MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Time Series and Density Estimation
13.12. bis 19.12.1981

Die Tagung fand unter der Leitung von Herrn Prof. Dr. L. Davies (Essen), Herrn Prof. Dr. Th. Gasser (Mannheim) und Herrn Prof. Dr. R. Reiß (Siegen) statt.

Im Bereich der Zeitreihenanalyse fanden u.a. Vorträge über optimale Vorhersage, Paramterschätzung bei Spektraldichtefunktionen, Identifikationsprobleme und über asymptotische Eigenschaften von Schätzern statt.

Im Bereich der Dichteschätzung behandelten die Vorträge u.a. Maximum-Likelihood-Schätzer, Kernschätzer, die Schätzung bei zeitabhängigen Dichten sowie die Effizienz bestimmter Dichteschätzer.

Daneben gab es einige Vorträge, die sich mit beiden Gebieten beschäftigten, beispielsweise die Anwendung von Splinefunktionen, sowie eine Reihe von Vorträgen aus verschiedenen Gebieten der mathematischen Statistik, die die Vorträge zu den Hauptthemen ergänzten.

Über 40 Teilnehmer aus den Vereinigten Staaten und 6 Europäischen Ländern nahmen an der Tagung teil. Neben den Vorträgen haben vor allem die zahlreichen Gespräche zu dem Gelingen der Tagung beigetragen.

Vortragsauszüge

K. BEHNEN

Two-sample rank tests with estimated score-functions

In the general two-sample testing problem, $X_1,\dots,X_m$ i.i.d. with continuous c.d.f. F, $Y_1,\dots,Y_n$ i.i.d. with continuous c.d.f. G hypothesis $H_0 : F = G$ versus alternative $H_1 : F \leq G$, $F \neq G$, it is shown that a good test should be based on the linear rank statistic with score-function $b=\bar{f}-\bar{g}$, where $\bar{f} = d(F\circ H^{-1})/d\lambda$, $\bar{g} = d(G\circ H^{-1})/d\lambda$, $H=(m/N)F + (n/N)G$, $N=m+n$.
This score-function is quite different from the usual (shift) score-function $-f'\circ F^{-1}/f\circ F^{-1}$, if there is some deviation from shift model. (This is the reason for the breakdown of adaptive tests based on an estimator of $-f'\circ F^{-1}/f\circ F^{-1}$, if the shift model is not exactly true, cf. Behnen, Commun. Statist. 1975).
Moreover, it is shown that the estimators of b should also be based on ranks, e.g. kernel type estimators with respect to the ranks of the first sample and the second sample, respectively. This part of the paper may be considered as a starting point of a systematic evaluation of adaptive procedures in general models.
Finally, a consistency result for kernel estimators based on ranks and a Chernoff-Savage type of result on asymptotic normality of rank statistics with estimated scores is given. Partially this is joint work with G. Neuhaus and F. Ruymgaart and partially the results are preliminary.

R. DAHLHAUS

On the asymptotic nomality of estimates of spectral functions

There exist a lot of articles on the asymptotic normality of the expression

$\sqrt{T}(A^{(T)}_{a_i b_i}(\phi_i)-A_{a_i b_i}(\phi_i))_{i=1,\dots,l}$ with $A^{(T)}_{a_i b_i}(\phi_i) = \int_{-\pi}^{\pi} \phi_i(\alpha)I^{(T)}_{a_i b_i}(\alpha)d\alpha$ and $A_{a_i b_i}(\phi_i) = \int_{-\pi}^{\pi} \phi_i(\alpha)f_{a_i b_i}(\alpha)d\alpha$, where $I^{(T)}_{ab}(\alpha)$ is the ordinary periodogram and ϕ is a function of bounded variation. We give a proof of this part in the situation, where not necessarily all moments of the underlying stationary process $\underline{X}(t)$ exist. We also reduce the assumption of stationarity in the strict sence to a certain form of stationarity in the weak sence. Further we allow that the periodogram is calculated from tapered data. Using this method we derive a lot of other results, for example a functional central limit theorem for the empirical spectral distribution function.

