak compactness are based on a characterization of conditional and relative compactness in the topology $\sigma(L_1(\mu, X), L_\infty(\mu, X'))$ for which we show that the notions relative compactness, relative countable compactness and relative sequential compactness coincide. Specific examples and counterexamples are given.

B. BEAUZAMY: Invariant subspaces for $C_{1.}$ contractions in Banach spaces

We study $C_{1.}$ contractions on Hilbert spaces, that is, operators satisfying $||T|| = 1$ and $T^n x \not\to 0, n \to \infty$, $\forall\, x \neq 0$, from the point of view of the existence of invariant subspaces.

S.F. BELLENOT: Transfinite duals

Let X be a quasi - reflexive space of order one and let X^{ω} be its ω-th dual. Then X^{ω}/X has a neighborly basis. Conversely, if Y has a neighborly basis, then (under slight restrictions) there is an X with X^{ω}/X isomorphic to Y. In particular, there are spaces X, Y so that $X^{\omega} \approx X \oplus c_0$ and $Y^{\omega} \approx Y \oplus l_1$. Define $s(X)$ to be the smallest ordinal α so that X^{α} is non-separable. Then $\{s(X) : X$ separable quasi-reflexive space of order one$\} =$ $\{\omega+1, \omega+2, 2\omega+1, 2\omega+2, \omega^2+1\}$. The structure of X^{ω}/X for general non-reflexive X may turn out to be useful to know. Indeed, if all such X^{ω}/X have uniformly complemented l_{p}^{n}'s, then so do all Banach spaces. A partial positive resultsays if X^{2}/X is a non-trivial reflexive space, then X has uniformly complemented l_p^n's.

Y. BENYAMINI: Small into isomorphisms between spaces of continuous functions

A well known theorem of D. Amir and M. Cambern says that if T is an isomorphism of $C(K)$ onto $C(S)$ with $||T|| \cdot ||T^{-1}|| < 1+\varepsilon$ for some $0 < \varepsilon < 1$, then $C(K)$ and $C(S)$ are, in fact, isometric. Their proofs show that if $||f|| \le ||Tf|| \le (1+\varepsilon)\, ||f||$ for all $f \in C(K)$, then there is an isometry $W: C(K) \to C(S)$ with $||T-W|| \le \varepsilon$.

We generalize this result to into-isomorphisms, and prove that if K is metrizable and $T: C(K) \to C(S)$ satisfies $||f|| \le ||Tf|| \le (1+\varepsilon)\, ||f||$, then there is an into-isometry W with $||T-W|| \le 3\varepsilon$. If K is not assumed to be metrizable, the result is no longer true. The novelty of the proof lies in the construction of a ω^*-continuous nearest-point map from $C(K)^*$ onto its unit ball.

S. V. BOCHKARIEV: On unconditional convergence

In this report the following results were proposed.

Theorem 1. There exists an unconditional basis in space H_1, consisting of the trigonometric polynomials $P_n(x)$, such that degr $p_n(x) \leq 2n$.

Theorem 2. Let $\rho(k)$ be an increasing sequence of numbers such that $\Sigma \frac{1}{k\rho(k)} = \infty$. Then there exists a sequence $\{C_k\}$ such that $\Sigma C_k^2 \rho(k) < \infty$, and there exists a rearrangement $\sigma(k)$ of the natural numbers such that the series $\Sigma C_{\sigma(k)} \psi_{\sigma(k)}(x)$ diverges almost everywhere.

Theorem 3. Suppose that the modulus of continuity $\omega(\delta)$ satisfies the condition $\sum_{n=2}^{\infty} \omega(\frac{1}{n})/n \sqrt{\log n} = \infty$. Then there exists a continuous function $f(x) \in H^{\omega}$, $f(x) \sim \sum_{k=1} C_k \psi_k(x)$, and there exists a rearrangement $\sigma(k)$ of the natural numbers for which the series $\Sigma C_{\sigma(k)} \psi_{\sigma(k)}(x)$ diverges almost everywhere.

