MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 47/1982

Mathematical Problems in the Kinetic Theory of Gases

22.11. bis 26.11.1982

Die Tagung wurde geleitet von H. Neunzert (Kaiserslautern) und D. C. Pack (Glasgow). Es wurde eine Vielfalt von Themen behandelt. Von großem Interesse war auch diesmal die Frage nach der Existenz von Lösungen der Boltzmann-Gleichung. Dieses Thema wurde unter mehreren Gesichtspunkten behandelt : Vom räumlich homogenen Fall über Lösungen mit geeigneten Anfangs- und Randbedingungen bis zur gestörten Boltzmann-Gleichung und Modell-Gleichungen hiervon. Es zeigte sich jedoch, daß man von einem allgemeinen globalen Existenzsatz nach wie vor weit entfernt ist.

Weitere Problemkreise waren unter anderen lineare Transportgleichungen (zum Teil mit sehr allgemein formulierten Stoßoperatoren) und Lösungen für das Knudsen-Gas unter allgemeinen Randbedingungen.

Vorgetragen wurde weiterhin über verschiedene Limites bei der Beschreibung von Vielteilchensystemen sowie über Lösungen von speziellen Transportproblemen.

Existenzsätze über die Euler- und Navier-Stokes-Gleichungen einerseits und die Vlasov-Gleichung andererseits bildeten den Abschluß der Tagung, deren Verlauf durch lebhaftes Interesse aller Beteiligten gekennzeichnet war. Dieses Interesse läßt sich unter anderem auch an dem Umstand ablesen, daß der weitaus größte Teil der Anwesenden zu den Vortragenden gehörte.

Welcher Impuls von dieser Tagung auf die Forschung in Deutschland ausgehen kann, zeigt die Tatsache, daß bei 11 deutschen Teilnehmern z. B. 6 Personen aus Italien und ebenso viele aus den USA und Kanada teilnahmen. Erfreulich war auch, daß zwei Wissenschaftler aus Japan die Tagung besuchten und daß ein Wissenschaftler aus der Sowjetunion nach Oberwolfach kommen konnte.

Vortragsauszüge

K. ASANO:

On the Existence of Global Solutions of the Initial Boundary Value Problem of the Boltzmann Equation in a Bounded Domain

Let $f = f(t,x,\xi)$ be the density distribution of gas particles at time t and position $x \in \Omega \subset \mathbb{R}^n$ with velocity $\xi \in \mathbb{R}^n$ $(n \geqq 3)$. If we put $f = g + g^{1/2}u$, $\sigma(\xi) = \exp\{-|\xi|^2/2\}$, then u satisfies the Boltzmann Equation formulated in the Grad Scheme:

$$(1)\quad \begin{cases} \dfrac{\partial u}{\partial t} = -\xi\cdot\nabla_x u + Lu + \Gamma[u,u], \\[2ex] u|_{t=0} = u_0(x,\xi) \quad \text{(the initial data)} \end{cases}$$

where L is the linearized Boltzmann operator and Γ is the modified collision operator. We assume that the scattering factor $q(|\xi-\eta|,\theta)$ is determined by the inverse k-th power intermolecular force $(k \geqq 5)$ with an angular cutoff, and the gas particles are confined in a bounded domain Ω in $\mathbb{R}^n$ with regular reflection boundary condition or diffuse boundary condition:

(2) $\gamma^+u = C(\gamma^-u)$ ... the incoming data determines the outgoing.

The initial boundary value problem (1) - (2) has the unique mild solution in the whole time interval for sufficiently small u_0.

H. BABOVSKY:

The Boltzmann Equation with Stochastic Boundary Conditions

A class of linear boundary conditions R of kinetic theory is defined to which belong all usual deterministic and stochastic boundary conditions. By a transformation solutions for a Knudsen-Gas contained in a bounded convex domain are described as solutions of an equality of the form $\varphi = R\varphi + \varphi_0$. Properties of R are given which guarantee uniqueness of the solution of this equation and for which the special solution $\varphi = \sum_{k=0}^{\infty} R^k\varphi_0$ satisfies the mass conservation law.

A method of Kaniel and Shinbrot is applied to a similar series to construct mild solutions of the non-linear Boltzmann Equation with boundary conditions described above and initial conditions which are bounded by a multiple of a Maxwellian. It is shown that for a modified diffuse reflection law and initial conditions small enough a global existence theorem for the Boltzmann Equation can be proven.

K. BÄRWINKEL:

Gas-Surface-Interaction and Some Related Mathematical Questions

Some questions of importance for the analysis of internal Knudsen flows under stationary boundary conditions are briefly illustrated: 1.) Existence of a stationary state and its determination as the unique fixed point of a suitably defined boundary operator.
2.) Time-dependence of the flow and its convergence to the stationary state.
Moreover, the transfer of a dynamical quantity from a monatomic gas to a solid wall is discussed in its linear dependence on gas-surface interaction via a matrix element of the corresponding scattering operator. Best bounds to such matrix elements are explicitly represented as functionals of the velocity-dependent functions involved. The theory is applied to energy accomodation and the problem of joint bounds is, in principle, solved. Results on the possible further optimization in the case of reflection invariance are also given.

