MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 27/1982

Riesz Spaces and Operator Theory

27.6. bis 3.7.1982

Die Tagung fand unter Leitung von Herrn Prof. W.A.J. Luxemburg (Pasadena) und Herrn Prof. H. H. Schaefer (Tübingen) statt. In 33 Vorträgen wurden viele neue Forschungsergebnisse vorgestellt und in einer Problemsitzung offene Fragen diskutiert. Die verbleibende Zeit nutzten die Teilnehmer zu einem regen Gedankenaustausch.

Die in den letzten Jahren zu beobachtende Entwicklung hin zur Operatorentheorie kam bei dieser Tagung deutlich zum Ausdruck. Der überwiegende Teil der Vorträge behandelte Aspekte der Operatorentheorie. In vielen Fällen waren es Probleme positiver Operatoren (Positivität und Kompaktheitseigenschaften, Irreduzibilität, Kernoperatoren, multilineare Operatoren) und mehrere Vorträge befaßten sich mit Orthomorphismen. Weitere operatorentheoretische Aspekte wurden in Vorträgen über die Fouriertransformation, die Hilberttransformation und die Darstellung lokalkompakter Gruppen behandelt. Einen weiteren Themenkreis bildeten Aspekte der abstrakten Maß- und Integrationstheorie (Vektormaße, Desintegration, Daniell - Stone - Integration). In zwei Vorträgen wurde gezeigt, wie man mit Hilfe der Theorie der Vektorverbände und der positiven Operatoren konkrete außermathematische Probleme (aus der Ökonomie bzw. der Transporttheorie) in neuartiger Weise behandeln kann.

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Vortragsauszüge

C. D. ALIPRANTIS:

Equilibria in Markets with a Riesz Space of Commodities

Using the theory of Riesz spaces, we present a new proof of the existence of a competetive equilibria for an economy having a Riesz space of commodities.

G. D. ALLEN:

Lipschitz Conditions for Operators

Let T be a bounded operator on $L_p[0,1]$, and suppose P_Δ is the projection induced by the measurable set $\Delta \subset [0,1]$. We consider those operators T on $L_p[0,1]$ for which one of the following conditions hold for all measurable $\Delta \subset [0,1]$:
(1) $\|TP_\Delta\|_p \leq \varphi(|\Delta|)$, (2) $\|P_\Delta T\|_p \leq \psi(|\Delta|)$, (3) $\|P_\Delta TP_{\Delta'}\|_p \leq \psi(|\Delta|)\varphi(|\Delta'|)$.
Here $|\Delta|$ is the measure of Δ and φ and ψ are continuous, increasing functions on $[0,1]$ for which $\varphi(0) = \psi(0) = 0$.

W. ARENDT:

An Order Theoretical Characterization of the Fourier transformation

Let G, G_1, G_2 denote locally compact groups. $C^b(G) \supseteq P(G)$ denotes the cone of all continuous positive definite functions on G, $B(G) := \operatorname{span} P(G) \subseteq C^b(G)$ the Fourier-Stieltjes-Algebra, $A(G) \subseteq B(G)$ the Fourier-Algebra and $L^1(G)$ is defined as usual via Haar measure. $X(G)$ $(X = A, B, L^1)$ is an ordered vector space for two cones: 1) the cone $X(G)_+$ of the pointwise positive functions in $X(G)$ and 2) $X(G)_p$ the cone of the positive definite functions in $X(G)$ in the case $X = A, B$; resp. $L^1(G)_p = \overline{\mathrm{co}}\,\{f * f : f \in L^1(G)\}$ in the case $X = L^1$.

<u>Theorem</u> Let $T : X(G_1) \longrightarrow X(G_2)$ be a bijective linear mapping, $X = A, B, L^1$. Then the following assertions are equivalent:
(i) $T X(G_1)_+ = X(G_2)_+$, $T X(G_1)_p = X(G_2)_p$.
(ii) There exists $c > 0$ and a topological group isomorphism or antiisomorphism $\alpha : G_2 \longrightarrow G_1$, such that $Tf = c \cdot f \circ \alpha$.

The theorem implies that the "biordered space" $(X(G), X(G)_+, X(G)_p)$ $(X = A, B$ or $L^1)$ is a complete isomorphism invariant. This is of particular interest for $X = B$, since the space $(B(G), B(G)_+, P(G))$ is very easy to define (note that Haar measure is not needed).

Corollary Let $F : L^1(G_1) \longrightarrow A(G_2)$ be bijective and linear. The following assertions are equivalent:

(i) $F(L^1(G_1)_+) = A(G_2)_p$, $F(L^1(G_1)_p) = A(G_2)_+$.

(ii) G_1 and G_2 are abelian and there exists $c > 0$ and a group isomorphism $\alpha : G_2 \longrightarrow \hat{G}_1$ such that $Ff = c \cdot \hat{f} \circ \alpha$ $(f \in L^1(G_1))$.

