MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 42/81

Funktionalanalysis: C*-Algebren

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Die jährliche Funktionalanalysistagung in Oberwolfach wurde in diesem Jahr unter das Thema "C*-Algebren" gestellt. Unter der Leitung von A. Connes (Paris), J. Cuntz (Heidelberg, z. Zt. Marseille) und R. Nagel (Tübingen) nahmen daran über 40 Mathematiker aus Europa und Amerika teil. Ein Interessenschwerpunkt bestand in der Untersuchung der Struktur solcher C*-Algebren, die mit dynamischen Systemen und geometrischen Objekten assoziiert werden können. Vor allem aber wurden die aktuellen Entwicklungen auf dem Gebiet der K-Theorie und allgemeinerer Homologie-Kohomologie-Theorien für nichtkommutative C*-Algebren und ihre Beziehungen zu topologischen Fragen diskutiert. Als Höhepunkt kann ohne Zweifel der mehrstündige Vortrag von A. Connes über "Spectral sequence and homology of currents for operator algebras" bezeichnet werden. Die angenehme "Oberwolfach-Atmosphäre" tat ein übriges, den Aufenthalt für alle Teilnehmer fruchtbar und angenehm zu gestalten. Daraus resultierte der Wunsch, daß diese bisher erste Oberwolfachtagung über C*-Algebren eine Wiederholung finden möge.

Vortragsauszüge

C.A. AKEMANN:

Perfect C^*-algebras

This talk describes joint work in progress with Fred Shultz. Let A be a C^*-algebra, z the central projection in A^{**} covering the pure states P(A) of A, $M = zA^{**}$ and $A_c = \{b \in M : b, b^*b, bb^*$ are continuous on $P(A) \cup \{0\}$ with the weak* topology$\}$. We say A is perfect if $zA = A_c$. Here are some facts about A_c: It is a C^*-algebra whose center is the same as the center of A and which is simple iff A is simple. If A is perfect, is a subalgebra of a C^*-algebra B and separates $P(B) \cup \{0\}$, then B = A. This explains our interest in perfect C^*-algebras. If H is a Hilbert space, B(H) is perfect and so is K, the compact operators on H. Perfection passes to hereditary C^*-subalgebras, and A_c is always perfect. If C is an abelian C^*-subalgebra of B(H), H is separable and C has uncountable spectrum, then $A = K \oplus C$ is not perfect.

A. BEHNKE:

A Class of AF Algebras

The work described was done jointly with G. Elliott. Let P be a countable partially ordered set. Then $G = \overset{\ell}{\underset{x \in P}{\oplus}} Z$, the lexicographic product of Z indexed by P is a dimension group. The aim is to classify all separable AF algebras A with dimension group G. Any $x \in P$ gives rise to a "unique" irreducible representation π_x of A. Then $U = \{x \in P \mid \pi_x(A) \text{ is unital}\}$ is order closed in P. Thus $P \setminus U$ determines uniquely an ideal I of A.

Theorem 1: P,U and $^A/_I$ are isomorphism invariants for A.

Thus it suffices to study algebras with U = P only. Such algebras are called locally unital.

Theorem 2: Let A be a locally unital algebra. Then there exists a function d on P, which is an isomorphism invariant of A modulo ideal preserving automorphisms of A.

A gives the coordinates of the identity locally. One obtains a further invariant also by extending d appropriately to all of P. It is believed that this invariant is complete.

For P totally ordered and related cases, a complete classification of all algebras is possible.

O. BRATTELI:

On generators of positivity-preserving semigroups

(Joint work with A. Kishimoto and D.W. Robinson, Commun. Math. Phys. 75 (1980), 67-84). Let $H = L^2(X;d\mu)$ for some positive measure μ, and let H_+ be the positive functions in H. An operator $A \in B(H)$ is said to be positivity-preserving, written $A \geq 0$, if $AH_+ \subseteq H_+$. If H,K are generators of self-adjoint, positivity-preserving semigroups e^{-tH}, e^{-tK} of contractions on H, the relation $e^{-tH} \geq e^{-tK}$ can be expressed by means of relations between H and K. As an application, we show that a nonnegative Borel-function $f : [0,+\infty) \to [0,+\infty)$ has the property that $e^{-tH} \geq e^{-tK} \geq 0$

implies $e^{-tf(H)} \geq e^{-tf(K)} \geq 0$ for all $t \geq 0$ and all $H = H^* \geq 0$, $K = K^* \geq 0$ if and only if $f(0) \leq f(0+)$ and the restriction of f to $(0,+\infty)$ is C^∞ with $(-1)^n f^{(n+1)}(x) \geq 0$ for all $x > 0$, $n = 0,1,2,..$.

