MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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# TOPOLOGIE

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Die Tagung fand unter der Leitung von Herrn L. Siebenmann (Orsay), Herrn Ch. Thomas (Cambridge) und Herrn F. Waldhausen (Bielefeld) statt. Es wurden Fragen aus unterschiedlichen Bereichen der Topologie erörtert.

## Vortragsauszüge:

J. AGUADÉ:

Some questions on realizing cohomology algebras

If $R$ is a commutative ring and $A^*$ is a graded $R$-algebra, is there a space $X$ such that $H^*(X;R) \cong A^*$ as graded $R$-algebras? This is the problem of realizing cohomology algebras. In the talk I have discussed three different topics:

1) polynomial algebras

Conjecture: If $U^*(X;\mathbb{Z})$ is a polynomial algebra, then $U^*(X;\mathbb{Z}) \cong H^*(Y;\mathbb{Z})$, where $Y$ is a product of spaces $BU(n)$, $BSp(m)$, $BSO(k)$.

Theorem: If there are no two generators in the same dimension, the conjecture holds true.

2) $\mathbb{Z}_p[x] \otimes E(y)$

I have presented some results on the realizability of $\mathbb{Z}_p[x] \otimes E(y)$.

Theorem: (p odd) 1) If $\mathbb{Z}_p[x] \otimes E(y)$ is realizable $\Rightarrow$ dim $X = 2n$ $n \mid p^2(p-1)$
2) if $n \mid p-1 \rightarrow \mathbb{Z}_p[x] \otimes E(y)$ is realizable
3) if $\mathbb{Z}_p[x] \otimes E(y)$ is realizable and $\beta = 0 \Rightarrow n \mid p-1$
4) if $\mathbb{Z}_p[x] \otimes E(y)$ is realizable and $\beta_4 \neq 0 \Rightarrow n \mid p-1$
5) if $4 \mid p(p-1) \rightarrow \mathbb{Z}_p[x] \otimes E(\beta x)$ is realizable.

Conjecture: The converse of 4) is true.

3) algebras over $\mathbb{Z}$ .
Roughly speaking, I showed that in the general case and for infinitely many primes p, $U^*(X;\mathbb{Z}_{(p)})$ has to have a big divisibility, at least as much as an algebra with divided powers, if $U^*(X;\mathbb{Q})$ is free.

J.L. DUPONT:

Hyperbolic scissors congruence

Let $X = \mathbb{R}^n$, $S^n$ or $H^n$, that is Euclidean, spherical or hyperbolic n-space. The generalized Hilbert's 3rd problem is to determine the structure of the scissors congruence group $P(X)$ defined as the abelian group generated by all proper polytopes $P$ of $X$ subject to the relations:

i) $P = P_1 \cup P_2$, $\dim P_1 \cap P_2 < n \Rightarrow [P] = [P_1] + [P_2]$ ,

ii) $[P] = [gP]$ $g \in G$ = group of isometries of X. This problem is related to the problem of calculating the Eilenberg-MacLane group homology of G considered as a discrete group. We show:

Theorem 1: $P(H^3)$ is a divisible abelian group.

Notice that the group of orientation preserving isometries of $H^3$ is $PSl(2,\mathbb{C})$. Thus at the same time we prove:

Theorem 2: $H_3(Sl(2,\mathbb{C}), \mathbb{Z})$ is a direct sum of a divisible abelian group and a group of finite 2-primary exponent.

T. GOODWILLIE:

A Homotopy Limit Theorem for Spaces of Embeddings, and some Computations in Concordance Theory

(1) Let $P^p$ and $N^n$ be smooth manifolds, with $P$ compact, $n - p \geq 3$ and for simplicity say $\partial p = \emptyset = \partial N$ . We study the space of smooth embeddings $E(P,N)$. The idea is to express the homotopy type of $E(P,N)$ in terms of homotopy-theoretic data, thus in a sense achieving for embeddings (in codimension $\geq 3$) what the Smale-Hirsch theorem does for immersions (in codimension $\geq 1$). In order to state the result I introduce the notion of continuous diagram, a generalization of the notion of a diagram of spaces. A continuous diagram $(\underline{\underline{I}}, \Phi, \mu)$ consists of a topological category $\underline{\underline{I}}$ , a sheaf of spaces $\Phi$ over the object space.