O. BURKINSHAW:

Positive Compact Operators

Let E be a Banach lattice, and let $T : E \longrightarrow E$ be a positive compact operator. The purpose of this paper is to study compactness properties of positive operators that are dominated be T. Specifically, we ask: If $S : E \longrightarrow E$ is an operator such that $0 \leq S \leq T$, then what effect does the compactness of T have on S? The following are the main results answering this question.

Theorem If $0 \leq S \leq T$ are operators defined as above with T compact, then S^3 is a compact operator.

Theorem If $0 \leq S \leq T$ are operators defined as above with T weakly compact, then S^2 is a weakly compact operator.

P. DODDS:

Riesz Spaces, Vector Measures and Conical Measures

Let μ be a conical measure on the lctvs X such that every conical measure ν with $0 \leq \nu \leq \mu$ has resultant $r(\nu)$ in X. Let $K_\mu = \{r(\nu) : 0 \leq \nu \leq \mu\}$. If L is a Riesz space with order unit e and if $A : L \longrightarrow X$ is a linear map for which $A([0,e])$ has $\sigma(X,X')$-compact closure, then there is a conical measure μ on X such that the $\sigma(X,X')$-closure of $A([0,e])$ is precisely K_μ. Conversely each K_μ is even the order continuous image of an order interval in a Dedekind complete Riesz space. The techniques used are from the duality theory of Riesz spaces and the results place the Kluvanek characterization of the range of a vector measure within a purely order theoretic setting.

K. DONNER:

On Positive Projections in Classical Banach Lattices

Let H be a closed linear subspace of an arbitrary Banach lattice E. Consider the following conditions:

i) H is a lattice with respect to the ordering induced by E on H.

ii) For every $e \in E$ the set $\{h \in H : h \leq e\}$ is bounded from above in H.

If there exists a positive projection from E onto H, then i), ii) hold. It is shown that the conditions i) and ii) are also sufficient for the existence of a positive projection in E with range H in each of the following cases:

a) E is an L^p-space, $1 \leq p < \infty$,

b) H is finite dimensional,

c) $E = C_o(X)$ for some locally compact space X and for any two points $x, y \in \{z \in X : \exists h \in H \text{ with } h(z) \neq 0\}$, $x \neq y$, $\varepsilon_x|_H$ is not a real multiple of $\varepsilon_y|_H$, where ε_x, ε_y denote the respective Dirac measures.

M. DUHOUX:

An Extension Theorem for Extended Orthomorphisms

Recently M.Meyer and myself have obtained the following result:

Theorem If E is an Archimedean f-algebra and F a quasi-unital Riesz subspace, then every $T \in \mathrm{Orth}^\infty(F)$ can be extended to $\tilde{T} \in \mathrm{Orth}^\infty(E)$ in the sense that $T = \tilde{T}$ on some order dense ideal of F. If F is order dense in E, super quasi-unital and the ideal generated by F is super order dense in E, then every $T \in \mathrm{Orth}^\sigma(F)$ has an extension $\tilde{T} \in \mathrm{Orth}^\sigma(E)$ such that $T = \tilde{T}$ on some super order dense ideal of F.

(F is (super) quasi-unital if the ideal generated in F by $\{x \in F : xy = x$ for some $y \in F_+\}$ is (super) order dense in F; $T \in \mathrm{Orth}^\sigma(E)$ if $T \in \mathrm{Orth}^\infty(E)$ and can be defined on a super order dense ideal of E).

Corollary 1 If $\mathrm{Orth}^\sigma(E)$ is order dense in $\mathrm{Orth}^\infty(E)$, then $\mathrm{Orth}^\sigma(\mathrm{Orth}^\sigma(E)) = \mathrm{Orth}(\mathrm{Orth}^\sigma(E)) = \mathrm{Orth}^\sigma(E)$.

Corollary 2 If $\mathrm{Orth}(E)$ is order dense in $\mathrm{Orth}^\infty(E)$, then $\mathrm{Orth}^\infty(\mathrm{Orth}(E)) \leq \mathrm{Orth}^\infty(E)$ and $\mathrm{Orth}^\sigma(\mathrm{Orth}(E)) \leq \mathrm{Orth}^\sigma(E)$.

Corollary 3 If E is uniformly complete and F any Riesz subspace, then every $T \in \mathrm{Orth}^{\infty}(F)$ has an "extension" $\tilde{T} \in \mathrm{Orth}(E)$.

P. VAN ELDIK:

Non-order Bounded Linear Operators in Riesz Spaces

We investigate the behaviour of different classes of linear operators $T: E \longrightarrow F$ from E into F (E and F Riesz spaces) which have the property that they are order bounded when considered as mapping into F^u, the universal completion of F. This abstract formulation is adequate to generalize some classical and recent results to the setting where no measure space is involved.

We report on two classes of operators in this general setting: Carleman operators and (extended) abstract kernel operators. This includes characterizations and important properties for these operators which have been obtained recently by J.J.Grobler and myself.

