MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 52/1981

Operatorungleichungen

6.12. bis 12.12.1981

Die dritte Tagung über "Operatorungleichungen" stand wieder unter der Leitung von N.W. Bazley und J. Schröder (Köln). Die Tagung sollte über Fortschritte auf dem Gebiet der Differentialungleichungen, Invarianzsätze, Berechnung von Fehlerschranken und dergleichen informieren und darüber hinaus den entsprechenden Theorien neue Anwendungsgebiete erschließen. In den Vorträgen wurden daher neben Themen aus Theorie und Anwendung der Operatorungleichungen auch anwendungsorientierte Themen aus verschiedenen benachbarten Gebieten behandelt. Insbesondere waren mehrere Spezialisten auf dem Gebiet der numerischen Behandlung gewöhnlicher Differentialgleichungen eingeladen worden. Es ergaben sich ausgedehnte Diskussionen und fruchtbare Kontakte zwischen Teilnehmern, welche auf verschiedenartigen Gebieten arbeiten.

The third meeting on "operator inequalities" was again organized by N.W. Bazley and J. Schröder of the University of Cologne. The conference was intended to report progress in the areas of differential inequalities, invariance theorems, computation of error bounds, etc., as well as corresponding theories for new areas of applications. In addition to topics from the theory and application of operator inequalities, the lectures covered application oriented themes from neighbouring disciplines such as the numerical treatment of ordinary differential equations. Fruitful contacts and extended discussions took place between the participants from different problem areas.

Vortragsauszüge

RAVI P. AGARWAL: Gronwall type inequalities

The purpose of the paper is to provide some Gronwall type inequalities which appear to be new.

U. ASCHER: Collocation methods for singular perturbation problems

The application of collocation methods for the numerical solution of singularly perturbed ordinary differential equations is investigated. Collocation at Gauss, Radau and Lobatto points is considered, for both initial and boundary value problems for first order systems without turning points. Particular attention is paid to symmetric schemes for boundary value problems; these problems may have boundary layers at both interval ends.

Our analysis shows that collocation schemes based on Gauss or Lobatto points do perform very well on such problems, provided that a fine mesh with steps proportional to the layers' width is used in the layers only, and a coarse mesh, just fine enough to resolve the solution of the reduced problem, is used in between. Ways to construct appropriate layer meshes are proposed. Of all methods considered, the Lobatto schemes appear to be the most promising class of methods.

C. BANDLE: Remarks on the free boundary in problems of plasma physics

Necessary and sufficient conditions are given for a free boundary to appear.

The method consists in studying a suitable variational problem and in deriving a priori bounds for the solutions. The conditions are expressed in terms of the flux of the solutions.

K. BÖHMER: Diskrete Newton Verfahren für Operatorgleichungen

Zu einem vorgegebenen tol > 0 soll für eine nichtlineare Operatorgleichung mit der Lösung z eine Näherung $\tilde{z}$ mit $\|z-\tilde{z}\| \approx$ tol bestimmt werden. Ausgehend von Differenzen- oder Quadraturverfahren niedriger Ordnung wird mittels Defektkorrekturen eine Familie von Verfahren höherer Ordnung (Diskrete Newton Verfahren) definiert. Die Auflösung der nichtlinearen Gleichungssysteme zur Bestimmung der diskreten Approximation niedriger Ordnung wird aufgrund der engen Zusammenhänge mit dem Diskreten Newton Verfahren zur Steuerung der Schrittweite, Ordnung und Gitterverteilung herangezogen. Ziel dieser Überlegungen ist die möglichst rechenzeitsparende Berechnung der endgültigen Approximation. Darüber hinaus werden Zuverlässigkeitstest und Beispiele diskutiert.

E. BOHL: Über ein finites Modell für exotherme chemische Reaktionen

Es wurde ein einfaches finites Modell vorgestellt, welches gleichzeitig als Differenzenverfahren einer Transportgleichung gedeutet werden kann. Das Verzweigungsdiagramm in Abhängigkeit verschiedener Kontrollparameter zeigt Zweige symmetrischer und nichtsymmetrischer Lösungen. Verzweigungspunkte werden aufgelöst und geben einen geschlossenen Zweig frei. Eine große Anzahl von Hysteresisschleifen wird beobachtet.