A. CONNES:

Spectral sequence and homology of currents for operator algebras

The transversal elliptic theory for foliations requires as a preliminary a purely algebraic work of computing for a non-commutative algebra A the homology of the following complex: n-cochains and multilinear fcts. $\varphi(f^1,..,f^n)$ of $f^1,..,f^n \in A$ with $(f^1,f^2,...,f^n,f^o) = (-1)^n \varphi(f^o,f^1,..,f^n)$ and the boundary is $b\varphi(f^o,..,f^{n+1}) = \varphi(f^o f^1,f^2,..,f^{n+1}) - \varphi(f^o,f^1 f^2,..,f^{n+1})$ $+ ... + (-1)^{n+1}\varphi(f^{n+1}f^o,..,f^n)$. The basic class associated to a transversally elliptic operator for A = the algebra of the foliation is given by

$$\varphi(f^o,..,f^n) = \text{Trace}\ (\varepsilon\ F\ [F,f^o][F,f^1]\ ...\ [F,f^n])\ ,\quad f^i \in A$$

where $F = \begin{bmatrix} 0 & Q \\ P & 0 \end{bmatrix}$, Q is a parametrix of P and $\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. An operator $S : H^n(A) \to H^{n+2}(A)$ is constructed as well as a pairing $K(A) \times H(A) \to \mathbb{C}$ where $K(A)$ is the algebraic K-theory of A, it gives the index of the operator from its associated class φ, moreover $\langle e,\varphi\rangle = \langle e,S\varphi\rangle$ so that the important group to determine is the inductive limit $H_g = \varinjlim H^n(A)$ for the map S being the tools of homological algebra the groups $H^n(A, A^*)$ of Hochschildt cohomology with coeff. in the bimodule A^* are easier to determine and the solution of the problem is obtained in two steps:

1. The construction of a map $B: H^n(A, A^*) \to H^{n-1}(A)$ and the proof of a long exact sequence

$$\ldots \to H^n(A, A^*) \overset{B}{\to} (H^{n-1}(A) \overset{S}{\to} H^{n+1}(A) \overset{I}{\to} H^{n+1}(A, A^*) \to \ldots$$

where I is the obvious map from the cohomology of the above complex to the Hochschildt cohomology.

2. The contruction of a spectral sequence with E_2 given by the cohomology of degree -1 differential $I \circ B$ on the Hochschildt group $H^n(A, A^*)$ and which converges strongly to a graded group associated to the above inductive limit.

This purely algebraic theory is then used for $A = C^\infty(V)$ one gets the de Rham homology of currents and for the pseudo torus, i.e. the algebra of the Kronecker foliation one finds that the Hochschildt cohomology depends on the deophactive nature of the rotation number while the above theory gives H_g^o of dim 1, H_g^1 of dim 2 and H_g^2 of dim 2 as expected, but by some remarkable cancellations.

G.A. ELLIOTT:

<u>Temperature-density state spaces of dynamical systems</u>

If $(A, \mathbb{R}^2, \alpha)$ is an almost periodic C^*-dynamical system and if whenever a nontrivial one-parameter subgroup (read "density") has an α-invariant ground state this is unique, then, at least if A is unital and each ideal of the fixed point algebra is generated by projections, the space of α-invariant ground states must be totally disconnected.