J. BATT:

Stationary Spherically Symmetric Models in Stellar Dynamics

A general existence theorem is proven for stationary spherically symmetric solutions of the Vlasov-Poisson system; this theorem is based on "Jean's Theorem" for which a rigorous proof is given (joint work with E. Horst and W. Faltenbacher).

N. BELLOMO:

The Discrete Velocity Models for Gas Mixtures: Theory and Applications

As it is known [1], by the Discrete Velocity Models in Kinetic Theory, the full nonlinear integro-differential Boltzmann Equation can be replaced by a suitable system semilinear partial differential equations. This communication deals with the mathematical theory for the construction of discrete velocity models for mixtures of various gases. The analysis is the natural extention of a recently proposed plane eight velocity model for binary gas mixtures to be presented by the author to the 13th R.G.D. Conf. As an application a $6 \cdot n$ regular space model is considered for a system constituted by n different gases and a problem of shock wave propagation is mathematically dealt with.
The Illner method [2] for the proof of the global existence of the initial value problem is here considered and applied to the derived class of equations.

[1] H. Cabannes, J. Mech., 17, (1979), 1 - 22

[2] R. Illner, Prep. n.26, Fach. Math., Un. Kaiserslautern, October (1981).

A. V. BOBYLEV:

Nonlinear Boltzmann Equation for Maxwell Molecules

Some years ago the Fourier transformation of the nonlinear Boltzmann Equation (BE) for Maxwell molecules was made and several new results were obtained for this equation. In particular a simple exact solution was found (Bobylev, 1975, Krock and Wu, 1976). Some related results obtained last years are briefly described below.

1. An infinite number of conservation laws and wide classes of special (so-called N-modal) solutions were constructed for nonlinear BE.

2. The high-energy asymptotic theory of the space uniform relaxation was developped. It was shown by a counter-example that the convergence in L_1-metric Hermits (Laguerre) series for the initial distribution function $f(\vec{v},0)$ in general case fails to produce the convergence of the series for the proper solution $f(\vec{v},t)$ of BE with all $t > 0$.

3. The equilibrium solution ρ = const, T = const, $\vec{u} = 0$ for Burnett equations (Maxwell molecules) is instable with respect to the small periodic perturbations with a wave length (period) $l < 2{,}5\, l_o$, where l_o is mean free path.

V. C. BOFFI and G. SPIGA:

Global Existence and Iterative Schemes for the Solution to the Nonlinear Boltzmann Equation

Some problems of existence and uniqueness for the solution to the nonlinear Boltzmann Equation are discussed by meaningful applications of the contracting mapping principle. The new problems arising when either the hypothesis of spatial independence and/or of the $1/|v|$-approximation for the cross sections are relaxed are critically examined, and an approach is illustrated along the way of identifying appropriate iterative schemes for the sought solution.

H. CABANNES:

Global Solution of the Discrete Boltzmann Equation, Cases of the Regular Spatial Models with 12 and 20 Velocities

Although the global existence of the initial value problem for the discrete Boltzmann Equation has been proved by Illner, when the initial densities are small, that same global existence has only been proved, when the initial densities are bounded, for certain models. The regular spatial models are tied to the convex regular polyhedrons and are therefore five in number. The model related to the tetrahedron is without interest, and those related to the hexahedron (cube) and octahedron have been already studied. We prove in this paper the global existence for the two most complex models: those related to the dodecahedron and to the icosahedron.

C. CERCIGNANI:

The Boltzmann-Grad Limit for a Gas of Soft Spheres

A system of soft spheres which can interpenetrate each other and have a certain probability density of scattering each other for each value of the distance of the centers is considered. In the limit when the probability is concentrated on a single distance d (the particle diameter) the traditional hard sphere gas is re-obtained. The factorization theorem of the reduced s-particle distribution function (fixed s) in the limit $N \to \infty$ for a finite mean free path and arbitrarily long times, provided the initial data are in L^1 with finite energy and finite H functional, modulo an uniqueness theorem. The one particle distribution function satisfies, in the limit, a mollified Boltzmann Equation.

T. ELMROTH:

On the Space-Homogeneous Boltzmann Equation with Forces of Infinite Range

In this seminar we present the following results on the space-homogeneous, non-linear Boltzmann Equation for molecular encounters with infinite range forces. For inverse k^{th}-power molecules with $k > 3$, there exists a weak solution which conserves mass, momentum and energy. This can be proved by a weak L^1-compactness property of the solutions in the cut-off case. If $k > 5$ the solution has globally bounded higher moments in time for all moments that exists initially. From studies of the derivative of the H-function for solutions in the cut-off case, we prove that the solution for infinite range forces converges weakly towards the corresponding Maxwellian. As a consequence all moments that exist initially, except the highest one, converge towards the corresponding moments of the Maxwellian. We also prove that the solution has bounded H-function.

W. GREENBERG:

Generalized Kinetic Equations

Boundary value problems are solved for the abstract differential equation $(Tf)' = -Af$ on a Hilbert space H. Here, T and A are assumed self-adjoint (not necessarily bounded) with T injective and A semi-Fredholm. The boundary condition is of the form $Qf(0) = f_o$, for Q a (maximal) projection associated with T. This includes, as special cases, a number of previously considered problems from neutron transport, electron transport, radiative transfer, and gas kinetics.

