MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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The Twentieth International Symposium on Functional Equations was held from August 1 to August 7, 1982 in Oberwolfach, Germany. The organizational committee consisted of J. Aczél (Waterloo), W. Benz (Hamburg), and J. Rätz (Bern). The meeting was opened by J. Aczél who also took the occasion to transmit to Professor E. Lukacs the best wishes of the participants of the meeting for his 75th birthday, and to commemorate the late Professor J. Kampé de Fériet.

The symposium was attended by 47 participants coming from 14 countries of Europe, Asia, America and Africa. We were pleased to see some mathematicains from Poland and Rumania.

Among the fields strongly represented were iteration, equations characterizing special functions, equations in one and several variables, conditional equations, equations for multiplace functions, vectors, operators, and functionals. Connections with analysis, algebra, geometry, and applications to physics, the behavioral and social sciences, probability and information theory were also discussed in depth. At the end of each of six sessions, there was time dedicated to remarks, new open problems, and solutions of old ones. These, as always, were very stimulating and successful.

The practice of scheduling special informal sessions, introduced at the 18th Symposium, was continued, mostly with new topics. There were three profitable sessions, devoted to the translation equation in conjunction with iteration, to

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applications of functional equations to geometric algebra and to information measures.

The usual creative and congenial atmosphere prevailed, which all newcomers to the Symposium found quite remarkable. At least two participants have discovered just on this Symposium that they have been working on the same topic and even proved (roughly) the same theorem.

An evening gathering was devoted to the celebration of the 20th anniversary of the first functional equations meeting in Oberwolfach. Professors Rätz and Aczél spoke there on the history and impact of these meetings through the years.

The meeting was closed by W. Benz, who expressed the thanks of the participants to the Institute for its hospitality. The Twenty-first International Symposium on Functional Equations will be held in Konolfingen, Switzerland from August 6 to August 13, 1983.

All participants expressed their desire to have the 22nd Symposium in Oberwolfach in 1984.

Abstracts of the talks in alphabetical order, problems and remarks in chronological sequence, and reports on the special sessions follow in three separate sections.

ABSTRACTS

J. ACZÉL:

Aggregation theorems for allocation problems

(Joint work with C. Wagner; an alternative proof and generalization by C.T. Ng.) Each of n individuals (say, members of a committee) distributes an amount σ among m decision variables (say, grant applications) allocating ζ_{ij} $(i=1,\dots,m;\ j=1,\dots,n)$ to the ith. These individual assignments are aggregated (say, by a chairman) in order to produce a 'consensual' value $\phi_i(\zeta_{i1},\dots,\zeta_{in})$. Thus

$$\sum_{i=1}^{m} \zeta_{ij} = \sigma \ (j=1,\dots,n) \Rightarrow \sum_{i=1}^{m} \phi_i\ (\zeta_{i1},\dots,\zeta_{in}) = \sigma$$

has to be satisfied. With $\zeta_{ij} \in \langle\alpha,\delta\rangle$ (open or closed interval; $\alpha \geq 0$, $\delta+(m-1)\alpha \leq \sigma \leq \alpha+(m-1)\delta$; more general domains are also permitted) we find all solutions, all solutions bounded from below on a set of positive measure, all solutions bounded by α from below and by δ from above and all solutions satisfying $\phi_i(\gamma,\dots,\gamma)=\gamma$ for at least one γ between α and δ ($\gamma=\sigma/m$ has a singular behavior). Here σ and m may be fixed ($m>2$ is more interesting for application).

C. ALSINA:

Bilinear triangle functions

We have solved several functional equations in the space of probability distribution functions by determining the values of the unknown function on the dense subspace of step-functions.

THEOREM 1. The unique continuous triangle function τ of L-class that satisfies the bilinear functional equation:

$$\tau(aF+bG,H)=a\tau(F,H)+b\tau(G,H),$$
$$F,G,H\in\Delta^{+},\ a,b\in[0,1],\ a+b\leq 1,$$

is $\tau=\sigma_{\mathrm{Prod},L}$, i.e., $\sigma_{\mathrm{Prod},L}(F,G)(x)=\iint_{L(u,v)\leq x} d(F(u)\cdot G(v))$.

This characterizes the convolution as the only bilinear triangle function of + - class.

THEOREM 2. A (T,L)-triangle function is distributive simultaneously with respect to the lattice operations in Δ^{+} if and only if $\tau=\tau_{T,L}$, i.e., $\tau(F,G)(x) =\sup\{T(F(u),\ G(v))\,|\,L(u,v)=x\}$.

Further applications of this method have been used in characterizations of the truncation function (C. Alsina, Proceedings S.W.C.M.S.M., Las Palmas, 1982) and strong negations on Δ^{+} (C. Alsina, Proceedings 1[er] Congrés Catalán de Logica Matemática, Barcelona, 1982).

K. BARON:

Non-homogeneous functional equations usually have no continuous solutions in the indeterminate case

The functional equation

(*) $\qquad \phi[f(x)] = g(x)\phi(x)+h(x),$

where ϕ is an unknown function, usually is considered under the following assumptions (cf. [3]):

(i) $f:I\to I$ is a continuous function, $I\subset\mathbb{R}$ is an interval and $0<(f(x)-\xi)/(x-\xi)<1$ for every $x\in I\setminus\{\xi\}$, where ξ is a point of I;

(ii) $g:I\to\mathbb{R}$ is a continuous function and $g(x)\neq 0$ for every $x\in I\setminus\{\xi\}$.

It is well known that if $|g(\xi)|\neq 1$ and the function f is strictly increasing in a neighborhood of the point ξ then for every continuous function $h: I\to\mathbb{R}$ equation (*) has a continuous solution $\phi: I\to\mathbb{R}$. The case $|g(\xi)|=1$ is called indeterminate. It turns out that in this case the existence of continuous solutions of (*) is rather an exception. In order to see this, let us consider the space C of all continuous mappings of the interval I into R with the topology of uniform convergence on all compact subsets of I, and its subspace C_0 of all functions from the space C vanishing at the point ξ. In both these spaces C and C_0, we have two σ-ideals; the σ-ideal of all subsets of the first category (let us denote them by $C_1(C)$ and $C_1(C_0)$, respectively) and the σ-ideal of all Christensen zero subsets (cf. [1] regarding the definition and denote them by $C_0(C)$ and $C_0(C_0)$, respectively). Putting

$$H = \{h\in C:\ V_{\phi\in C}(\phi\circ f=g\phi+h)\},\ H_0 = \{h\in C_0:\ V_{\phi\in C}(\phi\circ f=g\phi+h)\}$$

we have the following theorem, the topological part of which has been proved by W. Jarczyk (cf. [2]).

THEOREM. Assume (i) and let $g\colon I\to R$ be a continuous function.
If $g(\xi)=-1$, then $H\in C_1(C)\cap C_0(C)$ and $H_0\in C_1(C_0)\cap C_0(C_0)$.
If $g(\xi)=1$, then $H=H_0$ and $H_0\in C_1(C_0)\cap C_0(C_0)$.

References

1. P. Fischer and Z. Slodkowski, Christensen zero sets and measurable convex functions, Proc. Amer. Math. Soc. 79 (1980), 449-453.
2. W. Jarczyk, On a set of functional equations having continuous solutions, Glasnik Mat. (to appear).
3. M. Kuczma, Functional equations in a single variable, Monografie Mat. 46, PWN, Warszawa, 1968.

W. BENZ:

A functional equation in connection with derivations

All solutions $f,g\colon R^2\to R$ of

$$[f(x+z,y+\tfrac{1}{z})-f(x,y)]+z^2[g(x+z,y+\tfrac{1}{z})-g(x,y)]=0$$

(for all $x,y,z\in R$ with $z\neq 0$)

are given by

$$f(x,y)=d(x)+\alpha x+\beta, g(x,y)=d(y)-\alpha y+\gamma$$

where $\alpha,\beta,\gamma\in R$ are constants and where d: $R\to R$ is a derivation of R. This theorem is the main tool for solving

$$[F(x+z,y+\frac{1}{z})-F(x,y)]\cdot[G(x+z,y+\frac{1}{z})-G(x,y)]=1$$

in case of the Clifford numbers, thus characterizing Lorentz transformations in Hjelmslev's world.

B. CHOCZEWSKI:

Discontinuous solutions of an iterative functional equation

Consider the functional equation

(1) $\phi(f(x)) = g(x)\phi(x)$

in an interval $I:=(0,A)$ or $(0,A]$ such that the functions f,g: $I^*\to R$ are continuous in $I^*:=I\cup\{0\}$ and $0<f(x)<x$, $g(x)>0$ in I. Asymptotic properties (as $x\to 0+$) of solutions ϕ: $I^*\to R$ of (1), which are continuous in I but not necessarily in I^*, are studied. The formulae obtained improve slightly on those contained in the paper by M. Kuczma and the speaker (Zeszyty Naukowe Uniw. Jagello. Prace Mat. 22(1981), 119-123). However, we assume more on the behaviour of the function g at the origin.

I. COROVEI:

The sine functional equation for groups

We consider the functional equation

(1) f: $G\to K$, $f(xy)f(xy^{-1}) = f^2(x)-f^2(y)$

where G is a group and K is a field. Clearly the functions of the form

(2) $$f(x) = \frac{g(x)-[g(x)]^{-1}}{2b}$$

where g is a homomorphism of G into the multiplicative group of K, satisfy the equation (1).

Let K(b) denote the extension of field K by the element b and let $K^*(b)=K(b)\setminus\{0\}$. We prove the following theorem.