Conversely, given a totally disconnected set of densities T_∞, there exists an almost periodic system $(A,\mathbb{R}^2,\alpha)$ with A unital and simple such that α-invariant ground states exist only at densities in T_∞, and are unique. Such a system may be constructed so that the KMS states for various nonzero temperatures and various densities form a bundle over $\mathbb{R}^2$ isomorphic to the restriction to $\mathbb{R}^2$ of an arbitrary metrizable compact simplex bundle $S \to T$ with $T_\infty \subseteq T \subseteq \mathbb{R}^2 \cup T$. Here necessarily $T \setminus T_\infty \subsetneqq \mathbb{R}^2$.

D.E. EVANS:

Entropy of automorphisms of AF algebras

A notion of topological entropy for automorphisms of AF algebras is considered based on a corresponding measure theoretic entropy of Connes and Størmer for automorphisms of hyperfinite von Neumann algebras with an invariant trace. I show how to compute the entropy of the shift on certain AF-algebras associated with topological Markov chains.

TH. FACK:

Finite sums of commutators in C^*-algebras

For a C^*-algebra A, let A_o be the null space of all finite traces on the self-adjoint part of A. By a result of Cuntz and Pedersen, $A_o = \{x \in A \mid x = \Sigma\, x_n x_n^* - x_n^* x_n;$ norm convergence$\}$.

We show that A_o is spanned by finite sums of the above type for C^*-algebras with ≪ sufficiently many projections ≫ like infinite simple C^*-algebras or simple AF algebras (with unit).

P.A. FILLMORE:

Contractibility of certain unitary groups

An argument due to J. Mingo is sketched that proves that for any unital C^*-algebra A, the groups $U(M(A \otimes K))$ and $GL(M(A \otimes K))$ are contractible. The argument is actually somewhat more general in that $M(A \otimes K)$ can be replaced by any C^*-algebra B for which there is a unital $*$-monomorphism $\varphi : B(H) \to B$ such that the ideal $I(\varphi(K(H))$ is essential and φ is sequentially continuous from the $*$-strong topology to the strict topology on B relative to $F(\varphi(K(H))$.
As an application one has that the space $\mathfrak{F}_A$ of Fredholm elements (elements of $M(A \otimes K)$ that are invertible mod $A \otimes K$) is classifying for the Grothendieck function K_A of bundles with a finitely-generated projective A-module as fibre.

B. GRAMSCH:

Some homogeneous spaces in the operator theory and ψ-algebras

Def.: Let ψ be a Fréchetalgebra (of pseudo-differential operators) contained in $\mathcal{L}(E)$, E B-space, and ψ^{-1} the group of invertible elements of ψ with $\psi \cap \mathcal{L}(E)^{-1} = \psi^{-1}$, then ψ is called a ψ-algebra.

If furthermore E is a Hilbert space and $\psi = \psi^*$ then ψ is called a ψ^*-algebra. If Ω is a locally compact Lie group with a representation $g : \Omega \to \mathcal{L}(E)^{-1}$ then the algebra $C^\infty \mathcal{L} = \{a \in \mathcal{L}(E) : g(\cdot)^{-1} a g(\cdot) \in C^\infty(\Omega, \mathcal{L}(E))\}$ is a ψ-algebra.

I) For real analytic ψ-algebra Fredholm functions the division of distributions can be solved as in B. Gramsch, R. Wagner: Manuscripta Math. 21, 25-42 (1977).

II) For holomorphic functions with values in $C^\infty \mathcal{K}$ the Cartan-factorisation (L. Bungart, Topology 7, 55-68 (1968), Th. 2.1) is true.

III) The connected components of the set of (Semi-)Fredholm operators with fixed dimension of the kernel are algebraic homogeneous Fréchet-submanifolds of ψ. An analogy is true for the set of C^∞-vector bundles.

In the constructions for II and III the "implicit function theorem" is substituted by "algebraic" (explicit) methods.

U. GROH:

On the spectrum of uniformly ergodic operators on C^*-algebras

Theorem. Let T be a 2-positive irreducible and uniformly ergodic operator with spectral radius $r(T) = 1$ on a C^*-algebra. Then the peripheral spectrum is the group Γ_n of all n-th roots of unity for some $n \geq 1$ and every $\alpha \in \Gamma_n$ is a simple eigenvalue of T and a pole of order one of the resolvent $R(\lambda, T)$.