A. J. ELLIS:

Intersections of Ideals in Simplex Spaces

Let K be a compact convex set and let $A(K)$ denote the Banach space of all affine continuous real-valued functions on K with the supremum norm. A subset E of ∂K (the set of extreme points of K) is facially closed if $E = \partial F$ for some closed split face F of K. It is known that K is a simplex (equivalently $A(K)^*$ is a Riesz space) if and only if $f^{-1}(0) \cap \partial K$ is facially closed for every $f \in A(K)^+$.

K has the property that $f^{-1}(0) \cap \partial K$ is facially closed for every $f \in A(K)$ if and only if (P): the intersection of any family of near-lattice ideals (i.e. annihilators in $A(K)$ of closed split faces of K) is a near-lattice ideal.

Some years ago Gleit showed that (P) is equivalent to $A(K)$ being a Riesz space. Using some methods of Uttersrud and Rao we give extensions to and a simple proof of Gleit's result.

H. FISCHER:

Verallgemeinerung des Oettli/Prager - Satzes auf Riesz - Räume

Der klassische Oettli/Prager - Satz gibt Auskunft darüber, ob eine vorgegebene Näherungslösung für ein lineares Gleichungssystem $Ax = b$ aufgefaßt werden kann als exakte Lösung eines leicht geänderten Problems $\hat{A}x = \hat{b}$. Dieser Satz wird verallgemeinert auf lineare Operatoren A in Riesz - Räumen. Zum Beweis wird ein Banach - Hahn - Fortsetzungssatz für lineare Abbildungen verwendet.

G. GREINER:

Asymptotics in Linear Transport Theory

The central object in linear transport theory is the linear Boltzmann operator $B = -\sum_{i=1}^{3} v_i \frac{\partial}{\partial x_i} - M_\sigma + K_\chi$ (M_σ is a multiplication operator and K_χ is a special integral operator). B is the generator of a strongly continuous semigroup $\{S(t)\}$, which describes the time evolution of the particle density. One of the main problems is the study of the asymptotic behavior of $\{S(t)\}$ for large times t.

In the talk I consider two important cases, the so-called "reactor problem" and the "multiple scattering problem". Using abstract methods and general results from the theory of positive operators, we can show that every operator $S(t)$ is quasi-compact. This enables us to give a complete description of the qualitative behavior of $\{S(t)\}$ for large times.

J. J. GROBLER:

Fredholm Theory for Operators With a Trace

Let $(\mathcal{A}, \underline{A})$ be a quasi-normed operator ideal on the class of Banach spaces and let τ be a continuous trace on $(\mathcal{A}, \underline{A})$. Let T be an operator such that $T^p \in \mathcal{A}$ for some $p \in \mathbb{N}$. If $R(\lambda, T)$ denotes the resolvent of T for $\lambda \neq 0$ not an eigenvalue of T, let $R_p(\lambda, T) := \lambda^{2-p} T^p R(\lambda, T)$. The function $\delta(z) := \exp\left[\int_{\gamma(z)} \tau [R_p(\lambda, T)]\, d\mu\right]$, $\mu = \lambda^{-1}$ and $\gamma(z)$ a rectificable curve from 0 to z, is then a Fredholm divisor of T. We apply this to kernel operators which are completely of finite double norm on a Banach function space.

W. HACKENBROCH:

Some Order Theoretical Aspects of Disintegration

In two different situations it is shown how an abstract integration procedure leads to disintegration:

1. From Hahn-Jordan decomposition to disintegration:

Let $(P_t)_{t\in\mathbb{R}}$ be an increasing, right-continuous family in some Boolean σ-algebra $\mathcal{B}$ and let $\pi : (S,\Sigma) \longrightarrow \mathcal{B}$ denote the Loomis-Sikorski homomorphism. Then there is a function $f : S \longrightarrow \mathbb{R}$ with $P_t = \pi(\{f \leq t\})$, $\forall\ t\in\mathbb{R}$, and f is (essentially) given as $f = \int t\,\mu(dt)$ for some vector measure μ on $\mathbb{R}$ with values in the vector lattice of Σ-measurable functions on S.

This f gives immediately the desired disintegration in the Radon-Nikodym or the spectral theorem context (for Hilbert space-, Freudenthal-, Alfsen-Shultz-spectral theory).

2. A "non-commutative" Strassen theorem:

Let $(E,\leq)$ be a monotone σ-complete ordered vector space with weak order unit u ($\rightsquigarrow \mathcal{P}(E)$ Boolean σ-algebra of split projections); let (S,Σ) be a measurable space and $\pi : \Sigma \longrightarrow \mathcal{P}(E)$ a σ-homomorphism ($\rightsquigarrow$ E_+-valued vector measure $\pi(.)u$) ; let $\mu \in E_+^*$ be σ-order continuous ($\rightsquigarrow$ $\mathbb{R}_+$- valued measure $\sigma := \mu\circ\pi(.)u$).

