MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 26/1982

Algebraische Gruppen

20.6. bis 26.6.1982

An der von T. A. Springer (Utrecht) und J. Tits (Paris) geleiteten Tagung zum Thema "Algebraische Gruppen" nahmen 31 Mathematiker aus verschiedenen Ländern teil. In den Vorträgen wurde vorwiegend über neuere Ergebnisse und Entwicklungen berichtet; die Referate konzentrierten sich inhaltlich um die folgenden Schwerpunkte und Einzelthemen:

- Darstellungen von Weylgruppen und endlichen Chevalleygruppen; Anwendungen der Schnittkohomologie, Bestimmung der Green-Funktionen
- Geometrische Aspekte homogener Räume
- Algebraische Gruppen über Zahlkörpern und Schemata
- Invariantentheorie
- Darstellungen algebraischer Gruppen, Distributionen auf GL(n)
- Freie Untergruppen halbeinfacher Gruppen
- Kombinatorik von Wurzelsystemen
- Arithmetik algebraischer Gruppen: Präsentationen, Kohomologie arithmetisch definierter Gruppen
- Struktur von Kac-Moody Gruppen und Zusammenhänge mit der Deformation von Singularitäten

Die vorgestellten, meist noch unveröffentlichten Ergebnisse ersehe man aus den Vortragsauszügen.

Vortragsauszüge

R. HOTTA:

Invariant eigendistributions and holomorphic systems

In this talk, first Kashiwara's main theorem about the Harish-Chandra systems on semisimple Lie algebras were introduced. Let $\mathfrak{g}$ be a complex semisimple Lie algebra and $p_1,\ldots,p_r$ be generators of invariant polynomials on the dual of $\mathfrak{g}$. For $\lambda \in \mathfrak{h}^* =$ the dual of a Cartan subalgebra, we consider the system of LDE: $(p_i(\partial_x) - p_i(\lambda))u = 0$ $(i=1,\ldots,r)$, $L_A u = 0$ $(A \in \mathfrak{g})$ where L_A is the vector field whose value at $x \in \mathfrak{g}$ equals $[A,X]$. Let M_λ be the $D_{\mathfrak{g}}$-module given by this system (call it the Harish-Chandra system). Then it is easily seen that M_λ is a holonomic system in the sense of M. Sato. Let $\mathfrak{g}_{rs}$ be the set of regular semisimple elements in $\mathfrak{g}$. Then the local system $E_\lambda := \mathrm{Hom}_{D_{\mathfrak{g}_{rs}}}(M_\lambda|\mathfrak{g}_{rs}, \mathcal{O}_{\mathfrak{g}_{rs}})$ = the local solutions on $\mathfrak{g}_{rs}$, has a W-module structure (W = the Weyl group). The first theorem: $\mathbb{R}\mathrm{Hom}_{D_{\mathfrak{g}}}(M_\lambda, \mathcal{O}_{\mathfrak{g}}) \simeq \underline{IC}^{\bullet}(E_\lambda)[-\dim \mathfrak{g}]$ = the Deligne-Goresky-MacPherson middle intersection complex for E_λ. Thus we have an analytic construction of the Springer representation. The second theorem concerns the "Fourier transform" M_o^F of the Harish-Chandra system M_o for $\lambda = 0$. We have a quasi-isomorphism

$\mathbb{R}\mathrm{Hom}_{D_{\mathfrak{g}}}(M_o^F, \mathcal{O}_{\mathfrak{g}}) \simeq \underline{IC}^{\bullet}(E_\lambda)[-\dim \mathfrak{g} - \mathrm{rank}\, \mathfrak{g}]|_N$ where N is the nilpotent variety of $\mathfrak{g}$. The RHS was decomposed by Borho-MacPherson according to the W-action, but here, we have the decomposition: $M_o^F \simeq \bigoplus_{\chi \in \hat{W}} V_\chi \otimes M(\chi)^F$ which corresponds to the decomposition $M_o \simeq \bigoplus_{\chi \in \hat{W}} V_\chi \otimes M(\chi)$. Here the $D_{\mathfrak{g}}$-module $M(\chi)^F$ is a simple holonomic system supported by the closure of a single nilpotent orbit $\mathcal{O}(\chi) \subset N$ and corresponds to the $\underline{IC}^{\bullet}$ of some local system on $\mathcal{O}(\chi)$. This recovers the original Springer correspondence between $\hat{W}$ and the local systems on nilpotent orbits.

Secondly as applications, we can determine the Fourier transform of nilpotent orbital measures $\mu_{\mathcal{O}}$ for any nilpotent orbits $\mathcal{O}$ (This was discovered first by Barbasch-Vogan for "special" orbits.). This leads to some consequences which generalize some results by King and Joseph concerning the relations among the $\hat{W}$, nilpotent orbits and irreducible characters of infinite dim. representations of G. (For analogue over finite fields, see Brylinski's lecture given during the meeting 'Darstellungstheorie und ℓ-adische Kohomologie', Oberwolfach, 13. - 19.6.1982.)