Using this spectral theoretical theorem one can show that the following assertions are equivalent for an irreducible 2-positive operator on a C^*-algebra A

(a) T is uniformly ergodic.

(b) T is quasi-compact and the set $\{(\lambda - T)R(\lambda,T) : \lambda > 1\}$ is uniformly bounded.

(c) There exists a partially periodic operator $S \in L(A)$ such that $\lim_n \|T^n - S^n\| = 0$.

(d) $r(T)$ is a pole of the resolvent of order ≤ 1.

U. HAAGERUP:

Completely bounded maps on B(H)

Let M be a von Neumann algebra on a Hilbert space H. We prove that every completely positive M'-bimodule map T on B(H) can be approximated pointwise σ-weakly by maps of the form

$$x \longmapsto \sum_{i=1}^{n} a_i^* x a_i \ , \ a_i \in M \ , \ \sum_{i=1}^{n} a_i^* a_i = T(1).$$

Moreover every completely bounded M'-bimodule map T on B(H) can be approximated pointwise σ-weakly by maps of the form

$$x \longmapsto \sum_{i=1}^{n} a_i x b_i \ , \ a_i, b_i \in M \ , \ \|\sum_{i=1}^{n} a_i a_i^*\| \leq \|T\|_{CB}, \|\sum_{i=1}^{n} b_i^* b_i\| \leq \|T\|_{CB}$$

($\|T\|_{CB} = \sup\|T \otimes i_n\|$, where i_n is the identity on $M_n(\mathbb{C})$.)
These results combined with the Grothendieck inequality for bilinear forms on C^*-algebras are used to give "elementary" proofs of the following biimplications:

1) For any von Neumann algebra M:

M injective $\Longleftrightarrow$ M has property P

2) For any C^*-algebra A:

A nuclear $\Longleftrightarrow$ A amenable

The proofs are elementary in the sense, that they do not rely on the deep results due to Connes, that injectivity is equivalent to hyperfiniteness.

D. HANDELMAN:

Finite Group actions on UHF algebras

This is joint work with Wulf Rossmann. We study and classify outer product type (and non-product type) actions of a finite group G on a UHF algebra A. We observe that $K_o(A^G)$ is an ordered module over $K_o(G)$ (= the representation ring of G), and this structure in the product case, together with an additional datum is a complete invariant for stable conjugacy (i.e., conjugacy after tensoring with the regular representation of G). The product type actions yield rank 1-modules, and this classification resembles that of UHF algebras; non-product actions lead to larger ranks, and these correspond to AF algebras with Z replaced by $K_o(G)$. The numbers of traces can be written down, as can the traces themselves; this yields a complete determination of the number of generators of A as a (projective) A^G-module. In particular, A is (G)-generator if and only if $A^G \simeq A$, if and only if the action of G is conjugate to an infinite tensor product of multiples of the regular representation.

P. DE LA HARPE:

Acyclic groups of automorphisms

A discrete group Γ is said to be acyclic if the Eilenberg-MacLane homology groups $H_i(\Gamma)$ are zero for all $i \geq 0$. We show that certain groups, such as the groups of unitary or invertible bounded linear transformations of an infinite dimensional Hilbert space, are acyclic. This results from a joint work of the reporter and of Dusa McDuff.

J. KAMINKER:

On relations between the Novikov conjecture and $C^*(\pi)$

The relationship between $K_*(C^*(\pi))$, $Ext_*(C^*(\pi))$ and a certain topological problem, the Novikov conjecture, can be made precise using the Miscenko-Fomenko Index theorem for operators commuting with the action of a C^*-algebra. Specifically, we define a map $\beta : K_*(B\pi) \to K_*(C^*(\pi))$ whose injectivity implies the Novikov conjecture and show that it is dual to a map $\alpha : Ext_*(C^*(\pi)) \to K^*(B\pi)$. This is then extended to provide some other statements involving group C^*-algebras which would imply the Novikov conjecture.