THEOREM. Let G be a group whose elements are of odd order, K a skewfield of char $K\neq 2$ and $f: G\to K$ a non zero solution of the equation (1). Then f has the form (2), where g is a homomorphism from G into the multiplicative group of $K^*(b)$, or f is a homomorphism from G into the additive group of K.

C.C. COWEN:

<u>Commuting analytic functions on the unit disk</u>

Let f and g be non-constant analytic mappings of the unit disk, D, into itself such that no iterate of either is the identity. We say <u>f and g commute</u> if $f(g(z))=g(f(z))$ for all z in D. If g is an iterate of f, or vice versa, then f and g trivially commute. In fact, the converse is almost true! We define the <u>local iteration semigroup of f</u>, a semigroup of analytic functions on D that are generalized fractional iterates of f, and obtain the following.

THEOREM. If f and g (as above) commute, then f and g are both in the local iteration semigroup of $f\circ g$.

Let a denote the distinguished fixed point of f in D at which $|f'(a)|\leq 1$.

<u>Cor.1</u>. If f is univalent with $|f'(a)|<1$ and g commutes with f, then g is univalent.

<u>Cor.2</u>. If $0<|f'(a)|<1$ and g_1 and g_2 (as above) commute with f, then g_1 and g_2 commute with each other.

Examples can be given with $f'(a)=0$ and with $f'(a)=1$ in which the conclusion of corollary 2 is false. In certain cases, all g that commute with f have the same fixed point set as f.

S. CZERWIK:

Continuous solutions of a system of double functional inequalities

In this paper we investigate a system of multi-place functional equations and give a characterization of continuous solutions of such a system and the existence theorem for a system of double functional inequalities.

THEOREM. Let X be a topological Hausdorff space, $Y_i \in CC(R)=\{A\subset R: A$ is nonempty convex closed$\}$, $i=1,\dots,n$ (R is the set of real numbers), and $X\times Y_1^k\times\dots\times Y_n^k$ be a paracompact space. Let $h_i^1: X\times Y_1^k\times\dots\times Y_n^k\to Y_i\cup\{-\infty\}$, $i=1,\dots,n$, be upper semicontinuous functions and $h_i^2: X\times Y_1^k\times\dots\times Y_n^k\to Y_i\cup\{+\infty\}$, $i=1,\dots,n$ be lower semicontinuous functions such that $h_i^1\le h_i^2$ for $i=1,\dots,n$, and $f_m: X\to X$, $m=1,\dots,k$ continuous functions. Then $\phi_i: X\to Y_i$, $i=1,\dots,n$ is a continuous solution of the system of inequalities

$$h_i^1(x,\phi_1[f_1(x))],\dots,\phi_1[f_k(x)];\dots;\phi_n[f_1(x)],\dots,\phi_n[f_k(x)])\le\phi_i(x)\le$$

$$h_i^2(x,\phi_1[f_1(x)],\dots,\phi_1[f_k(x)];\dots;\phi_n[f_1(x)],\dots,\phi_n[f_k(x)]),$$

$$x\in X,\ i=1,\dots,n$$

if and only if there exist continuous functions $h_i: X\times Y_1^k\times\dots\times Y_n^k\to Y_i$, $i=1,\dots,n$ such that $h_i^1\le h_i\le h_i^2$, $i=1,\dots,n$ and

$$\phi_i(x)=h_i(x,\phi_1[f_1(x)],\dots,\phi_1[f_k(x)];\dots;\phi_n[f_1(x)],\dots,\phi_n[f_k(x)]),$$

$$x\in X,\ i=1,\dots,n.$$

Z. DARÓCZY:

Über gewichtbare Mittelwerte

Die Folge von Funktionen $M_n: I^n\to I$ ($n\in N, I\subseteq R$ Intervall) wird ein diskreter Mittelwert genannt, wenn die M_n symmetrisch und intern sind. Ein diskreter Mittelwert $\{M_n\}$ ist Q-gewichtbar, wenn die Gleichung

$$M_{kn}(\underbrace{x_1,\ldots,x_1}_{k};\ldots;\underbrace{x_n,\ldots,x_n}_{k}) = M_n(x_1,\ldots,x_n)$$

für alle $k,n \in N$ und $(x_1,\ldots,x_n) \in I^n$ gültig ist. Sind $r_i = \frac{k_i}{k}$ ($k_i \geq 0$ ganze Zahlen mit $\sum_{i=1}^{n} k_i > 0$ und $k \in N$) rationale Zahlen, so definieren wir die Funktion

$$\tilde{M}_n(x_1,\ldots,x_n;r_1,\ldots,r_n) := M_{\Sigma k_i}(\underbrace{x_1,\ldots,x_1}_{k_1};\ldots;\underbrace{x_n,\ldots,x_n}_{k_n}).$$

Hat $\tilde{M}_n$ eine stetige Fortsetzung $\bar{M}_n$ bezüglich der Gewichte r_i, so nennt man M_n gewichtbar. Es sei $\Delta_n := \{(p_1,\ldots,p_n) \mid p_i \geq 0, \sum_{i=1}^{n} p_i > 0\}$. - Die Folge $\bar{M}_n : I^n \times \Delta_n \to I$ wird monoton genant, wenn die Funktion $t \mapsto M_n(\underline{x}; t\underline{a}+\underline{b})$ auf $J_{\underline{a},\underline{b}} := \{t \mid t \in R,\ t\underline{a}+\underline{b} \in \Delta_n\}$ ($\underline{a},\underline{b} \in R^n$ sind beliebig) streng monoton oder konstant ist. - Die allgemeine Gestalt der gewichtbaren, monotonen diskreten Mittelwerten wird bestimmt.

T.M.K. DAVISON:

The dilogarithm and some of its functional equations

Leibniz and then Euler discussed the function $Li_2(x) = \sum_1^\infty \frac{x^n}{n^2}$ ($|x| \leq 1$); this is the dilogarithm. It is defined for all real x by $Li_2(x) = \int_0^x - \frac{\ell n(|1-t|)}{t}\, dt$. I list many functional equations satisfied by Li_2; these lead one to introduce $M(x) = Li_2(x) + \frac{1}{2}\ell n|x|\ell n|1-x|$. I prove that

$$M(xy) = M(x)+M(y) + M\left(\frac{x-xy}{x-1}\right) + M\left(\frac{y-xy}{y-1}\right) - \lambda(xoy)\, M(2)$$

where

$$\lambda(xoy) = \begin{cases} 2 \text{ if } xoy>1 \\ 0 \text{ if } xoy<1 \end{cases}, \text{ and } xoy = x + y - xy.$$

Finally I show that all the functional equations given follow from this, as do the equations of Abel, Rogers, and Gelfand-MacPherson.

J. DHOMBRES:

Finding subgroups

Let G,F be two nonempty sets and let * be an associative binary law on $G=G\times F$. We look for subsemigroups H of $(G,_*)$ which possess a convenient parametrization ("faithful"). Thus the problem is reduced to the solving of a functional equation of the form

$$(f(z),z) = (f(x),x) * (f(y),y)$$

When $(\alpha,\beta) * (\alpha',\beta') = (\alpha\alpha',\alpha\beta'+\beta)$, we get the subgroups of all proper affine transformations on a real topological linear space.

When $(\alpha,\beta) * (\alpha',\beta') = (\lambda\alpha\alpha',\alpha\beta'+\beta\alpha')$, we get a functional equation

$$f(xf(y) + yf(x)) = f(x)f(y)$$

whose solution gives the subsemigroups of the group of all matrices

$\begin{pmatrix} \alpha & 0 \\ \beta & \alpha \end{pmatrix}$, $\alpha\in \mathbb{R}\setminus\{0\},\beta\in \mathbb{R}$.

A generalization of some methods of functional equations is then explained.

B. EBANKS:

Some functional equations arising in the mixed theory of information on the open domain.

The following problem arises in the characterization of 3-symmetric, β-recursive inset measures of information on the open domain. Find all solutions of

$$(*) \qquad F(x\cup y,z)+F(x,y) = F(y,z)+F(x,y\cup z),\ F(x,y) = F(y,x)$$

for $(x,y,z)\in D_3:=\{(x_1,x_2,x_3)|\emptyset \neq x_i\in B,x_i\cap x_j = \emptyset \text{ for } i\neq j;\ i,j=1,2,3\}$, where B is a ring of sets and F is real-valued.

THEOREM. The general solution of the pair (*) of functional equations is given by

$$F(x,y) = f(x) + f(y) - f(x\cup y)$$

for all $(x,y)\in D_2:=\{(x,y)\,|\,x,y\in B\setminus\{\emptyset\},\ x\cap y=\emptyset\}$, where f is an arbitrary real-valued function on $B\setminus\{\emptyset\}$.

This theorem can be used to find all 3-symmetric, β-recursive inset entropies on the open domain, and to find all symmetric, branching inset entropies on the "classical" domain. The proof of the theorem relies on a result of K. Davidson and C.T. Ng.

W. EICHHORN:

A model of optimal economic growth yielding a functional equation

The arithmetic means of the investment ratios $\alpha_t=I_t/Y_t$ (=investment in capital goods of year t divided by the GNP of year t) of the years 1976 to 1980 were very different in different countries. Examples: 33% in Japan; 22 to 24% in France, Italy and West Germany (D); 20% in GB; 18% in the USA. Problem: Find sequences of optimal α_t's with respect to maximization of consumption. Model:

(i) $F_t: R_+\to R_+$ (production function of year t); $Y_t=F_t(K_{t-1})$, K_t=capital stock of year t.

