MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 31/82

Cohomologie der Gruppen

25.7. bis 31.7.1982

Die Tagung stand unter der Leitung von R.Bieri (Frankfurt a.M.) und K.W.Gruenberg (London) - wobei Karl Gruenberg krankheitshalber an der Tagung selber nicht dabeisein konnte.

Der überwiegende Teil der Vorträge (inkl. je ein Übersichtsvortrag) läßt sich einem der folgenden vier Themenkreise zuordnen:

1. Cohomologie mit Gruppenringkoeffizienten: Verfeinerungen und Verallgemeinerungen der Struktursätze von Hopf und Stallings über Gruppen G mit $H^1(G;\mathbb{Z}G) \neq 0$. Anwendungen auf Gruppen mit Poincaré-Dualität, insbesondere für die Dimension 2.
2. Relationenmoduln: Fragen betreffend Isomorphie und Zerlegbarkeit von minimalen Relationsmoduln.
3. Komplexität und Cohomologiering von Moduln: Weitreichende Verallgemeinerung der Sätze von Quillen über den Cohomologiering $H^*(G;K)$ einer endlichen Gruppe.
4. Endlichkeitsbedingungen: Virtuell endliche cohomologische Dimension sowie Existenz von endlichen Präsentierungen und endlich erzeugten projektiven Auflösungen.

Weitere Vorträge beschäftigten sich mit der Cohomologie von algebraischen und endlichen Gruppen, mit der Lyndon-Hochschild-Serre Spectralreihe und mit Fragestellungen aus der K-Theorie.

Die Gelegenheit zu intensivem Gedankenaustausch wurde eifrig benutzt - nur selten war einer der dafür vorgesehenen Räume nicht belegt. Und schon während der Tagung selbst manifestierte sich der erste sichtbare Erfolg! B.Eckmann und P.Linnell konnten zeigen, daß es keine perfekten 2-dimensionalen Poincaré-Dualität-Gruppen gibt. Damit ist jetzt die PD^2-Vermutung in voller Allgemeinheit bewiesen; d.h., es gilt

Satz. Jede 2-dimensionale Poincaré-Dualität-Gruppe ist isomorph zur Fundamentalgruppe einer geschlossenen Fläche.

Vortragsauszüge

H.ABELS: Finite presentability of generalized arithmetic groups

The problem which S-arithmetic groups are finitely presentable is solved for arbitrary algebraic groups G over a number field k by reducing it first to the question of compact presentability of G(K) for certain local fields K over k (Kneser, Crelle 1964), then passing to a maximal K-split solvable subgroup (Borel-Tits IHES 1965) and finally giving necessary and sufficient conditions for this case as follows.

Theorem. Let $G = T \ltimes U$ be a K-split solvable algebraic group, K a local field of characteristic 0. Then G(K) is compactly presentable iff the following two conditions hold

1) 0 is not a positive linear combination of two roots of T on u^{ab}
 (u = Lie algebra of U)

2) There is no K-split solvable algebraic group H such that

$$\overset{\circ}{N} \rightarrowtail H \twoheadrightarrow G$$

with $O \neq N$ central in H and $H/H'' \xrightarrow{\sim} G/G''$.

J.L.ALPERIN: Complexity of modules

The complexity of a module for a group algebra is a measure of the rate of growth of resolutions of the module. A survey is given of the theory that has developed around this definition and the applications of the theory. The dominant themes are calculation of this invariant in terms of elementary abelian subgroups and placing the theory in a geometrical context.