T. SHOJI:

Green polynomials of classical groups

Let G be a connected reductive group defined over $\mathbb{F}_q$, $A \in \underline{g}$ a nilpotent element of the Lie algebra of G. For a closed subvariety B_A of the variety of Borel subgroups B, Springers representation of the Weyl group $\overline{W}$ on ℓ-adic cohomology $H^i(B_A, \overline{\mathbb{Q}}_\ell)$ can be defined. For $A \in \underline{g}^F$ (F is Frobenius endomorphism), Green functions $Q_{T_w}(A)$ are described using Springer representations as follows: $Q_{T_w}(A) = \sum_{i \geq 0} (-1)^i \mathrm{Tr}(F^* r_i(w)^{-1}, H^i(B_A))$. In this talk, we give a systematic way of computing Green functions for Chevalley groups. This depends on the following three properties. (1) The Springer correspondence. (2) The Theorem of Borho-MacPherson. (3) For $\phi \in \widehat{C(A)}$ (where $C(A) = Z_G(A)/Z_G^o(A)$), if the ϕ-isotypic subspace of $H^{2d_A}(B_A)$ ($d_A = \dim B_A$) is zero, then the same is true for every $H^i(B_A)$.

Actually, for each case, the Springer correspondence is given explicitly (Shoji, Alvis, Lusztig, Spaltenstein), and the property (3) is serious in the case of classical groups. It is proved using the local triviality of the map $B_A \to P_A$ for a suitable subvariety of G/P and the classification of $\widehat{C(A)}_o = \{\phi \in \widehat{C(A)} \mid H^{2d_A}(B_A)_\phi \neq 0\}$. Green functions turn out to be polynomials in q whose coefficients are independent of p. Also, we get a basis of the space of uniform functions of G^F whose support are in the set of unipotent elements. In particular, characteristic functions on the sets $O(n)^F$ (n = unip., $O(n)$ = G-orbit in $\overline{\mathbb{F}}_q$) are uniform. These methods for computations are also applied to the non-split group of type D_n. (See also the talk of Spaltenstein during the meeting, Oberwolfach, 13. - 19.06.1982).

S. KATO:

On representations of Hecke algebras of affine Weyl groups

The study of unramified principal series representations of p-adic reductive groups are generalized by Matsumoto in terms of Hecke algebras. Let G be a connected semisimple group (for simplicity, of adjoint type) over $\mathbb{C}$ and T be a maximal torus. Then the Weyl group W of (G,T) naturally acts on $X(T)$, the character group of T and one can construct $\tilde{W}$, the semidirect product of W by $X(T)$. This group $\tilde{W}$ is an affine Weyl group (a Coxeter group). So one can define the Hecke algebra $H = H(W,q)$ with a parameter $q \in \mathbb{C}^\times$. Matsumoto constructed a nice family of H-modules (principal series representations) M_s parametrized by every element s of T. Concerning to this module M_s, one can give a criterion for irreducibility in terms of the parameter s. This criterion resembles the irreducibility criterion for spherical principal series representations of real groups given by Kostant. Also, this result suggests the connection between the set of equivalence classes of irreducible H-modules and the set $F(q) = \{(s.N) \mid s \in G$, semisimple; $N \in \mathrm{Lie}\, G$, nilpotent; $\mathrm{Ad}(s)N = q^{-1}N\}/\overset{G}{\sim}$ what is called Deligne-Langlands conjecture. The general parametrization of irreducible H-modules seems to be difficult. But in case $q = 1$ (hence $H = \mathbb{C}[\tilde{W}]$), one can construct $\tilde{W}$-representations on cohomology groups $H^{\bullet}(B_g)$ of the fixed point subvariety B_g $(g \in G)$ of the flag variety by using Lusztig's method. In exactly the same way of Springer representations, one can obtain all irreducible representations of $\tilde{W}$ in the top cohomologies. In this case, the connection of irreducible H-modules and $F(1) = \{$the set of conjugacy classes$\}$ are obvious.