J. HEJTMANEK:

Asymptotic Behavior of Transport Processes

The asymptotic behavior of the solution of the neutron transport eqution for large times is studied within the framework of the functional formulation of the equation in the Banach lattice $L^1(\Omega\times S)$. Results from the theory of strongly continuous semigroups of positive operators in L^1-lattices are applied. Then two problems are discussed: 1) to find sufficient conditions such that asymptotic part of the spectrum of the semigroup consists of isolated points of finite algebraic multiplicity and 2) to find sufficient conditions such that the asymptotic part of the spectrum of the transport operator has this property. Problem 2 can only be reduced to problem 1, if in the spectral mapping theorem for the exponential function the equality sign is true. For details see our monograph: Spectral Methods in Linear Transport Theory, Kaper, Lekkerkerker, Hejtmanek, Birkhäuser-Verlag.

E. HORST:

Weak p-Solutions of Vlasov's Equation

We present an existence theorem for weak solutions of the initial value problem for the non-linear Vlasov equation.

$$\frac{\partial}{\partial t}\Phi(t,x,v) + v\frac{\partial}{\partial x}\Phi(t,x,v) \pm \int\frac{x-y}{|x-y|^N}\Phi(t,y,w)\ d(y,w)\ \frac{\partial}{\partial v}\Phi(t,x,v) = 0,$$

$\Phi(0) = \varphi$; $x, v \in \mathbb{R}^N$, $t \in I$ (an interval with $0 \in I$).

A weak p-solution is a weakly continuous function $\Phi : I \to L_1(\mathbb{R}^{2N}) \cap L_p(\mathbb{R}^{2N})$ which satisfies the equation in a sort of distribution sense. A typical result is the following for the three-dimensional case:

Theorem. Assume $N = 3$, $\Phi \geq 0$, $\varphi \in L_1(\mathbb{R}^6) \cap L_p(\mathbb{R}^6)$, with $p \geq \frac{3}{11}(4+\sqrt{5})$, $\int v^2\Phi(x,v)d(x,v) < \infty$.

Then there exists a global non-negative p-solution of the initial value problem.

This is an extension of results by Arsen'ev (1975) and Illner & Neunzert (1979).

We have also generalized an explicit example by Kurth (1978) to all dimensions and the case of repulsing forces.

A. E. HURD:

Existence of Time Evolutions of Nonequilibrium States in Statistical Mechanics

Under rather general conditions, and in the context of continuous "infinite system" statistical mechanics we demonstrate the existence of time evolutions μ^t of states μ which are the limits of time evolutions μ_Λ^t of finite volume ensembles μ_Λ, the latter time evolutions being induced by finite volume flows T_Λ^t so that $\mu^t(F) = \mu_{\Lambda(T_\Lambda^{-t}(F))}$. Nonstandard analysis is used in the proof.

E. A. JOHNSON:

Couette Flow in a Strong Internal Field

The simple problem of free-molecular plane Couette flow is considered in the presence of an external force proportional to x, where x is perpendicular to the boundary planes and is measured from the midpoint between boundaries. The exact distribution function is obtained for diffuse-plus-specular reflection of molecules at boundaries. For forces of a strength analogous to those of interest in gas centrifuges, most of the gas in a given half-space is predicted to flow with a velocity essentially that of the nearest boundary plane.

H. G. KAPER:

Sturm-Liouville Eigenvalue Problems with Indefinite Weights Suggested by Transport Theory

Some standard problems of linear transport theory lead to differential equations of the form (*) $\frac{d}{dt} T\Phi(t) + A\Phi(t) = 0,\ t \in \Delta \subset \mathbb{R}$, for a vector-valued function $\Phi:\Delta \longrightarrow H$. Here, H is some (weighted) Hilbert space; T and A are selfadjoint operators on H; T is injective and A is positive or positive-definite. In our monograph [H.G.Kaper, C.G.Lekkerkerker, J.Hejtmanek, "Spectral Methods in Linear Transport Theory", Birkhäuser, Basel, 1982]we have given extensive discussions of this type of problems, where A is a compact perturbation of the identity operator in H.
Lately, we have become interested in problems of the type (*) where A is unbounded. In our talk we shall illustrate some of our results, using the simple example $H = L^2(-1,-1)$, $Tf(x) = \mathrm{sgn}(x)f(x)$, $Af(x) = -f''(x)$ on dom $A = \{f \in H\colon f'$ absol. cont., $f'' \in H,\ f(\pm 1) = 0\}$. In particular, we shall prove the following half-range completeness theorem: The restrictions of the eigenfunctions of $A^{-1}T$ associated with positive eigenvalues to the interval $(0,1)$ form a basis in $L^2(0,1)$.

I. KUŠČER:

Properties of Integral Transport Operators

A convex uniform body is considered, and the time-independent neutron distribution sought from the integral transport equation. After the general properties of the scattering operator $\hat{K}$ are listed, the integral transport operator $\hat{G}\hat{K}$ is investigated in the space C of bounded continuous functions, in a weighted L_2-space, and in the space $\mathcal{M}$ of measures. An attempt is made to show that the spectral radius $\mathcal{R}(\hat{G}\hat{K})$ is the same in all three spaces and that it is < 1, whichassures the convergence of iterative solutions. On the other hand, the norm $\|\hat{G}\hat{K}\|$ equals 1 in C-space and may exceed this bound in L_2 and $\mathcal{M}$-spaces. The operator $\hat{K}\hat{J}\hat{G}\hat{K}$, where $\hat{J}$ reverses the neutron velocity, is selfadjoint in L_2. This is used to establish a well known variational method.