R.ALPERIN: Cohomological dimension of linear groups

Let A be a finitely generated ring of characteristic zero. Let O be the ring of integers in the field K which is the algebraic closure of Q in the fraction field F of A. There are discrete valuations $v_1,\dots,v_m$ of F with valuation rings O_{v_i} so that

$$A \cap \bigcap_{i=1}^{m} O_{v_i} \subset O.$$

For any group $\Gamma \subset GL_n(A)$ we obtain by the action of Γ on Tits' buildings $T(F^n, O_{v_i})$ a decomposition of Γ which enables one to determine the virtual cohomological dimension of Γ as follows

$vcd(\Gamma) \leq m(n-1) + c_A + N_\Gamma + [K;Q]n^4$,

where $N_\Gamma = \sup\{vdc(\Gamma_u) \mid \Gamma_u$ a unipotent subgroup$\}$ and c_A is constant depending only on A, and $[K;Q]n^4$ is a bound on the virtual cohomological dimension of subgroups of $GL_n(A)$ all of whose elements have characteristic polynomials with algebraic integer coefficients.

G.AVRUNIN: Quillen stratification for modules

Let G be a finite group and k a fixed algebraically closed field of characteristic $p > 0$. If p is odd, let H_G be the subring of $H^*(G,k)$ consisting of elements of even degree; take $H_G = H^*(G,k)$ if $p = 2$. H_G is a finitely generated commutative k-algebra, and we let V_G denote its affine variety Max H_G. If M is any finitely generated kG-module, the cohomology variety $V_G(M)$ of M may be defined as the largest support in V_G of $H^*(G,L \otimes M)$, where L is any kG-module. A module L with each irreducible kG-module as a direct summand will do.

D.Quillen proved a number of beautiful results relating V_G to the varieties V_E associated with the elementary abelian p-subgroups E of G, culminating in his stratification theorem. This theorem gives a piecewise description of V_G in terms of the subgroup E and their normalizers in G. In this paper, we prove a stratification theorem for the cohomology variety $V_G(M)$. A key step in the argument is the proof of a conjecture of J.Carlson regarding $V_G(M)$ for E elementary. We are also able to generalize several of Quillen's other results to the module case.

F.R.BEYL: The spectral sequence of a group extension

There are various spectral sequences describing the cohomology of an extension group G in terms of the cohomology of a normal subroup N and the corresponding factor group G/N. These spectral sequences were believed to be the "same", although some properties are immediate with on construction and are hard to prove with another. We give explicit isomorphisms between the Cartan-Leray(-Serre), the Hochschild-Serre, and the Grothendieck constructions. The first two spectral sequences are isomorphic even as spectral rings. (An isomorphism of the additive structures has already been established by L.Evens, we give an independent proof.) The corresponding result for homology also holds and the proof carries over. The details have appeared in:Bull.Sci.Math.Sér.(2) 1o5 (1981), 417-434. D.W.Barnes has further results in this direction.

R.BIERI: On the cokernel of res: $H^1(G;\mathbb{Z}G) \to G^1(S;\mathbb{Z}G)$

Let G be a finitely generated infinite group, $\underline{S} = \{S_1,\ldots,S_m\}$ a finite family of finitely generated infinite subgroups of G, and let C denote the cokernel of the restriction map res: $H^1(G;\mathbb{Z}G) \to H^1(\underline{S};\mathbb{Z}G)$.

Theorem. If G is accessible then C is free-Abelian of rank 0, 1 or ∞, except when G is infinite cyclic in which case rkC can attain any positive integer.

(The freeness of C is due to Heinz Müller). One can give a complete classification of the three cases rkC = 0, 1, ∞ in terms of the structure of $(G;\underline{S})$.

K.S. BROWN: Finiteness conditions on groups

A group G is said to be of type FP_n if the G-module $\mathbb{Z}$ admits a projective resolution $(P_i)_{i\geq 0}$ with P_i finitely generated for $i \leq n$. Suppose X is a contractible G-CW-complex such that the isotropy group G_σ is of type $FP_{n-\dim\sigma}$ for each cell σ of X. If X/G has finite n-skeleton, it is well-known that G is of type FP_n. This is generalized as follows: If X is a directed union of G-invariant subcomplexes X_α ($\alpha \in D$) such that X /G has finite n-skeleton, then G is of type FP_n if for all $i < n$ the direct system $\{\tilde{H}_i(X_\alpha)\}_{\alpha\in D}$ of reduced homology groups is essentially zero. Examples and applications involving matrix groups are obtained via the theory of buildings.