$\mathrm{Ob}\,(\underline{\underline{I}})$ , and a morphism of sheaves $\mu : d_1^* \Phi \to d_0^* \Phi$ over the morphism space $\mathrm{Mor}(\underline{\underline{I}})$ . The triangle of sheaves over $\mathrm{Mor}_2(\underline{\underline{I}})$

$$\begin{tikzcd} d_2^* d_1^* \Phi \arrow[rr, "d_1^*\mu"] \arrow[dr, "d_2^*\mu"'] & & d_1^* d_0^* \Phi \\ & d_2^* d_0^* \Phi \arrow[ur, "d_0^*\mu"'] & \end{tikzcd}$$

is required to commute, where each $d_j$ is a face map in the nerve of $\underline{\underline{I}}$ .

$$\cdots\cdots \begin{smallmatrix}\longrightarrow\\ \longrightarrow\\ \longrightarrow\\ \longrightarrow\end{smallmatrix} \quad \mathrm{Mor}_2(\underline{\underline{I}}) \quad \begin{smallmatrix}\xrightarrow{d_0}\\ \xrightarrow{d_1}\\ \xrightarrow{d_2}\end{smallmatrix} \quad \mathrm{Mor}(\underline{\underline{I}}) \quad \begin{smallmatrix}\xrightarrow{d_0}\\ \xrightarrow{d_1}\end{smallmatrix} \quad \mathrm{Ob}(\underline{\underline{I}})$$

In the case where $\underline{\underline{I}}$ is a small category (with discrete topology) a continuous diagram is then just a diagram in the usual sense. Every continuous diagram $\mathcal{D} = (\underline{\underline{I}},\Phi,\mu)$ has an inverse limit

$$\underleftarrow{\lim}\, \mathcal{D} = \{\text{global sections } \varphi \text{ of } \Phi \text{ such that } \mu\, d_1^* \Phi = d_0^* \Phi \}$$

and also a homotopy inverse limit $\mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}$ defined much as in the case of ordinary diagrams (see Bousfield/Kan "Homotopy Limits, Completions and Localizations", Springer Lecture Notes) -

<u>Theorem:</u> Given P and N as above, for a certain continuous diagram $\mathcal{D}_\infty$ we have $E(P,N) \cong \underleftarrow{\lim}\, \mathcal{D}_\infty \xrightarrow{\simeq} \mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_\infty$ .
Here $\mathcal{D}_\infty = (\underline{\underline{F}}_\infty P;\, \Phi_\infty, \mu_\infty)$ where $\underline{\underline{F}}_\infty P$ has for objects the finite subsets of P (topologized by an obvious direct limit construction) and as a category is determined by the partial ordering "reverse inclusion". The sheaf $\Phi$ is such that the stalk at $\Sigma \in \mathrm{ob}(\underline{\underline{F}}_\infty P)$ is the space of germs of embeddings of P in N at $\Sigma \subset P$. The map $\mu$ is a restriction map.

One idea involved in the proof is to write $\mathcal{D}_\infty$ as a sort of colimit of continuous diagrams $\mathcal{D}_j$, $0 \leq j < \infty$ , $\mathcal{D}_j$ being defined using only the subsets of P with at most j points. Then there are fibrations

$$\text{----} \to \mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_j \to \mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_{j-1} \to \text{----}$$

with $\mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_\infty \cong \underleftarrow{\lim}\, \mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_j$ . The fiber of $\mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_j$ over $\mathrm{ho}\underleftarrow{\lim}\, \mathcal{D}_{j-1}$ has a connectivity which grows with j, and this fiber can be replaced (up to homotopy type) by a space of sections of a bundle, by a argument like the proof of the Smale-Hirsch theorem.

I am not quite prepared to claim the "Theorem" above in general, because I have not worked out some details involving $\pi_0$. I definitely claim it in the case $2p + 1 < n$.

(2) A similar statement holds for spaces $C(P,N)$ of concordance embeddings again with $n - p \geq 3$. These are embeddings

$$(I \times P;\ 0 \times P,\ I \times \partial P,\ 1 \times P) \to (I \times N;\ 0 \times N,\ I \times \partial N,\ 1 \times N)$$

coinciding with $\mathrm{id} \times f_0$ on $0 \times P \cup I \times \partial P$, where $f_0 : P \to N$ is a fixed embedding which we take to be an inclusion. We have:

Theorem: If $P^p \subset N^n$ is a compact submanifold with $n - p \geq 3$ then for a certain continuous diagram

$$C(P,N) \cong \varprojlim \mathcal{D}^c_\infty \simeq \operatorname{holim}_{\leftarrow} \mathcal{D}^c_\infty$$

Again the topological category in $\mathcal{D}^c_\infty$ is $\underline{\underline{F}}_\infty p$ .