G. LUKE:

The Mischenko Extension

Mischenko defined a map $K^0(C^*(\Gamma)) \to K^0(B\Gamma)$ which assigns to a Fredholm operator F which intertwines two Γ representations, modulo compact operators, a Γ invariant family of Fredholm operators on the covering space $E\Gamma$ which is constant and equal to F in the Calkin algebra. This construction leads to a proof of the Novikov conjecture when the map is surjective.

In this talk we give a natural construction of this map for the case when Γ is a discrete, torsionless, co-compact subgroup of a non-compact semi-simple Lie group. This involves the construction of a universal Fredholm operator on the square integrable forms on the associated symmetric space G/K. The operator is

$$d + d^* + (i + e)(rdr).$$

D. OLESEN:

The Connes spectrum for C^*-dynamical systems

Let (A,G,α) be a C^*-dynamical system, with G locally compact abelian, and denote by $(G\times_\alpha A,\hat{G},\hat{\alpha})$ its dual system. Let $\Gamma(\alpha)$ denote the Connes spectrum of (A,G,α). Assuming the action to be G-simple one obtains in a number of cases that the primitive ideal space $\mathrm{Prim}(G\times_\alpha A)$ is homeomorphic to $\hat{G}/\Gamma(\alpha)$ and that $(G\times_\alpha A,\hat{G},\hat{\alpha})$ is covariantly isomorphic to the induced system of continuous functions $y : \hat{G} \to B$ (where $(B,\Gamma(\alpha),\beta)$ is a C^*-dynamical system with B simple) satisfying $y(\sigma-\tau) = \beta_\tau(y(\sigma))$ for σ in $\hat{G}$ and τ in $\Gamma(\alpha)$, with $\hat{G}$ acting by translation.

W.L. PASCHKE:

K-Theory for actions of the circle group on C^*-algebras

Let B be a unital C^*-algebra, and $\rho : T \to \mathrm{Aut}(B)$ a continuous action of the circle group on B with fixed-point algebra A, satisfying $E_1^* E_1 = E_1 E_1^* = A$, where E_1 is the spectral subspace $\{x \in B : \rho_\lambda(x) = \lambda x \quad \forall\, \lambda \in T\}$. Given $x_1, \dots, x_n$ in E_1 such that $x_1^* x_1 + \dots + x_n^* x_n = 1$, a *-monomorphism $\gamma : A \to A \otimes M_n$ is defined by $\gamma(a) = \sum_{i,j} x_i a x_j^* \otimes e_{ij}$.

Theorem: There is a six-term cyclic exact sequence

$$\dots \to K_j(A) \xrightarrow{\mathrm{id} - \gamma_*} K_j(A) \xrightarrow{i_*} K_j(B) \longrightarrow K_{1-j}(A) \longrightarrow \dots \quad (j = 0,1),$$

where $i : A \to B$ is the inclusion map.

G.K. PEDERSEN:

Properly outer automorphisms of C^*-algebras

For an automorphism α on a separable C^*-algebra A the following conditions are equivalent:

(ii) For every invariant ideal I of A the distance from $\alpha|I$ to the (multiplier) inner automorphisms of I is 2.

(iv) For every hereditary C^*-subalgebra B of A the infimum of numbers $\|x\alpha(x)\|$ is zero, where x ranges over the positive unit sphere of B.

(vii) There is no invariant hereditary C^*-subalgebra B of A such that $\alpha|B = \exp \delta$ for some *-derivation δ of B.

(viii) The Borchers Spectrum for $\alpha|_B$ is non-zero for every invariant, hereditary C^*-subalgebra B of A.

(xi) There is no invariant ideal of A on which α is universally (or atomically) weakly inner.

M. PIMSNER:

K-groups of reduced crossed-products by free groups

Let $\mathbb{F}_n$ be the free group on n generators $g_1,\dots,g_n$, let A be a C^*-algebra with an automorphic action $\alpha: \mathbb{F}_n \to \mathrm{Aut}(A)$ and let $A \times_{\alpha r} \mathbb{F}_n$ be the reduced crossed product of $(A,\alpha,\mathbb{F}_n)$.