J.F. CARLSON: The cohomology ring of a module

Let G be a finite group and let K be an algebraically closed field of characteristic $p > 0$. Let M be a finitely generated KG-module. It is known that a homogeneous element in the cohomology ring $\mathrm{Ext}^*_{KG}(M,M)$, is nilpotent if and only if its restriction to every elementary abelian p-subgroup of G is nilpotent. We present an analog to this theorem for elementary abelian p-groups.

Suppose that $E = \langle x_1,\ldots,x_n\rangle$ is elementary abelian of order p^n. For any $\alpha = (\alpha_1,\ldots,\alpha_n) \in K^n$, let $u_\alpha = 1 + \Sigma\alpha_i(x_i-1)$. Then u_α is a unit of order p in KG. The variety of M, $V_E(M) \subseteq K^n$ is the set consisting of 0 and of all α such that M is not free as a $K\langle u_\alpha\rangle$-module. An element $u \in \mathrm{Ext}^t_{KE}(M,M)$ is nilpotent if and only if its restriction to $\mathrm{Ext}^t_{K\langle u_\alpha\rangle}(M,M)$ is nilpotent for all $\alpha \in V(M)$.

The theorem can be used to characterize the radical of $\mathrm{Ext}^*_{KG}(M,M)$ in terms of the restriction homomorphisms. The radical is proved to be a nilpotent ideal.

T.DIETHELM: Über die Cohomologie von Gruppen mit p-Länge 1

Sei $k = \mathbb{F}_p$, G eine endliche Gruppe mit $p\,|\,|G|$, und sei A ein einfacher kG-Modul.

Frage: Wann ist $H^n(G,A)$ nicht trivial?

Satz: Sei $G = O_{p'p\,p'p}\,G$, und sei A ein einfacher kG-Modul, dann existieren unendlich viele $n \in \mathbb{N}$ mit $H^n(G,A) \neq 0$.

Der Beweis beginnt mit dem Fall $G = O_{p',p,p'}G$ (p-Länge1).Dabei wird die Operation einer p'-Gruppe Q auf dem Cohomologiering einer p-Gruppe P untersucht. Das geschieht mit Hilfe der Lyndon-Hochschild-Serre Spektralreihe. Mit einer langen exakten Sequenz wird der Fall $G = O_{p',p\,p',p}G$ auf den Fall $G = O_{p',p\,p'}G$ zurückgeführt.

M.T. DUNWOODY: Ends of groups and graphs

Stallings proved that if G is a finitely generated group and $H^1(G;ZG) \neq 0$, then G splits over a finite subgroup.

Two generalizations of this theorem were presented. The first dealt with a group G acting on a graph X. As a consequence it can be shown that if $G\backslash X$ is finite and X has more than one end, then G splits over a subgroup which contains an edge stabilizer as a subgroup of finite index.

The second generalization dealt with infinitely generated groups. As a consequence of this generalization one can prove the following theorem.

THEOREM: For an arbitrary G, $H^1(G,ZG) \neq 0$ if and only if either G is countably infinite and locally finite or G splits over a finite subgroup.

B.ECKMANN: Groups and Poincaré Duality

Survey talk on the concept of Poincaré Duality for groups, patterned after Poincaré Duality for compact manifolds, and its generalization (dualizing module C instead of $\mathbb{Z}$).

Examples: Free proper actions of a group G on $\mathbb{R}^n$ with compact fundamental domain or such that $\mathbb{R}^n/G$ is the interior of a compact ∂-manifold.

Criteria for Poincaré Duality and general Duality, Endpointgroups $H^i(G;\mathbb{Z}G)$, extension theorems.

Problem: Is a Poincaré Duality group of dim n(PD^n) the fundamental group of an spherical manifold? Case n = 2 (Eckmann-Müller 1981): Yes, if either $\beta_1 > 0$ or Type FF is assumed. Application: Nielsen realization problem in a special case. - For the "virtual" analogue one has (Eckmann-Müller 1981): A virtual PD^2-gp with $\beta_1 > 0$ is a motion group of the Euclidean or Hyperbolic plane with compact fundamental domain (modulo a finite normal subgroup).