L. COLLATZ: Berechnung von Ableitungen mittels Monotonie bei vektorwertigen Operatoren

Verschiedene klassische Sätze für punktweise Monotonie (z.B. bei gewöhnlichen, elliptischen und parabolischen Differentialgleichungen) können aufgefaßt werden als Spezialfälle einer etwas allgemeineren Monotonie, bei der man aus einem System von Ungleichungen und Gleichungen für gewisse Größen ein System von anderen Ungleichungen erhalten kann. So kann man in verschiedenen Fällen monotones Verhalten für Funktionswerte und für Ableitungen zeigen.

Beispiele I) $y^{iv}(x) = p(x)$ im Intervall $J = (0,a)$ und $y(0) = y'(0) = y(a) = y'(a) = 0$. (Beiderseits eingespannter homogener Stab mit ungleichmäßiger Belastung.) Für eine Funktion $w(x)$ gelte $w^{iv}(x) \geq y^{iv}(x)$ und $w(0) = w'(0) = w(a) = w'(a) = 0$. Dann gelten die Monotonieaussagen: $w(x) \geq y(x)$ in J , $w''(0) \geq y''(0)$, $w''(\frac{a}{2}) \leq y''(\frac{a}{2})$. Das kann zur numerischen Einschließung von Ableitungswerten benutzt werden, wie an Beispielen erprobt worden ist. II) $-\Delta u = 1$ in einem zweidimensionalen Bereich B der x-y-Ebene, $u = 0$ am Rande ∂B (Torsionsproblem). Mit Hilfe der Monotonie können die Werte der Normalableitung $\frac{\partial u}{\partial \nu}$ am Rande in Schranken eingeschlossen werden.

P.W. DAVIS: Comparison results for combustion problems

Differential inequality techniques have provided useful qualitative information about the behavior of solutions of models of combustion phenomena. These include bounds on solutions, relations among critical parameters obtained from various types of approximations, and stability and decay estimates. The comparison results employed in these analyses often have an appealing physical interpretation that lends intuitive support to the mathematical arguments.

L.H. ERBE: Comparison theorems for operator-valued differential equations

Let B be a Banach lattice with order continuous norm, $L(B)$ the algebra of bounded linear operators. Let B_+ denote the positive cone induced by the lattice structure of B and $L_+(B)$ the corresponding positive cone in $L(B)$. We consider second-order operator-valued differential equations of the form $Y'' + Q(x)Y = 0$, where $Q: [a,+\infty) \to L(B)$ is continuous in the uniform topology and is such that $\int_x^\infty Q(t)dt \in L_+(B)$ for all $x \geq a$. Comparison theorems of Hille-Wintner type are obtained.

B.A. FLEISHMAN: Monotone iteration for some discontinuous nonlinear boundary value problems

Consider the BVP $Q_\varepsilon(h)$ $\{-\Delta u = f(r,u)$ in $D: 0 \leq r < 1$, $u(1,\theta) = \varepsilon h(\theta)$ for $0 \leq \theta \leq 2\pi\}$ where $\varepsilon \geq 0$, and h is continuous and 2π-periodic and satisfies $0 < h(\theta) < 1$. On $\{0 \leq r \leq 1, u \geq 0\}$ f is non-negative, C^1 in r for fixed u, and non-decreasing in u for fixed r. f is also piecewise-continuous in u for fixed r, experiencing jumps at $u = \mu_j$ $(j=1,2,\ldots)$; where $0 < \mu_1 < \mu_2 < \ldots$, $\mu_j \to \infty$ as $j \to \infty$ and the μ_j are independent of r. Finally, suppose $\exists\, \rho > 0$ satisfying $\max_{0\leq r\leq 1} f(r,\rho) < \rho$ and $\rho > \mu_N$ for some N, and suppose $\alpha = \min_{0\leq r\leq 1} f(r,\mu_1+)$ satisfies $0 < \mu_1 < \alpha/4e$.