H. LANG:

Some Analytical Results for Thermal Transpiration and Mechanocaloric Effect in a Cylindrical Tube

Analytical expressions for the phenomenological coefficients for thermal transpiration and mechanocaloric effects in a cylindrical tube are obtained by using the BGK model, and extensions of some previous work on Poiseuille flow and thermal creep flow. Some minor algebraic oversights of the previous work are corrected, and some new results for the creep flow and the heat transfer are reported. Newman iteration and variational techniques are used in the near free molecular regime and the method of elementary solutions is used in the near continuum regime. It is found that the analytical expressions provide results that are in remarkable agreement with the reported numerical results for all the phenomenological coefficients.

C. G. LEKKERKERKER:

Operators with Multiple Spectra

The multigroup approximation in linear transport theory leads to an equation involving an operator with a multiple spectrum. The operator has the form AT^{-1}, where T is the multiplicative coordinate operator in some space $H = L^2(-1,1)^n$ and A is a perturbation of a diagonal operator in H. It is a notoriuosly difficult operator to analyze, even in the case that scattering is isotropic; the analysis is hampered by the nonuniformity of the multiplicity of the spectrum of AT^{-1}. The nonuniformity reflects itself in pecularities of the full-range expansion formula relative to the operator AT^{-1}. We discuss the non-uniqueness of the coefficient function appearing in this formula, the inversion of the formula, and the interpretation of the formula as an eigendistribution expansion.

C. MARCHIORO:

Euler and Navier-Stokes Equations, Vortex Theory and Mean Field Limit

We consider the Euler equation for an incompressible fluid in $D \subset \mathbb{R}^2$. We show that such equation may be interpreted as a mean field equation (Vlasov-like limit) for a system of particles, called vortices, interacting via a logaritmic potential. More precisely, we chose a vortex system which approximates for large numbers a smooth fuction f. Then we solve this system via suitable hamiltonian equations; we obtain a good approximation for f(t), where f(t) is the solution of the Euler equation with initial datum f. We prove a similar result for Navier-Stokes equation in $\mathbb{R}^2$, when the vortices move in presence also of a brownian motion.

J. MIKA and A. PALCZEWSKI:

Application of the Asymptotic Expansion Method for Singulary Perturbed Equations of the Resonance Type in the Kinetic Theory

The singularly perturbed system of ordinary differential equations are treated by the asymptotic expansion method in two forms: standard and newly developed by the authors. Both procedures are applied to the model equations of the kinetic theory and shown to be related to the Hilbert and Chapman-Enskog expansions.

M. OBERGUGGENBERGER:

Propagation of Singularities for Semilinear Mixed Hyperbolic Systems

We consider semilinear strictly hyperbolic $(n \times n)$-systems

$$A_o(x,t)\partial_t u(x,t) + A_1(x,t)\partial_x u(x,t) = f(x,t,u(x,t))$$

on $x > 0$, $t > 0$, A_o, A_1, and f smooth, with initial data prescribed along the x-axis and (possibly nonlinear) boundary conditions prescribed along the t-axis. Given that the initial data are piecewise C^k, $k \geq 0$, or piecewise in the Sobolev space H^k, $k \geq 1$, we investigate the degree of regularity and location of singularities of the solution u. In particular, we derive conditions on the strength of a singularity after reflection at the boundary. We construct a general class of Banach spaces B of distributions, which includes the usual Sobolev spaces, and show that for semilinear (2×2)-systems with initial data piecewise in such a space B, regularity (measured in terms of the spaces B) is propagated along characteristics and reflected at the boundary.

V. PROTOPOPESCU:

Half-Range Completeness for the Fokker-Planck Equation

The stationary one dimensional Fokker-Planck equation is considered on the line $[0,\infty)$ with generalized boundary conditions

$$B_\alpha f(0,v) = f(0,v) - \alpha f(0,-v) \qquad v > 0;\ 0 \leq \alpha \leq 1 \ .$$

The problem is cast in the abstract form

$$T\frac{\partial f}{\partial x} = Af + S$$

and existence, uniqueness is proved for $\alpha \neq 1$. For $\alpha = 1$, the existence is assured under a supplementary condition on the source, but the solution is determined only up to an additive constant. This solves also the half-range completeness problem for the Fokker-Planck operator.

M. PULVIRENTI:

Propagation of Chaos for some Non-Linear Parabolic Equations

Consider a parabolic equation of the following kind

$$\frac{\partial p}{\partial t} + \operatorname{div} \underline{u}\, p = \frac{1}{2}\sigma^2 \Delta p \qquad 1)$$

where

$$\underline{u}(x,t) = \int \underline{k}(x,x')\, p(x',t)dx' \ , \quad \sigma(x,t) = \int \sigma_o(x,x')p(x',t)dx'$$

$x,\ x' \in \mathbb{R}^d$, σ_o, $\underline{k} \in C^1(\mathbb{R}^d \times \mathbb{R}^d)$, $\underline{k}$, $\underline{u} \in \mathbb{R}^d$.