P. DEHEUVELS

Strong limiting bounds for maximal spacings and applications

Let $X_1, X_2, \dots, X_n, \dots$ be a sequence of independent identically distributed random variables, with a uniform distribution on $(0,1)$; if M_n stands for the maximal spacing defined by $0,1,X_1,\dots,X_n$, we show that for an arbitrary $p \geq 4$, $\limsup_{n\to\infty} \{nM_n - \mathrm{Log}\, n - 2\mathrm{Log}_2 n - \mathrm{Log}_3 n - \dots - \mathrm{Log}_{p-1} n\}/\mathrm{Log}_p n = 1$ a.s..
This result generalizes to higher dimensions and also to order statistics of the maximal spacings.
Some applications of the preceding results are derived for density estimation and uniform consistency conditions.

M. DEISTLER

Parameterization of ARMA-Systems

In this lecture some toplogical and geometrical properties of the parameter-space and of the relation between transfer-functions and parameters for ARMA-systems are shown. We think that, especially in the multivariable case, for a deeper understanding of both inference problems and problems of numerical calculations, in addition to the problems of identifiability, the associated topological and geometrical structure has to be clarifed. In this sense this paper aimes to contribute to a general theory of inference in ARMAX-systems by means of an investigation of the relevant properties of the structure of such systems and to link identifiability and estimation. In particular the results of the paper show the connection between consistent estimation and dynamic specification (i.e. the specification of the maximum lag lengths). The consequences of different kinds of misspecification of these maximum lags are discussed and possibilities of detecting such misspecifications are indicated.

K. DŽJAPARIDZE

Asymptotic inference in spectra of time series

The problem of simultaneous estimation of regression and spectral parameters is considered in the special situation when the spectral density degenerates at some points of the spectrum, so that conventional methods of estimation, such as the least squares for regression and the Whittle method, fail.
An appropriate modification of the methods are suggested for asymptotically efficient estimation.

F. EICKER

Simultaneous regression on lagged and exogenuous variables: parameter identification and a moment free concsistency proof for the L.S.Es.

A scalar linear regression model is being studied with simultaneous regression on p(given) lagged (stochastic) observation vectors and q(given) exogenuous non-stochastic regressors. The emphasis in the weakness of the assumptions on the errors: heavy tails and dependencies are admitted, no moments are assumed to exist. Only the ordinary least squares estimators (LSEs) for the total parameter vector are being considered and their consistency derived under general assumptions. Under these assumptions non-identifiability of the regression parameters from the non stochastic substructure of the oberservation vector (e.g. its expectation if existent) is admitted and standard examples for this are provided. In such a situation consistent estimation (and thus identification) is only possible from the stochastic substructure due to its higher variability. The fairly straightforward proof consists almost exclusively of algebraic and geometric arguments. The conditionson the error sequence are rather involved and concern sample correlation type expression and weighted sums of the errors. Independent Cauchy distributed errors can be admitted for some of the estimates. The proposition given generalizes a Theorem by Willers 78 where quasiindependence of order 4 is assumed for the errors. The relation of the paper to other recent results on linear stochastics regression is mentioned. Alternative modelling and estimation approaches are hinted at to avoid the shortcoming of the present model.

G. ERPENBECK

Stochastic L^2-Integration with a locally compact time-region

The main purpose of this lecture is to generalize the time-region of stochastic integration, which is usually a subset of $\mathbb{R}$ to a locally compact time-region. To do this some Hilbert space analysis as well as measure theoretic methods are used.

In the last part of the lecture examples will show, that the stochastic integral with a locally compact time-region is an extension of the usual stochastic integral for square integrable martingales.

M. FALK

Relative eficiency and deficiency of kernel type estimators f smooth distribution functions

The problem is investigated whether it is possible to find a kernel type estimator of a distribution function at a single point which makes the empirical estimator asymptotically inefficient. The answer will be negative. A representation of the relative deficiency of the empircal distribution function with respect to a kernel type estimator is established which improves a result of Reiss (1981). The problem of finding optimal kernels is studied in detail.

J. FISCHER

Maximum Likelihood Density Estimation via Mathematical Programming

A maximum likelihood estimator is introduced as the solution of an optimization problem in an appropriate Hilbert space. By regarding the optimal value of a parametrized family of simpler optimization problems this solution can be characterized as a certain polynomial spline function. An important feature of this estimate is that its support interval results in a quite natural way from the formulation of the problem and is not fixed in advance. The estimator can be shown to be strongly and uniformly consistent for a certain class of density functions. Finally, this approach makes it possible to determine the optimal solution by solving finite - dimensional optimization problems.