V. V. DEODHAR:

A finer decomposition of Bruhat cells

Let G be a semisimple algebraic group over an alg. closed field k. Let $B \supseteq T$ be respectively a Borel subgroup and a maximal torus. Let $W = N(T)/T$ be the Weyl group. One then has the Bruhat decomposition of G/B into cells which are parametrized by elements of W; $G/B = \bigcup_{y \in W} By{\cdot}B$. Each Bruhat cell $By{\cdot}B$ is isomorphic to an affine space $k^{\ell(y)}$ where ℓ is the length function in W. This talk gave a further (and finer in some sense) decomposition of $By{\cdot}B$ into sets $\{A_{\underline{\sigma}}\}_{\underline{\sigma} \in \Sigma}$ (Σ is some indexing set) each of which is isomorphic to a product of an affine space $k^{m(\underline{\sigma})}$ and a torus $(k^*)^{n(\underline{\sigma})}$. Further, there exists a map $\pi : \Sigma \to W$ such that $A_{\underline{\sigma}} \subseteq w_oBw_o\pi(\underline{\sigma}) \cdot B$ (where w_o is the maximal element of W). This in particular gives a description of $By{\cdot}B \cap w_oBw_o x \cdot B$ for $x \leq y$; this intersection is of interest in several different contexts e.g. in Kazhdan-Lusztig polynomials. The set Σ is the set of certain special subexpressions of a fixed reduced expression of y. One further considers the closures (in $By{\cdot}B$) of $A_{\underline{\sigma}}$'s. One then gets a partial order $\leq$ in Σ such that $\overline{A_{\underline{\sigma}}} = \bigcup_{\underline{\sigma} \leq \underline{\tau}} A_{\underline{\tau}}$. One has an explicit description of this order $\leq$. The discussion so far is applicable to the case of an affine Weyl group with minor changes.

One now considers the situation in an arbitrary Coxeter group (W,S) . The set Σ , the map $\pi : \Sigma \to W$ and the order $\leq$ still makes sense and one looks at the applications in this case. The first application is to give an explicit description of the polynomials $R_{x,y}$ which occur in the context of Kazhdan-Lusztig polynomials, viz.

$$R_{x,y}(q) = \sum_{\underline{\sigma} \in \Sigma, \pi(\underline{\sigma}) = x} q^{m(\underline{\sigma})} \cdot (q-1)^{n(\underline{\sigma})}$$

(where $m(\underline{\sigma})$, $n(\underline{\sigma})$ are as mentioned befor.) Another application is to the L-shellability of the Bruhat ordering. One further proves that there is a subset Σ_o of Σ sucht that $\pi : \Sigma_o \to W(y)$ $(W(y) = \{x \in W \mid x \leq y\})$ is an isomorphism of posets. Thus the order $\leq$ on Σ 'covers' the Bruhat ordering on W .

C. PROCESI:

Compactifications of algebraic groups and symmetric spaces

Let G be a semisimple algebraic group over a field k , algebraically closed of characteristic $\neq 2$. Let $\Theta : G \to G$ be an involutive automorphism, H the fixpoint subgroup of Θ . Then one can find a projective variety X with the following properties:

1) $G/N(H) \hookrightarrow X$ as a dense open set and the action of G on $G/N(H)$ extends to an action of G on X .
2) Every orbit closure in X is smooth (in particular X is smooth).
3) $X - G/N(H)$ is a union of h hypersurfaces which are orbit closures, smooth and meet transversally; h = the rank of G/H .
4) The G-orbits of X corespond to the subsets of the set of restricted simple roots.

Further one can describe each orbit closure as a locally trivial fibration over a suitable G/P (P parabolic) with fibers a compactification of the same type for the adjoint group associated to the Levi component L of P and an induced involution Θ_L in L .

In the case $G = SL(4)$, $\Theta(x) = {}^t x^{-1}$ one can use the above compactification to establish rigorously Schubert's computation of the number 666.841.088 of quadrics in P^3 tangent to 9 quadrics in general position.

V. LAKSHMIBAI:

Cohen-Macaulayness for multicones over Schubert varieties

Let G be a semisimple, simply connected Chevalley group over a field k. Let T be a maximal torus of G and B a Borel subgroup T. Let W be the Weyl group of G relative to T. Let Q be a parabolic subgroup $\supset B$ and W_Q the Weyl group of Q. For $w \in W/W_Q$, let $X(w)$ (= Zariski closure of $BwQ \pmod Q$) with the canonical reduced structure) be the Schubert variety in G/Q. Let $Q = P_1 \cap \dots \cap P_r$, where P_i, $1 \leq i \leq r$ are maximal parabolic subgroups. Let L_i = ample generator of $\mathrm{Pic}(G/P_i)$, $i \leq i \leq r$ and $L = \bigotimes_{i=1}^{r} L_i^{a_i}$, $a_i \in \mathbb{Z}^+$ be a positive line bundle on G/Q.

Let $R(w) = \bigoplus_{L \geq 0} H^0(X(w),L)$. Then we have Theorem 1. For G of type A_n, the ring $R(w)$ is Cohen-Macaulay.