For such an equation it can be proved the "propagation of chaos" in the following form. Let $x_1^N(t) \ldots x_N^N(t)$ be an $\mathbb{R}^{Nd}$-valued stochastic process, solution of the differential stochastic equation

$$dx_i^N(t) = \frac{1}{N}\sum_{j=1}^N \underline{k}(x_i,x_j)dt + \frac{1}{N}\sum_{j=1}^N \sigma(x_i,x_j)db_i(t)$$

where b_i are N independent standard brownian motions. Suppose the process $x_1^N \ldots x_N^N$ to start almost surely from the points $x_1 \ldots x_N$ at time zero.

Then if $\frac{1}{N}\sum_{i=1}^N \delta_{x_i}(dx) \longrightarrow q(x)dx$ weakly, for $N \to \infty$, then the stochastic measures $\frac{1}{N}\sum_{i=1}^N \delta_{x_i^N(t)}(dx)$ converge weakly, with probability one, to $p(x,t)$ solution of 1) with initial condition $q(x)$.

The above theorem is interesting for fluid mechanical purposes. For instance if σ = const., $d = 2$, and $\underline{k}(x-y)$ is a mollified version of $(\underline{\nabla}^{\perp} g)(x-y)$, where $\nabla^{\perp} = (\frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_1})$ and g is the fundamental solution of the Poisson equation, then 1) is a (converging when mollifier is removed) approximation of the Navier-Stokes, and the above theorem provides a converging finite dimensional algorithm.

R. RAUTMANN:

On Optimum Regularity of Navier-Stokes Solutions at time $t = 0$.

Recently John Heywood and Rolf Rannacher have drawn attention to a compatibility condition, which is necessary, if we will get a very smooth local Navier-Stokes solution. They pointed out, that because of its non-linear and non-local nature, "this condition is virtually uncheckable for given data".

This talk gives an answer to the question, how smooth a Navier-Stokes solution can be at $t = 0$ without such compatibility condition: By A we denote Friedrichs' selfadjoint extension of the Stokes operator $-P\Delta$ considered in a smoothly bounded domain $\Omega \subset \mathbb{R}^3$, P being Weyl's orthogonal projection of $L^2(\Omega)$. For any $\alpha \geq 0$, $D_A\alpha$ stands for the domain of definition of the fractional power A^α, $D_A\alpha$ being equipped with the usual graph norm. For the local strong solution $u(t)$ of the Navier-Stokes evolution equation $(\partial_t + A)u = P(f - u\nabla u)$, $u(0) = u_o$ on a time interval $[0,T]$ we prove the

Theorem: (a) Assume $u_o \in D_A1+\varepsilon$, $\varepsilon \in [0,\frac{1}{4}]$, $Pf \in C_\mu([0,\infty),D_A\varepsilon)$ being strongly Hölder-continuous with exponent $\mu > 0$. Then $u \in C_o([0,T],D_A1+\varepsilon)$ holds.
(b) Assume $u \in C_o([0,T],D_A1+\varepsilon)$, $\varepsilon > \frac{1}{4}$, $f \in C_o([0,T],L^2(\Omega))$. Then the compatibility condition holds on $\partial\Omega$ in the sense of $H_{2\varepsilon - \frac{1}{2}}(\partial\Omega)$.

J. SCHRÖTER:

Comments on the Grad Procedure for the Boltzmann Equation

Grad's procedure can be characterized as an expansion of the molecular density f in terms of Hermite functions ϕ^n. One can expand about global and about local equilibrium. Here the first type of expansion is studied. Inserting $f = \sum \frac{1}{n!} (\phi^n,h^n)$ (*) into the Boltzmann equation (BE) one arrives at the so-called Hermits equations of transfer (HET):

$$\partial_t h^n + \nabla\cdot h^{n+1} + n(\nabla-K) \vee h^{n-1} = \sum (C^n_{lk},h^l \otimes h^k) .$$

This system of equation is truncated in Grad's procedure by setting $h^l = 0$, $l > k$ for a fixed k, and in addition by introducing a socalled closing relation $h^k = F^k(h^o,\ldots,h^{n-1})$. It is possible to choose $F^k = 0$. This procedure can be made rigorous for Maxwellian molecules on the stage of the HET because in this case in the collision term of HET the constraint $k + l = n$ occurs. Then the HET can be solved recursively and there are operators Q^n such that each h^n can be written in the form $h^k = Q^n(w^1,\ldots,w^{n-1}) + w^n$, $h^o = w^o$ and $\nabla\cdot w^n = 0$, $n > 1$. In addition one can prove that the recursion is possible if $w^i \in C^\infty$. This theorem also justifies the use of the closing relations. The problem whether the described procedure is compatible with BE can not be solved completely, but one can give sufficient conditions on the solutions of HET such that (*) is a classical solution of BE.

M. SHINBROT:

Some Candidates for Global Solutions of the Boltzmann Equation

There are a number of sequences which, if they converged appropriately, would converge to a global solution of the Boltzmann equation. Some of these are discussed, and it is even shown that there is a sequence which converges if and only there is a global solution of the equation.