(ii) $K_t=\alpha_t F_t(K_{t-1}) + q_t K_{t-1}$ (q_t=depreciation factor)

(iii) Given a planning horizon n_t and a capital stock K_{t-1} at (the beginning of) year t, find investment ratios $\alpha^*_{t+1},\ldots,\alpha^*_{t+n_t}$ that maximize the discounted consumption $\sum_{k=1}^{n_t} r_t^{(k)}(1-\alpha_{t+k})F_{t+k}(K_{t+k-1})$, where $r_t^{(k)}:=r_{t+1}r_{t+2}\cdots r_{t+k}$ (discounting factors).

THEOREM. Let F_t be twice differentiable and concave. Let F'_t be invertible in a neighborhood of $1/r$ and $(1-r_tq_t)/r_t$ and let $F''_t(a_t)\le 0$, $F''_t(b_t)\le 0$ at points a_t, b_t (whose values shall not be specified here). Then there exist optimal investment ratios α^*_{t+k} (formulas omitted here), and these depend only on F_{t+k}, F_{t+k+1} and K_{t-1}, the initial capital stock. Beginning with K_{t-1} one can calculate the optimal capital stocks $K^*_{t+k-1}(k=1,\dots,n_t)$.

PROOF. Bellman's method of backward dynamic programming using Bellman's functional equation.

SPECIALIZATION. $r_t=r$, $q_t=q$, $F_t(K)=F(K)=cK^\gamma$ ($\gamma<1$, c positive reals) for all $t=1,2,\dots$, planning horizon infinite, suitable convergence conditions. Then $(\alpha^*_1,\alpha^*_2,\alpha^*_3,\dots) = (\alpha^*_1,\alpha^*,\alpha^*,\dots)$ and $\alpha^*=(1-q)r\gamma|(1-rq)$. The 1980 investment ratio 0.229 of D is obtained for, e.g., $q=0.9$, $r=0.97$, $\gamma=0.3$, which are quite realistic values. Note that our specialization implies zero growth.

I. FENYÖ:

On a summability theorem connected with a functional equation

The following theorem is proved: If the function $f: R\to R$ has the property that $\Sigma f(x_n)$ is $(C,1)$ summable for every set $\{x_n\}$ for which Σx_n is $(C,1)$ summable, then $f(x)=cx$, where c is an arbitrary constant. The main idea of the proof is to show that f is continuous and satisfies the Cauchy functional equation. (Cf. W.B.Jurkat-B.L.R. Shawyer, Analysis 1 (1981), 209-210.)

G.L. FORTI:

On a Cauchy equation on a restricted domain

Consider the following functional equation

(1) $f(x+f(x)) = f(x)+f(f(x))$, $f: R\to R$.

This equation can be viewed as a Cauchy equation on a restricted domain: f must be additive on its graph.

The following theorem holds:

If f is a solution of (1) which is continuous on R and differentiable at zero, then it is linear.

Using the same techniques as in the previous case, analogous results can be proved for the equation

$$f(x+h(x)) = f(x)+f(h(x)),\ f\colon R\to R,\ h\colon R\to R.$$

where h is a given function.

H. FUNKE:

Algebraic methods for the solution of an aggregation problem for demand functions

The functional equation

$$(1)\qquad \frac{f_1(x_1)}{\sum\limits_{i=1}^{n} f_i(x_i)} = \lambda \frac{g_1(x_1)}{\sum\limits_{i=1}^{n} g_i(x_i)} + (1-\lambda) \frac{h_1(x_1)}{\sum\limits_{i=1}^{n} h_i(x_i)}$$

is important for solving an aggregation problem stated at the 19th Symposium on Functional Equations. Let X_i be arbitrary sets, the functions $f_i, g_i, h_i\colon X_i\to F$, $i=1,\dots,n$, where F is a field. Of course, all denominators have to be different from zero. Equation (1) is solved for arbitrary $\lambda \in F$, but $\lambda\notin\{0,1\}$, $0 \notin f_1(X_1) \cup \sum\limits_{i=2}^{n} f_i(X_i)$, $|f_i(X_i)| \geqq 4$ for $n = 2$ and $|f_i(X_i)| \geqq 3$ for $n \geqq 3$, $i=1,\dots,n$. The solution of (1) as a special case includes the solution of problem 31 stated at the last meeting.

W. GEHRIG:

The solution of Cournot's difference equation system

In his book "Recherches sur les Principes Mathêmatiques de la Thêorie des Richesses" [1838], A. Cournot (1801-1877) presented the first exact dynamic oligopoly model. From his assumptions about the form of the cost and demand functions and the behaviour of the oligopolists, the elementary difference equation system

$$(1)\qquad \underline{x}(t+1)=A\,\underline{x}(t)+\underline{b};\ t=0,1,2,\dots;\quad \underset{(n,n)}{A}=\begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix}$$

is derived. The complete solution of (1) is given by

(2) $$\underline{x}(t) = A^t \cdot (\underline{x}(0) - (I-A)^{-1}\underline{b}) + (I-A)^{-1}\underline{b}$$

with

(3) $$A^t = (1/2)^t \cdot \begin{pmatrix} \frac{n-1}{n} & & -1/n \\ & \ddots & \\ -1/n & & \frac{n-1}{n} \end{pmatrix} + \left(\frac{1-n}{2}\right)^t \cdot \begin{pmatrix} 1/n & & 1/n \\ & \ddots & \\ 1/n & & 1/n \end{pmatrix}$$

and

(4) $$(I-A)^{-1} = 2 \cdot \begin{pmatrix} \frac{n}{n+1} & & \frac{-1}{n+1} \\ & \ddots & \\ \frac{-1}{n+1} & & \frac{n}{n+1} \end{pmatrix} .$$

D. GRONAU:

Two functional equations for power series

We consider the two functional equations

$$\phi(x,\phi(y,z)) = \phi(x \cdot y,z) \text{ and } \phi(x,\phi(y,z)) = \phi(x+y,z)$$

as identities for $\phi(x,z) \in K[\![x,z]\!]^n$, the cartesian power of the ring of formal power series in the n+1 variables x and $z=(z_1,\dots,z_n)$ with coefficients from the field K. Some aspects of the solutions are discussed.

H. HARUKI:

A generalization of the cosine functional equation

We consider the following functional equation which is a generalization of the cosine functional equation (see [1]):

(1) $$(f(x+y) + f(x-y))(f(x)^2 + f(y)^2) = 2f(x)f(y)(f(x+y)f(x-y) + 1),$$

where $f: C \to C$ and x,y are complex variables.

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The purpose of this talk is to prove the following theorem:

THEOREM. If $f(z)$ is a meromorphic function of a complex variable z in $|z|<+\infty$, then the only solutions of (1) are $f(z)=\mathrm{cn}(\alpha z;k)$ and $f(z)\equiv 0$ where α, k are arbitrary complex constants. Here the symbol cn denotes one of the three Jacobi elliptic functions.

References

1. J. Aczél, Lectures on functional equations and their applications, Academic Press, New York and London, 1966, 117-128.

2. H. Haruki, A generalization of the cosine functional equation. The Seminar on Functional Equations, University of Waterloo, Waterloo, Ontario, Canada, November 6, 1980.

H.H. KAIRIES:

Principal solutions of Nörlund difference equations

Nörlund's principal solution

$$g_N(x) = \lim_{s\to 0+} \left[\int_1^\infty \phi(t)e^{-st}dt - \sum_{n=0}^\infty \phi(x+n)\, e^{-s(x+n)}\right]$$

of (D): $g(x+1)-g(x)=\phi(x)$ exists, if the following assumptions on ϕ are satisfied [$b\in(0,1)$, $P_m(x)=B_m(x-[x])$, B_m the m-th Bernoulli polynomial]: $\phi:[b,\infty)\to R$ m-times differentiable, $\phi^{(m)}|_{[b,c]}\in L_1[b,c]$ for all $c\geq b$, $\lim_{x\to\infty}\phi^{(m)}(x)=0$ and $\int_0^\infty P_m(t)\phi^{(m)}(t+x)dt$ uniformly convergent for $x\in[b,b+1]$ as an improper Lebesgue-integral.

In the sequel we assume ϕ to satisfy the above conditions.

From (D) we obtain for a normalized solution of (D) and for all $N\in\mathbb{N}$:

$$g(N+1) = \sum_{\nu=1}^{N} \phi(\nu),\quad g(1) = 0.$$

Application of the Euler-Maclaurin sum formula in the standard version

$$(S_1)\quad \sum_{\nu=1}^{N} \phi(\nu) = \int_1^N \phi(t)dt+\frac{1}{2}[\phi(N)+\phi(1)]+\sum_{r=2}^{m} \frac{B_r}{r!}[\phi^{(r-1)}(N)-\phi^{(r-1)}(1)]$$

$$+\frac{(-1)^{m+1}}{m!}\int_1^N P_m(t)\phi^{(m)}(t)dt$$

and replacement of $N \in \mathbb{N}$ by $x\in(b,\infty)$ yields

$$\tilde{g}(x+1) = \int_1^x \phi(t)dt + \frac{1}{2}[\phi(x) + \phi(1)] + \dots \quad .$$

It can be shown that generally $\tilde{g}$ is not a solution of (D).