Define: $x \leq y\ \mu$-a.e. iff $\mu(\pi(A)x) \leq \mu(\pi(A)y)$, $\forall\ A\in\Sigma$.

Let $p : F \longrightarrow E$ (F any $\mathbb{R}$- vector space) be sublinear mod "$\leq \mu$-a.e.", let $\phi \in F^*$: $\phi \leq \mu\circ p$.

Then ϕ can be disintegrated below, i.e. $\phi = \mu\circ\varphi$ for some linear $\varphi : F \longrightarrow E$ with $\varphi \leq p$ mod "$\leq \mu$-a.e." pointwise.

D. R. HART:

The Spectrum of Lattice Homomorphisms

We give a new constructive proof of a theorem of Lotz, which states that the approximate point spectrum of a lattice homomorphism is cyclic. We discuss problems involving the spectrum of aperiodic lattice homomorphisms and also extensions of the Schaefer - Wolff - Arendt theorem.

C. B. HUIJSMANS:

Unital Embeddings of f-Algebras

Let A be an Archimedean semiprime f-algebra. It is well-known that in this case A can be embedded as a Riesz subspace and as a Ring ideal in its unital Archimedean f-algebra Orth(A) of all orthomorphisms on A. In general, however, A is not embedded as an order ideal. In this connection the following theorem can be proved:

Theorem 1 Let A be a relatively uniformly complete Archimedean semiprime f-algebra. Then the following conditions are equivalent:

(i) A is (embedded as) an order ideal in Orth(A).

(ii) A has the M.D. property (i.e., if $0 \leq u \leq vw$, $0 \leq v \in A$, $0 \leq w \in A$ then $u = pq$ for some $0 \leq p \leq v$ and $0 \leq q \leq w$).

(iii) A has property (*) (i.e., if $0 \leq u \leq v^2$, $0 \leq v \in A$ then $u = wv$ for some $0 \leq w \in A$).

Each of the three conditions holds in a relatively uniformly complete Archimedean unital f-algebra. This theorem is heavily based on:

Theorem 2 Let A be a relatively uniformly complete Archimedean semiprime f-algebra. Then $\sqrt{uv}$ exists in A for all $0 \leq u,v \in A$.

The latter theorem generalizes a result of and Johnson, stating that $\sqrt{u}$ exists for all positive u of a relatively uniformly complete Archimedean unital f-algebra.

H. KÖNIG:

Daniell-Stone Integration and Abstract Hardy Algebra Theory

Let E be a vector space of real-valued functions on a set X, not supposed to be a lattice under the pointwise operations, and $I : E \longrightarrow \mathbb{R}$ a positive linear functional. LEINERT [Arch.Math. 38 (1982), 258 - 265] extended the Daniell-Stone integral extension procedure to this situation. The present author [Math.Ann. 258 (1982), 447 - 458] uses a fortified continuity condition on I and applies a different construction of $L^1(I|E) \supset E$ and $\hat{I} : L^1(I|E) \longrightarrow \mathbb{R}$. Our main result is the fact, familiar in the

case of a vector lattice E, that the functions in $L^1(I|E)$ can be represented in terms of limits of isotonic sequences of functions in E^+.
The new set-up and the main result had been inspired by a typical example arising in the abstract Hardy algebra theory in the sense of BARBEY - KÖNIG [LNM Vol.593]. Here $L^1(I|E)$ becomes the space of conjugable functions, to be defined in an appropriate sense.

H. E. LACEY:

On the Boundedness of the Hilbert Transform

Given a Banach space X, $L_p(\mathbb{R},X)$ is the space of all strongly measurable functions $f: \mathbb{R} \longrightarrow X$ such that $\|f\|_p := \left(\int \|f(r)\|^p dr\right)^{1/p} < \infty$. The Hilbert Transform is defined, when it exists, as the singular integral $(Hf)(t) = \lim_{\delta \downarrow 0} \left[\int_{-\infty}^{t-\delta} \frac{f(r)}{t-r}\, dr + \int_{t+\delta}^{\infty} \frac{f(r)}{t-r}\, dr \right]$.
It is now known that the existence and boundedness of H is characterized by the Burkholder condition: There is a biconvex function $\varphi : X \times X \longrightarrow \mathbb{R}$ such that (i): $\varphi(0,0) > 0$ and (ii): if $\|x\| \leq 1 \leq \|y\|$, then $\varphi(x,y) \leq \|x+y\|$.
We show that if H is well-defined as an operator, then X is superreflexive, and H is bounded.