This theorem has as one corollary a stability theorem:

Cor.: The suspension map $C(P,N) \to C(P \times I, N \times I)$ is $(2n-p-4)$-connected. It also makes it possible to compute some of the relative homotopy groups $\pi_i(P(X), P(Y))$ for 2-connected pairs $(X,Y)$, where is the stable pseudo-isotopy functor, and in general to write down the $E^2$-term of a spectral sequence converging to $\pi_i(P(X), P(Y))$.

H. IBISCH:

Classifying spaces for principal bundles with compact Lie group actions

In the smoothing theory of continuous actions of compact Lie groups one has to classify locally trivial principal bundles $\xi = \{p : E \to X\}$ with topological structure group $G$ and an additional action of a compact Lie group $\Gamma$ on $E$ and on $X$ compatible with $p$ and the right translations of $G$ on $E$. For a given $\xi$, define an equivariant homotopy $f : Y \times I \to X$ to be a $\xi$-homotopy if there is a G-homeomorphism $\varphi : E(f^*(\xi)) / \Gamma \xrightarrow{\approx} E(f_0^*(\xi)) / \Gamma \times I$ fibred over $Y/\Gamma \times I$. Then $f$ is a $\xi$-homotopy if and only if $f^*(\xi) \approx f_0^*(\xi) \times I$. Moreover we construct a numerable principal bundle $\mathcal{E}(\Gamma;G) \xrightarrow{\varepsilon} \mathcal{B}(\Gamma;G)$, and we establish a canonical bijection between the set of isomorphism classes of $(\Gamma,G)$-principal bundles over $X$ and the set of $\mathcal{E}$-homotopy classes of equivariant maps $X \to \mathcal{B}(\Gamma,G)$. The relation to isotopy of actions was discussed.

K. IGUSA:

Half functions and the pseudoisotopy stability theorem

I state the stability theorem and some corollaries, and the definition of a half function. All my manifolds are compact and smooth $(C^\infty)$ with corners (locally diffeo to $[o,\infty)^n$.)

Let $M$ be an n-manifold.

$C(M) = \mathrm{Diff}(M\times I \text{ rel } M\times 0 \cup \partial M\times I)$ = concordance space of M.

(concordance = pseudoisotopy)

$\Sigma : C(M) \to C(M\times I)$ is the suspension map of Hatcher-Wagoner as given in Astêrisque 6.

MAIN THEOREM: $\Sigma$ is k-connected if $n \geq 7k+10$ (It was previously "proved" that $\Sigma$ is $\sim n/6$-connected, but this "proof" was wrong.

Corollary 1: If $n \geq 7k+17$ $\pi_k\, C(M)$ is a homotopy invariant of $M$

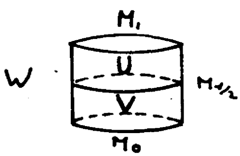
Corollary 2: Suppose $n \geq 7k+17$ and $M = N\times I$. Then

$$\pi_k\, \mathrm{Diff}(M \text{ rel } \partial M) \otimes \mathbb{Z}[\tfrac{1}{2}] \cong \pi_k\, C(M) \otimes \mathbb{Z}[\tfrac{1}{2}]^{\pm} \oplus \widetilde{\mathrm{Diff}}(M \text{ rel } \partial M) \otimes \mathbb{Z}[\tfrac{1}{2}]$$

where $\pm = \mathrm{sign}(-1)^n$ and $A^+ \oplus A^- = A$.

Note that $\pi_k\, \mathrm{Diff}(D^n \text{ rel } \partial D^n) \cong \Gamma_{n+k+1}$ (exotic n+k+1-spheres).

Suppose that $W^{n+1}$ is a cobordism from $M_0^n$ to $M_1^n$, rel. $\partial M_0$.




Suppose that $W$ is broken into two cobordisms $U,V$ as indicated. Let $V(W) = \{V\}$ given in this way. Let $V^s(W) = \{V | U \approx M_1 \times [1/2,1]\}$. Then

Prop.: $V^s(W) \simeq BC(M_1)$.

Let $f: V \to [o,\tfrac{1}{2}]$ be a smooth map such that $f^{-1}(i) = M_i$, $i = 0,\ 1/2$ and $f|\partial M_0 \times [o,\tfrac{1}{2}]$ is projection, and critical points of $f$ lie in $\mathrm{int}(V)$. The pair $(V,f)$ is called a half function (it is half of a function $W \to I$).