J. BOURGAIN: Some new Banach space properties of H^{∞}

We study projections, unconditional decompositions and certain topological sequence properties of the span $H^{\infty}(D)$ of bounded analytic functions on the disc.
If E is a finite dimensional complemented subspace of H^{∞}, then E contains uniform ℓ_n^{∞} - copies, where $n \sim \dim E$. If moreover E has an unconditional basis, then $E \sim \ell^{\infty}_{\dim E}$. If E is infinite dimensional and complemented, then $\ell^{\infty} \hookrightarrow E$.

It was known that $H_{\infty} \sim \oplus_{\ell^{\infty}} H_{\infty}$. In fact there are only unconditional decompositions of H^{∞} in ℓ^{∞} sense. H^{∞} has Dunford-Pettis property (and also all duals), $(H^{\infty})^*$ is weakly sequentially complete (and also the odd duals) and H^{∞} has Grothendieck property (as well as the even duals).

J. BOURGAIN, P. CASAZZA, J. LINDENSTRAUSS and L. TZAFRIRI:

Banach spaces with a unique (up to a permutation) normalized unconditional basis

It is known (Lindenstrauss and Zippin), that c_0, l_1, l_2 have a unique unconditional basis and (Edelstein and Wojtaszczyk) $c_0 \oplus l_1$, $c_0 \oplus l_2$, $l_1 \oplus l_2$, $c_0 \oplus l_1 \oplus l_2$ have a unique (u.t.a.p.) unconditional basis.

We show: (1) $(\Sigma \oplus l_2)_{c_0}$, $(\Sigma \oplus l_2^n)_{c_0}$, $(\Sigma \oplus l_1)_{c_0}$ $(\Sigma \oplus l_1^n)_{c_0}$, $(\Sigma \oplus l_2)_{l_1}$, $(\Sigma \oplus l_2^n)_{l_1}$, $(\Sigma \oplus c_0)_{l_1}$, $(\Sigma \oplus l_\infty^n)_{l_1}$ have a unique (u.t.a.p.) unconditional basis; (2) $(\Sigma \oplus l_1)_{l_2}$ $(\Sigma \oplus l_1^n)_{l_2}$, $(\Sigma \oplus l_\infty^n)_{l_2}$, $(\Sigma \oplus c_0)_{l_2}$ do not have a unique (u.t.a.p.) unconditional basis. Direct sums and finite iterations of the space in (1) also have this property. Related results are given.

F. FEHER: Weak-type interpolation on Banach function spaces

The interpolation theorem of M. Riesz/G. Thorin states that any linear operator T which is simultaneously a bounded operator from L_{p_i} into L_{q_i} (i = 1, 2; L_{p_i}, L_{q_i} Lebesgue spaces on $(0,\ell)$, $1 \le p_i, q_i \le \infty$) is also a bounded operator from L_q provided $1/p_1 < 1/p < 1/p_2$ and $1/q_1 < 1/q < 1/q_2$.

The purpose of this talk is to generalize this theorem to the following setting: (1) The Lebesgue spaces of functions on $(0,\ell)$ are replaced by rearrangement invariant Banach function spaces of μ-measurable functions on Ω, where (Ω,Σ,μ) is a σ- finite measure space with a nonatomic measure . (2) Linear bounded operators in the hypotheses of the theorem are replaced by sublinear operators of weak type. (3) The conditions $1/p_1 < 1/p < 1/p_2$, and $1/q_1 < 1/q < 1/q_2$ are replaced by conditions upon the Boyd indices of the r.i. spaces involved. As a tool, the Luxemburg representation of a r.i. space is used as well as two integral operators. On one hand these operators are closely related to the Boyd indices and, on the other hand, to the Calderón operator of the respective interpolation segment. Finally, some applications are given.

T. FIGIEL and S. KWAPIEN: Discontinuous invariant functionals

Let $(E, ||\cdot||)$ be a symmetric Banach sequence space, i.e. $l_1(\mathbb{N}) \subseteq E \subseteq l_\infty(\mathbb{N})$ and, if $f \in E$, then $f \cdot \pi \in E$ and $||f\cdot\pi|| = ||f||$ for each permutation π of $\mathbb{N}$. Assume that E is solid, i. e. if $f \in E$, $g \in l_\infty(\mathbb{N})$ and $|g| \leq |f|$, then $g \in E$.