H. P. LOTZ:

Quasi-kompakte positive Operatoren auf $C(X)$

Ein beschränkter linearer Operator auf einem Banachraum E heißt quasi-kompakt, falls es eine natürliche Zahl m und einen kompakten Operator K gibt, mit $\|T^m - K\| < 1$. Ein Banachraum E heißt Grothendieckraum, falls jede $\sigma(E',E)$-konvergente Folge in E' auch für die Topologie $\sigma(E',E'')$ konvergiert.
Theorem Es sei T ein positiver linearer Operator auf $C(X)$, X kompakt und $C(X)$ ein Grothendieckraum.
Ist der Spektralradius $r(T) \leq 1$, $\sup_{\lambda>1} (\lambda-1)\|R(\lambda,T)\| < \infty$ und der Fixraum $\{\mu \in M(X) : T'\mu = \mu\}$ von T' norm-separabel, dann ist T quasi-kompakt und $\left(\frac{1}{n}\sum_{i=0}^{n} T^i\right)$ konvergiert gleichmäßig gegen einen Operator endlichen Ranges.

Korollar Es sei T ein positiver linearer Operator auf $C(X)$. Ist $r(T) \leq 1$, $\sup_{\lambda > 1} (\lambda - 1)\|R(\lambda,T)\| < \infty$ und der Fixraum von T''' norm-separabel, so ist T quasi-kompakt und $(\frac{1}{n}\sum_{i=0}^{n} T^i)$ konvergiert gleichmäßig gegen einen Operator endlichen Ranges.

W. A. J. LUXEMBURG:

Disjointness Preserving Operators

We shall again return to the question when band preserving operators are order bounded. In addition, some more representation of Orth(L) will be discussed. In particular, it will be shown that if L is the Banach lattice of the harmonic functions in the unit circle $D = \{z : |z| < 1\}$, then Orth(L) is Riesz isomorphic and isometric to $C^{**}\{z : |z| = 1\}$.

M. MEYER:

Extended Orthomorphisms and Completions of Riesz Spaces

Extended orthomorphisms on an Archimedean Riesz space E are band preserving order bounded linear operators defined on an order dense ideal of E. The space $Orth^{\infty}(E)$ of all extended orthomorphisms on E is a laterally complete f-algebra. Constructing some embeddings from E into $Orth^{\infty}(E)$, it is possible to compare $Orth^{\infty}(E)$ to the lateral and universal completions of E and to compare the space $Orth^{\alpha}(E)$ of all T in $Orth^{\infty}(E)$ with super order dense domain and countably generated range to the lateral and universal σ-completions of E.

R. NAGEL:

On the Asymptotic Behavior of Positive Semigroups

We show what order structure and positivity can do for stability theory. In particular we investigate strongly continuous, irreducible semigroups $\{T(t)\}_{t \geq 0}$ of bi-Markov operators on $L^1(X,\mu)$ and ask what conditions on the spectrum of the generator A (e.g.: (a) 0 is isolated in $P\sigma(A) \cap i\mathbb{R}$; (b) 0 is isolated in $\sigma(A) \cap i\mathbb{R}$; (c) 0 is a pole of the resolvent) imply the existence of a partially periodic, positive semigroup $\{S(t)\}_{t \geq 0}$ such that $\lim_{t\to\infty}(T(t) - S(t)) = 0$ for one of the standard operator topologies (e.g.: (A) weak operator topologiy; (B) strong operator topology; (C) operator norm).

B. DE PAGTER:

Components of Positive Operators

Let L and M be Dedekind complete Riesz spaces and denote by $\mathcal{L}_b(L,M)$ the Riesz space of all order bounded linear operators from L into M. For any $0 \leq T \in \mathcal{L}_b(L,M)$ the Boolean algebra of components of T is denoted by $\mathcal{B}_T$. Components of the form $\bigvee_{i=1}^{n} Q_i T P_i$ (P_i and Q_i order projections in L and M resp.) will be called simple components. The collection of all simple components of T will be denoted by $\mathcal{A}_T$, which is a subalgebra of $\mathcal{B}_T$.

We introduce some notation: Let $\mathcal{B}$ be a Boolean algebra and X a sublattice of $\mathcal{B}$. Put $X^{\uparrow} := \{b \in \mathcal{B} : \exists x_\alpha \in X \text{ s.t. } x_\alpha \uparrow b\}$ and similary define $X^{\downarrow}$. Furthermore $X^{\uparrow\omega} := \{b \in \mathcal{B} : \exists x_n \in X \text{ s.t. } x_n \uparrow b\}$. and in like manner define $X^{\downarrow\omega}$. the main result is as follows:

Theorem Let L and M be Dedekind complete Riesz spaces with $^{\perp}(M_n^{\sim}) = \{0\}$. For any $0 \leq T \in \mathcal{L}_b(L,M)$ we have $\mathcal{B}_T = \mathcal{A}_T^{\uparrow\omega\downarrow\uparrow}$, and if T is in addition order continuous, then $\mathcal{B}_T = \mathcal{A}_T^{\uparrow\omega\downarrow\omega}$.