RESULT. For sufficiently small $\varepsilon > 0$ there exist:

a) closed curves $\Gamma_j: r = \gamma_j(\theta)$, $0 \leq \theta \leq 2\pi$, $j=1,\ldots,N$, where $\gamma_j \in C^1$ and $0 < \gamma_N(\theta) < \gamma_{N-1}(\theta) <\ldots< \gamma_1(\theta) < 1$; and

b) a function $u(r,\theta) \in C^0(\bar{D}) \cap C^1(D) \cap C^2(D\setminus\Gamma)$ (where $\Gamma = \bigcup_{j=1}^{N} \Gamma_j$) satisfying $0 \le u \le \rho$, such that u is a solution of $Q_\varepsilon(h)$ in $D\setminus\Gamma$ and $u(\gamma_j(\theta),\theta) = \mu_j$, $0 \le \theta \le 2\pi$, $j=1,\dots,N$.

CO-AUTHORS: ROSS GINGRICH and THOMAS J. MAHAR.

D.W. FOX: Bounds for sloshing eigenvalues by conformal mapping

For a large class of two-dimensional regions, sloshing eigenvalues can be bounded rigorously by transforming the region G conformally onto a rectangle R. The resulting problems for the eigenvalues λ have the form $\Delta\phi = 0$ in R, $\partial\phi/\partial n - \lambda|\zeta'_1|\phi = 0$ on $\partial_1 R$ and $\partial\phi/\partial n = 0$ on $\partial_2 R$ where $\partial_1 R$ is one edge of R, $\partial_2 R$ is the remainder of the boundary of R, and ζ' is the derivative of the conformal map that takes R onto G. When ζ' is bounded on $\partial_1 R$, lower bounds are obtained by intermediate problem methods after reformulating the transformed problem as $Su - \lambda(I - T^2)u = 0$ in $L^2(\partial_1 R)$. Here S is a spectrally resolvable unbounded self-adjoint operator, and T is bounded symmetric with $\|T\| \le 1$. Upper bounds are obtained by Rayleigh-Ritz or by intermediate operators. Excellent numerical bounds have been computed for a number of regions.

W.M. GREENLEE: Convergence of variational methods of eigenvalue approximation

Consider the relative eigenvalue problem

$$(1) \qquad Au = \lambda Bu ,$$

where A and B are selfadjoint operators in a complex Hilbert space, with A lower semibounded, B positive definite, and $\operatorname{dom} A \subset \operatorname{dom} B$. Further assume that the lower portion of the spectrum of (1) is discrete. Nondecreasing sequences of lower bounds for the lower eigenvalues of (1), complementary to

Rayleigh-Ritz upper bounds, are obtained from the eigenvalues of

$$A_n u^{(n)} = \lambda^{(n)} B_n u^{(n)} \quad , \tag{2}$$

where the operators of (2) have the properties of those of (1), and $A_n \uparrow A$, $B_n \downarrow B$ in appropriate senses as $n \to \infty$. Recently Brown, Weidman, and the speaker have obtained convergence results for various methods of the form (2) when (1) has nontrivial essential spectrum. New general convergence theorems unifying previous results will be presented. The implications for various particular methods, including the use of constraints, will be discussed.

K.P. HADELER: Invariant sets for reaction diffusion equations and solutions to elliptic boundary value problems

Using an invariance principle of Weinberger (1975) for a class of semilinear parabolic equations the existence of stationary solutions under Dirichlet boundary conditions is proved. The result can be applied to various concrete problems, where the underlying ordinary differential equation admits convex positively invariant sets (joint work with F. Rothe, H. Vogt).

J. HERNÁNDEZ: Positive solutions of reaction-diffusion systems and the fixed point index

We obtain some results concerning existence of positive solutions of some reaction-diffusion systems of elliptic type. For that, we prove first a global bifurcation theorem for cones, which is a variant of some previous ones by Rabinowitz, Amann and Dancer. The essential tool for the proof is the fixed point index. This theorem, together with *a priori* estimates for positive solutions of the system gives our existence results. Of course, the main point is to be able to get in each particular case the necessary *a priori* estimates. We apply this method to two examples arising, respectively, in chemical reaction theory and mathematical biology.