E.M. FRIEDLANDER: Descrete cohomology of topological groups

We describe conjectures predicting the cohomology of various groups with finite, constant coefficients. The simplest class of groups which we consider are real Lie groups: what is the relationship between the cohomology of such a group viewed first as a topological group and then as a discrete group. We next consider the rational points of a reductive algebraic group over an algebraically closed field. Further, much more general groups are also considered.

D.GILDENHUYS: The cohomology of solvable groups

My talk will be a report on my joint work with Ralph Strebel on the cohomological dimension $cd_R G$ of a solvable group G over a commutative ring R with $0 \neq 1$. If G is countable and its Hirsch number hG equals $cd_Q G$, then $h\bar{G} = cd_Q \bar{G}$ for every homomorphic image $\bar{G}$ of G. If $hG = cd_Q G < \infty$ and G is nilpotent-by-abelian, then G is finitely generated. We conjecture that if G is torsion-free and $hG = cd_Q < \infty$, then G must be constructible. We have succeeded in reducing this conjecture to the case where G is a semi-direct product of a torsion-free abelian group A of finite rank d-1 and a free abelian group $\langle t_1,\dots,t_n\rangle$ on finitely many generators $t_1,\dots,t_n$. We have derived, for this case, an explicit formula for $H^{n+d}(G,X)$, where $X = Q(G/\langle t_1,\dots,t_n\rangle)$ and $R = Q$, by means of which we prove that $cd_Q G > hG$ if G is Abels' finitely presented group, or if G is nilpotent-by-infinite cyclic and not constructible.

J.R.J.GROVES: Metabelian groups with finitely generated integral homology

It has become clear in recent years that the relationship between the structure of a group and the structure of its homology is, in the more general cases tenuous. For sufficiently restricted classes of groups, however, there seems to be more hope for relationships between these structures. In this paper the structure of finitely generated metabelian groups for which the integral homology groups of every dimension are finitely generated, is investigated. Under the condition that the group is a split extension of one abelian group by another, it is shown that the group has finite abelian-section rank. It is unknown whether this condition of being a split extension is necessary.

K.J.HORADAM: The cup coproduct and cup product of a combinatorially aspherical group

It is shown that there exists a cup coproduct which gives the homology modules of certain groups with trivial coefficients in suitable cyclic groups the structure of a commutative graded co-ring. For combinatorially aspherical presentations, this cup coproduct is calculated from a diagonal approximation on the Lyndon resolution. It is computable in terms of first and second order Fox derivatives alone. As a corollary the cup product for aspherical groups with these coefficients is obtained.

H.SCHNEEBELI : 1.Teil
J. HOWIE: 2.Teil
: Homological properties of locally indicable groups

The class of (locally) indicable groups was introduced by Higman in his study of units and zero divisors in group rings. The class of conservative groups was defined by Adams in his work on Whitehead's asphericity problem. Finally the class of D-groups was introduced by Strebel to study derived series by homological means.

We show that these three classes all coincide. We also prove a mod p version, and for any abelian group A we classify groups conservative over A in terms of local p-indicability for various primes p.

We apply these results to prove the non-existence of perfect projective $\mathbb{Z}$ G-modules for a large class of groups G.

There are also topological applications, mainly centering around the problem of Whitehead mentioned above.

N.HABEGGER: Relative cohomology of group extensions

Let $1 \to N \to G \to Q \to 1$ be an extension of groups. Let A be a G-module, A^N the set fixed by N. Let $C^*(G,A)$ denote the normalized inhomogeneous cochain complex. Hochschild-Serre, Cohomology of group extensions, TAMS 1953, define a map $l: C^n(G,A) \to \bigoplus_{p+q=N} C^p(G,C^q(N,A))$ using shuffles which gives rise to

$$\bar{l}: \frac{C^n(G,A)}{C^n(Q,A^N)} \to \frac{\bigoplus_{p+q=n} C^p(G,C^q(N,A))}{C^n(G,A^N)}.$$

Denote by $L^*(G,N;A)$ = image $\bar{l}$.