Definition. A linear functional $\phi \in E$ is said to be invariant, $\phi \in \mathrm{Inv}(E)$, if

$$\phi(f\cdot\pi) = \phi(f)$$

for $f \in E$ and each permutation π.

For $f \in l_\infty(\mathbb{N})$ we define $f \otimes h \in l_\infty(\mathbb{N} \times \mathbb{N})$ by $(f \otimes h)(n, m) = f(n)/m$.

Theorem 1. Let E be as above. TFAF

a) $\mathrm{Inv}(E) = \{0\}$,

b) if $f \in E$, then $f \otimes h \in E(\mathbb{N} \times \mathbb{N})$,

c) the lower Boyd index $p_E > 1$.

Theorem 2.

If $E = l_p$, $0 < p \leq \infty$, then $\mathrm{Inv}(E) \subset E^*$ if and only if $p \neq 1$.

In the lecture some of these results were proved and their analogs for symmetric function spaces on [0, 1] were discussed as well.

G. GODEFROY: Isometric theory of duality

The main problems of the isometric theory of duality are the following: existence of preduals, unicity of preduals, smoothness properties of dual norms, inversion of the functor (*)-. By elementary methods, one can show that many Banach spaces are unique preduals of their dual, and then you can inverse the functor (*) for bijective isometries of E^*. We can better the known results about loss of smoothness and strict convexity in duals of high order, and obtain necessary and sufficient conditions for a Banach space to be isometric to a dual space: as an example, an Asplund space is a dual space iff it is 1-complemented in its bidual.

R. HAYDON: Some more about Darboux integrability in Banach spaces

At a previous Oberwolfach meeting (Functional Analysis, October 1980), A. Pelczynski spoke about some results, due to himself and to GC da Roch Filho, concerning Darboux operators and Banach spaces with the Darboux Property. I present here an extension of one of these results and a counterexample to a natural conjecture in this area. First some definitions and basic results:

Definitions. Let $f : [0,1] \to X$ be a bounded function, taking values in the Banach space X. We say that f is Riemann integrable if there exists $x \in X$ such that for all $\varepsilon > 0$, we can find a partition of [0,1] into intervals $T_1,\dots,T_n$ such that $\|x - \sum_{j=1}^{n} m(I_j)f(t_j)\| < \varepsilon$ for all choices of $t_j \in T_j$.

We say that f is Darboux integrable if $\{t \in [0,1]: f$ is not continuous at $t\}$ is a null set.

For any X, if a function f is Darboux integrable then it is Riemann integrable. We say that X has the Darboux Property if the converse holds for X-valued functions.

One notices that c_o and l_p $(p \neq 1)$ do not have the Darboux Property. For if we let (t_n) be a sequence dense in [0,1],

and define $f(t_n) = e_n$ (the n^{th} vector in the usal basis)

$f(t) = o$ other values of t,

we find that f is every where discontinuous although it is R-integrable with integral 0. One can show similarly that if X has the Darboux Property then every spreading model of X is isomorphic to l_1.

A Lemma due jointly to the author and E. Odell shows that a "bad" function of the above type exists for every non-Darboux space.

Lemma. Suppose that X fails the Darboux Property. Then there exists in X a normalized basis sequence (x_n) such that the function f defined by

$$f((k+1/2)2^{-n}) = x_{2^n+k} \qquad (0 \leq k < 2^n,\ n \geq 0)$$

$$f(t) = 0 \qquad (t \text{ not a dyadic rational})$$

is Riemann integrable.

The following theorem extends a result of Pelczynski and da Roch Filho from the class of subspaces of L_1 to the class of "stable" spaces, recently introduced by Krivine and Maurey. There is a similar result for Darboux operators on stable spaces.

Theorem. For a stable space X, the following are equivalent:

a) X has the Darboux Property;

b) X has the Schur Property (every weakly convergent sequence is norm convergent);

c) every spreading model of X is isomorphic to l_1.