Theorem 1: There is an exact sequence:

$$\begin{array}{ccccc} K_o(A) & \xrightarrow{id - \alpha_{g_{n*}}^{-1}} & K_o(A \times_{\alpha' r} \mathbb{F}_{n-1}) & \longrightarrow & K_o(A \times_{\alpha r} \mathbb{F}_n) \\ K_1(A \times_{\alpha r} \mathbb{F}_n) & \longleftarrow & K_1(A \times_{\alpha' r} \mathbb{F}_{n-1}) & \xleftarrow{id - \alpha_{g_{n*}}^{-1}} & K_1(A) \end{array}$$

when $\alpha' : \mathbb{F}_{n-1} \longrightarrow \mathrm{Aut}(A)$ is the restriction of α to the subgroup $\mathbb{F}_n$ generated by $g_1,\dots,g_{n-1}$.

Theorem 2: There is an exact sequence

$$\begin{array}{ccccc} K_o(A)^n & \xrightarrow{\beta_o} & K_o(A) & \longrightarrow & K_o(A \times_{\alpha r} \mathbb{F}_n) \\ K_1(A \times_{\alpha r} \mathbb{F}_n) & \longleftarrow & K_1(A) & \xleftarrow{\beta_1} & K_1(A)^n \end{array}$$

when $\beta_i(\gamma_1 \oplus \dots \oplus \gamma_n) = \sum_j (\gamma_j - \alpha_{g_{j*}^{-1}}(\gamma_j))$.

Corollary 1: For $C_r^*(\mathbb{F}_n)$ the reduced C^*-algebra of $\mathbb{F}_n$ we have

$$K_o(C_r^*(\mathbb{F}_n)) = \mathbb{Z} .$$

$$K_1(C_r^*(\mathbb{F}_n)) = \mathbb{Z}^n .$$

Corollary 2: (Kadison's conjecture). There are no nontrivial projections in $C_r^*(\mathbb{F}_n)$.

R.J. PLYMEN:

Bundles of elementary C^*-algebras: the Weyl and Clifford bundles

Let E be a Euclidean vector bundle over the space X of even fibre dimension, and let C(E) be the complex Clifford bundle, whose fibre at x is the complex Clifford algebra over E_x. Let (F,ω) be a symplectic vector bundle over X, and let W(F) be the Weyl bundle, whose fibre at x is the Kastler-Weyl algebra. Then C(E) and W(F) are bundles of elementary C^*-algebras. Let $\delta(A)$ be the Dixmier-Donady obstruction, given such a bundle A. When $\delta(A) = 0$, there exists an underlying Hilbert bundle S with definite A-module structure. The characteristic class $\kappa(A)$ is defined as the mod 2 reduction of the first Chern class of the complex Hermitian line bundle $\mathrm{Hom}_A(S,S^*)$. When $\delta(C(E)) = 0$ the underlying bundle S is the spinor bundle; for the Weyl bundle S is the bundle of symplectic spinors. Let ω_2 denote the second Stiefel-Whitney class, β the integral Bockstein of the coefficient sequence $0 \to \mathbb{Z} \to \mathbb{Z} \to \mathbb{Z}/2 \to 0$. We prove that

(i) $\delta(C(E)) = \beta\omega_2(E)$

(ii) $\kappa(C(E)) = \omega_2(E)$

(iii) $\delta(W(F)) = 0$

(iv) $\kappa(W(F)) = \omega_2(F)$.

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S. POPA:

Maximal abelian *-subalgebras in factors

Let M be a separable factor and $N \subset M$ a type II factor such that there exists a normal conditional expactation onto N. We prove the following

Theorem. If $N' \cap M = \mathbb{C}$, then N contains a maximal abelian subalgebra of M.

In the nonseparable case the statement is not true: take M a non Γ II_1-factor; then $N' \cap M^{\omega} = \mathbb{C}$ but there exist no maximal abelian subalgebras of N^{ω} contained in N.

We also exhibit a method that can be used to compute the normalizer of certain subalgebras. This has as corollary the fact that M has no regular maximal abelian subalgebras (for any II_1 M).