We mention some applications of this theorem. Let L and M be Banach lattices. For any $0 \leq T \in \mathcal{L}_b(L,M)$ let $(T)^{-}$ be the closure in the r-norm of the set of all operators of the form $\sum R_i T S_i$ (where $S_i \in \mathcal{L}_b(L)$ and $R_i \in \mathcal{L}_b(M)$, $i = 1, 2, .., n$).

Theorem Let L and M be Dedekind complete Banach lattices with $^{\perp}(M_n^{*}) = \{0\}$. If $0 \leq T \in \mathcal{L}_b(L,M)$ with order continuous norm, then it follows from $0 \leq S \leq T$ in $\mathcal{L}_b(L,M)$ that $S \in (T)^{-}$.

Corollary Let L and M be Dedekind complete Banach lattices with L^* and M having order continuous norms. Suppose $0 \leq S \leq T$ in $\mathcal{L}_b(L,M)$.

(i) If T is compact, then $S \in (T)^{-}$, in particular S is compact (a result due to P. G. Dodds and D. H. Fremlin).

(ii) If T is Dunford - Pettis, then $S \in (T)^{-}$, in particular, S is likewise Dunford - Pettis.

E. SCHEFFOLD:

Über Faltungsoperatoren und Multiplikatoren

Der Vortrag befaßt sich mit der Darstellung von Markoff-Operatoren. Das zentrale Ergebnis lautet:

Theorem Es sei S eine kompakte abelsche Halbgruppe mit der Eigenschaft, daß die stetigen Semicharaktere die Punkte trennen. Ferner existiere in S ein Punkt s_o mit $\chi(s_o) \neq 0$ für alle stetigen Semicharaktere χ. Es sei T ein Markoff-Operator auf $C(S)$ mit der Eigenschaft, daß jeder Semicharakter eine Eigenfunktion von T ist. Dann gibt es auf der Translationshülle S^* von S ein Wahrscheinlichkeitsmaß μ, so daß T wie folgt als Faltungsoperator dargestellt werden kann:

$(Tf)(s) = \int_{S^*} f(st)\, d\mu(t)$ für alle $f \in C(S)$ und $s \in S$.

Als Anwendung wird gezeigt, daß gewisse Multiplikatoren "lokal" als Faltungsoperatoren betrachtet werden können.

A. R. SCHEP:

Factorization of Positive Multilinear Operators

Nikišin's and Maurey's factorization theorems for positive linear operators are extended to positive multilinear operators. The main result can be summarized as follows:

Theorem If $B : L_{p_1} \times \ldots \times L_{p_n} \longrightarrow L_q$ $(q \geq 0)$ is a positive n-linear operator and $r \geq 1$ is such that $r^{-1} = \sum p_k^{-1}$ and $r \geq q$, then there exists $\rho \in L_s$ with $\rho > 0$ a.e., where $s^{-1} = q^{-1} - r^{-1}$, such that $\frac{1}{\rho} B(L_{p_1} \times \ldots \times L_{p_n}) \subseteq L_r$.

The proof of this theorem involves the positive projective tensor product of Banach lattices.

K. D. SCHMIDT:

The Jordan Decomposition for Vector Measures

Two methods are presented which yield Jordan decomposition theorems for vector measures with values in a Banach lattice. The first method is based on a common approach to vector measures and linear operators whereas the second one relies on factorization theorems which reduce the decomposition of vector measures to that of linear operators.

S. SIMONS:

Variational Inequalities and Directional Derivatives

A proof was given of a variational inequality that depends only on the Hahn - Banach theorem in finite dimensional space. A new definition of directional derivative was discussed and a characterization was given of the directional derivatives of convex hemicontinuous functions. A generalization of Fan's minimax inequality was given which can be used to improve the result given in a previous talk by Aliprantis on the existence of free disposal equilibrium prices in the model of a Walrasian economy based on a Riesz space of commodities.

J. TRIAS PAIRÓ:

Lattice - Isometries in Riesz Spaces

It is the purpose of this contribution to develop the theory of lattice - isometries, that is, modulus preserving mappings, in the frame of Riesz space theory, partially focussing it into those aspects concerned with operator theory. We first relate the lattice-theoretic properties of a Riesz space E with the Boolean properties of H(E), the Boolean algebra of homogeneous 1-isometries. We proceed then to relate H(E) with the space of order-bounded operators and orthomorphisms of E. In a second part we study 1-isometries in Dedekind complete and σ-Dedekind complete Riesz spaces and recover the multiplicative pointwise expressions of 1-isometries that are valid for f-rings with unity (in terms of idempotents). Lattice - isometries in Riesz spaces with weak units are also dealt with.

C. T. TUCKER:

Mappings on Certain Specialized Riesz Spaces

If L is a Riesz space then the property that if each of f and g belong to L then there is a finite disjoint subset A of L such that each of f and g is a linear combination of the points of A is equivalent to the property that if p is a positive linear functional defined on a subspace of L, then p can be extended to L.

Suppose X is a topological space which is a perfectly normal

Baire space or ccc compact or a P-space then bounded pointwise convergence of sequences implies order convergence.