The only new part is the implication (c) ⇒ (a); it was remarked above that (a) ⇒ (c) for arbitrary X, and (b) ⇒ (c) for arbitrary X too; the implication (c) ⇒ (b) for stable spaces is a result of S. Guerre and J.-Th. Lapresté.

It was shown by da Roch Filho that the Figiel-Johnson version of the Tsirelson space, usually called T, has the Darboux Property. Since T is reflexive, (a) does not imply (b) in general. The following example shows that (b) does not imply (d).

Example. There is a space X (a "somewhat Tsirelson - like tree - space", in fact) which has the strong Schur Property but not the Darboux Property.

R. C. JAMES: Subbushes and extreme points in Banach spaces

An asymptotic subbush of a bush B is a subset B_0 of $\overline{co}(B)$ which is a bush and for which there is a one-to-one correspondence with a subset of B that preserves partial ordering and for which $\lim ||x_n - y_n|| = 0$ if some branch of B_0 contains all $\{x_n\}$, x_n is in the n^{th} column of B_0, and $\{y_n\}$ is the sequence of corresponding members of B. It is shown that if B is an asxmptotic subbush with separation constant ε and $0 < \bar{\varepsilon} < \varepsilon$, then there is an asymptotic subbush for which no $\frac{1}{4}\bar{\varepsilon}$-ball contains more than finitely many bush elements, and an asymptotic subbush for which all wedge-intersections out branches are empty. These imply the Huff-Morris (and Bourgain) theorems that a Banach space has the Radon-Nikodým property if each bounded closed (weakly closed) nonempty set has an extreme point, and also give serious restraints on the nature of any counterexample for the conjecture that KMP implies RNP.

H. JARCHOW: Hahn - Banach extensions for ideals of operators

Let $[A,\alpha]$ be a complete quasi-normed operator ideal in the sense of Pietsch. A Banach space Y is said to have the A-Hahn-Banach property (A-HBP) if for every choice of Banach spaces E,F with $E \subset F$ every $S \in A(E,Y)$ extends to some $\tilde{S} \in A(E,Y'')$ such that $\alpha(\tilde{S}) \leq C_Y \cdot \alpha(S)$, where C_Y is a constant which depends only on Y. By using Pietsch-Persson duality for operator ideals it can be shown that Y has the A-HBP if $I_Y \in [A \cdot A^{\Delta\ \mathrm{inj}}]^{\Delta}$; the converse holds at least when Y' has the metric approximation property. Here A^{Δ} is the ideal which is conjugate to A in the sense of Gordon-Lewis-Retherford, and "inj" refers to the formation of the injective hull. For some concrete ideals A, this allows an easy determination of the spaces with A-HBP.

B. S. KACHIN: On certain properties of operators in a finite dimensional Hilbert space

Let $w = \{x_j\}_{j=1}^{n}$ be a sequence of unit vectors in a Hilbert space. We say that w is (3,2)-orthogonal, if whenever $1 \leq j_1 < j_2 < j_3 \leq n$, two of the vectors $x_{j_1}, x_{j_2}, x_{j_3}$ are orthogonal.

In the lecture we presented some properties of (3,2)-orthogonal systems obtained in a joint work with S. V. Konyagin. This problem is related to some extremal problems for trigonometrical polynomials with a prescribed spectrum.

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N. J. KALTON: An example of a symplectic Banach space

We give an example of a Banach space X and a symplectic form Ω on X so that X cannot be split in the form $X = M \oplus N$ where Ω vanishes identically on both M and N. This answers a question raised by Weinstein.

The example is the space Z_2 introduced by Kalton and Peck in 1979, with its "natural" symplectic form.

This work was joint with R. C. Swanson.

S. V. KISLIAKOV: Cotype properties of L^1/H^1

Let C_A be the disc algebra. a) If $T \in L(C_A^*, l^2)$ and rank $T \leq n$ then $\pi_1(T) \leq C(\log n)||T||$. b) If $T \in L(C_A, L^p)$, $1 \leq p \leq 2$ then $\pi_2(T) \leq C(\log n)\,||T||$ provided rank $T \leq n$.
c) The space C_A^* is of cotype q-Rademacher for every $q > 2$; moreover for the cotype 2 constant defined by n vectors we have the inequality $K^{(2,n)}(C_A^*) \leq C \log n$.