We show: If we use the Euler-Maclaurin sum formula in the following form:

$$(S_2)\quad \sum_{\nu=1}^{N} \phi(\nu) = \int_1^N \phi(t)dt + \frac{1}{2}\phi(N) + \sum_{r=2}^{m} \frac{B_r}{r!} \phi^{(r-1)}(N) + \frac{(-1)^m}{m!} \int_N^\infty P_m(t-N)\phi^{(m)}(t)dt-\gamma .$$

where

$$\gamma = \frac{(-1)^m}{m!} \int_1^\infty P_m(t)\phi^{(m)}(t)dt + \sum_{r=1}^{m} \frac{B_r}{r!} \phi^{(r-1)}(1),$$

the above substitution procedure yields

$$\hat{g}(x+1) = \int_1^x \phi(t)dt + \frac{1}{2} \phi(x) + \sum_{r=2}^{m} \frac{B_r}{r!} \phi^{(r-1)}(x) + \frac{(-1)^m}{m!} \int_x^\infty P_m(t-x)\phi^{(m)}(t)dt-\gamma$$

and we have $\hat{g}(x) = g_N(x) - \gamma$.

PL. KANNAPPAN:

<u>On some functional equations from additive and non-additive measures</u>

Let $\Gamma_n=\{P=(p_1,\dots,p_n): p_i\geq 0, \Sigma p_i=1\}$. From the algebraic properties of symmetry, expansibility and branching of the measures results the sum representation of the measures, which together with the property of additivity produces the 'sum' functional equations. Usually these functional equations are solved when the equations hold for all m, n or for some particular pair. Here we determine the 'measurable' solutions of

$$(1)\qquad \sum_{i=1}^{n}\sum_{j=1}^{m} g_{ij}(p_iq_j) = \sum_i h_i(p_i) + \sum_j f_j(q_j),\ P \in \Gamma_n,\ Q \in \Gamma_m ,$$

$$(2)\qquad \sum_{i=1}^{n}\sum_{j=1}^{m} g(p_iq_j,r_is_j) = \sum_i g(p_i,r_i) + \sum_j g(q_j,s_j),\ P,R \in \Gamma_n,\ Q,S \in \Gamma_m ,$$

holding for some arbitrary but fixed pair m,n ($m\geq 2$, $n\geq 3$) and show that the solutions depend upon m, n.

M. LACZKOVICH:

<u>On the continuous solutions of the multiplication formula</u> $f(\frac{x}{k})f(\frac{x+1}{k})\dots f(\frac{x+k-1}{k})=f(x)$.

It is well-known that certain elementary and transcendental functions satisfy a so-called replicative formula

$$(1)\qquad f(\tfrac{x}{k})+f(\tfrac{x+1}{k})+\dots+f(\tfrac{x+k-1}{k}) = a_k f(x)+b_k \qquad (k\in\mathbb{N},\ x\in(0,1)),$$

where (a_n) and (b_n) are sequences of real numbers depending on the function in question. Our aim is to determine the class $M(a_n)$ of functions f which are measurable and locally summable on (0,1) and satisfy (1) with a given sequence (a_n) and $b_n=0$.

THEOREM 1. If $(a_n)\neq(n^s)(s=2,3,\dots)$ then there are functions $p,q\in M(a_n)$ such that $M(a_n)=\{ap+bq;\ a,b,\in R\}$.

This extends some results of H.H. Kairies, M. Yoder and P. Schroth concerning solutions of (1) which are continuous, Riemann-integrable or summable on [0,1].

THEOREM 2. Let $f: (0,1)\to R$ be a continuous solution of the multiplication formula given in the title. Then either $f\equiv 0$ or there are real numbers a,b such that $a>0$ and

$$f(x) = a^{x-\frac{1}{2}}\cdot(2\sin \pi x)^b \text{ for every } x \in(0,1).$$

K. LAJKÓ:

Functional equations in probability theory

Let X be an n-dimensional continuous random variable with density function $p: R^n \to R$. Let $\underline{f}: \Omega_{\underline{x}} (\subseteq R^n) \to \Omega_{\underline{y}} (\subseteq R^n)$ be a one-to-one transformation and denote by $\underline{g}$ its inverse transformation. Let us define a new n-dimensional random variable by $Y=\underline{f}(X)$.

A possible characterization of a class P of probability distributions can be formulated as follows:

The independent random variables $X_1,\ldots,X_n$ have distributions in P if and only if the random variables Y_1 and $(Y_2,\ldots,Y_n)$ are independent.

In the proof of such characterization theorems functional equations of the form

$$q(\underline{y}) = p(\underline{g}(\underline{y}))\,|F(\underline{y})|\,(\underline{y}\in\Omega_{\underline{y}})$$

can be used, where p and q are the unknown density functions of X and Y respectively, and F is the Jacobian of $\underline{g}$.

In the lecture we studied the cases when P is the class of normal distributions or the class of gamma distributions.

L. LOSONCZI:

Functional equations of sum form

The functional equation

$$(1)\qquad \sum_{i=1}^{k}\sum_{j=1}^{\ell}\Big[f_{ij}(p_iq_j) - \sum_{t=1}^{N} g_{it}(p_i)h_{jt}(q_j)\Big] = 0 \quad ((p_1,\ldots,p_k)\in\Gamma_k,(q_1,\ldots,q_\ell)\in\Gamma_\ell)$$

is investigated, where $k,\ \ell\geq 3$ are fixed integers, $f_{ij}, g_{it}, h_{jt}: [0,1]\to R$ $(i=1,\ldots,k;\ j=1,\ldots,\ell;\ t=1,\ldots,N)$ and

$$\Gamma_n = \{(p_1,\ldots,p_n)\,|\ p_i\geq 0,\ \sum_{i=1}^{n} p_i=1\}.$$

If $f_{i1}, f_{1j}, g_{it}, h_{jt}$ are measurable for all possible values of i,j,t then

(1) is satisfied if and only if

$$(2)\quad \bar{f}_{ij}(pq) - \sum_{t=1}^{N} \bar{g}_{it}(p)\bar{h}_{jt}(q) = 0 \quad (p,q \in [0,1];\ i=1,\dots,k;\ j=1,\dots,\ell)$$

hold, where

$$\bar{f}_{ij}(p) = f_{ij}(p) - f_{ij}(0) + p \sum_{r=1}^{k} \sum_{s=1}^{\ell} f_{rs}(0)$$

$$\bar{g}_{it}(p) = g_{it}(p) - g_{it}(0) + p \sum_{r=1}^{k} g_{rt}(0)$$

$$\bar{h}_{jt}(p) = h_{jt}(p) - h_{jt}(0) + p \sum_{s=1}^{\ell} h_{st}(0) \quad (p \in [0,1], i=1,\dots,k; j=1,\dots,\ell; t=1,\dots,N).$$

Possibilities for solving equation (2) and a method for finding the general solution of (1) are also discussed.

E. LUKACS:

<u>On a problem concerning two differential equations</u>

The following problem arose in connection with a question in probability theory. Suppose that a function $f_1(t)$ satisfies a certain differential equation while a second function $f_2(t)$ satisfies a differential equation whose coefficients are close to the coefficients of the equation satisfied by $f_1(t)$. Are then the functions $f_1(t)$ and $f_2(t)$ close to each other?

In the case considered (the stability of a characterization of the rectangular distribution) $f_1(t)$ satisfies the equation

$$\text{(a)}\qquad t f_1'' + 2 f_1' + t f_1 = 0$$

while $f_2(t)$ satisfies

(b) $(t-\epsilon_1)f_2'' + 2(1-\epsilon_2)f_2' + (t-\epsilon_1)f_2 = 0$, where ϵ_1 and ϵ_2 are small.

D. LUTZ:

An approximation theorem for operator cosine functions

Let X denote a Banach space and B(X) the algebra of bounded linear operators on X. Let C denote a strongly continuous operator cosine function on R with values in B(X) and let A be its infinitesimal generator with domain D(A). Suppose that $||C(t)||\leq 1$ for all $t\in \mathbb{R}$. Then all positive reals are in the resolvent set $\rho(A)$ of A. Let $R(\lambda,A)$ denote the resolvent operator of A. Define for $t\in\mathbb{R}$, $t\neq 0$, $H_m(t)\in B(X)$ by

$$H_m(t)x = [\sum_{j=0}^{m} \binom{2m}{2j}\left(\frac{t}{2m}\right)^{2j}A^j]\,[(\frac{2m}{t})^2R((\frac{2m}{t})^2,A)]^{2m}x,\ x\in X.$$

Then,

$$\lim H_m(t)x = C(t)x$$

for all $x\in X$, and

$$||H_m(t)x - C(t)x||\leq \frac{t^2}{\sqrt{m}}\,||Ax||\ \text{for all } x\in D(A).$$

G. MAKSA:

Some functional equations on rings of sets

Let B be a ring of subsets of a given set. In this talk we present the general solution of the following functional equations (separately).

$$F(x) = F(y)$$

$$F(x,y) = F(x,z)$$

$$F(x\cup y,z) + F(x,y) = F(x\cup z,y) + F(x,z)\ ,$$

holding for all pairwise disjoint $x,y,z\in B\setminus\{\emptyset\}$.

We apply these results for determining the semi-symmetric and recursive inset entropies of all degrees, excluding zero probabilities and empty sets.

L. MIAO:

A family of Dirichlet series with functional equation

We consider the product $\eta_{gh}\eta$ of the generalized Dedekind functions $\eta_{gh}(\tau;N)$ (B. Scheneberg, Elliptic Modular Functions, Springer 1974) and the Dedekind function $\eta(\tau)$, find relations between $\eta_{gh}\eta$ and the division values of the theta function $\theta_1(v|\tau)$, and give Fourier expansions for $\eta_{gh}\eta$. Following Maak, we use the theory of almost automorphic forms, and apply the Mellin transform to $\eta_{gh}(\tau;N)\eta(\tau)$. We obtain then a family of Dirichlet series $\phi(s)$ with the functional equation

$$\Gamma(\tfrac{1}{2}-s)\phi(\tfrac{1}{2}-s) = \Gamma(s)\phi_T(s),$$

where $\phi_T(s)$ is the transformed Dirichlet series of $\phi(s)$ by the modular transformation $T=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$. If we write

$$\phi(s) \sim (2\pi)^{-s} \sum_{\lambda>0} \frac{a(\lambda)}{\lambda^s},$$

we note that the λ-terms are rational numbers and the coefficients $a(\lambda)$ are in the cyclotomic field $Q(e^{2\pi i/N})$. As a special case we have, for N=2, g=h=1, the well-known functional equation for the Riemann ζ-function.