H. SPOHN:

Vlasov Limit for the Equilibrium Distribution of Classical Particles

Solutions of

$$[- v\frac{\partial}{\partial q} - \int dq'dv' \ \rho(q',v') \ F(q-q')\frac{\partial}{\partial v} + \frac{\partial}{\partial v}v + \frac{1}{\beta}\frac{\partial^2}{\partial v^2}]\rho(q,v) = 0$$

are considered with $(q,v) \in \Lambda \times \mathbb{R}^d$, $\Lambda \subset \mathbb{R}^d$ bounded and connected, $\partial\Lambda$ smooth, $F = -\text{grad}\, V$ and $V \in C^\infty$. The set of solutions are exactly of the form $\rho(q,v) = \rho(q)\ (\beta/2\pi)^{d/2} \exp(-\frac{1}{2}\beta v^2)$ with $\rho(q)$ stationary point of the free energy functional

$$L^1 \cap L^\infty \ni \rho \to F(\rho) = \frac{1}{2}\int_\Lambda dq' \int_\Lambda dq \ V(q-q')\rho(q)\rho(q') + \beta \int_\Lambda dq\rho(q) \ \log\rho(q) \quad ,$$

$\rho(q) \geq 0$, $\int dq\rho(q) = 1$. In a finite particle approximation one considers the probability measure $Z^{-1} \ \exp[-\frac{\beta}{2N}\sum_{i\neq j=1}^{N} V(q_i - q_j)]dq_1\ldots dq_N$ on Λ^N and induces by $(q_1,\ldots,q_N) \to \frac{1}{N}\sum_{j=1}^{N}\delta_{q_j} \in M$, the space of probability measures on Λ, the probability measure $\nu^{(N)}$ on M. Jointly with Messer it is shown that any weak*-limit ν of $\{\nu^{(N)}|N = 1,2,\ldots\}$ has a support which is contained in set of absolute minima of F. For the example $\Lambda = [-\pi L,\pi L]$, $V(q) = -\cos q$ the stationary points of F are discussed.

L. TRIOLO:

Hydrodynamical Limit for Infinite Systems (One-dimensional Harmonic Oscillators)

Many efforts have been recently spent on the rigorous foundations of the hydrodynamic limit.

This programm has been successful for stochastic models of interacting particles and for essentially free deterministic systems. Here some results on a deterministic system of harmonic oscillators are presented. In the hydrodynamic limit are found conservation laws related to the (infinite) stationary states of the system.

S. UKAI:

Stationary Solutions of the Boltzmann Equation

We discuss the exterior problem for the nonlinear Boltzmann equation which describes a gas flow having a prescribed constant velocity $c \in \mathbb{R}^n$ at infinity and passing by an obstacle $O \subset \mathbb{R}^n$, and specifically show for $n \geq 3$ that if c is small, then stationary solutions exist and are asymptotically stable in time. The domain O is assumed to be bounded and convex with piecewise smooth boundary ∂O, the intermolecular potential employed is cutoff hard potential of Grad, and the assumption on the boundary condition at ∂O is general enough to include the specular, reverse and diffuse reflection laws of gas molecules by the wall ∂O.

H. D. VICTORY:

On the Development of a Convergence Theory for Multigroup Methods in Steady-State Linear Transport

The stability, convergence, and consistency properties of the steady-state multigroup model are investigated for planar and higher dimensional media. These concepts are investigated in an L^1-setting, in which the norm of the angular flux turns out to be the collision density integrated over phase space. Ideas are indicated for extending the L^1-analysis to a function space setting where the norm of the angular flux is the maximum, over position and angle, of the energy-integrated angular flux. Such a Banach space setting is motivated by the use of the multigroup method in shielding analyses. Convergence of the approximates is assured when the maximum fluctuations in the total cross section, and in the expected number of secondary particles arising from each energy level, tend to zero as the energy mesh becomes finer.

J. VOIGT:

Boundary Conditions Generating a Time Evolution

We present some boundary conditions for which the initial boundary value problem for the "free streaming equation" $\frac{\partial f}{\partial t}(t,x,\xi) = -\xi\cdot\mathrm{grad}_x f(t,x,\xi)$ $(t \geq 0,\ x \in D = \mathring{B} \subset \mathbb{R}^n,\ \xi \in \mathbb{R}^n\setminus\{0\})$ is solvable as an evolution equation $f' = Tf$ in $L_1(D \times V)$. An operator T is associated with the differential expression $-\xi\cdot\mathrm{grad}_x$ and the boundary condition.

The closure of T is the generator of a strongly continuous semigroup in the following cases: 1) specular reflection; 2) Maxwell's boundary condition with constant temperature; 3) Maxwell's boundary condition with variable temperature and accomodation coefficient one; 4) Maxwell's boundary condition with temperature bounded from above and below, if D satisfies an additional condition.

J. WICK:

Some Remarks to the Carlemann Model

Die Funktionen

$$u = \frac{1}{2}\,\frac{t+x}{\alpha t^2+(1-\alpha)x^2}\,\chi_{(-t,t)}(x), \qquad v = \frac{1}{2}\,\frac{t-x}{\alpha t^2+(1-\alpha)x^2}\,\chi_{(-t,t)}(x),$$

wobei $\chi_{(-t,t)}(x)$ die charakteristische Funktion des Intervalls $(-t,t)$ bezeichnet, genügen für jedes $\alpha > 0$ dem verallgemeinerten System der Carlemann-Gleichungen

$$\int_0^\infty \int_{-\infty}^\infty (\varphi_t u + \varphi_x u + \varphi(v^2 - u^2))\,dx\,dt + \int_{-\infty}^\infty \varphi(0,x)u_0(x)\,dx = 0$$

$$\int_0^\infty \int_{-\infty}^\infty (\varphi_t v - \varphi_x v - \varphi(v^2 - u^2))\,dx\,dt + \int_{-\infty}^\infty \varphi(0,x)v_0(x)\,dx = 0$$

für alle $\varphi \in C_0^1(\mathbb{R}^2)$.