If $\tilde{F}(W)$ is the space of all half functions and $F^s(W) = \{(V,f) | U \approx M_1 \times [\tfrac{1}{2},1]\}$ then

Prop.: $\tilde{F}^s(W) \simeq V^s(W) \simeq BC(M_1)$. The suspension map can be defined by $\Sigma': \tilde{F}(M\times I) \to \tilde{F}(M\times I^2)$. I use quasifibrations type arguments to show that $\Sigma'$ is highly connected. Thus I decompose the spaces and show $\Sigma'$ is highly connected on each piece and glue together.

A typical theorem on one piece:

Theorem: Let $\tilde{M}_j(W) = \{(V,f) | f$ has exactly one critical point which is Morse of index j.$\}$ Then

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$\tilde{M}_j(W) \simeq \mathrm{Emb}((D^j,S^{j-1}),(W,M_0))/O_j$ . By embedding theory there is a highly connected map to the homotopy theoretic space $\mathrm{Map}((D^j,S^{j-1}), (W,M_0))\times EO_j/O_j$. Thus $\Sigma'$ is highly connected on this piece.

ST. JACKOWSKI:

The Krull dimension of group cohomology with twisted coefficients

For any finite group $G$ and a $G$-module $A$ we examine the growth of a graded cohomology group $H^*(G;A)$. We try to answer a question posed by D.Quillen, who solved the problem for constant coefficients. It turns out that the growth of $H^*(G;A)$ is not faster than that of the group $H^*(T;A)$ for some elementary abelian subgroup $T$ of $G$. As a corollary we prove that a G-module has periodic cohomology iff all its restrictions to elementary abelian subgroups of $G$ do have periodic cohomology. The main result can be generalized from finite groups to groups of finite virtual cohomological dimension.

J.D.S. JONES:

The Kervaire Invariant of Immersions

This is joint work with Ralph Cohen and Mark Mahowald. The object of this work is to study and compute E.H. Brown's generalized Kervaire invariant, defined, under suitable hypotheses, for manifolds immersed in euclidean space. The main examples we consider are (i) closed compact n-manifolds immersed in $R^{n+1}$ (ii) closed compact oriented n-manifolds immersed in $R^{n+2}$. The prototype for our theorems is W. Browder's theorem that the Kervaire invariant of framed manifolds vanishes unless the manifold has dimension $2^{k+1} - 2$, $k \geq 1$. Browder's theorem also gives a necessary and sufficient condition for the existence of a framed $(2^{k+1}-2)$-manifold with non-zero Kervaire invariant - this condition has been verified for $k=1, 2, 3, 4, 5$, the remaining cases are unsettled.
Our theorems are (i) for n-manifold immersed in $R^{n+1}$, then the Kervaire invariant vanishes unless $n=2^{k+1}-2$; there is once more a necessary and sufficient condition for the existence of $(2^{k+1}-2)$-dimensional manifolds with non-zero Kervaire invariant, but the condition is no simpler to verify (possibly it is equivalent) than the conditions occuring in Browder's theorem.

Next we show (ii) for oriented n-manifolds immersed in $R^{n+2}$, once more the Kervaire invariant vanishes unless $n=2^{k+1}-2$. In this case, however, we are able to show that there is in each dimension $2^{k+1}-2$ an oriented manifold which immerses in $R^{2^{k+1}}$ with non-zero Kervaire invariant. Our methods involve an adaptation and modification of Browder's methods for proving his theorem, together with an application of a construction, due to Mahowald, of an infinite family of non-zero homotopy classes $\eta_j \in \pi^S_{2^j}(S^0)$ .

U. KOSCHORKE:

The span and the stable span of a manifold

Given a connected, closed, smooth n-dimensional manifold M, we compare its span (i.e. the maximum number of linearly independent tangential vector fields) with stable span(M) := $\max\{k \mid TM \oplus \underset{\sim}{R}$ allows k-1 linearly independent sections}. Using the singularity approach, we show that in most cases the span of M coincides either with the stable span or with a certain well defined number s(M).E.g. s(M) = 0 iff n is even, s(M) = 1 iff $n \equiv 1(4)$ and $w_1(M)^2 = 0$, etc. In general s(M) is not smaller than and often exceeds span $(S^n)$. As an example we exhibit a family of n-manifolds of the form $M = P^r \times S^q$ such that span(M) = s(M) = 4 is different from the stable span of M, from the Hurwitz-Radon number span($S^n$), and from the span of the factor sphere $S^q$. This disposes of a conjecture of Eagle and Kusinski.