M. HOFFMANN-OSTENHOF: On L^2-lower bounds for solutions of Schrödinger - type equations

This is part of a recent work done in collaboration with T. Hoffmann-Ostenhof, R. Froese and I. Herbst. We study the asymptotic decay of solutions $\phi \in W^{2,2}(\Omega_R)$, $\Omega_R = \{x \in \mathbb{R}^n / |x| > R > 0\}$ of the equation $(-\Delta + V(x) - E)\phi(x)=0$ where $E \in \mathbb{R}$ and the multiplicative operator V is relatively $-\Delta$-bounded with bound smaller 1 in Ω_R . Amongst other results it is derived: If $E < 0$ (resp. $E > 0$) and $|V| \leq c\, r^{-1}$ in Ω_R , $(c > 0$, $r = |x|)$, then $r^k \exp(\sqrt{|E|}\, r)\phi \notin L^2(\Omega_R)$ (resp. $r^k\phi \notin L^2(\Omega_R)$) for some $k > 0$. If $E = 0$ and $|V| \leq cr^{-2-\delta}$ in Ω_R $(c,\delta>0)$, then $r^k\phi \notin L^2(\Omega_R)$ for some $k > 0$. The following result applies especially to the Schrödinger equation of a many body system with Coulomb potential: Let $(-\Delta + V(x) - E)\,\phi(x) = 0$, $E < 0$, $\phi \in W^{2,2}(\mathbb{R}^n)$, V relatively $-\Delta$-bounded with bound < 1 and let V be homogeneous of degree β with $0 > \beta > -2$, then $\exp(\sqrt{|E|}\, r)\phi \notin L^2(R^n)$. The proofs are based on considerations similar to those used to prove unique continuation theorems.

F.A. HOWES: Maximum principles for parabolic and elliptic systems

We consider some extensions of the standard results on linear and nonlinear weakly coupled systems of parabolic and elliptic partial differential equations which are contained, for example, in Szarski's and Protter and Weinberger's well-known treatises. Our results are motivated by (and applied to) certain singularly perturbed systems of this type.

R. KANNAN: Numerical methods for Neumann problems

We study numerical methods for nonlinear partial differential equations arising in radiation, following the Boltzmann law. The equation governing the problem may be represented by

$$\Delta \varphi = 0 \quad , \text{ in } \Omega$$

$$- k \frac{\partial \varphi}{\partial n} = \sigma e \varphi^4 - \gamma Q \quad , \quad \text{on } \partial \Omega$$

where Ω is the interior of a convex solid with boundary $\partial \Omega$, k , σ , e, γ are physical constants and Q is the energy input function.

By applying ideas from the theory of operator inequalities we are able to obtain numerical algorithms for the corresponding discrete problems. These ideas are also applied to finding periodic solutions of parabolic partial differential equations.

H.B. KELLER: Geometrically isolated nonisolated solutions and their approximation

A solution $x = x^o$ of $F(x) = 0$ is "isolated" if the Fréchet derivative $F'(x^o)$ is nonsingular. It is "geometrically isolated" if no other solution exists in $||x-x^o|| \leq \rho$ for some $\rho > 0$. Isolated solutions are always geometrically isolated but the converse is not true. Sufficient conditions are obtained to insure that a nonisolated solution is geometrically isolated.

We then study approximation methods, $F_h(x_h) = 0$, to approximate geometrically isolated nonisolated solutions. Under strong consistency conditions multiple solutions of accuracy $O(h^{P/N})$ can be obtained for a solution of multiplicity N.

V. LAKSHMIKANTHAM: Monotone iterative technology in abstract cones

Consider the IVP (*) $u' = f(t,u)$, $u(0) = u_0$ in a Banach space E where $f \in C[I \times E,E]$ and $I = [0,T]$. Suppose that $v_0, w_0 \in C^1[I,E]$ such that $v_0' \leq f(t,v_0)$, $w_0' \geq f(t,w_0)$ with $v_0 \leq w_0$ on I. Here the inequalities are induced by a cone K in E. Let f be quasi-monotone relative to K. Then does there exist a solution of (*) in the sector $[v_0,w_0] = \{u \in C[I,E]: v_0 \leq u \leq w_0\}$? In case f is not quasi-monotone, v_0,w_0 have to satisfy inequalities in terms of linear functionals from K^*, namely for $\varphi \in K^*$,

$$\varphi(v_0'-f(t,\sigma)) \leq 0 \quad \text{for all } \sigma \text{ such that } v_0 \leq \sigma \leq w_0 \text{ and } \varphi(v_0-\sigma) = 0,$$

$$\varphi(w_0'-f(t,\sigma)) \geq 0 \quad \text{for all } \sigma \text{ such that } v_0 \leq \sigma \leq w_0 \text{ and } \varphi(w_0-\sigma) = 0.$$