Dabei sind die Anfangswerte

$$u_0(x) = v_0(x) = \gamma\delta(x) \text{ mit } \gamma = \begin{cases} (\alpha(\alpha-1))^{-1/2}\operatorname{Artgh}\sqrt{\frac{\alpha-1}{\alpha}} & \alpha > 1 \\ 1 & \alpha = 1 \\ (\alpha(\alpha-1))^{-1/2}\operatorname{arctg}\sqrt{\frac{1-\alpha}{\alpha}} & 0 < \alpha < 1 \end{cases}$$

W. WIESER:

An Existence Theorem for a Perturbed Boltzmann Equation

Motivated by the difficulties that arise from the existence problem for Boltzmann's equation (BE), when the initial value f_0 is restricted only by the natural assumptions

(1) $f_0 \gtrsim 0$; $(1 + v^2)f_0$, $f_0 \log f_0 \in L^1(\Omega \times \mathbb{R}^3)$,

global existence of a nonnegative solution is shown for the perturbed Boltzmann Equation

$$f_t^\varepsilon + v\cdot\nabla_x f^\varepsilon - \varepsilon(1 + v^2)\Delta_x f^\varepsilon = Qf^\varepsilon, \quad f^\varepsilon|_{t=0} = f_0$$

with periodic boundary conditions in the space variable $x \in \mathbb{R}^2$ and f_0 satisfying (1). The theorem is proved by first constructing a sequence (f^n), solutions of "approximating" equations. f^ε then results as the limit of a subsequence by a compactness argument. The latter uses only the perturbed analogues of the moment and entropy estimates. Within the scope of the weakly nonlinear theory of (BE), the perturbation is shown to be a regular one: in this case the f^ε converge to f, the solution of

(BE). (Of course, the usual smoothness and smallness of f_o has to be assumed here as well.)

T. YTREHUS:

Asymmetries in Evaporation and Condensation Half-Space Problems

It has been assumed in the past that linear evaporation and condensation half-space problems are, in a certain gaskinetic sense, antisymmetric to each other. As a result of this assumption, the solution to the condensation problem obtained so far are compatible only with very special physical properties of the substance. In this communication it is demonstrated that linear evaporation and condensation are indeed asymmetric phenomena from the gaskinetic point of view in the general case, and that the solution to the condensation half-space problem depends upon an additional substance parameter which does not enter into the evaporation problem. Only for one exceptional value of this parameter are the two problems antisymmetric to each other. At all other values does the condensation solution present features that are distinct from evaporation. The analysis is based upon the linearized Maxwell-Boltzmann transport equation for a monatomic gas, and the results furthermore indicate non-existence of solutions to the steady condensation half-space problem if the substance parameter is below a certain critical value.

F. ZWEIFEL:

An Abstract Version of the Transport Equation

Consider an equation of the form

$$h(\mu)\,\frac{\partial f}{\partial x}(x,\mu) + Af(x,\mu) = q(x,\mu)$$

where $x \in [a,b] \subset \mathbb{R}$, $\mu \in S$. S is an arbitrary set equipped with a measure dm, and $f \in L^2(S,dm)$. $h \in L^\infty(S,dm)$ and $A: L^2(S,dm) \to L^2(S,dm)$ is a positive operator. $q \in L^2(S,dm)$ is a given function. We consider the boundary conditions required for Eq. (1) to be well posed and prove existence and uniqueness of solutions. If $h(\mu) \neq 0$, $\mu \in S$, Eq. (1) is a standard semi-group problem. However, if the set $S^o = \{\mu \in S \mid h(\mu) \neq 0\} = \phi$, "forward-backward" boundary conditions are required, and existence theory for Eq. (1) is a generalization of so-called "partial-range" completeness theorems of transport theory.