It also may be used to give an example of a subfactor R_1 of R such that $R_1 \neq R$ and such that the normalizer of any nonatomic subalgebras of R_1 is in R.

J. RENAULT:

Virtual Groups without Measure Theory

The notion of virtual group introduced by G. Mackey in the theory of induced representations has been developed almost exclusively within a measure-theoretical framework. However, when correspondences are substituted to groupoid homomorphisms, most ideas and results of the theory find a natural way within a topological context. Let us call equivalence an invertible correspondence. The main result presented here is that an equivalence between two locally compact groupoids canonically defines an equivalence bimodule between their C^*-algebras. In fact, when correspondences are used as morphisms, the construction of the C^*-algebra of a groupoid becomes functorial.

N. RIEDEL:

<u>On the Dixmier property of simple C^*-algebras</u>

The following problem is considered: For a simple unital C^*-algebra A is it true that A admits at most one tracial state if and only if the Dixmier property is satisfied, i.e. for each $x \in A$ the closed convex hull $\delta(x)$ of the set $\{u^*xu \mid u \in A \text{ unitary}\}$ intersects the center $\mathbb{C}1$? It is shown that A has at most one tracial state if and only if A satisfies the following condition which is called the weak Dixmier property: For each $x \in A^+ \setminus \{0\}$ the closed linear space generated by the set $\delta(x)$ contains $\mathbb{C}1$. For a strictly ergodic C^*-dynamical system (A,G,α), G abelian, it is shown that the crossed product $A \bullet_{\alpha} G$ has the Dixmier property.

J.E. ROBERTS:

<u>Connection and Curvature in Algebraic Field Theory</u>

The basic object is a net of C^*-algebras $\mathfrak{A}$ over open bounded sets in space-time. A self-adjoint element of the net is an observable but the corresponding derivation has a different physical interpretation: it is an infinitesimal operation. The support properties of the derivations give a new local structure to the C^*-inductive limit of the net $\mathfrak{A}$ leading to a structural sheaf φ of C^*-algebras over space-time characteristic of the underlying physical theory. Unobservable fields can be studied in terms of φ-modules. Gauge groups of the first kind typically yield Poincaré covariant bihermitian modules which are locally free finite rank modules both as left and right φ-modules. The situation with gauge theories is still unclear but formal evidence strongly suggests the appearance of Hermitian φ-modules which, although not even translation covariant, allow a notion of parallel transport. In this sense these modules have a non-zero curvature.

N. SALINAS:

Open sets of eigenvalues for n-tuplas of operators

We give a natural extension to several complex variables of some of the results obtained by M.J. Cowen and R.G. Douglas in their pioneer work "Complex Geometry and Operator Theory", Acta Mathematica 141, 187-261, 1978. We also illustrate our results with various canonical examples involving Toeplitz Operators on Bergman spaces associated with strongly pseudoconvex domains and Reinhard domains.

J.M. SCHWARTZ:

Representations of Kac algebras. C^*-Kac algebras.

The purpose of the talk is to give a sketch of the Kac algebra theory. It should be mainly thaught of them as generalizations of locally compact groups. We shall undertake to see whether, and how, they are relevant to cope with harmonic analysis, dynamical systems and representation theory.
After a short exposition of the historical background, we shall give a brief description of Kac algebras including the generalizations of the unicity of Haar measure, of the Plancherel theorem, of the Pontrjagin duality theorem, and the intrinsic characterization of von Neumann algebras of the type $L^\infty(G)$, where G is a locally compact group. Examples of applications are then given. In harmonic analysis, elicitation of Walter's, Wendel's and Johnson's isomorphism theorems. For dynamical systems, the construction of crossed products is given, as well as general versions of theorems such as Takesaki's double crossed product. The notion of representation of a Kac algebra allows us to present another definition of an action of a Kac algebra on a von Neumann algebra which can then be expressed in terms of Connes' correspondances.
The definition and some properties of C^*-Kac algebras, including the intrinsic characterization of C^*-algebras of the form $C_o(G)$, due to J.M. Vallin, conclude the talk.