Z. MOSZNER:

Les prolongements des homomorphismes et des solutions de l'équation de translation

On fait quelques remarques au sujet des théorèmes sur le prolongement des homomorphismes d'un groupe à un sur-groupe, donnés par A. Grzaślewicz (Aequationes Math. 17 (1978), 119-207) et on compare ces théorèmes à un théorème sur le prolongement de la solution de l'équation de translation (Z. Moszner, Tensor 26 (1972), 239-242). Ces résultats de K. Dankiewicz et de moi-même paraitront dans la revue

Rocznik Naukowo-Dydaktyczny WSP Kraków.

G. OPRIS:

Certaines équations fonctionnelles en recherche opérationnelle

L'étude de certains processus stochastiques conduit au systeme

$$(1)\quad \begin{cases} P_n(t,u+v) = \sum_{k=0}^{n} P_k(t,u)P_{n-k}(t+u,v);\ n>0 \\ \\ P_0(t,u+v) = P_0(t+u,v)\ . \end{cases}$$

La solution générale s'exprime avec une fonction génératrice $h: R_+ \times C \to C$ par l'expression

$$(2)\qquad P_n(t,u) = H^{-1}\left[\frac{h(t+u,z)}{h(t,z)}\right]$$

ou H est un opérateur satisfaisant à l'équation de convolution

$$(3)\qquad H(f*g) = H(f).H(g)\ .$$

On caractérise ensuite certains processus particuliers par la fonction h. On détermine aussi la solution d'un modele d'attente sans hypothèse de différentiabilité quant aux fonctions.

P. PLAUMANN:

The natural domain of definition of $\phi(x^{\phi(y)}y)=\phi(x)\phi(y)$.

If T is a subset of a group G, we call a map ϕ: $T\to G$ a coupling if: (a) $x^{\phi(y)}y \in T$ and (b) $\phi(x^{\phi(y)}y)=\phi(x)\phi(y)$ for all $x,y \in T$. The functional equation (b) is essentially the equation of Gołab and Schinzel In the present context it was first considered by H. Karzel (Archiv. Math. 16 (1965), 247-256) and employed in the construction of Dickson near-fields.

THEOREM. Let G be a group. There is a bijection ϕ between

(a) the pairs (H,η) where 1) H is a subgroup of G 2) η: $H \to G$ is an antihomorphism 3) $\eta(H) \cap H = 1$ and

(b) the pairs (T,K) where 1) T is a subset of G 2) K: $T \to G$ is a coupling 3) $K(T) \cap T = 1$

This bijection is given by ϕ: $(H,\eta) \to (T,K)$ where $T = \{\eta(h)h \mid h \in H\}$ and $K(t) = \eta(h)^{-1}$ if $t \in T$ has the representation $t = \eta(h)h$, $h \in H$.

F. RADÓ:

On the generalization of Lorentz transformations

Consider in the affine plane K^2 (K being a field) the Lorentz-Minkowski distance of two points $P=(p_1,p_2)$ and $Q=(q_1,q_2)$ defined by $\overline{PQ} := (q_1-p_1)(q_2-p_2)$. Generalizing results of W. Benz the following are obtained.

THEOREM 1. Let char $K \notin \{2,3,5\}$ or $K = GF(5^m)$, $m \geq 2$. Then every map σ: $K^2 \to K^2$ satisfying the condition

$$(1) \qquad \overline{PQ} = 1 \Rightarrow \overline{P^\sigma Q^\sigma} = 1$$

is of the form $\sigma = \tau \circ \hat{f}$, where τ is an affine isometry and $\hat{f}$: $K^2 \to K^2$ is given by $(x_1,x_2) \mapsto (f(x_1), f(x_2))$ with f a monomorphism of the field K.

THEOREM 2. If char $K=5$, then a map σ: $K^2 \to K^2$ satisfying (1) and $\sigma(0,0)=(0,0)$ is an endomorphism of the vector space K^2 over the prime-field K_0 of K.

J. RÄTZ:

A completeness criterion for inner product spaces in terms of orthogonally additive functions

Throughout this note, X denotes a real inner product space with $\dim_R X \geq 2$. A mapping f: $X \to R$ is called orthogonally additive if it satisfies the conditional Cauchy functional equation

(*) $f(x_1+x_2) = f(x_1) + f(x_2)$ for all $x_1,x_2 \in X$ with $x_1 \perp x_2$.

Fact 1: f is a solution of (*) if and only if there exist additive mappings ℓ: R→R and h: X→R with

$$f(x) = \ell(||x||^2) + h(x) \quad (\forall\, x \in X)$$

([2], Corollary 3).

Fact 2: f: X→R is a solution of (*) and bounded below if and only if

(i) f vanishes identically, or

(ii) there exist $c \in R_+^*$ and h: X→R continuous and linear such that

$$f(x) = c||x||^2 + h(x) \quad (\forall\, x \in X).$$

(This was proved independently by P. Fischer (oral communication)).

Using a result of [1] (p.435, Corollary 3.6),we obtain:

THEOREM. For a real inner product space X with $\dim_R X \geq 2$, the following statements are equivalent:

(a) Every solution f of (*) which is bounded below has a minimum on X,

(b) X is a Hilbert space.

References

1. Gudder, S., and Strawther, D., Orthogonally additive and orthogonally increasing functions on vector spaces. Pacific J. Math. 58 (1975) 427-436.

2. Ratz, J., On orthogonally additive mappings. Proceedings of the 18th International Symposium on Functional Equations, Waterloo, Ontario, 1980, pp.22-23.

L. REICH:

Über algebraische Relationen zwischen additiven und multiplikativen Funktionen

Wir nennen eine Funktion f: C→C additiv, wenn sie Lösung der Funktionalgleichung $f(u+v)=f(u)+f(v)$ ist, und eine Funktion $g: C\to C$, $g\neq 0$, multiplikativ, wenn sie Lösung der Funktionalgleichung $g(u+v)=g(u)g(v)$ ist. Es seien $f_1,\ldots,f_n (n\geq 0)$ gegebene additive Funktionen, $g_1,\ldots,g_m (m\geq 0, n+m>0)$, gegebene multiplikative Funktionen. Im Vortrag werden mit Hilfe des Systems der linearen Relationen (über C) zwischen $f_1,\ldots,f_n$ und des Systems der Relationen in der von $g_1,\ldots,g_m$ erzeugten abelschen Gruppe (multiplikativer Funktionen) diejenigen Polynome $R(Y_1,\ldots,Y_n, Z_1,\ldots,Z_m)$ über C charakterisiert, für die

$$R(f_1,\ldots,f_n,g_1,\ldots,g_m) = 0, \text{ d.h., } R(f_1(t),\ldots,f_n(t),g_1(t),\ldots,g_m(t)) = 0 \text{ gilt.}$$

Speziell folgt aus diesen Sätzen :

(1) Die Funktionen $f_1,\ldots,f_n$, $g_1,\ldots,g_m$ sind algebraisch unabhängig über C genau dann, wenn $f_1,\ldots,f_n$ linear unabhängig und $g_1,\ldots,g_m$ unabhängig sind (in der von ihnen erzeugten Gruppe).

(2) Die Funktionen $t\mapsto t$, $t\mapsto e^{\lambda_1 t},\ldots,t\mapsto e^{\lambda_n t}$ sind algebraisch unabhängig genau dann, falls ausedner Relation $\alpha_1\lambda_1+\ldots+\alpha_n\lambda_n=0$ mit ganzen Zahlen stets $\alpha_1=\ldots=\alpha_n=0$ folgt.

M. SABLIK:

Some remarks on the functional equation $2h(x)=h(x+\phi(x))+h(x-\phi(x))$. (Joint work with J. GER.)

The equation $h(x+\phi(x))=h(x)+h(\phi(x))$; where $\phi: [0,+\infty)\to[0,+\infty)$ is a homeomorphism, has only linear solutions under the condition that they are differentiable at 0 (Zdun [1] and Dhombres [2]). We consider the equation

$$(*) \qquad 2h(x) = h(x+\phi(x)) + h(x-\phi(x)) ,$$

assuming moreover that $0<\phi(x)<x$ for $x>0$. Although (*) is similar to the first equation,existence of $h'(0)$ is not sufficient for the uniqueness of solutions. In fact,for every ϕ a convenient example can be constructed. We prove also the following.

THEOREM. If $h'(0)$ and $\lim_{x\to\infty} h(x)/x$ (finite or not) exist, then every solution h of (*) is of the form $h(x)=h'(0)x+h(0)$.

References

1. M.C. Zdun, on the uniqueness of solution of the functional equation $\phi(x+f(x))=\phi(x)+\phi(f(x))$, Aequationes Math. 8 (1972), 229-232.