L. WEIS:

On the Representation of Order Continuous Operators by Random measures

Using the representation $Tf(y) = \int f \, d\nu_y$, where (ν_y) is a random measure, we characterize some interesting bands in the lattice of all order-continuous operators on a space of measurable functions. For example, an operator T is (lattice-) orthogonal to all integral operators (i.e. all ν_y are singular) or belongs to the band generated by all Riesz homomorphisms (i.e. all ν_y are atomic) if and only if T satisfies certain properties which are modeled after the Riesz homomorphism property, the Maharam property and continuity with respect to convergence in measure. On the other hand, for operators orthogonal to all Riesz homomorphisms (i.e. all ν_y are diffuse) we give characterizations analogous to the characterization of Dunford-Pettis and Bukvalov for integral operators. The latter results are related to Enflo-operators and to a result of J. Bourgain on Dunford-Pettis operators.

A. W. WICKSTEAD:

Non Order Bounded Disjointness Preserving Operators on Uniformly Complete Riesz Spaces

Partial answers are given to the question 'which uniformly complete Riesz spaces have a non order bounded disjointness preserving operator defined on them?' Such a space must contain a non-trivial non-atomic principal projection band which is σ-inextensible. In particular it cannot support a locally convex locally solid Hausdorff topology. We also give a description of the inextensible Riesz spaces in this class and note that it does not include all the non-atomic ones.

M. WOLFF:

On the Spectrum of Representations of Groups by Positive Operators

We give a review of results concerning the spectrum of a bounded representation of the group G as a group of lattice isomorphisms on a Banach lattice. These results may be found in the paper: "Group actions on Banach lattices and applications to dynamical systems" in I. Gohberg(ed.): Toeplitz Centennial, 501 - 524 , Birkhäuser 1982 . We add some results concerning the Fredholm part of the spectrum. In particular we prove :

Theorem Let U be a bounded strongly continuous representation of G on the Banach lattice E such that all U_t are positive. The following assertions are equivalent provided that U is non-degenerate:

(a) There exists a bounded positive measure μ such that the spectral radius $r(U_\mu^1)$ is an isolated Fredholm point in the spectrum of U_μ^1 ($U_\mu^1 x = \int U_t x \, d\mu(t)$) .

(b) The operators U_f^1 are compact for all $f \in L^1(G)$.

(c) E splits into the direct sum of finitely many mutually disjoint bands E_j such that each E_j is U-invariant, $U_j : t \longmapsto U_{t|E_j}$ is irreducible and $U_j(G)$ is compact.

J. D. M. WRIGHT:

Outer Automorphisms of Dedekind Completions of Operator Algebras

Dedekind completions of vector lattices have an analogue in C^*-algebras, the regular completion. Each automorphism α of a C^*-algebra A has a unique extension to an automorphism $\hat{\alpha}$ of its regular completion $\hat{A}$. When A is simple and $\hat{\alpha}$ is inner in $\hat{A}$ then K. Saitô and I can show that α is inner. More precisely, when A is unital α is implemented by a unitary in A , when A is not unital α is implemented by a unitary in the multiplier algebra of A .

A. C. ZAANEN:

A Problem About Irreducible Operators

Let L be a Dedekind complete Riesz space, $L^{\sim}$ its order dual and $L_n^{\sim}$ the band of all order continuous members of $L^{\sim}$. Furthermore, let $\mathcal{L}_r(L)$ be the Dedekind complete space of all regular linear operators in L and $(L_n^{\sim} \otimes L)^{dd}$ the band in $\mathcal{L}_r(L)$ of all "abstract" kernel operators in L. The positive operator T in L is called irreducible (band-irreducible) if T leaves no band invariant except $\{0\}$ and L and T is called strongly irreducible if, for every $u > 0$ in L, the image Tu is a weak unit in L. Furthermore, for $0 \leq T \in (L_n^{\sim} \otimes L)^{dd}$ we shall call T superirreducible if T is a weak unit in $(L_n^{\sim} \otimes L)^{dd}$. For non-abstract kernel operators we have: superirreducible $\Longrightarrow$ strongly irreducible $\Longrightarrow$ irreducible, and none of the conclusions in the converse direction holds. Furthermore, if the positive kernel operator T is strongly irreducible, then T^2 is superirreducible. The proof of the same for"abstract" kernel operators requires heavy machinery (L is represented as a space of measurable functions). The problem is to find a direct proof for this result.

Probleme

C. D. ALIPRANTIS and O. BURKINSHAW:

1. Let $S, T: E \longrightarrow E$ be two positive operators on a Banach lattice satisfying $0 \leq S \leq T$.

a) How the various "pieces" of the spectrum of S are related to those of T?

b) If T has a measure of non-compactness (see [7]), then what measures of non-compactness does S have?

References: [2], [3], [4], [9] and [12].

2. It is known [8] that every weakly compact operator between two Banach spaces factors through a reflexive Banach space.