<u>Theorem</u> $\frac{C^*(G,A)}{C^*(Q,A^N)} \to L^*(G,N;A)$ induces an isomorphism on homology.

It is easy to see that the group $XExt_G(N,A)$ of crossed extensions, studied by John Ratcliffe and Johannes Huebschmann, is isomorphic to $H^2(L^*(G,N;A))$ and hence, by the theorem, to $H^2(Q,G;A)$, the homology of $\frac{C^*(G,A)}{C^*(Q,A^N)}$

Theorem. There is a spectral sequence

$$E_2^{p,q} = \begin{cases} H^p(Q,H^q(N,A)) & q > 0 \\ 0 & q = 0 \end{cases} \Rightarrow H^{p+q}(Q,G;A).$$

J.HUEBSCHMANN: Cohomologie of $GL(\mathbb{F}_q)$ and other classical groups

Let $\mathbb{F}_q$ denote the field with $q = p^n$ elements, where p is a prime, and denote by $GL(\mathbb{F}_q)$ the union of the ascending sequence $GL_1(\mathbb{F}_q) \subset GL_2(\mathbb{F}_q) \ldots$. Furthermore let Ψ^q denote the q'th Adams operation in complex K-theory, and denote by $F\Psi^q$ the fibre of the map Ψ^q-1: BU $\to$ BU. It has been proved by Quillen that there is a map $BGL(\mathbb{F}_q) \to F\Psi^q$ which extends to a homotopy equivalence $BGL(\mathbb{F}_q)^+ \to F\Psi^q$; here "+" is Quillen's plus construction. The purpose of the talk is to present a description of the cohomology of $F\Psi^q$ with coefficients in a large class of rings R, including the integers $\mathbb{Z}$ and the rings $\mathbb{Z}/l$, where l is an arbitrary number. The method is to construct a small model for the (singular) chains of $F\Psi^q$, and then to construct cocycles in the small model. This leads to explicit formulas for the cohomology of $F\Psi^q$ and hence that of $GL(\mathbb{F}_q)$.
Similar results can be obtained for the group $SL(\mathbb{F}_q)$, and in the sympletic and orthogonal cases; in the orthogonal cases the methods developed so far only apply if 2 is invertible in the ground ring R.

W.KIMMERLE: Bounds for the gap of a finite group

Let G be a finite group. The gap of G is the difference between d(G), the minimal number of generators of G, and $d_G(\mathfrak{g})$, the minimal number of generators of its integral augmentation ideal $\mathfrak{g}$.
K.W.Roggenkamp has shown that gap(G) coincides with the presentation rank of G. The following cohomological criterion is due to K. Gruenberg.
$\mathrm{gap}(G) \geq n \iff |H^1(G,S)| \leq |S|^{d(G)-n-\zeta_S}$ $\forall$ simple $\mathbb{Z}G$-modules S, where ζ_S is 1 or 0 according as S is not, or is a trivial module. Moreover the simple modules that have to be tested all arise as split abelian chief factors

within G itself. The results in this direction depend on the cohomology sequence determined by a group extension $1 \to N \to G \to G/N \to 1$. In contrast to this the following results are obtained using cohomology relative to arbitrary subgroups.

Denote by $d(G,H)$ the minimal number of elements, which is needed to generate G together with the subgroup H, and by $\pi(G)$ the set of primes dividing the order of G.