Berichterstatter: H. Babovsky

Liste der Tagungsteilnehmer

Prof. K. Asano	Inst. of Math., Osaka City University, Sugimoto-cho, Sumiyoshi-ku OSAKA 558, Japan
H. Babovsky	Fachbereich Mathematik, Univ. Kaiserslautern Postfach 3049, 6750 KAISERSLAUTERN
Prof. Dr. Bärwinkel	Phys. Inst., Univ. Osnabrück 4500 OSNABRÜCK
Prof. Dr. J. Batt	Math. Inst., Univ. München, Theresienstr. 39, 8000 MÜNCHEN
Prof. Bellomo	Istituto Physica, Università die Torino, 10100 TORINO, Italien
Prof. A. W. Bobylev	The Keldish Inst. of Appl. Math., Ac. of Sc. Miussakaya Sq. 4, I 25047, MOSCOW A - 47, USSR
Prof. V. Boffi	Laboratorio Ingegneria Nucleare, Via dei Colli, 16 40136 BOLOGNA, Italien
Prof. Dr. H. Cabannes	Mécanique Théorique, Université Paris VI, Tour 66, 4 Place Jussieu, 75230 PARIS cedex 05, Frankreich
Prof. C. Cercignani	Istituto di Matematica, Politecnico di Milano, Piazza Leonardo da Vinci, 20133 MILANO, Italien
Dr. B. Cole	Dpt. of Math., Univ. of Strathclyde, Livingstone Tower, Richmond Street, GLASGOW 1xH, Schottland
Dr. T. Elmroth	Dept. of Math., Chalmers Univ. of Technology, Fack, 40220 GÖTEBORG, Schweden
Prof. W. Greenberg	Istituto Matematico "U.Dini", Univ. degli Studi, Viale Morgagni 67/A, 50134 FIRENZE, Italien
Prof. Dr. J. Hejtmanek	Inst. f. Math., Universität Wien, Strudlhofgasse 4, 1090 WIEN, Österreich
Dr. E. Horst	Math. Inst., Univ. München, Theresienstr. 39 8000 MÜNCHEN
Prof. A. E. Hurd	Dept. of Math., Univ. of Victoria, P. O. Box 1700, VICTORIA, B. C., V8W 2Y2, Kanada
Dr. E. A. Johnson	Dept. of Physics, Univ. of Surrey, GUILDFORD Gu25XH, England
Prof. Dr. H. Kaper	Argonne Nat. Laboratory, 9700 South Cass Avenue, ARGONNE, Ill. 60439, USA
Prof. Dr. I. Kuščer	Univerza v Ljubljani, VTO ODDELEK ZA FIZIKO, Jadranska c. 19, 61001 LJUBLANA, Jugoslawien
Dr. H. Lang	Max-Planck-Inst. für Strömungsforschung, 3400 GÖTTINGEN

Prof. C. G. Lekkerkerker	Math. Inst., Univ. of Amsterdam, Roettersstraat 15, AMSTERDAM, Niederlande
Prof. C. Macchioro	Dipartimento di Matematica, 38050 (Povo) TRENTO, Italien
Prof. Dr. J. Mika	Fachbereich Mathematik, Univ. Kaiserslautern, Postfach 3049, 6750 KAISERSLAUTERN
Prof. Dr. H. Neunzert	Fachbereich Mathematik, Univ. Kaiserslautern, Postfach 3049, 6750 KAISERSLAUTERN
Dr. M. Oberguggenberger	Inst. f. Math. und Geom., Univ. Innsbruck, Technikerstr. 13, INNSBRUCK, Österreich
Prof. D. C. Pack	Dept. of Math., Livingstone Tower, 26 Richmond Street, GLASGOW G1 1XH, Schottland
Dr. A. Palczewski	Wydziat Matematizki, Univ. Warszawski, Palac Kulturz i Nauki, Pietro 9, 00-901 WARSZAWA, Polen
Dr. V. Protopopescu	Inst. Reactor Physics, Chalmers Univ. Tech., 41296 GÖTEBORG, Schweden
Prof. M. Pulvirenti	Istituto Matematico dell', Universita di Roma, 00100 ROMA, Italien
Prof. Dr. G. V. Ramanathan	Dept. of Math., University of Illinois, Box 4348, CHICAGO, Ill. 60680, USA
Prof. Dr. R. Rautmann	Fachbereich Mathematik, GH Paderborn, Warburgerstr. 100, 4790 PADERBORN
Prof. M. Shinbrot	Dept. of Math., Univ. of Victoria, P. O. Box 1700, VICTORIA, B. C., V8W 2Y2, Kanada
Prof. Spiga	Laboratorio Ingegneria Nucleare, Via dei Colli, 16 40136 BOLOGNA, Italien
Dr. H. Spohn	Dept. of Math., The State Univ. of New Jersey, NEW BRUNSWICK, N. J. 09803, USA
F. J. Schwarz	Fachbereich Mathematik, Univ. Kaiserslautern, Postfach 3049, 6750 KAISERSLAUTERN
Prof. Dr. J. Schröter	Fachbereich Physik, GH Paderborn, Warburgerstr. 100, 4790 PADERBORN
Dr. L. Triolo	Istituto di Matematica Applicata, Via A. Scarpio, 00161 ROMA, Italien
Prof. S. Ukai	Dept. of Appl. Math., Osaka City Univ., Sugimoto-cho, Sumiyoshi-ku, OSAKA 558, Japan
Dr. H. D. Victory	Texas Tech. Univ., Dept. of Math., Box 4319 LUBBOCK, Texas 79409, USA
Dr. J. Voigt	Math. Inst., Univ. München, Theresienstr. 39, 8000 MÜNCHEN

Prof. Dr. J. Wick	Fachbereich Mathematik, Univ. Kaiserslautern Postfach 3049, 6750 KAISERSLAUTERN
W. Wieser	Inst. f. Angew. Math., Universität Bonn Wegelerstr. 5300 BONN
Dr. T. Ytrehus	Appl. Mechanics, The Norwegian Inst. of Tech., 7034 TRONDHEIM-NTH, Norwegen
Prof. Dr. P. Zweifel	Virg. Polytechnic Inst. and State Univ., BLACKSBURG, Virginia 24061, USA

Gefördert durch
DFG Deutsche Forschungsgemeinschaft