HEINZ KÖNIG: An extended Mooney-Havin Theorem.

The "classical" Mooney-Havin theorem (about H^∞ on the unit circle) had been extended to the abstract Hardy algebra situation (e.g. in the sense of Barbey-König, Lechere Notes in Mathematics Vol. 593) independently by Barbey and Cnop-Delbaen; they obtained the theorem in the case that the set M of representing functions of φ: $H \to ¢$ is $\sigma(L^1(m), L^\infty(m))$ compact. In the present talk it is shown that the methodical idea of Cnop-Delbaen can be carried further and leads to a perfectly general result. Define $S \subset H'$ to consist of the $\lambda \in H'$ such that

$$\forall \varepsilon > 0 \, \exists c(\varepsilon) > 0 \text{ with } |\lambda(u)| \leq \varepsilon ||u||_{L^\infty(m)} + c(\varepsilon)\, \theta\, (|u|) \; \forall u \in H.$$

Its is always $L^1(m)/H^\perp \subset S$, and $L^1(m)/H^\perp = S$ if M is weakly compact. The idea of Cnop-Delbaen then gives the

Theorem. Let $\lambda_n \in S$ $(n = 1,2,\ldots)$ be such that $\lim_{n\to\infty} \lambda_n(u)$ exists $\forall u \in H$. Then $\forall \varepsilon > 0 \; \exists \; c(\varepsilon) > 0$ with

$$|\lambda n(u)| \leq \varepsilon \, ||u||_{L^\infty(m)} + c(\varepsilon)\, \theta\, (|u|) \; \forall u \in H \text{ and } n \geq 1.$$

Corollary. S is always weakly sequentially complete. In the case that M is weakly compact this is the Mooney-Havin theorem.

HERMANN KÖNIG: Trace theorems, resolvent estimates and eigenvector completeness

Let A_i be operators on a complex Banach space with $1/i$ - summable approximation numbers, $i = 1,\ldots n$. Consider the operator polynomial $p(\lambda,A_i) := \lambda^n I - \lambda^{n-1} A_1 \ldots . - A_n$. Then (as for $n = 1$) $\{\lambda \in \; | \; p(\lambda,A_i)^{-1}$ is not invertible$\}$ is a null sequence of eigenvalues λ_i. It is shown that $\sum_i \lambda_i = tr(A_1)$, generalizing a result of Sigal.

The linearization method of the proof can be used to derive a result on the completeness of eigenvectors of the operator differential equation $y^{(n)}(t) - A_1 y^{(n-1)}(t) \ldots - A_n y(t) = 0$, $y:[0,1] \to X$. A sufficient assumption is e. g. $A_i = T_i B^i$, $i = 1,\ldots n$, where T_i are compact maps, and B satisfies $\alpha_j(B) = o\,(j^{-1/\rho})$, $\rho \geq 1$ and $||(I - \lambda B)^{-1}||$ is bounded as $|\lambda| \to \infty$ outside of a suitable domain consisting of sectors of angle less than π/ρ.

D. R. LEWIS: The distance from subspaces of L_p-spaces to l_r^n spaces

Theorem 1 If $2 \leq p < \infty$, $1/p + 1/q = 1$ and $E \subset L_p(\mu)$ is any n dimensional subspace, there are maps $u: l_q^n \to E$ and $v: L_p(\mu) \to l_2^n$ such that vu = inclusion, $||v|| \leq 1$ and $||u|| \leq cp\,(\log n)^{1-2/p}$; further, the norm of u on l_2^n is at most $n^{1/2 - 1/p}$.

Theorem 2 If $1 \leq r \leq 2 \leq p < \infty$, $1/p + 1/q = 1$ and $E \subset L_p(\mu)$ is n dimensional, then

$$d(E, l_r^n) \leq cp \begin{cases} (\log n)^{1 - 2/p}\, n^{1/r - 1/2}, & \text{if } 1 \leq r \leq q \\ (\log n)^{2/r - 1}\, n^{1/2 - 1/p}, & \text{if } q \leq r \leq 2. \end{cases}$$

Similar results hold for $1 < p \leq 2$ and $1 < p \leq 2 \leq r \leq \infty$.