M. KRECK:

Manifolds with Unique Differentiable Structure

An n-dimensional manifold is called of type BSO if $\tau_* : \pi_i(M) \to \pi_i(BSO)$ is an isomorphism for $i \leq (n-1)/2$, and surjective for $i \leq (n+1)/2$, and for n odd, some extra conditions are fulfilled. For $n \neq 3$ every oriented manifold is bordant to one of this type. The stable diffeomorphism type of those manifolds is classified by the bordism class of the middle Betti number. This result can be used to show the existence of manifolds with unique differentiable structure, $n \neq 4$, M of type BSO. Then there exists an $r \in \mathbb{N}$ such that $M \# rS$ has unique differentiable structure up to diffeomorphism, where $S = S^k \times S^k$, or $= S^k \times S^{k+1}$ .

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P. LÖFFLER:

## The Kahn-Priddy theorem, the Hopf construction and surgery on manifolds with involution

We saw various result on involutions on (homotopy) spheres. For instance:

Proposition: Suppose $k < 2n - \varepsilon$. Suppose further that $\Sigma^k$, a sphere, embeds in $R^{n+k}$ with trivial normal bundle. Then modulo surgery obstructions there in an involution on $S^{n+k}$ fixing $\Sigma^k$.

W. MEIER:

## Derivations in the cohomology of homogeneous spaces and compact fibrations

A conjecture of S.Halperin states that the rational Serre spectral sequence of any orientable fibration collapses, if the fibre $X$ is a finite simply connected space with evenly graded rational cohomology. This conjecture is shown to be equivalent to the vanishing of all derivations of negative degree in $H^*(X;\mathbb{Q})$. Then this algebraic statement is verified in many cases, including several classes of homogeneous spaces of compact connected Lie groups. As a first application a computation of $\pi_i(\widetilde{\mathrm{Diff}}\, M) \otimes \mathbb{Q}$, $i \geq 1$, is given, where $\widetilde{\mathrm{Diff}}\, M$ denotes the (simplicial) group of block automorphisms of a manifold M, and $M$ is taken to be a homogeneous space as above As a second application we obtain a strong restriction on the possible compact fibrations of complex (or quaternionic) Grassmann manifolds. In fact, we obtain new examples of such spaces which can't be fibred by a compact base and fibre.

B. MORIN:

## The Kühnel triangulation of $\mathbb{C}P^2$

Let the vertices $1,\dots,9$ of the 8-simplex $\Delta_8$ be arranged in an array as follows: $\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$, and let the group $G = \{\alpha,\beta,\gamma;\ [\alpha,\beta],\ \beta[\alpha^{-1},\gamma^{-1}],\ \alpha^3 \beta^3,\gamma^3\}$ act on $\Delta_8$ by $\alpha\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}=\begin{pmatrix} 4 & 7 & 1 \\ 5 & 8 & 2 \\ 6 & 9 & 3 \end{pmatrix}$, $\beta\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 1 & 4 & 7 \end{pmatrix}$, $\gamma\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 7 \\ 3 & 4 & 8 \\ 1 & 5 & 9 \end{pmatrix}$.

Let $K_0$ be the subcomplex of $\Delta_8$ generated by the orbit $G\{1,2,4,5,9\}$.

Theorem 1: The geometric realization $|K|$ of $K = K_0 \cup K_1$ is homeomorphic to $\mathbb{C}P^2$.

Theorem 2: (W.Kühnel): If $K'$ is a subcomplex of $\Delta_8$, containing the 2-skeleton of $\Delta_8$ such that $|K'|$ is a 4-dimensional manifold, then $K'$ is of the form $\sigma(K)$, where $\sigma$ is a permutation of the vertices of $\Delta_8$.

Proof of Theorem 1: Let $h : S^5 \to \mathbb{C}P^2$ be the Hopf map and $\pi : \mathbb{C}P^2 \to \Delta_2$ be defined by the map $S^5 \to \Delta_2$ given by $(z_1,z_2,z_3) \mapsto (|z_1|^2,|z_2|^2,|z_3|^2)$. Let a,b,c be the vertices of $\Delta_2$; a',b' and c' the midpoints of (bc),(ac) and (ab) respectively. Then the inverse images under $\pi$ of each of the triangles shaded in figure 1 are homeomorphic to $B^4$

