MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 28/1983

Maßtheorie

26.6. bis 2.7.1983

Die Tagung, an der 51 Wissenschaftler aus 18 Ländern teilnahmen, stand unter der Leitung von D. Kölzow (Erlangen) und D. Maharam-Stone (Rochester). In ihrem Verlauf wurden insgesamt 41 Vorträge gehalten; daneben gab es noch zwei "Problem Sessions"

Es ist geplant, einen Tagungsbericht zu veröffentlichen, wenn möglich wieder in den 'Lecture Notes in Mathematics' des Springer-Verlages.

Die Tagungsteilnehmer möchten sich an dieser Stelle beim Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. Barner, und seinen Mitarbeitern für die große Unterstützung bedanken, die den erfolgreichen Verlauf der Tagung möglich machte.

Vortragsauszüge

Allgemeine Maßtheorie

J.P.R. CHRISTENSEN

Critical exponents in the classical theory of moments

Let P be the space of all polynomials.

The main result was:

For each $\lambda \in [1,2]$ there exists a probability measure u on $\mathbb{R}$, with moments of all orders such that

for $1 \leq p < \lambda$ P is dense in $L^p(u)$, and

for $p > \lambda$ P is not dense in $L^p(u)$.

That is, each such λ may occur as a critical exponent.

Several ralated problems were discussed, in particular whether λ can be strictly larger than 2.

P. ERDÖS

Combinatorial and geometric problems in measure theory

Let $x_1 > x_2 > \dots > x_n$ be a sequence of positive numbers tending to zero. Is is true that there always is a set E of positive measure which does not contain a sequence similar to the given sequence?

A reward of 100 Dollars was offered for a proof or a disproof.

Is it true that there is an absolute constant C such that every plane set E of (finite) measure bigger than C contains the vertices of a triangle which has area 1? If E has infinite measure the result is easy.

Let C be given and $r > r_o(c)$. Let E be a subset of the circle of radius r with measure bigger than $c \cdot r^2$. Is it then true that E contains the vertices of an equilateral triangle of sidelength bigger than 1?

Perhaps Furstenberg proved this. Straus further asked: Does the conclusion remain true if we assume that the measure of E is larger than $r \cdot f(r)$, e.g. for $f(r)=c \cdot r$?

Székely, a young Hungarian Mathematician conjectured that if E is a set such that the intersections of E with a circle of radius r has measure bigger than $c \cdot r^2$ for all $r > r_0$ then the set E realizes all sufficiently large diameters (i.e. for each positive d there are point z_1, z_2 in E with distance d) ?

W.F. PFEFFER

(reporting on joint work with R.J. GARDNER)

When Radon measures are saturated

Let X be a Hausdorff space and μ be a Radon measure on X. A concassage of μ is a family $\mathfrak{D}$ of compact subsets of X such that $\operatorname{supp} \mu|D = D$ for all $D \in \mathfrak{D}$ and such that $\mu(B) = \sum_D \mu(B \cap D)$ holds for all Borel sets B.

Theorem: Let X be metacompact or let X be meta-Lindelöf and assume that (MA)+NON(CH) holds. Then $Y = \bigcup \{D : D \in \mathfrak{D}\}$ is Borel and paracompact whenever Y is regular; furthermore μ is saturated.

An example was given to show that the result becomes false, in general, for meta-Lindelöf spaces if (CH) is assumed.

Applications to the decomposability of Radon measures were given.

R.J. GARDNER

(reporting on joint work with W.F. PFEFFER)

Radon measures

The first part of the talk gave consequences of the results announced in W.F. Pfeffer's talk. These are:

Theorem: Let X be (i) weakly Θ-refinable or (ii) meta-Lindelöf and assume (MA)+NON(CH). Then every complete Radon measure on X is decomposable and every Radon measure on X is MAHARAM.

(decomposable = strictly localizable; MAHARAM = localizable)

Example (CH): There is a meta-Lindelöf space X and a Radon measure on X which is not MAHARAM.

The second part compared various approaches to find sufficient conditions for a (say completely regular) space to be a Radon space. There were

(a) If every open subset of X is Souslin-K (this in fact implies that every open subset is σ-compact) then X is Radon

(b) If X is Souslin then X is Radon (see Schwartz's book)

(c) If X is hereditarily weakly Θ-refinable, has no discrete subsets of measurable cardinality and is universally Radon measurable then X is Radon.

(a) and (b) are not directly related, but are subsumed by (c), which is the most general result presently known. In some situations (e.g. when dealing with Eberlein compacts) its full generality is needed.