Å. LIMA: Extreme operators on Banach spaces

Let X be a Banach space and let B(X) denote the unit ball in the space of bounded linear operators on X. We shall discuss the problem: Which spaces X have the property that every extreme point of B(X) maps extreme points of the unit ball of X into extreme points.

W. LUSKY: A remark on rotations in separable Banach spaces

We show that for every separable Banach space X there is a separable

Banach space $Y \supset X$ and a contractive projection $P : Y \to X$ such that Y has the following property:

There is a dense subset D of the unit sphere of Y such that for all $x,y \in D$ there is an onto isometry $T : Y \to Y$ with $Tx = y$.

S. HEINRICH and P. MANKIEWICZ: On Lipschitz and uniform classifiction of Banach spaces

Theorem 1. If a Banach space X is Lipschitz homeomorphic to a reflexive Orlicz space on $[0,1]$ then X is isomorphic to it.

Theorem 2. If a Banach space X is uniformly homeomorphic to a Banach space Y then they have Lipschitz homeomorphic ultrapowers.

Theorem 3. If in addition to the assumption in Th. 2. Y is superreflexive and the pair X,Y satisfies Decomposition Scheme then for some ultrafilter U the ultrapowers $(X)_U$ and $(Y)_U$ are isomorphic.

Remark. It is worth mentioning that technique developed to obtain Thms. 1 - 3 easily yields majority of known results on Lipschitz and uniform classification of Banach spaces (e.g. Ribe's Theorems).

B. MAUREY: Fixed points for non expansive mappings on certain weakly compact subsets of L^1

We say that a Banach space X satisfies the weak fixed point property (WFPP) if every non expansive mapping T defined on an arbitrary weakly compact convex subset of X admits a fixed point. It is proved that the following spaces satisfy the WFPP: every reflexive subspace of L^1, the Hardy space H^1 and c_0.

V. MILMAN: Some remarks about embeddings of l_1^k in finite dimensional spaces.

Let $\{x_i(t)\}_1^n$, be real valued functions on a set T and $|x_i(t)| = 1$. We consider span $\{x_i\}_1^n = X_n$ in supremum norm on T. The following two theorems are proved.

Theorem 1. Let $\underset{\varepsilon_i = \pm 1}{\text{Average}} \|\sum_1^n \varepsilon_i x_i\| = M_n$. Then (for n sufficiently large) there exists $A \subset [1,\dots,n]$ such that $|A| \geq \frac{M_n^2}{5n \ln n}$ and span $\{x_i\}_{i \in A} = l_1^{|A|}$.

Theorem 2. Assume that X_n cannot be embedded in l_∞^N. Then there exists $A \subset [1,\dots,n]$ such that $k = |A| > {}^{\ln N}/_{\ln n}$ and span $\{x_i\}_{i \in A} = l_1^k$.

Some corollaries follow. For example: Let $\dim X_n = n$, $\varepsilon > 0$ and X_n have a cotype q with cotype constant K_q. Then there exists $m = m(n;\varepsilon;K_q;q) \to \infty$ $(n \to \infty)$ such that X_n contains $(1 + \varepsilon)$-isometric and $(1 + \varepsilon)$-complemented copy of l_1^M.

Theorem 1 was inspired by Pisiers's result which considered the case of a compact Abelian group T and a set of characters $\{x_i(t)\}_{i=1}^n$.

N. J. NIELSEN: The uniform approximation property in Banach lattices and operator ideals

In the lecture we shall investigate, when certain tensor products of Banach lattices have the uniform approximation property (u.a.p.) provided both factors have this property. We then use these results to prove that if a superreflexive Banach lattice has the u.a.p., then the approximating operators can be chosen with controlled moduli. The results are also used to prove that certain spaces of absolutely summing operators have the u.a.p.