J. FRANKE

Linear Prediction of Time Series in the Presence of Correlated Noise

Let (X_n) be a discrete time series satisfying $X_n=S_n+R_n$, where (S_n,R_n) is a bivariate weakly stationary zero-mean process. The spectral density s of (S_n) is known, the spectral density g of (R_n) is unknown except: $\int g(\omega)d\omega\leq\varepsilon$. The cross-spectral density c is contained in a known convex subset K of $L_c^1[-\pi,\pi]$. Let D be the set of spectral densities of (X_n) which may arise under these conditions on g,c. Under rather weak conditions on K we show that there exists a minimax robust linear predictor P^0 for X_0 given X_k, k<0, i.e. a predictor which minimizes (in the set of all predictors) the maximum(taken over all spectral densities in D) of the predition error. Moreover P^0 is the classical Wiener-Kolmogorov-predictor with respect to a certain spectral density f^0 in D.
We determine f^0 and consequently P^0 explicitly for the three examples: $K=\{0\}$, $K=L_c^1[-\pi,\pi]$ and $K=\{c;\int\exp(-in\omega)c(\omega)/s(\omega)d\omega=0 \text{ for all } n<0\}$ representing no, arbitrary and causal correlation between $(S_n)$and (R_n). For the last example we get that under suitable conditions f^0 is the spectral density of time series $S_n+\sqrt{\varepsilon}V_n$, where (V_n) is the normed innovation process of (S_n). This gives an intuitive interpretation of the robust predictor and implies that robustifying against causally correlated noise has other important advantages as well.

Th. Gasser

Model identification by autoregressive processes

The basic model for EEG data is that of a vector time series. Some empirical work relates to the further assumptions of stationarity and Gaussianity. Autoregressive processes are considered for obtaining parameters which characterize the EEG not just individually but for a sample of subjects. The choice of the order following the criteria by AKAIKE, HANNAN and RISSANEN is investigated on real data. Furthermore, a comparison is made of fitting the full frequency domain or a subinterval of central interest in EEG-research. Work done jointly with Pham Dinh Tuan (Grenoble), H.Heinberg, J. Franke.

W. GAWRONSKI

Strong laws for density estimators of Bernstein type

Starting from the classical theorem of Weierstraß (and its modifications) on approximation of continuous functions by means of Bernstein polynomials a smoothed histogram type estimator is developed for estimating probability densities and its derivatives. Consistency results are obtained in form of various strong laws. In particular one gets estimates for the rates of pointwise and uniform strong convergence of estimators for the derivatives. Moreover, for approximating the density itself the exact order of consistency is established. This is done by a law of iterated logarithm for pointwise approximation and a law of logarithm in case of uniform approximation.

F. GÖTZE

Asymptotic expansions for sums of dependent random vectors

Let X_t, t=1,2,... denote k-dimensional random vectors which are strong mixing with exponential rate. Edgeworth expansions for the distribution of

$S_n = n^{-\frac{1}{2}} (X_1+\ldots+X_n)$ can be proved under moment conditions on X_t and under conditions (including smoothness) on the conditional distribution of X_p given X_t, $t \neq p$. The expansion is based on the cumulants of S_n. Expansions for expectations of smooth functions hold under moment conditions only.

W. HÄRDLE

Robust smoothing of curve data

An important question in data analysis is to estimate the regression curve $m(x)=E(Y/X=x)$, for stochastic design, or $m(x)=\int yf(y;x)dy$, for fixed design. We propose an estimator which is less sensitive to outliers as follows: Let $m_n(x)$ be the solution of $n^{-1}\sum\delta_n(x-X_i)\psi(Y_i-\Theta)=0$ (stochastic design) and $\bar{m}_n(x)$ the solution of $\sum\alpha_i(x)\psi(Y_i-\Theta)=0$ (fixed design), where ψ is monotone and bounded. We show (strong) consistency and as.normality. The asymptotic variance is similar to Huber (1964) so the same minimax results hold.

S. HEILER

On Filtering Time Series

A survey on methods of extracting smooth curves and/or seasonal variations with slowly changing seasonal pattern is given. In the time domain approach these methods include kernel procedures, smoothing splines and robust versions of smoothing splines. A general setup to construct moving averages for a simultaneous treatment of smooth and seasonal component is presented. For the same problem a combination of polynomial and trigonemetric splines can be used. The latters have been introduced by H.Hebbel. In the frequency domain approach one is interested in designing recursive or non-recursive filters which have characteristics that come close to the characteristics (gain, phase, group delay) of an imagined ideal filter.