2. J. Dhombres, Some aspects of functional equations, Chulalongkorn University, Bangkok, 1979.

W. SANDER:

3-symmetric and α-recursive inset measures

Let B be a ring of sets,

$$\Omega_n = \{(x_1,\dots,x_n): x_i \in B,\ x_i \cap x_j = 0 \quad \text{if } i\neq j;\, i,j=1,2,\dots,n\},$$

$$\Gamma_n = \{(p_1,\dots,p_n): \sum_{i=1}^{n} p_i=1, p_i\geq 0; i,j=1,2,\dots,n\} \quad n\geq 2,$$

$$\overset{\circ}{\Gamma}_n = \{(p_1,\dots,p_n): \sum_{i=1}^{n} p_i=1, p_i>0; i,j=1,2,\dots,n\} \quad n\geq 2.$$

An inset measure is a sequence

$$S_n: \Omega_n^k \times \Gamma_n \times (\overset{\circ}{\Gamma}_n)^{m-1} \to R \quad (k=1,2,\dots;\ m,n=2,3,\dots).$$

We present the general form of all symmetric, α-recursive inset information functions and thus the form of all 3-symmetric, α-recursive inset measures, where

$$\alpha = (\alpha_1,\alpha_2,\dots,\alpha_m) \text{ with } \alpha_1 \neq 0.$$

Our result contains some recent results of the mixed theory of information, introduced by Aczél and Daróczy.

For example (in the special case k=1), if $\alpha_1 \neq 1$ or $\alpha_1 = 1$ and $(\alpha_2,\dots,\alpha_m) \neq 0$ the general form of S_n is given by

$$S_n\begin{pmatrix} x_1,\dots\dots,x_n \\ p_{11},\dots,p_{1n} \\ p_{m1},\dots,p_{mn} \end{pmatrix} = g\left(\bigcup_{i=1}^{n} x_i\right) - \sum_{i=1}^{n} p_{1i}^{\alpha_1}\, p_{2i}^{\alpha_2} \dots p_{mi}^{\alpha_m}\, g(x_i),$$

where g is a function from B into R.

W. SCHEMPP:

Euler-Frobenius-Polynome

Die nach Euler und Frobenius benannten Polynome $(p_m)_{m\geq 1}$ wurden von L. Euler 1755 eingeführt und von F.G. Frobenius im Jahr 1910 im Zusammenhang mit den Bernoulli-Zahlen näher untersucht. Frobenius stellte u.a. die Funktionalgleichung

$$h^{m-1} p_m\left(\frac{1}{h}\right) = p_m(h) \qquad (h \neq 0,\ m \geq 1)$$

fest.

Ziel des Vortrags ist es, auf der Grundlage der Theorie der kardinalen Spline-Interpolation für die Polynome $(p_m)_{m\geq 1}$ eine komplexe Kurvenintegraldarstellung (mit nicht kompaktem Integrationsweg) anzugeben und mit dieser einen neuen Zugang zu den Eigenschaften der Euler-Frobenius-Polynome zu eröffnen.

P. SCHROTH:

Principal solutions of difference equations with periodic forcing functions

Concerning the principal solution (cf. [1]) of the difference equation

$$\forall\ x \geq 0:\ \frac{1}{y}\,(g(x+y,y) - g(x,y)) = \phi(x),$$

where $y>0$ and ϕ: $[0,\infty) \to R$ has period $\omega > 0$ and $\phi|_{[0,\omega]} \in L_1([0,\omega])$, the following results hold:

1) If ϕ is not constant a.e., then the principal solution does not exist for $y=k.\omega$, $k \in N_+$.

2) If ϕ is continuously differentiable and for almost all $y>0$ there is a $\delta(y)>0$ with $\int_0^\omega \phi'(t)\exp(-2\pi in\frac{t}{y})dt=O(n^{-\frac{1}{2}-\delta(y)})$, then the principal solution exists for almost all $y>0$.

3) If ϕ is m times continuously differentiable, $m\geq 2$, then the set of those $y>0$, for which the principal solution fails to exist, has Hausdorff dimension less or equal $\frac{4}{2m+1}$.

Reference

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J. SCHWAIGER:

On the matrix equation $A(t+s)=A(t).A(s)$

Regarding the matrix equation

$$A(t+s)=A(t).A(s),\ s,t\in C, A(t)\in Gl(n,C) \tag{1}$$

the following holds:

THEOREM 1. Given nilpotent, upper triangular matrices $C_1(t), C_2(t),\dots,C_r(t)$ with $C_i(t+s)=C_i(t)+C_i(s)$ and $C_i(t)C_i(s)=C_i(s)C_i(t)$ for $s,t\in C$ and $1\leq i\leq r$ and given r non-vanishing generalized exponential functions $f_1,f_2,\dots,f_r$ the matrix function B with $B(t)=B_1(t)\oplus B_2(t)\oplus\dots\oplus B_r(t)$, where $B_i(t)=f_i(t).\exp(C_i(t))$, is a solution of equation (1).

THEOREM. Every solution A of equation (1) is of the form $A=TBT^{-1}$, where T is a constant non-singular matrix and B is a solution of (1) of the form described in Theorem 1.

REMARK. A solution was also given by M.A. McKiernan in [1] and [2], but the proof here is different from that in [1] and [2].

References

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H. SHERWOOD:

<u>Dominates on equivalence classes of operations</u>

Fix a partially ordered set S and let $\mathcal{O}$ be the set of all associative binary operations on S having e in S as an identity. For any H,G in $\mathcal{O}$ we say H <u>dominates</u> G and write H>>G if, for all a,b,c,d in S,

$$H(G(a,b),G(c,d)) \geq G(H(a,c),H(b,d)).$$

The set M of all order-preserving bijections α: S→S with $\alpha(e)=e$ is a group under composition. For H in $\mathcal{O}$ and α in M, define H_α: $S^2 \to S$ via

$$H_\alpha(a,b) = \alpha^{-1}H(\alpha(a), \alpha(b)).$$

Each H_α is in $\mathcal{O}$. For H, G in $\mathcal{O}$ write $H \sim G$ if there is an α in M such that $G=H_\alpha$. Observe that $\sim$ is an equivalence relation and let [H] denote the equivalence class determined by H.

PROPOSITION. $H_\alpha >> H_\beta$ if and only if $H_{\alpha\circ\gamma} >> H_{\beta\circ\gamma}$.

PROPOSITION. "Dominates" is transitive on [H] if and only if, whenever $H_\alpha >> H$ and $H_\beta >> H$ then $H_{\alpha\circ\beta} >> H$.

F. STEHLING:

<u>Stability conditions for special linear difference equation systems</u>

Numerous models in mathematical economics (e.g. in dynamical price theory) lead to systems of first order linear difference equations

(*) $p(t+1) = A.p(t) + b \qquad (t=0, 1,2,\ldots),$

where $p(t)\in R^n_{(+)}$, $b\in R^n_{(+)}$, A a real nxn-matrix. Many conditions are known ensuring the stability of the solutions of equation (*) for a nonnegative matrix A. The following theorem can be proved:

THEOREM. Let $A = (a_{ij})$ be of the type

$$(M)\quad \begin{cases} a_{ij} = d_i.e_j & (i \neq j) \\ a_{ii} = s_i & (i = 1,\ldots,n). \end{cases}$$

(i) Then, if either $d_i.e_i \geq 0$ $(i=1,\ldots,n)$ or $d_i.e_i \leq 0$ $(i=1,\ldots,n)$, all eigenvalues of A are real numbers.

(ii) If additionally $|s_i - d_i.e_i| < 1$ $(i=1,\ldots,n)$, then the conditions

$$\sum_{i=1}^{n} \frac{d_i.e_i}{1-s_i+d_i.e_i} < 1 \quad \text{for } d_i.e_i \geqq 0 \ (i=1,\ldots,n) \quad ,$$

resp.

$$\sum_{i=1}^{n} \frac{-d_i.e_i}{1+s_1-d_i.e_i} < 1 \quad \text{for } d_i.e_i \leqq 0 \ (i=1,\ldots,n)$$

are necessary and sufficient for the eigenvalues of A to be less then 1 in absolu value. This generalizes a result of Hosomatsu (1969) and Neudecker (1970).

L. SZÉKELYHIDI:

Exponential polynomials

Exponential polynomials on Abelian groups are investigated and a number of regulari theorems concerning them are proved. One typical result states that, on a locally compact Abelian group generated by any neighborhood of the origin, the set of all zeros of a nonidentically zero exponential polynomial has measure zero.

G. TARGONSKI:

Directed graphs and the structure of chaos: results of U. Burkart

The pseudostochastic behaviour of iterative sequences of continuous functions that arises under suitable conditions has been termed chaos by Li and Yorke. If one looks more globally on this behaviour, one finds that there exists a macroscopic structure in the chaos that can be obtained by considering directed functional graphs. A graph theoretic condition for chaos to arise is given and connections to other chaos conditions are discussed for continuous functions of R^n to R^n.

M.A. TAYLOR:

Some varieties of quasigroups which are abelian group isotopes

Quasigroups that satisfy balanced identities and are group isotopes have been completely determined by Krapež, and some general types of identities which give rise to group isotopes have been found by the author.

By using the closure condition

$$x_1y_2 = x_2y_1,\ x_1y_4 = x_2y_3,\ x_3y_3 = x_4y_1, \text{ implies } x_3y_4 = x_4y_2,$$

and related theorems, it is possible to identify certain varieties of quasigroups as abelian group isotopes.

The results presented in this paper are generalizations of the following:

THEOREM. Let $(Q,\cdot)$ be a quasigroup which satisfies a balanced identity of the form

$$-xy - = -t\cdot y -$$

or

$$-xy - = -x\cdot t -$$

where t is a subword which does not involve the variable x and – denotes a (possibly empty) string of variables and brackets. Then $(Q,\cdot)$ is isotopic to an abelian group.

P. VOLKMANN:

Bedinguugen, unter welchen $f(x)=x$ die einzige Lösung der Funktional-Ungleichung $f(x+y)\geq f(x)+f(y)$ ist.