S. GRAF and R.D. MAULDIN

Random homeomorphisms

(this is an abstract of two talks given by the two authors)

Several methods of constructing random homeomorphisms of the unit interval onto itself were discussed. Two of these were specifica ly investigated. The first construction is as follows: First the value at 1/2 is chosen according to uniform distribution over (0,1). Next, the value at 1/4 is chosen according to uniform distribution over the interval from 0 to the value already chosen for 1/2 and, idependently, the value at 3/4 is chosen according to uniform distribution over the interval from that value to 1. Continue this process. This defines a probability P on the set of monotonely increasing homeomorphisms of the unit interval.

The second method can be derived from the first by taking the average right translate of P with respext to P: $P_a(E) := \int_H P(E_g)dP(g)$. It turns out that the measure P_a is also derived from a "point" process. So, both P and P_a have the important feature that one can make computer experiments to obtain information about their properties. P as well as P_a give non-empty open sets positive measure. Besides this they have certain "natural" invariance properties. They are both invariant under "time reversal". P_a is invariant under inversion whereas P is not. Most importantly, P is invariant under scaling between $i/2^n$ and $i+1/2^n$. This means that one gets P back when one scales the conditional distribution of P given the values at $i/2^n$ and $i+1/2^n$ onto the homeomorphism of the interval. On the other hand P_a does not have this property anywhere.

It was shown that P-almost all homeomorphisms have derivative 0 at 0, whereas P_a-almost all homeomorphisms have upper derivative ∞ at 0 and lower derivative 0 at 0. Finally, the structure of the fixed point set of P and P_a homeomorphisms was discussed. Several computer print-outs generated by P and P_a random homeomorphisms were exhibited.

D. MAHARAM

On the planar repsesentation of a measurable subfield

This talk sketched a simpler proof of a slightly sharper version of the planar representation theorem of Rokhlin (1949) and Maharam (1950). Let $(\Omega, \mathfrak{B}, m)$ be a Polish measure space, with a σ-finite completed Borel measure m, and let $\mathfrak{A}$ be a countably generated σ-subfield of the Borel field $\mathfrak{B}$. Then there is a measure preserving isomorphism of almost all of Ω onto a certain Borel subset Z of the plane, taking $\mathfrak{B}$ to the relative Borel subsets of Z and $\mathfrak{A}$ to the relative vertical Borel cylinders, on which the measure is relative planar Lebesgue measure on an ordinate set together with a sequence of linear sets, the measure being absolutely continuous with respect to linear Lebesgue measure, and countably many atomic points.

The proof uses two devices (both known): (a) the use of two Marczewski functions to embed Ω suitably in the plane, (b) the fact that a function f of two variables x and y that is Borel measurable in x for fixed y and monotone, continuous from the right in y for fixed x is Borel measurable.

G. MÄGERL

(reporting on joint work with S. GRAF)

Isometries of measure algebras

Let $(X,\mathcal{A},\mu)$ be a measure space, $\mathfrak{A} = \mathcal{A}/\mu$ the associated measure algebra and $\mathcal{E} = \{a\in\mathfrak{A} : \mu(a)<\infty\}$. For $a,b\in\mathcal{E}$ the Nikodym distance of a and b is given by $d(a,b) = \mu(a\,\triangle\, b)$. Question: What are the isometries of the metric space $(\mathcal{E},d)$ and of certain subspaces of that space ?

Theorem 1: If μ is σ-finite, then $T:\mathcal{E}\to\mathcal{E}$ is an isometry iff there is a measure preserving Boolean automorphism Φ of $\mathfrak{A}$ and $a_o\in\mathcal{E}$ such that $T(a) = a\triangle a_o$. Using a theorem of von Neumann one gets Theorem 2: If X is Polish and μ a σ-finite Borel measure on X then $T:\mathcal{E}\to\mathcal{E}$ is an isometry iff there is a bimeasurable measure preserving bijection F of X and a Borel set A_o of finite measure such that $T([A])=[F(A)\triangle A_o]$, for all A. Now let X be Polish, μ be a locally finite Borel measure (necessarily a σ-finite Radon measure), $\mathcal{K}=\{K\subseteq X:\ K \text{ cpt.}\}$, $\mathcal{F}=\{F\subseteq X:\ F \text{ closed}, \mu(F)<\infty\}$ and $\mathfrak{K} = \mathcal{K}/\mu$, $\mathfrak{F} = \mathcal{F}/\mu$. Call a Borel isomorphism F of X an almost isometry if there is an F-invariant Borel set Y with $\mu(X\setminus Y) = 0$, such that F restricted to Y is a homeomorphism of Y. Then one has Theorem 3: Suppose that μ is diffuse. (a) $T:\mathfrak{F}\to\mathfrak{F}$ is an isometry iff there is a measure preserving almost homeomorphism F of X such that $T([A])=[F(A)]$.