E. JOLIVET

Density estimation for the moments of point processes

Let $\underline{P}$ be a stationary point process on $\mathbb{R}^d$ and suppose that its reduced moment measure $\nu^{(k)}$ admits a density with respect to the Lebesgue measure on $\mathbb{R}^{d(k-1)}$. Let $p^{(k)}$ be that density.
Brillinger (1975) and Krickeberg (1981) proposed estimators for $p^{(k)}$. Let $\hat{p}_r^{(k)}$ be such an estimator, r being an index tending to infinity when the oberservation region tends to $\mathbb{R}^d$.
Studying the cumulants of $\hat{p}_r^{(k)}$, we can overestimate $E_{\underline{P}}|\hat{p}_r^{(k)}-p^{(k)}|^h$.
The same speed as for i.i.d. random variables is founded.

D.W. MÜLLER

Inference by means of total variation statistics

Two problems are discussed where (differential) total variation statistics appear to be useful. The problem of estimating $\|F-G\|_1$ (F and G distributions on the line) arises when individual treatment effects are studied, but joint measurements of the quantities X, Y before and after treatment is impossile (as in the case in life time analysis). If the treatment is such that $L(Y|X=x)$ always gives mass unity to $[x,+\infty)$, then the minimum proportion of the population benefitting from the treatment (i.e. $x<y$) can be computed as $\frac{1}{2}\|F-G\|_1$. The estimation of this quantity from independent marginal samples is studied, a histogram type estimator is considered.

The second problem concernes cluster analysis on the line; it is shown that the testing problem of ($\leq k$) clusters versus ($\geq k+1$) clusters can be treated by means of a differential total variation statistic whose asymptotic properties are studied.

H.G. MÜLLER

Kernel estimation of regression functions with an application to growth curves

Consider the model $y_i=g(t_i)+\varepsilon_i$, $i=1\ldots n$, where y_i are measurements of a function g at fixed (nonrandom) points $0<t_1<\ldots<t_n<1$, which are contaminated with i.i.d noise terms ε_i. Kernel estimates for g and its derivatives are proposed and investigated.
Under appropriate conditions, we gives rates of convergence of Integrated Mean Square Error (IMSE) and for a.s. uniform convergence. Choice of kernels optimal in the ISME-sense is discussed. An application to longitudinal growth data of children shows that estimating derivatives via this nonparametric method not only makes sense, but also yields new insights into the dynamics of children's growth.

G. NEUHAUS

H_0-contiguity in nonparametric testing problems and sample Pitman efficiency

The notation of H_0-contiguity is introduced for certain nonparametric testing problems. This concept is at the same time more general and easier to characterize than the usual contiguity concept and, furthermore, simplifies the derivation of local asymptotic properties of rank tests. These assertions are exemplified by treating the two sample testing problem for "randomners" versus "positive stochastic derivation of the first sample". Finally, a sample definition of the asymptotic relative Pitman efficiency is given on the basis of H_0-contiguity results. It turns out that this sample efficiency concept coincides with the more traditional concept of asymptotic relative efficiency based on asymptotic translations of the test statistics. Therefore, a sample interpretation of all results concerning the traditional efficiency concept can be given.

F. PUKELSHEIM

L_p-differentiable distributions

Suppose $P = \{P_\Phi | \Phi \in H\}$ is a k-parameter family of distributions, and one is interested in studying its behaviour locally at an interior point Φ_0 of H. Hajek and LeCam proposed to investigate the derivative, in $L_2(P_{\Phi_0})$, not of the likelihood ratio itself, but of its second root. In particular, this concept does not depend on choosing smooth versions of the likelihood ratio, nor on imposing any extra integrability assumptions. Here we show that $L_1(P_{\Phi_0})$- differentiability of P suffices for discussing locally most powerful (rank-)tests, and that Cramér-Rao type inequalities require $L_p(P_{\Phi_0})$-differentiability for some $p > 1$, only. These considerations yield a simple proof of the Hajek inequality for the special case of squared error loss.