1) Assume that each nonabelian compositionfactor of G is generated by two of its 2-Sylowsubgroups and that there exists an odd prime q such that $d_G(\mathfrak{g}) = d_G(\mathfrak{g}/q\mathfrak{g})$, then, if $S \in Syl_2(G)$,
   $d(G) - d(G,S) \geq gap(G) \geq d(G) - d(G,S) - 1.$
2) Assume there exists $p,q \in \pi(G)$ such that each nonabelian compositionfactor of G is generated by two of its p-Sylowsubgroups and generated by two of its q-Sylowsubgroups, then exists $T \in Syl_p(G) \cup Syl_q(G)$ such that
   $d(G) - d(G,T) \geq gap(G) \geq d(G) - d(G,T) - 1.$

P.A.LINNELL: Accessibility of groups

Let G be a finitely generated group and suppose $H'(G,\mathbb{Z}G) \neq 0$. Then a celebrated theorem of Stallings states that G splits over a finite subgroup; that is we may write $G = A_{*_F}B (A \neq F \neq B)$ or $G = A_{*_F}$ (HNN extension) with F finite. If $H^1(A,\mathbb{Z}A) \neq 0$, then A itself splits over a finite subgroup. I will be concerned with whether this process of splitting must always come to a stop after a finite number of steps, and the proof of the following theorem will be sketched.

Theorem. Let G be a finitely generated group. If the finite subgroups of G have bounded order, then G is accessible.

G.Mc HARDY: Endlichkeitseigenschaften S-arithmetischer Gruppen

Sei K ein globaler Funktionenkörper, $O_S \subset k$ ein S-arithmetischer Ring, G eine reduktive algebraische k-Matrizengruppe und $\Gamma = G(O_S)$ die zugehörige S-arithmetische Gruppe. Thema des Vortrages ist die endliche Präsentierbarkeit von Γ bzw. die $(FP)_n$-Eigenschaften $(n \in \mathbb{N})$.Ergebnisse: $SL_2(O_S)$ e.präs. $\Leftrightarrow |S| \geq 3$; $n \geq 4 \Rightarrow GL_n(O_S)$ e.präs. $(|S| = 1)$; G eine einfache Chevelleygruppe, nicht von Typ G_2: Rang $G \geq 3 \Rightarrow G(\mathbb{F}q[t])$ ist endlich präsentiert; Rang $G \geq 2 \Rightarrow G(\mathbb{F}q[t,t^{-1}])$ ist endlich präsentiert; Rang $G = 2 \Rightarrow G(\mathbb{F}q[t])$ nicht endlich präsentiert. Ferner gilt: Wenn $S = \{\omega\} (|S| = 1)$ und k_ω die Komplettierung von k an der Stelle

ω ist, G klassisch und $\text{Rang}_k\ G = \text{Rang}_{k_\omega}\ G = 2 \Rightarrow G(O_s)$ ist nicht endlich präsentiert.
Stuhler zeigte: $PGL_2(O_s)$ ist vom Typ $(FP)_{|S|-1}$, aber nicht vom Typ $(FP)_{|S|}$.
Für die Gruppen SL_3 gilt: $SL_3(\mathbb{F}q[t])$ ist endlich erzeugt, aber nicht fast-endlich präsentiert, wenn $\mathbb{F}q$ die Dimension $r \geq 2$ über seinem Primkörper hat.

B.PARSHALL: Cohomology of algebraic and finite groups

This lecture will survey the work done on the cohomology of algebraic groups and related finite groups. Emphasis will be on recent results of Friedlander-Parshall.

J.G.RATCLIFFE: Euler characteristics of 3-manifold groups and discrete subgroups of SL(2,C).

In this paper, it is shown that every finitely generated 3-manifold fundamental group G has a rational Euler characteristic $\chi(G)$; moreover, it is shown that $\chi(G)$ measures the size of G in some sense. This is applied to show that every finitely generated discrete subgroup of $SL(2,\mathbb{C})$ or $PSL(2,\mathbb{C})$ has a Euler characteristic.
Also, it is shown that if Γ is a finitely generated discrete subgroup of $SL(2,\mathbb{C})$, then $SL(2,\mathbb{C})/\Gamma$ has finite invariant volume if and only if $\chi(\Gamma) = 0$ and Γ is not virtually abelian.
Also, it is shown that every finitely generated 3-manifold group G with $\chi(G) < 0$ is SQ-universal, that is, every countable group can be embedded as a subgroup of a quotient of G. This implies that every finitely generated nonelementary Kleinian group is SQ-universal.