G. SKANDALIS:

Wrong way functoriality

Let X and Y be smooth manifolds. To any K-oriented map $f : X \to Y$ are associated two elements of the Kasparov group $KK(X,Y) \overset{def}{=} KK(C_o(X),C_o(Y))$: a topological one $f_!^t$, and an analytic one $f_!$, which coincides with the class in KK(X,pt) of the Dirac operator when f is the projection from X to p^t. We then show that the two definitions coincide, by proving the functoriality $(g \circ f)_! = f_! \otimes g_!$ of the analytic $f_!$ and showing that it coincides with $f_!^t$ in some particular cases. When $Y = p^t$ the equality $f_! = f_!^t$ is equivalent to Atiyah-Singer index theorem.

An important fact about this wrong way functoriality is that the construction still works when Y is the space ${}^M/_F$ of the leaves of a foliation. Then take $M \to {}^M/_F$ be the natural projection. The equality $f_! = f_!^t \in KK(M,{}^M/_F) \overset{def}{=} KK(C_o(M),C^*({}^M/_F))$ is a longitudinal index theorem for foliation. (Joint work with A. Connes)

E. STØRMER:

Positive linear maps on C^*-algebras

A positive linear map Φ from a C^*-algebra A into B(H) is called decomposable if it can be written in the form $\phi(x) = v^*\pi(x)v$, where v is a bounded linear operator from the Hilbert space H into another K, and π is a Jordan *-homomorphism from A into B(K). If $A \subset B(H)$ and $\Phi^2 = \Phi$ and Φ positive and unital then the self-adjoint part of $\Phi(A)$ is often a Jordan algebra, e.g. always when Φ is faithful. It is noted that this Jordan algebra is reversible (i.e. closed under products $a_1a_2...a_n + a_n...a_2a_1$) iff Φ is decomposable. A direct extension of the Stinespring theorem on completely positive maps to decomposable maps is also discussed.

G. WITTSTOCK:

Hahn-Banach methods for completely bounded linear maps

Let A be a C^*-algebra, M an injective W^*-algebra, $X \subset A$ a subspace and $\varphi : X \to M$ a completely bounded linear map. Then φ has an extension $\tilde{\varphi} : A \to M$ preserving its completely bounded norm $\|\cdot\|_{cb}$. If $X = X^*$, $\varphi(x)^* = \varphi(x^*)$ then φ has a decomposition $\varphi = \varphi_+ - \varphi_-$, where $\varphi_\pm$ are completely positive, disjoint and $\|\varphi_+ + \varphi_-\| = \|\varphi\|_{cb}$. If in addition $X = A$ is a W^*-algebra and φ is continuous with respect to the weak topologies then φ_+, φ_- are normal. If φ is a module homomorphism with respect to some *-subalgebras of A and M then $\tilde{\varphi}, \varphi_+, \varphi_-$ can be chosen as module homomorphisms too.

The proof is based on the usual Hahn-Banach theorem applied to suitable sublinear functionals on $A \otimes M_*$. Moreover this is a special case of a general Hahn-Banach theorem for matrix sublinear functionals with values in an injective C^*-algebra.

L. ZSIDO:

Absence of spectral projections in operator algebras

Let (M,α) be a one-parameter W^*-dynamical system. We denote by Z the centre of M and by Ω the maximal ideal space of Z. Then there are equivalent (G.A. Elliott - L. Zsidó):

(i) there exists a bounded linear projection P from M onto the spectral subspace $M((-\infty,0])$:

(ii) for ω in a dense open subset of Ω, denoting by $[\omega]$ the norm closed ideal generated by ω in M and puting $M_\omega = M/[\omega]$, α invariates $[\omega]$, it induces a one-parameter C^*-dynamical system (M_ω,α_ω) and

$$\mathrm{card}(\sigma(\alpha_\omega)) \leq c < +\infty ,$$

where the constant c does not depend on ω.

Moreover, if (i) holds then in (ii) one can take

$$c = 16 \exp(2\pi \|P\|) ,$$

and if (ii) holds then in (i) one can choose P such that

$$\|P\| \leq c.$$

A similar statement holds also for one-parameter C^*-dynamical systems.

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