Let now G be of type A_n, B_n, C_n or D_n. Let P = the maximal parabolic subgroup of G corresponding to the simple root α_i. Call $X(w) \subset G/B$ a Kempf variety, if (1) $\pi_{|X(w)} : X(w) \to \mathrm{Im}\, X(w)$ (where $\pi : G/B \to G/P$ is the canonical projection) is equidimensional and (2) fibers are Kempf varieties in lower rank. Then we obtain a characterization of Kempf varieties by means of standard monomials and we have

Theorem 2: For G of type A_n, B_n, C_n or D_n and $X(w)$ a Kempf variety in G/B, the ring $R(w)$ is Cohen-Macaulay.

R. MACPHERSON:

Partial resolutions of nilpotent varieties (joint work with W. Borho)

Let $\pi : \tilde{N} \to N$ be the Springer resolution of the nilpotent variety N of a reductive algebraic group G with Weyl group W. In previous work, we showed that the endomorphism ring of $R\pi_* \underline{\underline{\mathbb{Q}}}_{\tilde{N}}$ is naturally isomorphic to the group ring of W. Now we consider the partial resolution $\xi : \tilde{N}^P \to N$ obtained by replacing the Borel subgroups in the construction of $\tilde{N}$ by parabolic subgroups conjugate to P. If $\eta : \tilde{N} \to \tilde{N}^P$ is the projection, we have $\mathrm{End}\, R\eta_* \underline{\underline{\mathbb{Q}}}_{\tilde{N}}$ is naturally isomorphic to the group ring of the Weyl group of the Levi part of P. As a corollary, we compute that the cohomology of the Steinberg fiber $\xi^{-1}(x)$ is the W-invariant part of the cohomology of the Springer fiber $\pi^{-1}(x)$.

T. A. SPRINGER:

Representations with a free algebra of invariants

Let G be a complex connected semi-simple linear algebraic group and $\pi : G \to GL(V)$ a rational representation. Assume that π does not contain the trivial representation. In the talk the proof, due to V. L. Popov (Izv. Akad. Nauk, SSSR, 46 (1982), 347 - 371), of the following result was discussed. If the algebra of invariants $\mathbb{C}[V]^G$ is free, i.e. is a polynomial algebra with homogeneous generators, then there are, for a fixed G, only finitely many possibilities for the isomorphism class of V. Popov's proof gives explicit bounds. It uses properties of Poincaré series.

A. BOREL:

Free subgroups of semi-simple groups

In connection with various developments arising out of the Hausdorff paradox (1914), R. J. Dekker asked (1956-58) whether there exists a free (non-commutative is always understood) subgroup F of $SO(n+1)$ which acts freely on S^n for n odd and with commutative isotropy groups for n even. He observed this was certainly the case for $n \equiv -1 \ (4)$ in the first case, for $n \neq 4$ in the second case. The reason for this was that Dekker had proved (among other things) that if a free group F acts on a space X with commutative isotropy groups, then there is a partition of X into four subspaces X_i such that $X_1 \equiv X_2 \equiv X_1 \cup X_2$ and $X_3 \equiv X_4 \equiv X_3 \cup X_4$, where $\equiv$ means: transforms of one another by an element of F. There exists then $x,y \in F$ such that

$$X = x \cdot X_1 \cup X_3 = y \cdot X_2 \cup X_4 ,$$

which is a strong form of the Hausdorff paradox (proved for S^2 first by R. M. Robinson). In answer to Dekker's first question, Deligne and Sullivan showed the existence of a free subgroup F of $SU(n)$ which acts freely on S^{2n-1}, using the existence of division algebras with involution of the second kind, any degree, over quadratic imaginary fields. As a generalization of this, which also answers Dekker's second question, I showed that a compact connected semi-simple group G contains a free subgroup F which acts freely on G/U if U is a closed subgroup of rank $\mathrm{rk}\, U < \mathrm{rk}\, G$ and one F' which has commutative isotropy groups on G/U if $\mathrm{rk}\, U = \mathrm{rk}\, G$. (The conditions on the rank are respectively equivalent to $\chi(G/U) = 0$ and $\chi(G/U) > 0$ by a theorem of Hopf-Samelson.) For F' one needs only to take a free subgroup of a principal three dimensional subgroup. The main point for the existence of F is the following theorem, in which now G is a connected semi-simple group over an arbitrary field k :

Theorem: Let $m \geqq 2$ and $w(X_1,\dots,X_m)$ be a non-trivial element in the free group over $X_1,\dots,X_m$. Let $f_w : g = (g_i) \mapsto w(g_1,\dots,g_m)$ be the natural map $G^m \to G$ associated to w . Then f_w is dominant.

This is first proved for SL_n by induction on $n \geqq 2$, using the existence of a division algebra of degree n over some infinite field of the same characteristic as k , and then for general G by induction on $\dim G$.

From this and from the fact that G is unitational over any field of definition (Grothendieck), one derives notably that if k has infinite transcendence degree over its prime field, then $G(k)$ contains a free subgroup F such that any $x \in F - \{1\}$ generates a subgroup which is Zariski dense in a maximal torus of G . It follows then that F acts freely on $G(k)/U(k)$, whenever U is a closed k-subgroup of rank $<$ rk G .