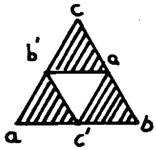


Figure 1

We use $K_0$ to triangulate these balls. It then remains to show, that $\pi^{-1}(a'b'c')$ is homeomorphic to $|K_1|$. This is done by studying the inverse image of a pint $p \in (a'b'c')$ under the map $\bar{\pi} : |K_1| \to (a'b'c')$ defined by $\bar{\pi}(1) = \bar{\pi}(2) = \bar{\pi}(3) = a'$, $\bar{\pi}(4) = \bar{\pi}(5) = \bar{\pi}(6) = b'$, $\bar{\pi}(7) = \bar{\pi}(8) = \bar{\pi}(9) = c'$. When p belongs to the interion of (a'b'c'), $\bar{\pi}^{-1}(p)$ is the union of 27 parallelograms matching together into a torus. When p is the center, the parallelograms become rhombic as shown in figure 2.

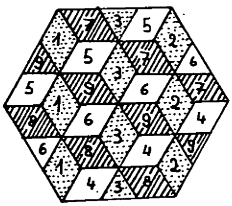

Figure 2

The proof is completed by studying how this picture degenerates, when p tends to the boundary of (a'b'c').

Proof of Theorem 2: It seems that up to now the proof still depends on computer work.

R. OLIVER:

## A construction for a transfer map

Peter May's construction of a transfer map was described - a natural homomorphism $H_*(X/G) \to H_*(X/H)$ for any compact Lie group $H \subset G$ and G-CW complex X. The key step is a form of "algebraic desuspension": homomorphisms $f_{\#} : H_*(X/G) \to H_*(Y/G)$ are constructed, induced by maps $f : \Sigma^V X \to \Sigma^V Y$ (where $\Sigma^V$ denotes suspension with respect to a representation V). This follows from (and is motivated by) the existence of RO(G)-graded homology extending the usual ($\mathbb{Z}$-graded) equivariant singular homology; but can also be proved directly.

C. OLK:

Immersions of Manifolds

First we use singularity obstructions to study immersions of (closed, $C^\infty$) manifolds up to (oriented) bordism and get the following results:

Theorem A: If M is an n-manifold, $k \in \mathbb{N}$, $k < n$, $k < 8$ and, for ($n=2^i+3$ and $k \geq 6$), M is decomposable in $N_*$, M immerses into $R^{2n-k}$ up to bordism if and only if for all $i \in \mathbb{N}$ and all partitions $i_1, \dots, i_\alpha$ of k-i, $w_{i_1}, \dots, w_{i_\alpha} \bar{w}_{n-k+i}(M) = 0$;

Theorem B: Let M be an n-manifold, $k \in \mathbb{N}$, $2k-1 \leq n$, $k \leq 8$, M oriented. If $n \not\equiv 0(4)$, M immerses up to orientes bordism into $\mathbb{R}^{2n-k}$, if

$$\forall i \forall i_1, \dots, i_\alpha \; : \; w_{i_1}, \dots, w_{i_\alpha} \bar{w}_{n-k+i}(M) = 0$$

if $n \equiv 0(4)$, the results are more complicated; for example $M \underset{\text{or.b.}}{\looparrowright} \mathbb{R}^{2n-3}$ iff $w_2\bar{w}_{n-2}(M) = 0$ and a certain Pontrjagin number $S_n(M)$ is $\equiv 0(4)$.

One can use this to prove existence theorems for immersions themselves, for example

Theorem C: Let M be an n-manifold, $n \equiv 0(4)$, $n \geq 8$.
If $\bar{w}_{n-2}(M) = 0 = \bar{w}_{n-4}(M)$, $H_1(M;\tilde{\mathbb{Z}}_{TM})$ has no 2-torsion and $S_n(M) \equiv 0(4)$, then $M \looparrowright \mathbb{R}^{2n-3}$.

D. REPOVŠ:

Topology of generalized 3-manifolds with zero-dimensional singular set

In early spring of 1977 J.W.Cannon conjectured that topological manifolds (without boundary) are precisely generalized manifolds satisfying a minimal

amount of general position. For $n \geq 5$ he suggested the dispoint disks property (DDP). By summer of 1978 his conjecture was settled in the affirmative by results of R.D.Edwards and F. Quinn. (At about the same time H. Toruńczyk proved a similar characterization for Q-manifolds.) We seek an analogue of DDP in dimension three.
(Little is known about dimension four, while every generalized ($n \leq 2$)-manifold is an n-manifold.)
We say that a space $X$ has the map separation property (MSP) if given any collection $f_1,\dots,f_k : D^2 \to X$ of Dehn disks (not necessarily PL) such that if $i \neq j$ then $f_i(\partial D) \cap f_j(D) \neq \emptyset$, and given a neighborhood $U \subset X$ of $\bigcup_{i=1}^{k} f_i(D)$ there exist mappings $F_1,\dots,F_k : D^2 \to U$ such that for each $i$: $F_i|\partial D = f_i|\partial D$ and if $i \neq j$, then $F_i(D) \cap F_j(D) = \emptyset$.