Satz 1: Sei A ein Ring Einselement, und für

$$f: A\to R \text{ gelte } f(x+y) \geq f(x) + f(y),\ f(xy) \geq f(x)f(y).$$

Dann ist f additiv und multiplikativ. – Zusatz (M. Rǎdulescu 1980): Im Falle $A=R$ ist $f(x)\equiv 0$ oder $f(x)=x$. –Satz 2: Für $F: R\to R$ gelte $f(x+y) \geq f(x)+f(y)$, $(*)\, f(x^2) \geq f(x)^2$, $f(1)=1$. Dann ist $f(x)=x$. –Statt $(*)$ genügt $f(g(|x|))\geq g(|f(x)|)$ mit einer stetigen, injektiven Funktion $g: [0,\infty)\to[0,\infty)$, für welche $g(0)=0, g(1)=1$ gilt.

M.C. ZDUN:

On a formula connected with regular iteration semigroups

We can prove the following theorem closely related to some results of L. Berg [1] and M. Kuczma [2]

THEOREM. Let f be a continuous function defined in $<0,a>$, strictly increasing in $<0,b>$ with the absolute maximum at b and $0<f(x)<x$, for $x\in(0,a>$. If, in a neighborhood of zero, f fulfills one of the following conditions:

1^o f is convex or concave,

2^o f is of class C^1 and $f'(x)=q+O(x^n)$, $n>0$ and $0<f'(0)=:q<1$,

then there exists the limit

$$(1) \qquad \lim_{n\to\infty}(f_{|<0,b>})^{-n}(q^t f^n(x))=: f_t(x),\ t \geq 0,\ x \in <0,a>$$

and $\{f_t|t>0\}$ is a regular iteration semigroup.

By this theorem we get the following

COROLLARY. If a function f of class C^1 fulfills the assumption of the theorem and $f'>0$ in $\langle 0,b)$, then the equation

$$g^N = f$$

has a unique C^1 solution g in $\langle 0,a\rangle$. This solution is given by the formula (1), where $t=1/N$.

References

1. L. Berg, Iterationen von beliebiger Ordnung. Z. Angew. Math. Mech. 40 (1960), 215-229.

2. M. Kuczma, A remark on commutable functions and continuous iterations. Proc. Amer. Math. Soc. 13 (1962), 847-850.

Problems and Remarks

Remark 1.

If (X,M), (Y,N) are uniform spaces and $(\tilde{X},\tilde{M})$, $(\tilde{Y},\tilde{N})$ are completions therof, $f:X\to Y$ is called Cauchy-regular or a Cauchy morphism if it satisfies one and therefore all of the following equivalent conditions:

(I) f preserves Cauchy filterbases.

(II) f preserves Cauchy nets.

(III) There exists a continuous extension $\bar{f}:\tilde{X}\to\tilde{Y}$ of f.

We then obtain

$$f \text{ uniformly continuous } \not\Longleftarrow\!\!\Longrightarrow f \text{ Cauchy morphism } \not\Longleftarrow\!\!\Longrightarrow f \text{ continuous}$$

and the following decomposition of Heine's theorem into two parts:

$$X \text{ complete, } f \text{ continuous} \Longrightarrow f \text{ Cauchy morphism,}$$

$$X \text{ precompact, } f \text{ Cauchy morphism} \Longrightarrow f \text{ uniformly continuous.}$$

Examples of not necessarily uniformly continuous Cauchy morphisms in connection with topological groups X, X_1, X_2, Y and their respective right (left) uniformities are:

a) The group operation $(x_1,x_2)\to x_1\cdot x_2$ on X.

b) Every continuous biadditive mapping $f:X_1\times X_2\to Y$ where $(Y,+)$ is commutative.

c) Every quadratic functional $q:X\to Y$ with $q(x_1x_2x_3) = q(x_2x_1x_3)$ $(\forall x_1,x_2,x_3\in X)$ where $(Y,+)$ is commutative and uniquely 2-divisible and $y\mapsto \frac{1}{2}y$ is continuous.

(III) now provides extension theorems for these examples of mappings.

References

1. M. Hosszú, A remark on the square norm. Aequationes Math. 2 (1969), 190-193.

2. R. F. Snipes, Cauchy-regular functions. J. Math. Anal. Appl. 79 (1981), 18-25.

(J. RÄTZ)

Remark 2.

A few months ago L. Dubikajtis gave an elegant axiomatic description of a certain geometry; the axioms involve the so called orthogonality function whose first derivative f turns out to satisfy the following functional equation

$$(*) \qquad \frac{f(x)+f(y)}{f(x+y)} + \frac{f(x)-f(y)}{f(x-y)} = 2.$$

The solution of this equation has recently been given by R. Ger under some regularity assumptions on f which are completely natural for the problem. By solving the equation (*) one obtains different geometries determined by the respective orthogonality functions derived in this way.

(M. SABLIK)

Problems 1,2.

Which identities on loops are inherited by isotopic loops? I look for necessary and/or sufficient conditions. It may be necessary that there should be (at least) three variables in the identity (except for trivial identities like xy=x), but this is certainly not sufficient (xx.yz=(xx.y)z is Pickert's counterexample). Or in the theory of webs: some closure conditions are equivalent to an identity of one kind being satisfied on all isotopic loops or to an identity of another kind to be satisfied on (at least) one isotopic loop (and then on all). What distinguishes these two kinds of identities?

(J. ACZÉL)

Remark 3.

Für die Differenzengleichung $g(x+y,y)-g(x,y) = y\,\phi(x)$ sind Hauptlösungsbegriffe von Nörlund (1924), Krull (1949), Schroth (1978) eingeführt worden. L. Büsing hat in seiner Dissertation [Clausthal,1982] bewiesen, daß alle diese Hauptlösungen übereinstimmen, falls die Störfunktion ϕ den Voraussetzungen des Krullschen Existenz-Satzes genügt. Weiter sind in seiner Arbeit eine auf den Krullschen Ergebnissen basierende einheitliche Theorie der von Bendersky untersuchten Gammafunktionen vorgestellt.

(H. H. KAIRIES)

Remark 4.

George M. Bergman has found a discontinuous automorphism ϕ of the field of complex numbers $\mathbb{C}$ which does not take any nonzero real number to a pure imaginary number (to be published in Aequationes Math. as solution to P. 178). So, according to a remark of mine (v. 20(1980), p. 304), there exists a discontinuous function $f:R\to R$ satisfying $f(x+y) = f(x) + f(y)$ for all $x,y\in R$ and $0<f(x)f(\frac{1}{x})\leq 1$ for all $x\neq 0$, answering a question of F. Rothberger (see P 178) in the negative. (W. BENZ)

Remark 5.

The Pexider analogue of the Gołab-Schinzel functional equation easily reduces to the Gołab-Schinzel equation. Namely, if $f(x+yg(x)) = g(x)h(y)$; $x,y\in F$; $f,g,h: F \to R$, where F is a real linear space, then with a,b in R and λ in F,

$$f(x) = aH(x-\lambda)b$$

$$g(x) = aH(x-\lambda) \qquad \text{except if } b = 0 \text{ where } g \text{ is arbitrary}$$

$$h(x) = H(xa)b \qquad \text{except if } a = 0 \text{ where } h \text{ is arbitrary}$$

and

$$H(x+yH(x)) = H(x)H(y).$$

(J. DHOMBRES)

Problems 3,4.

Consider the equation

$$A(x)\cdot A(y) = A(x\cdot y),$$

where $A(x) = (a_{ij}(x))$ is a $n\times n$ matrix with $a_{ij}(x)\in\kappa[\![x]\!]$, i.e. the ring of formal power series in one variable with coefficients of the field κ. If a solution $A(x)$ is supposed to be a triangular matrix, the elements of $A(x)$ are polynomials in x. Is this always true?

Find all vectors of formal power series $\phi(z)\in\kappa[\![z]\!]^n := \kappa[\![z_1,\dots,z_n]\!]^n$ (κ a field) such that

$$\phi(\phi(z)) = \phi(z).$$

(D. GRONAU)

Remark 6.

Let F be a one-way (or two-way) real flow, i.e., a family $\{f_t\}$ of functions indexed by the non-negative reals (or all reals) such that

$$f_s o f_t = f_{s+t}$$

for all indices s,t. For any x in Ran f_o, let p_x, the path or trajectory of x, be the function defined by

$$p_x(t) = f_t(x)$$

for all indices t such that x is in Dom f_t. For any p_x such that all indices t are in Dom p_x, let P(x) denote the set $\{t|t>0,\ p_x(t) = x\}$. A. Sklar has proved the following.

Lemma. If P(x) is not empty, then exactly one of the following three possibilities holds, and each possibility can be realized in an actual flow:

(a) p_x is constant;

(b) p_x is not constant, but P(x) is dense in the non-negative reals;

(c) P(x) contains a least element $t_o>0$.

Theorem. If there is a trajectory p_x such that case (c) of the lemma holds, then for each positive integer n, the function $f_{t_o/n}$ in the flow has infinitely many n-cycles.

(C. ALSINA)

Remark 7.

The problem of constructing a theatre of minimum height in which we can see the screen from any seat leads to the functional equation

(*) $$f(x+1)-f(x) = \frac{1}{x}.$$

The general solution of (*) is

$$f(x) = \psi(x)+p(x),$$

$\psi(x) = \dfrac{\Gamma'(x)}{\Gamma(x)}$, $p(x)$ an arbitrary periodic function with the period 1. From the parameters of the theatre hall we can calculate p.

(I. FENYÖ)

Problem 5.