B. WEISFEILER:

On Zariski-dense subgroups of simple algebraic groups

The following strong approximation result was stated and its corollaries discussed.

Theorem. Let k be an algebraic number field, G an absolutely almost simple simply connected algebraic group defined over k . Let Γ be a Zariski-dense subgroup of $G(k)$. Then there exist a subfield k_1 and a finite set S of primes of k containing all archimedean ones, such that

G is defined over k_1 and

the closure of Γ in $\prod^{res}_{v \notin S} G(k_v)$ contains an open subset of $\prod^{res}_{v \notin S} G(k_{1,v})$.

Parts of the proof of this result and its applications were obtained in collaboration with Ch. Matthews and L. Vaserstein. The proof uses classification of finite simple groups.

H. BEHR:

Existence and non-existence of finite presentations for some classes of arithmetic groups over global function fields.

Let G be an almost simple algebraic group, defined over a global function field k with ring O_S of S-integers and Γ a S-arithmetic subgroup of G. Denote by s the number of primes in S $(0 < s < \infty)$, by r the rank of G and by $\hat{r}_i$ the rank of $\hat{G}_i = G \otimes_k k_{v_i}$ for $v_i \in S$, if $s = 1$ we write only $\hat{r}$.

Then the following list of results is known:

I. Γ is not finitely generated $\Longleftrightarrow$ $s = 1$, $r = \hat{r} = 1$.

II. 1) $r = 0$: Γ is always finitely presented (f.p.).

2) $r = 1$, $s \geq 2$: Γ is f.p. $\Longleftrightarrow$ $\sum_{i=0}^{s} \hat{r}_i \geq 3$ (for $G = SL_2$ this is due to U. Stuhler).

$r = 1$, $s = 1$, $\hat{r} = 2$: There exist examples of not f.p. groups Γ.

3) $r = 2$, $s = 1$, $\hat{r} = 2$: Γ is not f.p. (for classical G).

4) $r = 2$, G split, $G \neq$ type G_2, $O_S = \mathbb{F}_q[t,t^{-1}]$ $(s = 2)$: Γ is f.p. (Hurrelbrink)

5) $r \geq 3$, G split, $O_S = \mathbb{F}_q[t]$ $(s = 1)$: Γ is f.p. (Rehmann-Soulê)

The ideas of the proofs for case 2 and 3 were given.

S. DONKIN:

Tensorproducts and filtrations for rational representations of algebraic groups

Let G be a connected affine algebraic group over k, an algebraically closed field. Let B be a Borel subgroup containing T, a maximal torus and X the character group of T. For $\lambda \in X$ we denote also by λ the one dimensional k-space on which B acts, T acting with weight λ. For $\lambda \in X$, $Y(\lambda) = \mathrm{Ind}_B^G(\lambda)$ - the induced G module. A G module V has a good filtration (g.f.) if there is a filtration $0 = V_0, V_1, V_2, \ldots$ of V s.t., for each i, V_i/V_{i+1} is either zero or $Y(\lambda_i)$ for some $\lambda_i \in X$.

Conjecture. If V has a g.f. then $V|_P$ has a g.f. for any parabolic subgroup P of G, moreover, if V' is also a G-module with a g.f. then $V \otimes V'$ has a g.f. We prove the conjecture for $p > 41$ (for arbitrary p if G has type A_ℓ, B_ℓ, C_ℓ, D_ℓ ; for $p > 2$, F_4, E_6 ; for $p > 19$ for E_7 and $p > 41$ for E_8). That $V \otimes V'$ has a g.f. for G of type A_ℓ or p large compared with the Coxeter number was proved by Wang Jian-pan.

K. POMMERENING:

Invariants of unipotent groups

A regular unipotent subgroup U of GL_n is given by a subset Ψ of the root system $\phi = \{(i,j) \mid 1 \leq i,j \leq n \;,\; i \neq j\}$ such that Ψ is a strict ordering of the set $\Omega = \{1,\ldots,n\}$. Conjecture: If GL_n acts on $k[X] = k[X_{ij} \mid 1 \leq i \leq n \,,\, 1 \leq j \leq N]$ in the natural way, then $k[X]^U$ is generated by the invariant minors of the matrix X . A necessary and sufficient condition is given for the stronger property that $k[X]^U$ is spanned by the invariant standard bitableaux; this proves the conjecture in many cases. By Grosshans' criterion, in these cases, the U-invariants are finitely generated whenever GL_n (or SL_n) acts rationally on a finitely generated k-algebra. This gives a positive (characteristic-free) answer to Hilbert's 14^{th} problem in many cases.