G. PISIER: K-convexity and related topics.

The main results are the following two theorems:

Theorem 1. A Banach space X is K-convex iff X does not contain l_n^1's uniformly. Moreover, this happens iff X and every space finitely representable in X contain uniformly complemented l_n^2's.

Theorem 2. Let X be a Banach space with a basis (or merely with the bounded approximation property).
Assume that X is of cotype q and that X^* is of cotype q_*.

Then if $\frac{1}{q} + \frac{1}{q_*} > \frac{1}{2}$, X does not contain l_n^1's uniformly.

I believe that theorem 2 is true without the restriction concerning q and q_*.

In the case $q = q_* = 2$, it was previously known that, in the situation of theorem 2, X must be isomorphic to a Hilbert space.

H. ROSENTHAL: Martingale proofs of some geometrical results in Banach space theory

Bushes and "approximate bushes" are precisely martingales and "quasi-martingales" respectively. Using this simple equivalence and results from classical martingale theory, we obtain a new proof of the theorem of T. Bourgain - R.R. Phelps that a closed bounded convex subset K of a Banach space is the closed convex hull of its set of strongly exposed points provided K has the Radon-Nikodym property. The proof employs a new "stopped-martingale" result as well as some geometrical lemmas with simple formulations and proofs transparent through martingales. (The work presented is joint with Ken Kunen)

W. SCHACHERMAYER: Norm attaining operators on some classical Banach spaces

We construct an operator from $L^1[0,1]$ to $C[0,1]$ which may not be approximated by norm attaining operators in the operator norm. Apparently this is the first example of a pair of classical Banach spaces for which the norm attaining operators are not dense.

We also show that the norm attaining operators are dense in $B(C(K), L^1)$ and $B(C(K), l^2)$.

C. SCHOTT: On volume ratio for tensor produets

Let E be an n-dimensional Banach space and B_E the unit ball of E. The volume ratio of E is given by

$$vr(E) = \left(\frac{vol(B_E)}{vol(\mathcal{E})}\right)^{\frac{1}{n}}$$

where $\mathcal{E}$ denotes the ellipsoid of maximal volume contained in B_E.
It was proved that

$$vr(l^r_n \otimes_\pi l^s_n) \sim \begin{cases} 1 & \text{for } 1 \le r,s \le 2 \\ n^{\frac{1}{2}-\frac{1}{r}} & \text{for } 2 \le r \le s' \\ n^{\frac{1}{s}-\frac{1}{2}} & \text{for } 2 \le s' \le r \\ 1 & \text{for } 2 \le r,s \le \infty \text{ and } \frac{1}{2} \le \frac{1}{r}+\frac{1}{s} \\ n^{\frac{1}{2}-\frac{1}{r}-\frac{1}{s}} & \text{for } \frac{1}{r}+\frac{1}{s} \le \frac{1}{2} \end{cases}$$

M. TALAGRAND: On Banach spaces which contain $l^1(\tau)$

We prove a combinatorial lemma about independent families of sets. Two consequences are the following.

Theorem 1. Let E be a Banach space and τ a cardinal with $cof(\tau) > \aleph_o$. Then TFAE:

i) E contains a subspace isomorphic to $l^1(\tau)$.

ii) Each total set A of E contains a subset equivalent to the natural basis of $l^1(\tau)$.

iii) The unit ball of E', provided with the w*-topology, maps continuously onto $[o,1]^\tau$.

Theorem 2. Let T be a set and τ a cardinal such that card $(T) < \mathrm{cof}(\tau), \aleph_0 < \mathrm{cof}(\tau)$. Let $(K_j)_{j \in T}$ be compact spaces. If $\prod_{j \in T} K_j$ maps continuously onto $[0,1]^\tau$, there is a $j \in T$ such that K_j maps continuously onto $[0,1]^\tau$.

We also prove that if $\tau = 2^\lambda$, the condition of Th 1 implies that E has a quotient isometric to $l^\infty(\lambda)$.