In this case, it is known that there may not exist solutions of (*) in $[v_0,w_0]$. What further assumption guarantees existence of solutions of (*) in $[v_0,w_0]$? This question will be discussed in relation to monotone iterative technique. The extension of such results to boundary value problems and initial and boundary value problems will also be discussed.

J. LORENZ: Inverse monotonicity for problems with shock-layers and discrete analogues.

We consider boundary value problems of singular perturbation type where typically the solution u_ε has a derivative of order ε^{-1} in small regions.

A typical example is

$$T_\varepsilon u \equiv -\varepsilon u'' + uu' + u = 0 \quad , \quad Ru \equiv (u(0),u(1)) = \gamma \ .$$

Here $T'_\varepsilon(u_\varepsilon)w = -\varepsilon w'' + u_\varepsilon w' + (1+u'_\varepsilon)w$ and the sign of the zero order coefficient $1 + u'_\varepsilon$ cannot be controlled. But nevertheless (T_ε,R) is inverse monotone. For appropriate discrete analogues a similar result holds true.

I. MAREK: <u>Perron-Frobenius theory for rectangular pencils</u>

An adequate generalization of the Perron-Frobenius theory of square matrices with nonnegative elements is presented for "rectangular" pencils of linear operators $\{A,B\}$, where A and B map a Hilbert space H_1 into another Hilbert space H_2. It is assumed that H_1 is generated by a closed normal cone K_1 and that the operators A and B satisfy the following condition

(*) $$B^* x^* \in K_1^* \Rightarrow A^* x^* \in K_1^* \ ,$$

where K_1^* is the dual cone with respect to K_1 and A^* and B^* denote the adjoint maps of A and B respectively.
The theory is based on the following fundamental result.

<u>Lemma</u>. Let a pencil $\{A,B\}$ satisfy condition (*) . Then there is a bounded linear operator Z mapping H_1 into H_1 and such that

(i) $A = BZ$

and

(ii) $ZK_1 \subset K_1$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

The proof of the Lemma is constructive, so that one can verify eventual further properties concerning Z , e.g. its K-irreducibility etc.

K. NICKEL: Schranken für die Lösungen impliziter Gleichungen

Es wird die implizite Gleichung

$$f(x,y) = 0$$

untersucht. Dabei sei etwa $f : \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^t$. Gesucht werden Lösungen $y(x)$ oder $x(y)$. Mit Hilfe von Operator-Ungleichungen werden beidseitige Schranken für die Menge aller Lösungen angegeben. Daraus folgen auch Eindeutigkeits- und Existenz-Aussagen.

F. ROTHE: Uniform bounds from bounded L_p-functionals in semilinear parabolic equations

For many reaction-diffusion systems arising in applications like the Brusselator, the Meinhardt-Gierer model or ecological models bounds for the solutions of the parabolic initial-boundary value problems cannot be found using invariant convex sets and comparison theorems. In some cases one has a Ljapunov functional and in many cases some functional can be bounded a priori. Conditions involving the growth of interaction terms and the functional are given under which uniform bounds and global existence can be shown. If one has a Ljapunov functional the compactness of the trajectory can be used to get results about asymptotic behavior.

R.D. RUSSELL: Spline basis selection for solving differential equations

(Joint work with U. Ascher and S. Pruess)

The suitability of B-splines as a basis for piecewise polynomial solution representation for solving differential equations is challenged. Two alternative local solution representations are considered in the context of collocating ordinary differential equations: "Hermite-type" and "monomial". Both are much easier and shorter to implement and somewhat more efficient than B-splines.