V. P. PLATONOV:

Birational properties of varieties of semi-simple groups

Let f be a non-degenerate quadratic form on n-dimensional vector space V over a field K , $\mathrm{char}(K) \neq 2$. Let $\mathrm{Spin}(f)$ be the spinor group over K of the form f . At the Congress in Helsinki P. Deligne has formulated the following question: Is the variety $\mathrm{Spin}(f)$ K-rational, in particular for $K = \mathbb{R}$? For a long time it was conjectured that the varieties of simply connected groups are always rational. However, the author showed that the varieties $SL_n(D)$ determined by the group $SL(n,D)$, where D is a division ring of finite K-rank, can not be K-rational. With the connection Deligne's question is proved following theorems

Theorem 1. Let $f(z) = z_1^2 + x_1 z_2^2 + \ldots + x_{n-2} z_{n-1}^2 + (x_1 \cdot x_2 \cdot \ldots \cdot x_{n-2}) z_n^2$ be a quadratic form in n variables over $K = \mathbb{Q}(x_1, x_2, \ldots, x_{n-2})$. Then the variety $\mathrm{Spin}(f)$ is not rational over K for $n \equiv 2 \pmod 4$, $n \geq 6$.

Theorem 2. If $f(z) = \sum_{i=1}^n z_i^2$ then $\mathrm{Spin}(f)$ is rational over an arbitrary field K .

Theorem 3. The variety $\mathrm{Spin}(f)$ is K-rational for an arbitrary locally compact (non-discrete) field K .

J. TITS:

Groups associated with Kac-Moody algebras

Let $\mathcal{R}$ be the data consisting of a free abelian group Λ, a finite system $(\alpha_i)_{i \in I}$ of elements of Λ and a system $(h_i)_{i \in I}$ (in 1 - 1 correspondence with the previous one) of elements of the $\mathbb{Z}$-dual $\Lambda^{\vee}$ of Λ, such that the matrix $A = (A_{ij}) = (\langle\alpha_j, h_i\rangle)$ is a generalized Cartan matrix, i.e. $A_{ii} = 2$, $A_{ij} \in \mathbb{Z}$, $A_{ij} \leq 0$ if $i \neq j$ and $A_{ij} = 0 \Rightarrow A_{ji} = 0$.

If A is "definite" (i.e. product of a positive definite symmetric matrix by an invertible diagonal matrix), the theory of Chevalley associates to such a data $\mathcal{R}$ a group scheme over $\mathbb{Z}$ (the Chevalley scheme), hence a functor $G_{\mathcal{R}}$ from the category of rings to the category of groups. Can one extend that to an arbitrary system $\mathcal{R}$?

The case where A is "semi-definite" suggests that one must rather try to define two functors $G_{\mathcal{R}}$ and $\hat{G}_{\mathcal{R}}$ from the category of rings to the category of topological groups, where $\hat{G}_{\mathcal{R}}(R)$ is the completion of $G_{\mathcal{R}}(R)$ (whenever the latter is defined: it is not clear that $G_{\mathcal{R}}(R)$ will have a natural meaning for an arbitrary ring R). For example, suppose that $I = \{-,+\}$, $\Lambda = \mathbb{Z}$ identified with its dual in the obvious way, $h_{\pm} = \pm 1$, $\alpha_{\pm} = \pm 2$, hence $A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$. The corresponding Kac-Moody algebra over $\mathbb{C}$ is known to be $sl_2(\mathbb{C}) \otimes \mathbb{C}[T,T^{-1}]$, hence the natural guess $G_{\mathcal{R}}(\mathbb{C}) = SL_2(\mathbb{C}[T,T^{-1}])$. On the other hand, the Iwahori-Matsumoto-Bruhat-T. theory associates A to the group $SL_2(\mathbb{C}((T)))$, which will be $\hat{G}_{\mathcal{R}}(\mathbb{C})$ in this case.

The above program has been carried out to the following extent: to every $\mathcal{R}$ are naturally associated two functors $G_{\mathcal{R}}$, $\hat{G}_{\mathcal{R}}$ from the category of principal ideal domains to the category of topological groups. The case of fields had been treated earlier under various characteristic restrictions and for special choices of Λ by Moody, Teo and Marcuson, and for matrices A of "affine type" by Garland.

In the lecture, the definition of $G_{\mathcal{R}}$ and $\hat{G}_{\mathcal{R}}$ was sketched and various questions concerning them were discussed, such as: BN-pairs in $G_{\mathcal{R}}(K)$ and $\hat{G}_{\mathcal{R}}(K)$ (K a field) and the corresponding Bruhat decompositions (the decompositions BNB and $B^{-}NB$, which are "equivalent" in the classical case, become here essentially different); elementary description of $\hat{G}_{\mathcal{R}}(R)$ in the "semi-definite case"; algebra-geometric structure of $\hat{G}_{\mathcal{R}}(\mathbb{C})$ (an ind-pro-variety); Schubert varieties (over $\mathbb{C}$) and Demazure desingularization.