What is the general solution of

$$f(xf(y)+yf(x)-xy) = f(x)f(y) \ ?$$

(I. FENYÖ)

Problem 6, Remark 8.

Solve the functional equation

$$(1) \qquad f(x+f(x))-f(x) = \phi(x),$$

where $\phi:[a,b]\to R$ is a given continuous function and $\phi(a) = \phi(b) = 0$. We look for a continuous f such that $f(a) = f(b) = 0$ and $x+f(x)$ is increasing on $[a,b]$.

If (1) is solvable for every ϕ which is "small enough" in some neighborhoods of a and b, then it is possible to construct a continuous function F such that

$$F(x+F(x)) = F(x)+F(F(x)) \qquad (x\in R)$$

and F is not linear in any one sided neighborhood of 0. The existence of this function was conjectured by Professors Dhombres and Forti.

(M. LACZKOVICH)

Problem 7, Remark 9.

Soient I, J des sous-ensembles non vides de R^n où J est supposé convexe. Soit $f:I\to J$ une homéomorphie. On definit la moyenne (quasi-arithmétique) associée à f selon $M(x,y) = f^{-1}\left(\dfrac{f(x)+f(y)}{2}\right)$.

Une fonction $g:I\to I$ est dite f-affine s'il existe une nxn-matrice [A] et un vecteur

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a de R^n de sorte que pour tout x de I on ait $f(g(x)) = [A]f(x)+a$.

Une fonction $g:I\to I$ est dite M-affine si pour tous x,y dans I on a $M(g(x),g(y)) = g(M(x,y))$.

On dispose de la caractérisation suivante (cf, [1]), laquelle revient au fond à généraliser le théorème de régularité de Darboux concernant l'équation de Cauchy.

THÉORÈME. Soit I un ouvert convexe non vide de R^n, où $n > 1$, et M la moyenne quasiarithmétique associée a une homéomorphie f de I sur I. Une condition nécessaire et suffisante pour que toute fonction M-affine soit f-affine est que I soit inclus dans un demi plan ouvert, cependant distinct d'une bande ouverte.

Le problème ouvert est d'obtenir une condition nécessaire et suffisante pour que toute fonction M-affine soit f-affine lorsque les ensembles I et J sont plus généraux (par exemple lorsque I est un ouvert connexe).

Reference

1. J. Dhombres, Moyennes. Publ. Sem. Analyse de l'Université de Nantes, Anneé 1982-1983 (à paraître).

(J. DHOMBRES)

Remark 10.

Let $(X,|\cdot|)$, $(Y,|\cdot|)$, $(Z,|\cdot|)$ be normed spaces and let $U\subset X$ with $0\in U$. Denote by $F(U,Y)$ the vector space of all functions $\phi:U\to Y$ and by $Lip(U,Y)$ the subspace of all functions ϕ such that

$$\|\phi\| := |\phi(0)| + \sup_{x\neq\bar{x}} \frac{|\phi(x)-\phi(\bar{x})|}{|x-\bar{x}|} < \infty$$

where the supremum is taken over all $x,\bar{x}\in U$.

Every function $h:U\times Y\to Z$ generates the so called Nemytski operator $N:F(U,Y)\to F(U,Z)$ defined by the formula

(1) $\quad (N\phi)(x) := h(x,\phi(x)), \qquad x\in U.$

Denote by $L(Y,Z)$ the normed space of all linear and continuous mappings $A:Y\to X$. J. Matkowski has proved the following.

THEOREM. *Suppose that* U *is convex and* N *maps* $\mathrm{Lip}(U,Y)$ *into* $\mathrm{Lip}(U,Z)$. *If* N *is a Lipschitz map, i.e. there is an* $\ell \geq 0$ *such that*

$$\| N\phi_1 - N\phi_2 \| \leq \ell \, \| \phi_1 - \phi_2 \| , \qquad \phi_1,\phi_2 \in \mathrm{Lip}(U,Y),$$

then

$$h(x,y) = A(x)y + b(x), \qquad x\in U,\ y\in Y,$$

where $A\in F(U,L(Y,Z))$ *and* $b\in \mathrm{Lip}(U,Z)$. *If moreover* Y *is a Banach space then* $A\in \mathrm{Lip}(U,L(Y,Z))$.

(M. C. ZDUN)

Problem 8.

Let f be a continuous and strictly increasing function in $\langle 0,a\rangle$ and $0 < f(x)<x$ in $(0,a\rangle$. Suppose that f possesses the iteration group $\{f_t \mid t\in(-\infty,\infty)\}$ such that all functions f_t are differentiable at zero and $0 < f'(0) = q < 1$. Does there exist the limit $\lim\limits_{n\to\infty} f^{-n}(q^t f^n(x)) = f_t(x)$ for $x\in\langle 0,u\rangle$ and $t > 0$?

(M. C. ZDUN)

Problem 9.

Suppose that the function $f:R_+\to R$ satisfies the functional equation

$$(1) \qquad f(x + \tfrac{1}{x}) = f(x) + f(\tfrac{1}{x}) \qquad (x\in R_+)$$

I think that the general power series solution of (1) is of the form

$$(2) \qquad f(x) = ax^2 + bx + 2a \qquad (x\in R_+).$$

Under what conditions is (2) the only solution of (1)?

(K. LAJKÓ)

Remark 11.

Studying Schur-convexity and Schur-concavity of t-norms, copulas and triangle functions for probability distribution functions, we have proved the following results:

Theorem 1. Any associative copula is Schur-concave. If T is a strict t-norm, then T is a copula if and only if T is Schur-concave. There are Schur-concave non-strict Archimedean t-norms which are not copulas.

Theorem 2. If T is a non-strict Archimedean t-norm with concave additive generator t, then T is Schur-convex and, therefore, $T(x,y) \leq Max(x+y-1,0)$. $W(x,y) = Max(x+y-1,0)$ is the unique t-norm which is at the same time Schur-convex and Schur-concave.

Theorem 3. The Schur-concavity for triangle functions: $\tau(F,G) \leq \tau(\alpha F+(1-\alpha)G, (1-\alpha)F+\alpha G)$ $(F,G\in\Delta^{+},\alpha\in[0,1])$, holds for any triangle function Π_C or $\tau_{C,L}$ if C is an associative copula and $L\in\mathcal{L}_0$. There exists no copula C such that $\sigma_{C,L}$ is a Schur-concave triangle function.

Theorem 4. If a triangle function τ is convex then $\tau(F,G)(x) \leq Max(F(x) + G(x) - 1,0)$.

(C. ALSINA)

Reports of the Special Sessions

I. Special session: Characterization theorems in geometric algebra

This special session was moderated by F. Radó (Cluj).

Considering a preeuclidean plane over the field K (char K $\neq$ 2), F. Radó has established geometrical conditions for K to be euclidean or pythagorean.

W. Benz (Hamburg) has presented a comprehensive review on the results concerning characterization of Lorentz-transformations in different settings. Minimal conditions and physical aspects were emphasized.

M. A. Taylor (Wolfville) spoke about the identity closure condition for varieties of groupoids with a neutral element and the corresponding general closure conditions. He posed the problem of determining all loop identities such that the general form of the identity closure condition holds in the given loop.

W. Schempp (Siegen) has shown by means of harmonic analysis on the real nilpotent Heisenberg group $\tilde{A}(R^n)$ that the radar ambiguity surface admits the real symplectic group Sp(n,R) as its group of energy preserving linear invariants.

(F. RADÓ)

II. Special session on the theory of the translation equation and on iteration theory

The special session was moderated by Z. Moszner (Kraków). In his introductory remarks he pointed out the fact that the translation equation plays an important role in various branches of mathematics (geometry, real and complex analysis, algebra etc.). He then sketched some results of his school and also some new fields of research on the translation equation: 1) Description of the general solution, 2) continuation of the solution if the semigroup of parameters is extended to a group, 3) generalization of the translation equation (e.g., related to the theory of abstract automata).

G. Targonski (Marburg/L.) recalled some concepts, results and problems of the "New Iteration Theory," e.g., the concepts of chaos, topological entropy, strange attractors, bifurcation, results of Feigenbaum. He presented the following open problems: 1) Invariant curves of self mappings of the plane. 2) The analytic iteration problem for a given homeomorphic self mapping of the plane considered as "inverse problem for autonomous differential systems." 3) Connections between the iterative properties of a continuous real function ω and its substitution operator Ω: $\Omega f = f \circ \omega$.

L. Reich (Graz) reported on the work of the Graz school in iteration theory in rings of power series. The main problem here is now to find a criterion for iterability of formal power series without any regularity conditions on the dependence on the parameter.

(L. REICH)

III. Special session on information measures

This special session was moderated by G. Maksa (Debrecen).

Much of the discussion was about three main topics. The problem of determining all (semi-) symmetric recursive measures of multiplicative type, depending upon several probability distributions has now been completely solved (Aczél-Ng), even when 0-probabilities are excluded.

The same is true for semisymmetric measures (depending only upon one probability distribution, but also upon events, again under exclusion of impossible events and 0-probabilities) recursive of degree a (Maksa, Ebanks), but the problem is still open for recursive inset measures of multiplicative type. The main difficulty lies in the solution, on open domains, of the generalization of the one dimensional fundamental equation of information, containing four unknown functions and an arbitrary multiplicative function. Dr. Maksa has presented his ideas on how to reduce this latter problem.

Finally, Drs. Losonczi (Lagos) and Kannappan (Ramnad) have presented results and open problems (and the reasons for their difficulty) concerning additive and generalized additive measures of sum form (on 'closed' and open domains).

(G. MAKSA)

Compiled by: B. R. Ebanks (Lubbock)

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