Theorem: Let $g$ be a cell-like closed 0-dimensional usc decomposition of a 3-manifold $M$. Then $M/G$ is a 3-manifold if and only if $M/G$ has the MSP.

Theorem: Let $C$ be the class of all generalized 3-manifolds $X$ with $\dim S(X) \leq 0$ and let $C_0 \subset C$ be the subclass of all compact $X \in C$ with $S(X) \subset \{pt\}$ and $X \simeq S^3$. TFAE:

(i) Poincaré conjecture in dimension three is true

(ii) For every $X \in C$, $X$ has the MSP $\Rightarrow S(X) = \emptyset$

(iii) For every $X \in C_0$, $X$ has the MSP $\Rightarrow S(X) = \emptyset$.

Theorem: Let $X$ be a generalized 3-manifold satisfying Kneser finiteness and having the MSP (in fact, it suffices to assume MSP only for pairs of disks). Then X has no isolated singularities. The last result is about resolutions of $\mathbb{Z}_2$-homology 3-manifolds.

Theorem: Let $f : M \twoheadrightarrow X$ be a closed monotone mapping from a 3-manifolds $M$ onto a locally simply connected $\mathbb{Z}_2$-homology 3-manifold and suppose there exists a zero-dimensional set $Z \subset X$ such that $\check{H}^1(f^{-1}(x);\mathbb{Z}_2) = 0$ for all $x \in X - Z$. Then

(i) the set $C = \{x \in X | f^{-1}(x)$ is not cell-like$\}$ is locally finite in $X$;

(ii) $X$ has a resolution (hence in particular, $X$ is a generalized manifold).

L. SCHWARTZ:

On $(\mathbb{K}\mathbf{F}_q)_*$ $(MU(n))$ and $(\mathbb{K}\mathbf{F}_q)_*$ $(MU(n), \mathbb{Z}/1)$

Let $(\mathbb{K}\mathbf{F}_q)_*(-,Z)$ be the homology theory defined by the infinite loop space $\mathbb{Z} \times BGL^+(\mathbb{F}_q)$. Then Hurewicz map $\pi_*^S(MU(n)) \to (\mathbb{K}\,\mathbb{F})_*(MU(n))$ gives a test for stable homotopy of MU(n). In fact there is an isomorphism from $(\pi_*^S(MU(2))/Tor) \otimes \mathbb{Z}_{(l)}$ into $(\mathbb{K}\,\mathbb{F}_q)_{ev}(MU(2)) \otimes \mathbb{Z}_{(l)}$ up to dimension $2(l^4+l^3)$, l an odd prime.

At the prime 2 there is the following fact:
$(\pi_{12}^S(MU(2))/Tor) \otimes \mathbb{Z}_{(2)}$ (which is isomorphic to $\mathbb{Z}_{(2)}^3$) does not surject on $(\mathbb{K}\,\mathbb{F}_q)_{12}(MU(2)) \otimes \mathbb{Z}_{(2)}$ (isomorphic to $\mathbb{Z}_{(2)}^3$). These results are indications in the direction of a conjecture of N. Ray and V. Snaith which say that $\pi_*^S(BU)/Tor$ maps isomorphically via the Hurewicz homomorphism onto the primitives (for the coaction) in $k_*(BU)$. (k(-) connective k-homology). The result is positive but partial in the first case (l odd prime), negative in the second case.