R.D. Reiß

Maximum penalized estimators based on initial estimators

A modified concept of maximum penalized estimators is introduced where the definition is based on the likelihood function, a penalty function and an initial estimator. The consistency of these modified maximum likelihood estimators is proved by applying the classical idea of Wald. Because of this general approach it is possible to carry out the investigation within the frame-work of minimum penalized contrast estimators. This method can e.g. be applied to the nonparametric density estimation problem if the given probability densities form a subset of a reflexive, proper functional Banach space.

J. RICE

Deconvolution problems

The problem of deconvolution in the presence of noise is discussed with some examples. The similarity of the method of "regularization" to the method of smoothing splines is noted and it is shown that in a particular case rather artificial boundary effects dominate the integrated mean square error. An alternative method is proposed and is illustrated by a bio-medical example. This example concernes the measurement of cell DNA content by a process called microfinorometry. Microfinorometry produces a histogram of DNA levels which must be deconvolved to recover an estimate of the true DNA distribution.

P.M. ROBINSON

Probability density estimation from time series data

Let X_t be a strictly stationary strongly mixing process on the integers. Kernel estimators of the marginal probability density function of X_t, of the joint density of $(X_t, X_{t+j_1}, \ldots, X_{t+j_n})$, and of conditional densities, are considered. The estimators are of value in assessing the proximity of X_t to a Gaussian process, and potentially provide a more detailed description of non-Gaussian processes than does spectral analysis. They may also be useful in numerical study of the properties of the non-linear time series models that are currently of considerable interest. Multivariate central limit theorems are established for the estimators. It is found that constraints on the rate of decrease of the bandwidth parameters, as series length increases, are imposed by the rate of decay of the strong mixing coefficient, by the smoothness of the density, and by the choice of kernel.

M. ROSENBLATT

Remarks on Smoothing Splines

Some of the joint work of John Rice and myself in this area is reported on. Let $y_i = f(i/n) + \varepsilon_i$, $i = 0,1,\ldots,n-1$ where f is a smooth unknown function (to be estimated) and the ε_i's are orthogonal random variables with mean zero and variance σ^2. Let $s_n(t)$ be a smoothing spline fitted to the data y_i with nodes at i/n, $i = 1,\ldots,n$. The asymptotic optimal behaviour of the integrated mean square error of the spline is determined and it is shown that there are generally (except, for example, in the case of periodic f) skin effects near the boundary points that dominate the asymptotic behavior. This suggests caution in using techniques like cross-validation here. Corresponding remarks can be made in the analogous situation in time series analysis where smoothing splines are sometimes used in estimating spectra.

R.J. SERFLING

On kernel-type density estimators with kernel having several arguments

Consider nonparametric estimation of a probability density function f by an empirical probability density function f_n based on a sample of independent observations $X_1,\dots,X_n$ from f_n. For f_n of kernel-type with step-function kernel, and f Lipschitz, it has been shown previously by the author that (*) $\sup_x |f_n(x)-f(x)| =_{wpl} O(n^{-1/3}(\log n)^{1/3})$. The current investigation extends to (i) the case of a more sophisticated kernel structure,

$$K(x;x_1,\dots,x_m)=\frac{1}{c_n}K\left(\frac{x-g(x_1,\dots x_m)}{c_n}\right) I[h(x_1,\dots,x_m)< d_n],$$

where $g(x_1,\dots,x_m)$ is a summary measure of location of $x_1,\dots,x_m$, $h(x_1,\dots,x_m)$ is a summary measure of dispersion, c_n is a bandwidth, and d_n is a tolerance, and (ii) the case that $\{X_i\}$ is a stationary time series. Questions in U-statistics and on empirical processes for dependent variables arise.

A.N. Shiryayev

A general Poisson approximation theorem - a martingale approach

Let $N^n=(N_t^n, F_t^n)$, $n\geq 1, t\geq 0$ be a sequence of point processes with compensators $A^n=(A_t^n,F_t^n)$ and $N=(N_t,F_t)$ be a point process with a deterministic compensator A. Then

1) If for all $t\geq 0$ $A^n \xrightarrow{P} A_t$ and $\sum_{s<t}(\Delta A_s^n)^2 \xrightarrow{P} \sum_{s<t}(\Delta A_s)^2$ then $N^n \xrightarrow{D} N$ where "$\xrightarrow{D}$" means weak convergence.