K.U.ROGGENKAMP: Isomorphisms and automorphisms of integral group rings

(A preliminary report on joint work with L.L.Scott) Let G be a finite p-group, and consider the following implications: ($\hat{\mathbb{Z}}_p$ are the p-adic integers)

IP: $\hat{\mathbb{Z}}_p G \simeq \hat{\mathbb{Z}}_p H \Rightarrow G \simeq H$

ZC: If $\alpha \in \text{Aut}(\hat{\mathbb{Z}}_p G)$, there is conjugation by a unit in $\hat{Q}_p G$ followed by a groupautomorphism.

SR: If $\alpha \in \text{Aut}(\hat{\mathbb{Z}}_p G)$, then α is conjugation by a unit in $\hat{\mathbb{Z}}_p G$ followed by a groupautomorphism.

Proposition: Given an exact sequence $1 \to A \to G \to \bar{G} \to 1$ with A abelian. Assume

IP, ZC and SR for $\bar{G}$, then G has ZC and IP. (These hypotheses are satisfied for $\bar{G}$. Dihedral 2-groups (Endo, Miyata Sekiguchi, R-S) Quaternion groups (R-S) and Galois action groups $C_{p^n} \rtimes C_{p^m}$ (Sekiguchi, R-S). By induction the problem is reduced to showing that a map of non-abelian cohomology groups is trivial.

Conjecture 1 (official): SR holds for nilpotent class 2 groups

Conjecture 2 (inofficial): SR holds for p-groups. (We hope to make this official soon).

L.SCOTT: 1) Cohomology rings of infinitesimal groups , 2) Transfer and stability for nonabelian cohomology

In 1) I discuss recent work of myself and a student, Ronnie Crane, toward understanding the cohomology rings $H^*(U,k)$ where U is the algebraic group of uppertriangular $n \times n$ matrices with 1's on the diagonal, or one of its infinitesimal subgroups. I hope also to compare and contrast this theory with corresponding theories for discrete groups. The coefficient domain k here is in general an algebraically closed field of characteristic p.

2) forms part of some joint work with Michael Aschbacher on maximal subgroups of finite groups. A theorem analogous to Shapiro's lemma for abelian cohomology is established in the nonabelian case, as well as a related result characterizing the image of a restriction map in terms of conjugation stability. There are direct and indirect applications to the theory of maximal subgroups.

R.STREBEL: A geometric invariant for modules over an abelian group

Let R be a commutative ring with $1 \neq 0$, let Q be a free abelian group of rank $n \geq 1$ and let A denote a finitely generated RQ-module. Identify Q with the standard lattice $\mathbb{Z}^n \subseteq \mathbb{R}^n$ via $\theta: Q \xrightarrow{\simeq} \mathbb{Z}^n$. Given a unit vector $n \in S^{n-1} \subseteq \mathbb{R}$, put $Q_u = \{q \in Q \mid \langle u, \theta q\rangle \geq 0\}$. This Q_u is an abelian monoid; let $RQ_u \subseteq RQ$ denote its monoid algebra. Then the geometric invariant $\Sigma_A \subseteq S^{n-1}$ is defined by

$$\Sigma_A = \{u \in S^{n-1} \mid A \text{ stays f.g. when viewed as an } RQ_u\text{-module}\}.$$

This geometric invariant has been introduced by R.Bieri (Frankfurt) and myself (Proc.LMS(3) 42 (198o), 439-64) to characterize finitely presented metabelian groups. The invariant has been further studied by the same authors in (Crelle 322 (1981), 17o-89) and (J.Pure Appl.Alg., to appear) and also by R.Bieri and J.Groves (Proc.LMS; to appear). It is closely related to the logarithmic limit set introduced by G.M. Bergmann (Trans.AMS 157 (1971),459-69).