Can a weakly compact positive operator between two Banach lattices be factored (with positive factors) through a reflexive Banach lattice ?

Note: If the range has order continuous norm, then the answer is affirmative. (The method in [8] works in this case.)

3. It is known [10] , [11] that every compact operator between Banach spaces factors through a reflexive Banach space with compact factors.

Can a positive compact operator be factored (with positive factors) through a reflexive Banach lattice ?

4. Give an example of two positive compact operators S and T between Banach lattices such that $S \vee T$ is not compact.

Note: Since $0 \leq S \vee T \leq S + T$ must hold, it follows that in this case $(S \vee T)^3$ is compact, $(S \vee T)^2$ is Dunford-Pettis and weakly compact and $S \vee T$ is weak Dunford-Pettis; see [2],[3],[4] .

5. Let $R : E \longrightarrow E$ be a positive operator on a Dedekind complete Riesz space. Consider the positive operator $T \longmapsto RT$ from $\mathcal{L}_r(E)$ into $\mathcal{L}_r(E)$.

a) If $T \longmapsto RT$ is a Riesz homomorphism, is then R a Riesz homomorphism ?

b) If R is a Riesz homomorphism, is then $T \longmapsto RT$ a Riesz homomorphism ?

c) If $T \longmapsto RT$ is a Riesz homomorphism, is then R order continuous ?

Note: In [5] (a) was shown to be true when $E^{\sim} \neq \{0\}$, and (b) whenever R is an order continuous Riesz homomorphism.

References: [5] and [6] .

6. Let L be an AM-space with unit e , and let L' be an ideal of $L^{\sim}$ (the order dual) separating the points of L . Put $\Delta = \{p \in L^{\sim}_{+} : \|p\| = p(e) = 1\}$ and $S = \{p \in L' \cap \Delta : p \gg 0\}$. Assume that the convex set S is w^*-dense in the convex w^*-compact set Δ .

A function $E : D \longrightarrow L$ is said to be an excess demand function, whenever its domain D is a convex subset of Δ satisfying the following properties:

a) D is w^*-dense in Δ;

b) there exists a locally convex topology t on L that makes the members of D t-continuous and such that $E:(D,w^*) \longrightarrow (L,t)$ is continuous;

c) if a net $\{p_\alpha\} \subseteq D$ satisfies $p_\alpha \xrightarrow{w^*} q \in \Delta \sim D$, then $\overline{\lim}\, p(E_{p_\alpha}) > 0$ holds for some $p \in D$;

d) $p(E_p) = 0$ holds for all $p \in D$.

Give examples of excess demand functions whose domain is S. (Especially for the Riesz dual systems $\langle \ell_\infty, \ell_1 \rangle$, and $\langle L_\infty[0,1], L_1[0,1] \rangle$.)

Reference: [1].

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W. ARENDT:

1. Let T be a bounded operator on a Banach lattice. The following assertions are equivalent:

(i) $x \in E_+, x' \in E'_+$, $\langle x, x' \rangle = 0$ implies $\langle Tx, x' \rangle \geq 0$.

(ii) $T + \lambda I \geq 0$ for some $\lambda \in \mathbb{R}$.

(iii) $e^{tT} \geq 0$ for all $t \geq 0$.

(see D.E.Evans, H.Hancke-Olsen: J.Funct.Anal. 32(1979),207-212)

It is well known that a positive linear mapping on a Banach lattice is automatically bounded. Is the same true for linear mappings satisfying the minimum principle (i) ?

Note: It is true if $E = C(X)$, X compact.(See: W.Arendt, P.Chernoff, T.Kato: J. Operator Theory 8 (1982), 167 - 180 .)

2. It is well known that the space of all compact operators on a separable Hilbert space H is the only non-trivial closed ideal in $\mathcal{L}(H)$. An order theoretical analogue is the following problem:

Let $E = L^2(X, \Sigma, \mu)$. The space $(E \otimes E)^{\perp\perp}$ of all kernel operators in $\mathcal{L}^r(E)$ is a Banach lattice algebra. The closure $E \tilde{\otimes}_e E$ of $E \otimes E$ in $\mathcal{L}^r(E)$ is a closed lattice and algebra ideal in $(E \otimes E)^{\perp\perp}$. Is that the only non-trivial one ?

Note: This is true, if $E = \ell^2$. (W.Arendt, A.Sourour: Ideals of regular operators on ℓ^2. Preprint 1981).

H. P. LOTZ:

Gibt es Banachverbände die Grothendieckräume sind und die Dunford-Pettis Eigenschaft besitzen, die aber nicht verbands-isomorph zu AM-Räumen sind ?

R. NAGEL:

Kishimoto - Robinson (J. Austral. Math. Soc.(Ser. A) 31 (1981), 59 - 76) proved that every strongly continuous semigroup of positive operators on $L^\infty(X, \mu)$ is uniformly continuous. Does the same hold without the positivity assumption ?

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