P. SLODOWY:

Adjoint quotients for Kac-Moody groups and deformations of singularities

Let G be the group associated to a Kac-Moody algebra with simply connected root datum of rank r (cf. the talk of Tits). Using the traces of the fundamental representations of G we define an adjoint quotient $\chi : G \to \mathbb{C}^r$ on the set of trace class elements of G. We analyse the fibers of the restriction of χ to the subset G^s of elements conjugate into a Borel subgroup B, and we obtain a complete classification of the conjugacy classes in G^s. These results allow a partial embedding of the semi-universal deformation of simply elliptic or cusp singularities of degree ≤ 5 into the map χ. The base of these deformations was described by Looijenga as a partial compactification of a certain orbit space of the Weyl group W. An analysis of the restriction of χ to the normalizer N of T relates the boundary components of this compactification to the cosets of T in N.

J. BERNSTEIN:

P-invariant distributions on GL(n)

Let F be a local non-archimedean field, $G_n = GL(n,F)$, P_n-subgroup of matrices with last row equal to $(0,\dots,0,1)$. Consider adjoint action of the group G_n on itself.

Theorem. If Q is a distribution on G_n invariant under the action of group P_n, then it is invariant under the action of the whole group G_n.
This theorem is important in representation theory of group G_n. For instance it implies:

Theorem. Let (π,E) be an irreducible smooth representation of G_n in a (complex) vector space E, $(\tilde{\pi},\tilde{E})$ - contragradient representation. Then each P_n-invariant bilinear pairing $B : \tilde{E} \times E \to \mathbb{C}$ is proportional to the scalar product $B(\tilde{e},e) = c \cdot \langle\tilde{e},e\rangle$.

Corollary. Restriction on P_n of a unitary topologically irreducible representation of G_n is also irreducible.

Note that all these theorems become false over finite field F.

J. SCHWERMER:

Cohomology of arithmetic subgroups of SL_3 and automorphic forms

Let Γ be a torsionfree arithmetic subgroup of a semisimple algebraic $\mathbb{Q}$-group $\underline{G}$ with $\mathrm{rk}_{\mathbb{Q}}\, \underline{G} > 0$. The real Liegroup $G = \underline{G}(\mathbb{R})$ operates on the associated symmetric space $X = G/K$ (K maximal compact subgroup of G) resp. on the space $\Omega^*(X)$ of differential forms on X . The Eilenberg-MacLane cohomology $H^*(\Gamma,\mathbb{C})$ of Γ may be identified with the cohomology $H^*(\Gamma,X;\mathbb{C})$ of the sub-complex $\Omega^*(X)^{\Gamma}$ of Γ-invariant elements in $\Omega^*(X)$. We discussed the attempt to relate these cohomology groups with the theory of automorphic forms, there, in particular, with the theory of Eisensteinseries (as developed by Selberg and Langlands). For this purpose one studies the natural restriction $r_P : H^*(\Gamma\backslash\overline{X};\mathbb{C}) = H^*(\Gamma,X;\mathbb{C}) \to H^*(e'(P),\mathbb{C})$ of the cohomology of the Borel-Serre compactification $\Gamma\backslash\overline{X}$ of $\Gamma\backslash X$ on the cohomology of a face $e'(P)$ in the boundary $\partial(\Gamma\backslash\overline{X})$ (P denotes a proper parabolic $\mathbb{Q}$-subgroup of G). We described the conditions under which one can associate to a given cuspidal class in $H^*(e'(P),\mathbb{C})$ a non-trivial class in $H^*(\Gamma,X;\mathbb{C})$, which is represented by the value of a suitable Eisensteinseries at a special point Λ_o . Various methods were indicated to decide if the Eisensteinseries in question has a pole at this point Λ_o or not. In some cases this question is related to the problem of unitarizibility of Langlands' quotients in the theory of irreducible admissible representations of G . The results one can obtain by these methods give us a complete picture in the case $\underline{G} = SL_3$.

Theorem: $\Gamma = \Gamma(m) \subseteq SL_3(\mathbb{Z})$ a full congruence subgroup , $m \geq 3$. One has a direct sum decomposition

$$H^*(\Gamma,X;\mathbb{C}) = H^*_{cusp}(\Gamma,X;\mathbb{C}) \oplus H^*_{Eis}(\Gamma,X,\mathbb{C})$$

in the cusp cohomology and a space which is generated by Eisenstein cohomology classes (Roughly spoken, these classes have a closed, harmonic representative which is either a value of a suitable Eisensteinseries or a redidue of such at a point Λ_o). $H^*_{Eis}(\Gamma,X;\mathbb{C})$ restricts isomorphically onto the image of the natural restriction $H^*(\Gamma\backslash\overline{X};\mathbb{C}) \to H^*(\partial(\Gamma\backslash\overline{X});\mathbb{C})$.