The other results are the computation of $(\mathbb{K}\,\mathbb{F}_q)_*(MU(n),\mathbb{Z}/l)$ for $n = 1,2$, l an odd prime, and indications how to do it for all n. These results are more or less equivalent to the computation of the homotopy groups of the K-theoretic localization of the suspension spectrum of MU(n) (reduced mod l). The computation involves study of actions of finite groups on polynomial algebras. For $MU(1) = \mathbb{C}P^\infty$ the result is stated in terms of the theory $(Ad_q)_*(-\mathbb{Z}/l)$ (non-connective theory associated to $\mathbb{K}\mathbb{F}_g(-\mathbb{Z}/l)$ is
$(Ad_q)_*(\mathbb{C}P^\infty,\mathbb{Z}/l) \cong (Ad_q)_*(-\mathbb{Z}/l)\ [T_i,\ i \geq o]/T_i^l = \alpha T_i,\ T_iT_j = 0,\ i \neq j,\ \delta T_i = 0$,
where we suppose $l \mid q-1$, $l^2 \mid q^{l-1}-1$, and we have
$(Ad_q)_*(\mathbb{Z}/l) \in \mathbb{F}_l[\alpha,\alpha^{-1}] \otimes E(\delta)$, $\alpha \in (\mathbb{K}\mathbb{F}_q)_{2(l-1)}(-\mathbb{Z}/l)$, $\delta \in (\mathbb{K}\mathbb{F}_q)_{2l-3}(-\mathbb{Z}/l)$.

W. SINGHOF:

## Parallelizability of homogeneous spaces

We consider the following problem: Let G be a compact connected Lie group and H a closed connected subgroup of G. When is G/H parallelizable?
It suffices to study the question of stable parallelizability, since the difference between stable parallelizability and parallelizability is controlled by the Euler characteristic of the semi-characteristic.
A proof of the following result is indicated:
Let $G = SU(n)$ and let H be isomorphic to $SU(k_1)\times\ldots\times SU(k_r)$; H may be embedded in SU(n) in an arbitrary way. Then G/H is stably parallelizable if and only if it is diffeomorphic to a Stiefel manifold, or to $SU(n)/(SU(2)\times\ldots\times SU(2))$, or to $SU(4)/H_0$ with

$$H_0 = \left\{ \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \middle| A \in SU(2) \right\} .$$

The proof is an application of the classical work of Borel and Hirzebruch on homogeneous spaces and characteristic classes, combined with information stemming from the Eilenberg-Moore spectral sequence and its K-theoretic analog.

S. ZARATI:

Quadruple points of codimension one immersion of three dimensional manifolds

Let M and N be two $C^\infty$ 3-manifolds such that $\partial N = \partial M = \emptyset$ and N compact, and let $\alpha : N \looparrowright M \times \mathbb{R}$ be a generic immersion with an oriented normal bundle $\nu_\alpha \simeq \varepsilon^1$. The cobordism group of these immersions is identified with $[\hat{M}, QS^0]$ the group of homotopy classes of maps from $\hat{M}$ = the one point compactification of M to $QS^0 = \lim_n \Omega^n S^n$ .
Let X be an oriented surface in M dual to $w_1(M)$. We have the following diagram

$$\begin{array}{ccc} N & \overset{\alpha}{\looparrowright} & M \times \mathbb{R} \\ \uparrow & & \uparrow \\ Y & \overset{\beta}{\looparrowright} & X \times \mathbb{R} \end{array}$$

We suppose $\beta$ generic and we call it the Bockstein of $\alpha$ . We define the Kervaire invariant of $\beta$ : We choose a stable trivialization over X and we put:

$$K(\beta,t) = K(Y,b^*t)+q^t(h) + \sum_{\substack{X'\text{conn.cp.}\\ \text{comp.of } X}} d(X') \cdot K(X',t') .$$

where $b : \nu_Y \to \nu_X$ is induced by $\beta$, $q^t: H^1_c(X;\mathbb{Z}/2) \to \mathbb{Z}_2$ is a quadratic form defined by t, h is dual to the element representing the surface of double points, K(?) is the Kervaire invariant and $d(X') = \mathrm{diag}(Y' \to X')$ induced by the composition $Y \underset{\beta}{\looparrowright} X \times \mathbb{R} \underset{\pi}{\to} X$. We prove that

$$K(\beta) = < \alpha^*_\beta \, \Delta \, w_2(\Gamma);[X] >$$

(independent of t).
where $w_2(\Gamma) \in H^2(\sigma,\mathbb{Z}/2)$ and $\Delta : H^*(\sigma) \to H^*(QS^0) \cong H^*(\sigma)^{\mathbb{Z}}$ is the diagonal embedding.
Theorem: Let $x \in H_2(M; \mathbb{Z}/2)$ represent the surface of double points of $\alpha_j(S_\alpha)$, then the number of quadruple points of $\alpha$ is equal mod 2 to the sum of the Euler characteristic of $S_\alpha$ and the triple intersections of $S_\alpha$ and the Kervaire invariant of $\beta$ : the Bockstein of $\alpha$ .

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