2) If $E \sup_{s<t}|A_s^n-A_s| \to 0$ then $\sum_{k=0}^{\infty}|P(N_t^n=k)-P(N_t=k)|\leq K(A_t)\; E\sup_{s<t}|A_s^n-A_s|$ where $K(A_t)=(2+(6+4A_t))\varepsilon_t(2A)$ and $\varepsilon_t(B)=e^{B_t}\prod_{s<t}(1+\Delta B_s)e^{-\Delta B_s}$.

B.W. SILVERMAN

On the estimation of a probability density function by the maximum penalized likelihood method

A class of probability density estimates can be obtained by penalizing the likelihood by a functional which depends on the roughness of the logarithm of the density. The limiting case of the estimates as the amount of smoothing increases has a natural form which makes the method attractive for data analysis and which provides a rationale for a particular choice of roughness penalty. The estimates are shown to be the solution on an unconstrained convex optimization problem, and mild natural conditions are given for them to exist. Rates of consistency in various norms and conditions for asymptotic normality and approximation by a Gaussian process are given, thus breaking new ground in the theory of maximum penalized likelihood density estimation.

U. STADTMÜLLER

Asymptotic results for nonparametric curve fitting based in approximation operators

Consider the following model (regression model with fixed design.

$$y_i = m(t_i) + \varepsilon_i \qquad o \leq i \leq N$$

where: i) m: $[0,1] \to \mathbb{R}$ is a unknwon function, belonging to a class of smooth functions which cannot be described by finitely many parameters.

ii) $(t_i)_{i=0}^N$ are the (fixed) points, where the values of m(.) are measured with errors:

iii) $(\varepsilon_i)_{i=0}^N$, described by random variables with $E(\varepsilon_i) = 0$ $E(\varepsilon_i \varepsilon_i) = \sigma_\varepsilon^2 \delta_{ij}$.

Based on the approximation operator

$$L_n(m) := \sum_j m(t_{jn})\, p_{jn}(t), \qquad n \in \mathbb{N}$$

with interpolation points $(t_{jn})_j$ and nonnegative functions $(p_{jn})_j$ we define the estimator : ($n = n(N)$ is a design parameter)

$$\hat{m}_{nN}(t) := \sum_j p_{jn}(t)\, b_{jn}^{(N)} \sum_{k(j)} Y_k$$

where $b_{jn}^{(N)} \sum_{k(j)} Y_k$ is the average of the values of those Y_k which belong to the interval $[t_{jn} - \frac{t_{jn} - t_{j-1,n}}{2},\ t_{jn} + \frac{t_{j+1,n} - t_{jn}}{2}),\ n \in \mathbb{N}$.

We give several limit theorems (e.g. exact rates of convergence, limit distribution for the global deviation), which imply results for many special estimators, as there are kernel estimators, regressograms and smoothed regressograms.

W. STUTE

Sequential confidence intervals for a density

In this paper sequential fixed-width confidence intervals for a non-parametric density function are derived. There are formed from certain values of a kernel density estimator. The efficiency of such procedures is measured in terms of the expected stopping time.

E. TARTER

Weighted nonparametric estimation and applications to the estimation of long term survivial probability

When a survival or other statistical curve is estimated, not all subregions of the curve's domain are of equal interest to a researcher. In particular, assessment of long term survival, which is associated with the right tail of certain distributions, is generally of more importance than assessment over other subsets of a distribution's support. In this paper it is shown that the mean integrated square error metric used in conjunction with most kernel and series estimation procedures can be generalized to weight support subregions non-uniformly. Results obtained by means of a series of experiments with simulated data are reported. These experiments tended to show that the new weighting techniques do enhance estimator efficiency over tail regions at the expense of tolerable degradation of efficiency over other regions. In the principal experiment the new procedures were used to estimate survival probability from samples of mixed complete and incomplete survival data. Both the tail estimation improvement of certain new nonparametric estimators of survival and superiority of the new procedures to concentional clinical life table methods with optimal abridgement were demonstrated.