A new condition number estimate for the B-splines and Hermite-type representations is presented. One choice of the Hermite-type representation is experimentally determined to produce roundoff errors at most as large as those for B-splines. The monomial representation is shown to have a much smaller condition number than the other ones, and correspondingly produces smaller roundoff errors, especially for extremely nonuniform meshes. The operation counts for the two local representations considered are about the same. It is concluded that both representations are preferable, and the monomial representation is particularly recommended.

B. RUTTMANN: Comparison of splines and polynomials in collocation methods

(Joint work with J. Schröder)

Our main aim is to get an a-posteriori error bound for a solution of BVP in ODE . In order to get it, one can proceed in the following way:

1) calculation of an approximation u_0 ,
2) calculation of an error bound for the defect of u_0 ,
3) calculation of an error bound for $|u^*-u_0|$ (u^* an exact solution).
   Here we use results about operator inequalities.

In 1) we calculate u_o with collocation at Chebyshev points with once or twice integrated Legendre-polynomials as basis functions. For our error estimation this approximation has the advantage that usually u_o has small defect. But we have seen also that for many problems u_o is a good approximation, also in comparison with spline approximation. Therefore, we compare splines and polynomials in collocation methods.

M. SCHATZMAN: A nonlinear convolution equation arising from a developmental model in neurophysiology

Consider the evolution problem

$$(1) \qquad \begin{cases} \frac{\partial u}{\partial t}(x,t) = (w*u)(x,t)(1-u^2)(x,t)1_\Omega(x,t) \\ u(x,0) = u_o(x) \in [-1,1] \quad \forall x \ , \end{cases}$$

where Ω is a bounded set of $\mathbb{R}^N$, $*$ is the spatial convolution, u is a real function which vanishes outside of Ω and w is the difference of two given non degenerate gaussian functions. We assume that w is negative for large $|x|$.

We prove existence, uniqueness, and give results on the asymptotic behavior, moreover if we consider the system

$$(2) \qquad \begin{cases} \frac{\partial u}{\partial t}(x,t) - (w*u)(x,t)(1-u^2(x,t))1_\Omega(x,t) + \beta(u(x,t)) \ni 0 \\ u(x,0) = u_o(x) \ , \end{cases}$$

where β is the monotone operator given by $\beta(r) = 0$ if $|r| < 1$, $\beta(1) = [0,\infty)$, $\beta(-1) = (-\infty,0]$, then existence and uniqueness hold, and the asymptotic behavior is the same as for (1).

System (2) was proposed by Swindale as a model of organisation of the visual cortex into stripes tensitive either to left-eye or to right-eye stimuli.

M.A. SNEIDER: On the minimal disk containing all the eigenvalues of a Fredholm integral operator.

A Fredholm integral operator T with an L^2-kernel $K(x,y)$ is considered. It is shown that the area $\sigma(K)$ of the smallest disk containing all the eigenvalues of T does not exceed $\frac{\pi}{2}\|K(x,y)\|^2$. That is the best obtainable estimate for $\sigma(K)$.

I. STAKGOLD: Gas-solid reactions

An important class of gas-solid reactions can be modeled by a pair of coupled nonlinear partial differential equations. For reactions of a special type, the system can be reduced to a scalar reaction-diffusion equation for the cumulative gas concentration. By dropping the terms involving the small porosity, a simpler, elliptic problem is obtained in which time appears only as a parameter in the boundary condition. This is known as the pseudo-steady-state problem.

The relation among the various formulations is analyzed. Existence and uniqueness proofs are supplied and it is shown that the pseudo-steady-state solution provides a uniform approximation to the exact cumulative concentration. Bounds are also calculated for the fraction of solid converted to products up to time t.