CH.B.THOMAS: K-theory of classifying spaces for arithmetic groups

For the arithmetic group $SL(n,\mathbb{Z})$ it is an easy consequence of the congruence subgroup theorem that the subring of $H^{even}(SL(n,\mathbb{Z}),\mathbb{Z})$ generated by Chern classes is actually generated by $\{c_{2k}(\pi_n)\}$, for the natural representation π_n of $SL(n,\mathbb{Z})$ in $SL(n,\mathbb{C})$, and by classes inflated up from finite quotients. Since the Chern ring "almost equals" the infinite cycles in the Atiyah-Hirzebruch spectral sequence, this example suggests that the compact part of $K(B\Gamma)$ may be easier to calculate than $H^*(B\Gamma,\mathbb{Z})$ for "suitable" arithmetic groups. The purpose of this talk is to explore analogues of the classical isomorphism $\alpha^n: R(\Gamma)^n \xrightarrow{\sim} K(B\Gamma)$ (Γ finite) and of Tate-Farrell cohomology in K-theory.

T.WILLIAMS: Relation modules of finite groups

(Report on joint work with W.Kimmerle)

Let G be a finite group and denote by $\bar{R}_o$ a minimal relation module. If $d(G) \geq 3$ or $\mathbb{Z}G$ satisfies the Eichler condition, and $(\overset{n}{\underset{1}{\oplus}} \bar{R}_o) \hat{=} A \oplus \mathbb{Z}G$ for some n, then there is only the isomorphism class of minimal relation modules. Let $\mathfrak{C}$ denote the class of groups for which $(\overset{n}{\underset{1}{\oplus}} \bar{R}_o) \hat{=} A \oplus \mathbb{Z}G$ for some n.

Theorem. (Kimmerle-Williams) If G is a finite simple group such that $G = \langle x_1 \ldots x_n | x_1^2 = 1, \ldots \rangle$, G nonabelian than $G \in \mathfrak{C}$.

This result forms part of the basis of the following result, under the assumption of the simple group classification.

Theorem. (Aschbacher-Guralnick) G finite, M faithful irreducible module then $|H^1(G,m)| < |m|$.

Corollary. If G is simple group, then $G \in \mathfrak{C}$.

The most general result can be expressed as: -

Theorem. (Kimmerle-Williams). Suppose $G \supset P$, $P \neq 1$, $P \lhd G$, $P = [P,P]$ and every abelian chief factor of P is cyclic, and either $G/P \in \mathfrak{C}$ or $d(G) > d(G/P)$, then $G \in \mathfrak{C}$.

This result covers Alt(n), Sym(n), all finite Chevellay groups and all $L \subseteq Aut(simple)$ $L \supset$ simple, such that L/simple is cyclic. If the abelian chief factors of P are not cyclic there is a theorem of Kimmerle about the existence of infinitely many perfect groups $\notin \mathfrak{C}$.

P.T.WEBB: Relation modules of infinite groups

We give a survey of recent results concerning the isomorphism and decomposition problems for relation modules of infinite groups.

These are problems

1. given two relation modules $\bar{R}_1$ and $\bar{R}_2$ arising from presentations of a group G by a fixed free group F, do we have $\bar{R}_1 \cong \bar{R}_2$ as $\mathbb{Z}G$-modules?
2. If $\bar{R}$ is a minimal relation module for G, can we have a $\mathbb{Z}G$-module decomposition $\bar{R} = A \oplus B$?

P.T.WEBB: Calculating cohomology from local subgroups

We obtain formulae reducing the cohomology of a finite group G to the cohomology of certain of its local subgroups. The general expression involves P.Hall's Möbius function for the lattice of subgroups of G. In particular cases one has, in effect, to determine some of its values, and but for straightforward cases this can be quite laborious. The best application I have made so far is a calculation of the 2-part of the cohomology of PSL(3,q), q odd. In addition there are formulae for some easier groups.

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