H. WALK

On recursive estimation of the mode

A recursive method for the estimation of a mode (maximum point) of a density function $f: R^k \to R_+$, which is due to Fritz (1973), is investigated for the case of a special linear combination of nonnegative kernel functions. For the recursion sequence (X_n) a.s. convergence of $\nabla f(X_n)$ to zero is proved using only differentiability and boundedness assumptions on f. In the case of a unique mode θ with $X_n \to \theta$ a.s., under the assumptions of $f \in C^3$ in a neighborhood of θ and $\nabla^2 f(\theta)$ negative definite, a functional central limit theorem for $X_n - \theta$ with convergence order $n^{-2/(k+6)}$ is proved where the limit Gaussian Markov process has no drift.
For a sequence of recursive estimates Y_n of $f(\theta)$ according to Carroll(1976), but with the above linear combination of kernels, one obtains a corresponding result with convergence order $n^{-2/(k+4)}$, also for $f \in C^2$, and

asymptotic independence of the standardized pair of estimates. This yields the possibility to construct asymptotic fixed width confidence domains for $(\theta,f(\theta))$.

M. WALKER

Existence of stationary time series satisfying bilinear model equations

The general bilinear model for a discrete time parameter process $\{X_t, t=0,\pm1,\dots\}$ is defined by a set of non-linear difference equations of the form

$$X_t + \sum_{j=1}^{p} a_j X_{t-j} = \sum_{j=0}^{q} c_j e_{t-j} + \sum_{j=1}^{r} \sum_{k=1}^{s} b_{jk} X_{t-j} e_{t-k},$$

where $\{e_t\}$ is a sequence of independent and identically distributed random variables with finite variance (strict white noise process) and the a_j, c_j, b_{jk} are constants (with $c_0=1$). In anordance with notation introduced by recent authors, we call this a BL(p,q;r,s) model.

It is shown, that for a wide class of these models, a strictly stationary process $\{X_t\}$ satisfying the model equations for all t exists (and is unique) almost surely, under specified conditions, as the limit of a certain series. In order of complexity the models considered are BL(1,0;1,1), for which the proof is very simple, BL(p,0;p,1), and BL(p,0;p,p) with $b_{jk}=0$ for $k>j$ (which has been called a 'lower triangular' bilinear model).

The relation of these results to recent work of C.W.Granger and A.P.Andersen, T.Subba Rao, and M.M.Gabr is briefly discussed.

W. WERTZ

Invariant Curve Estimation

Let f be a probability density on a σ-finite measure space (X,F,μ), G a locally compact σ-compact group acting on (X,F), let $\mu(gA)=\chi(g)\mu(A)$ for every $g \in G$ and $A \in F$. We consider the statistical model $(X,F,\{P_g : g \in G\})$, where P_g stands for the pr.measure with density $g \to \chi(g^{-1})f(g^{-1}x)$ resp. μ. For every g let $D_g \in L_p(X,\mu)(p \geq 1)$ and satisfy $D_g(x)=\phi(g^{-1})D(g^{-1}x)$ $(D:=D_e)$. The problem is to estimate D_g by estimators $f \in \bigcap_{g \in G} L_p(X^n \times X, F^n \otimes F, P_g^n \otimes \mu)$, which are ϕ-invariant, i.e. $\hat{f}(x^n,x)= \phi(g)\hat{f}(gx^n,gx)$. The risk is defined by $R_p(\hat{f},g)= \int_{X^n}\int_X |\hat{f}(x^n,x)D_g(x)|^p d\mu(x)\, dP_g^n(x^n)$. f_0 is called ϕ-invariantly optimal estimator of D_g (IOE) if it minimizes $R_p(\hat{f},g)$ over the class of all ϕ-invariant estimators for every g. Under some regularity conditions, the IOE's are characterized for $p \geq 1$. Explizit solutions are given for $p=1$ and 2, for which cases the IOE is shown to be a generalized Bayes estimator.

H. WOLFF

Estimation of time-dependent densitites

In this paper we attack the problem of estimating densities varying in time. For the one-dimensional case we propose to use a generalization of Tukey's lambda distribution given by Ramberg and Schmeiser. This family of distributions was recently used by Ramberg, Dudewicz, Tadikamalla and Mykytka for fitting curves (in the stationary case).

The application of Kormann's generalization of the author's expectation-tracking-procedure yields very nice results even if the four parameters of the considered family follow a trend of an unknown but restricted form. The higher dimensional case is yet unsolved.

W.R. VAN ZWET

Chernoff-Savage versus Chernoff-Gastwirth-Johns

It has long been known that the problems in showing asymptotic normality of rank statistics (Chernoff-Savage theorems) and of linear functions of order statistics (Chernoff-Gastwirth-Johns) are very similar. It is shown here that under mild conditions, the asymptotic normality of two-sample rank statistics follows from that of a certain linear combination of a bounded function of uniform order statistics.

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