L. WEIS: On the Reprensentation of L_1-operators

N. Kalton has shown that every bounded linear operator T: $L_1[0,1] \to L_1\ [0,1]$ can be represented in the following way:

$$Tf(s) = \int f(t)\, d\nu_s(t) + \sum_{n=1}^{\infty} a_n(s)\, f(\sigma_n(s)) \qquad \text{a.e.}$$

where ν_s are continuous measures and a_n: $[0,1] \to C$, $\sigma_n : [0,1] \to [0,1]$ are measurable functions. The main result (assume that also $T' : L_1 \to L_1$)

TFAE: a) T has a continuous representation $(a_n \equiv 0)$

b) For all $\varepsilon > 0$ there is a E, $\mu(E^c) \le \varepsilon$, s.th.

$$\sup_{f \in L_1(E), ||f|| \le 1} \int_B |T'f|\, d\mu \to 0 \quad \text{if} \quad d(B) \to 0$$

($d(B)$ is the diameter of B in $[0,1]$)

c) $0 \le f_n \le f$, $d(|f_n| > \delta) \to 0$ for all $\delta > 0$, then $Tf_n(y) \to 0$ a.e.

TFAE: a) T has atomic representation $(\nu_s = 0)$

b) For all $f_n \in L_1$, $||f_n|| \le 1$, $f_n \xrightarrow{\mu} 0$ we have $Tf_n \xrightarrow{\mu} 0$

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c) For all subalgebras Σ without atoms we have $F_n^{\Sigma}(x) \xrightarrow{\mu} 0$ where F_n^{Σ} is the martingale representing $T|_{L_1([0,1],\, \Sigma,\, \mu|\Sigma)}$

The main step in the proof is

Theorem. If T has continuous representation and $\varepsilon > 0$, then there is a subalgebra Σ without atoms, such that $T|_{L_1([0,1],\, \Sigma,\, \mu|\Sigma)}$ is a compact integral operator

There is a similiar result for the decomposition into an integral operator and an operator with singular representation. As an application of the Theorem one can give a short measure-theoretic proof of a theorem of Don: every multiplier from singular measures on an abelian group to singular measures has to be an atomic measure.

J.H.M. WHITFIELD: Normal measures and smooth points

Let $\mathcal{B}$ be the σ -algebra of Borel subsets of T, a compact Hausdorff space. $N(T, \mathcal{B}) = N^+(T,\mathcal{B}) - N^+(T,\mathcal{B})$ where $N^+(T,\mathcal{B})$ is the closed proper cone in $rca(T,\mathcal{B})$ consisting of positive normal measures.

Theorem. If T is hyperstonian, $\mu_0 \in N(T,\mathcal{B})$, $||\mu_0|| = 1$ then the following are equivalent: (1) the (variation) norm is Gateaux differentiable at μ_0; (2) $T = S(\mu_0) = S(\mu_0^+) \cup S(\mu_0^-)$ and $S(\mu_0^+) \cap S(\mu_0^-) = \emptyset$ ($S(\mu)$ is the support of μ); (3) $\nu \ll \mu_0$ for all $\nu \in N(T,\mathcal{B})$.

An example of an uncountable compact Haudorff space T for which the unit ball $N(T,\mathcal{B})$ has an abundance of smooth points is given. This contrasts with a result of Bilyeu and Lewis: if T is compact Hausdorff and the unit ball of $ca(T,\mathcal{B})$ has a smooth point, then T is countable. (This is a joint paper with I. E. Leonard.)

V. ZIZLER: $C^{(k)}$-smooth norms on Banach spaces

A preliminary announcement of results of three joint works by M. Fabian, K. John, J. Whitfield, L. Zajicêk and V. Zizler.

In the first paper we study properties of spaces which admit a real valued function φ with bounded support and φ locally Lipschitzian. We prove e.g. that such spaces are already of type 2, provided they contain no subspace isomorphic to c_0. This gives e.g. an extension of known smooth characterizatins of spaces isomorphic to Hilbert space. In the second work we construct C^K-smooth partitions of unity on WCG Banach spaces which admit C^K-smooth function with bounded support.

In the third note we prove the residuality of the set R of all to rotund norms in the space of all equivalent norms on a given Banach space X, with the metric of uniform convergence on the unit ball of X, under the assumption that $R \neq \phi$.

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