P. VOLKMANN: Bilineare Funktionen mit Hilbertraum-Operatoren als Veränderlichen.

Sei H ein komplexer Hilbertraum und $L(H)$ die Algebra der stetigen, linearen $A:H \to H$; $\|A\|_\infty$ bezeichne die Operatornorm. Ist A kompakt und $1 \leq p < \infty$, so werde $\|A\|_p = \sqrt[p]{\Sigma\, \mu^p} \leq \infty$ gesetzt, wobei über alle nichtverschwindenden (ihrer Vielfachheit entsprechend oft gezählten) Eigenwerte μ des positiven Operators $(A^*A)^{\frac{1}{2}}$ summiert wird. Für nicht-kompaktes A sei $\|A\|_p = \infty$ $(1 \leq p < \infty)$. Dann gilt nach gemeinsam mit R.M. Redheffer gemachten Überlegungen der Satz:

Ist $M = (a_{mn})_{m,n=1}^N$ eine Hermitesche Matrix and γ das Maximum der Beträge der Eigenwerte von M , so hat man für $S = \sum_{m,n=1}^N a_{mn} A_m B_n$ (mit $A_1,\ldots,A_N$, $B_1,\ldots,B_N$ aus $L(H)$) die Ungleichungen

$$\|S\|_p \leq \gamma \sqrt{\sum_{m=1}^N \|A_m\|_p^2 \sum_{n=1}^N \|B_n\|_2^2} \quad (1 \leq p < 2)$$

und

$$(*) \|S\|_p \leq \gamma \sqrt{\sum_{m=1}^N \|A_m\|_p^2 \sum_{n=1}^N \|B_n\|_\infty^2} \quad (2 \leq p \leq \infty) \ .$$

Beispiele zeigen, daß (*) zumindest für $1 \leq p < (\log 4)/\log(2+\frac{8}{\pi^2})$ falsch ist.

Berichterstatter: U. Gärtel

M. Plum

Liste der Tagungsteilnehmer

Agarwal, Prof. Dr. R.P., Istituto Matematico, c/o Professor Roberto Conti, Viale Morgagni, 76/A, I-50134 Firenze, Italy

Albrecht, Prof. Dr. J., Lehrstuhl C für Mathematik der Universität, Erzstr. 1, 3992 Clausthal-Zellerfeld, F.R. Germany

Ascher, Prof. Dr. U.M., Department of Computer Science, University of British Columbia, 2075 Wesbrook Mall, Vancouver, British Columbia, Canada, V6T 1W5

Bandle, Prof. Dr. C., Mathematische Anstalt der Universität Basel, Rheinsprung 21, CH-4051 Basel, Switzerland

Bazley, Prof. Dr. N.W., Mathematisches Institut der Universität zu Köln, Weyertal 86-90, 5000 Köln 41, F.R. Germany

Böhmer, Prof. Dr. K., Fachbereich Mathematik, Universität Marburg, Lahnberge, 3550 Marburg/Lahn, F.R. Germany

Börsch-Supan, Prof. Dr. W., Institut für Angewandte Mathematik der Universität, Saarstr. 21, 6500 Mainz, F.R. Germany

Bohl, Prof. Dr. E., Fakultät für Mathematik, Universität Konstanz, Postfach 5560, 7750 Konstanz, F.R. Germany

Bulirsch, Prof. Dr. R., Mathematisches Institut der Technischen Universität, Arcisstr. 21, 8000 München 2, F.R. Germany

Butler, Prof. Dr. G.J., Department of Mathematics, The University of Alberta, Edmonton, Alberta, Canada, T6G 2G1

Collatz, Prof. Dr. L., Institut für Angewandte Mathematik, Universität Hamburg, Bundesstraße 55, 2000 Hamburg 13, F.R. Germany

Davis, Prof. Dr. P.W., Mathematics Department, Worcester Polytechnical Institute, Worcester, Massachusetts 01609, USA

Erbe, Prof. Dr. L.H., Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada, T6G 2G1

Fichera, Prof. Dr. G., Istituto Matematico, Università di Roma, Via Pietro Mascagni 7, I-00199 Roma, Italy

Fleishman, Prof. Dr. B.A., Rensselaer Polytechnical Institute, Department of Mathematics, Troy, New York 12181, USA

Fox, Prof. Dr. D.W., Milton S. Eisenhower Research Center, Applied Physics Laboratory, The Johns Hopkins University, Johns Hopkins Road, Laurel, Maryland 20810, USA.