For the sake of completeness we mention here that there is a non-vanishing theorem for the cusp cohomology $H^*_{cusp}(\Gamma,X;\mathbb{C})$. (This was proved in joint work with R. Lee.)

G. HARDER:

Cohomology of arithmetic groups and special values of L-functions

In the theory of modular symbols one obtains information concerning the special values of L-functions attached to modular forms by integrating the modular forms against certain cycles. The result can be interpreted as an intersection number and this yields the algebraicity of the value $L(f,1)$ after dividing it by a transcendental period. This method goes back to Eichler, Shimura and Manin.

In my talk I presented this method and some variations from a more abstract and unified point of view. One uses the theory of representations of the group of finite adeles $GL_2(A_f)$ to interprete the intersection numbers in terms of an intersection intertwining operator between two irreducible $GL_2(A_f)$ modules. Then the L-values enter as normalizing factors between this intertwining operator and another one constructed from local data. This method allows us to prove results that are more general and more precise than those previously known. There is also some hope that we may generalize this to some higher dimensional groups.

Y. A. NISNEVICH

Arithmetic and cohomology of reductive group schemes over local regular rings of dim ≥ 1

Let R be a local, regular integral ring, K the quotient field of R, $\underline{m} = \max(R)$ the maximal ideal of R, G a reductive group scheme over R, E a principal homogeneous space for G, locally trivial in etale topology (= G-torsor). E is called rationally trivial if $E(K) \neq \emptyset$.

J.-P. Serre (1958) and Grothendieck (1958, 1966) conjectured that rationally trivial G-torsors are trivial. The following theorem partially proves this conjecture:

Theorem 1. Assume that either

(i) $\dim R = 1$ and the residue field $k = R/\underline{m}$ is perfect, or

(ii) $\dim R = 2$, k is infinite and G is quasi-split over R, or

(iii) R is an equicharacteristic henselian ring. Then any rationally trivial G-torsor is trivial.

For the proof of Theorem 1 several approximation and arithmetic properties of $G(R)$ are established. In particular, we have the following decomposition:

Theorem 2. Let $u \in \underline{m}-\underline{m}^2$ be a regular element, $\hat{R}$ the completion of R in (uR)-adic topology, $R_u = R[u^{-1}]$, $\hat{R}_u = \hat{R}[u^{-1}]$. Assume that one of the conditions (i) or (ii) of Theorem 1 are satisfied. Then $G(\hat{R}_u) = G(\hat{R})G(R_u)$.

I. G. MACDONALD:

Some conjectures for root systems and finite Coxeter groups

A brief account of some conjectures generalizing those of Dyson and Mehta, and the evidence in their favour.

(C1) Let R be a reduced (finite) root system. For each root $\alpha \in R$ let e^α be the corresponding formal exponential. Then the constant term in the Laurent polynomial $\prod_{\alpha \in R} (1-e^\alpha)^k$ (k a positive integer) should be equal to $\prod_{i=1}^{\ell} \binom{kd_i}{k}$, where ℓ is the rank of R and d_i are the degrees of the fundamental invariants of the Weyl group. (Dyson's conjecture is the case where R is of type A_n.) (C1) is true for R of classical type and all R, by virtue of an integral formula of Selberg; also for all R and $k = 1,2$.

More generally:

(C2) with R etc. as above, let q be an indeterminate. Then the constant term (i.e. not involving any e^α) in $\prod_{\alpha \in R^+} \prod_{i=1}^{k} (1-q^{i-1}e^{-\alpha})(1-q^i e^\alpha)$ should be $\prod_{i=1}^{\ell} \begin{bmatrix} kd_i \\ k \end{bmatrix}$, where $\begin{bmatrix} n \\ r \end{bmatrix}$ is the Gaussian polynomial (or q-binomial coefficient) which reduces to $\binom{n}{r}$ when $q = 1$.

(C3) Let W be a finite group of isometries of $\mathbb{R}^n$ generated by reflections. For each reflection $r \in W$ let $h_r(x) = 0$ be the equation of the reflecting hyperplane, and let $P(x) = \prod_r h_r(x)$. Then with $P(x)$ suitably normalized, the integral $\int_{\mathbb{R}^n} e^{-|x|^2/2} |P(x)|^{2k} dx$ should be equal to $(2\pi)^{+n/2} \prod_{i=1}^{n} \frac{(kd_i)!}{k!}$, with d_i the degrees of the fundamental invariants of W acting on $\mathbb{R}^n$. Mehta's conjecture is the case W = symmetric group, acting by permuting the coordinates in $\mathbb{R}^n$. (C3) is true for W of type A, B, D or dihedral.

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