Gärtel, U., Mathematisches Institut der Universität zu Köln, Weyertal 86-90, 5000 Köln 41, F.R. Germany

Greenlee, Prof. Dr. W.M., Department of Mathematics, University of Arizona, Tucson, Arizona 85721, USA

Hadeler, Prof. Dr. K.-P., Mathematisches Institut der Universität Tübingen, Auf der Morgenstelle 10, 7400 Tübingen 1, F.R. Germany

Hernández, Prof. Dr. J., Department of Mathematics, Universidad Autonoma, c/R. Herrera 11, Madrid 16, Spain

Hoffmann-Ostenhof, Prof. Dr. M., Institut für Theoretische Physik, Universität Wien, Boltzmanngasse 5, A-1090 Wien, Austria

Hoffmann-Ostenhof, Prof. Dr. Th., Institut für Theoretische Chemie, Universität Wien, Währingenstr. 17, A-1090 Wien, Austria

Howes, Prof. Dr. F., Department of Mathematics, University of California at Davis, Davis, CA 95616, USA

Jeltsch, Prof. Dr. R., Lehrstuhl für Mathematik und Institut für Geometrie und Praktische Mathematik, Universität Aachen, Templergraben 55, 5100 Aachen, F.R. Germany

Kannan, Prof. Dr. R., Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

Keller, Prof. Dr. H.B., Applied Mathematics 217-50, California Institute of Technology, Pasadena, CA 91125, USA

Lakshmikantham, Prof. Dr. V., Department of Mathematics, University of Texas at Arlington, Box 19408, Arlington, Texas 76019, USA

Lange, Prof. Dr. H., Mathematisches Institut der Universität zu Köln, Weyertal 86-90, 5000 Köln 41, F.R. Germany

Lehnigk, Dr. S., Chief, Physics & Math. Branch, European Research Office, 223 Old Marylebone Road, London, NW1 5TH, England

Lorenz, Prof. Dr. J., Abteilung Mathematik, Universität Konstanz, Postfach 7733, 7750 Konstanz, F.R. Germany

Marek, Prof. Dr. I., Katedra Numerické Matematiky, Na Matematicko-Fyzikalni Fakulté, University Karlovy, Malostranské námesti c. 25, CSSR-11800 Praha 1, Czechoslovakia

Nickel, Prof. Dr. K., Institut für Angewandte Mathematik, Universität Freiburg, Hermann-Herder-Str. 10, 7800 Freiburg/Breisgau, F.R. Germany

Pascali, Dr. D., Fachbereich Mathematik 4, Technische Hochschule Darmstadt, Schloßgartenstr. 7, 6100 Darmstadt, F.R. Germany

Plum, M., Mathematisches Institut der Universität zu Köln, Weyertal 86-90, 5000 Köln 41, F. R. Germany

Rothe, Prof. Dr. F., Mathematisches Institut der Universität Tübingen, Auf der Morgenstelle 10, 7400 Tübingen 1, F.R. Germany

Russell, Prof. Dr. R.D., Department of Mathematics, Simon Fraser University, Burnaby (2), British Columbia, Canada V5A 1S6

Ruttmann, B., Mathematisches Institut der Universität zu Köln, Weyertal 86-90. 5000 Köln 41, F.R. Germany

Schatzman, Dr. M., Analyse Numérique, Université Pierre et Marie Curie, Tour 55-65, 5ème étage, 4, Place Jussieu, F-75230 Paris 5ème, France

Schröder, Prof. Dr. J., Mathematisches Institut der Universität zu Köln, Weyertal 86-90, 5000 Köln 41, F.R. Germany

Sneider, Prof. Dr. M.A., Istituto Matematico "Guido Castelnuovo", Via A. Torlania 12, Università di Roma, I-00100 Roma, Italy

Stakgold, Prof. Dr. I., Department of Mathematical Sciences, University of Delaware, Newark, Delaware 19711, USA

Volkmann, Prof. Dr. P., Mathematisches Institut, Universität Karlsruhe, Englerstraße, 7500 Karlsruhe, F.R. Germany

Walter, Prof. Dr. W., Mathematisches Institut 1, Fakultät für Mathematik, Universität Karlsruhe, Englerstraße, 7500 Karlsruhe, F.R. Germany

Witsch, Prof. Dr. K., Mathematisches Institut der Universität Düsseldorf, Universitätsstraße 1, 4000 Düsseldorf, F.R. Germany