(b) If, in addition, X is locally compact then $T:\mathfrak{K}\to\mathfrak{K}$ is an isometry iff there is a measure preserving almost homeomorphism F of X such that $T([A])=[F(A)]$.

R.M. SHORTT

Complementation and conjugation for Borel structures

Let $(X, \mathcal{B})$ be a measurable space and let $\mathcal{C}$ and $\mathcal{D}$ be sub-σ-algebras of $\mathcal{B}$. $\mathcal{D}$ is a conjugate of $\mathcal{C}$ if $\mathcal{C} \cap \mathcal{D} = \{\emptyset, X\}$; $\mathcal{D}$ is a complement for $\mathcal{C}$ if also $\mathcal{B}$ is the σ-algebra generated by $\mathcal{C}$ and $\mathcal{D}$. A characterization of minimal complements for structures generated by a finite partition was given and also an example showing that this characterization fails for countable partitions. This characterization was reformulated for the case of two-fold partitions involving Borel embeddings. It also was applied to the problem of determining when the union of Blackwell sets is again a Blackwell set.

The analogous problem for maximal conjugates was also considered, and some partial results involving 0-1 transition kernels and measureable selections were presented.

M. TALAGRAND

Separate and joint measurabilty

Let (Ω, Σ, μ) be a complete probability space and $(Y, \mathcal{B}, \nu)$ a Radon probability space. Let $f: \Omega \times Y \to \mathbb{R}$ be a function, measurable in the first variable and continuous in the second. Problem : When is f measurable with respect to the product measure ? An example of D. Fremlin (under (CH)) shows that f may fail to satisfy Fubini's theorem; on the other hand, for small Y (e.g. Y metrizable), f is measurable. Consider the set $Z_f := \{f(\cdot, y) : y \in Y\}$ which is a pointwise compact set of measurable functions on Ω.

Main result: f is jointly measurable if Z_f is stable, i.e. if for all $\alpha < \beta$ and $A \in \Sigma$ with $\mu(A) > 0$, there is $n > 0$ such that $(\mu^{2n})^*(\{(s_1, \ldots s_n, t_1, \ldots, t_n) \in A^{2n} : f(s_i) \leq \alpha,\ f(t_i) \geq \beta\}) < (\mu(A))^{2n}$.

If one assumes the following Axiom (L):"The unit interval cannot be covered by less than continuum many Lebesgue null sets" then stability can be checked using Theorem: (Axiom L) Suppose that (Ω, Σ, μ) is perfect and that Y is the support of ν. Then Z_f is stable if the map $\omega \mapsto f(\omega,.)$ is measurable from Ω to $L^1(\nu)$. Note that the assumption is equivalent to the existence of a measurable map g on $\Omega \times Y$ such that for all ω and ν-almost all $t \in Y$ we have $g(\omega,t) = f(\omega,t)$. The point is that these neglegible sets depend a priori wildly on ω. As a consequence of the theorem and further properties of jointly measurable maps a result about maps and their images under invariant liftings on compact groups were derived.

Grundlagen

R. FRANKIEWICZ

(reporting on joint work with R. ANISÇZYK)

Some remarks on σ-fields and measurable functions

B.V. Rao proved that a countably generated σ-field always has a minimal generator and asked whether a σ-field without a minimal genarator exists. This question was answered by the following Theorem: The σ-algebra generated by the non stationary subsets of ω_1 has no minimal generator. Assume (CH). Then the following σ-algebras have no minimal generator: the power set of ω_1, the Lebesgue measurable subsets of $\mathbb{R}$ and the subsets of R having the Baire property. Also the following results were proved: Theorem: Assume that ZFC is consistent. Then ZFC + $MA_{\sigma\text{-linked}}$ + $\diamondsuit_{c=\omega_2}$ + " the Boolean algebra of Lebesgue sets mod null sets is not embeddable in the power set of ω mod finite sets" is also consistent. The same is true if the σ-algebra of Lebsgue subsets mod null sets is replaced by the Borel subsets of the Cantor set. Theorem: Under (MA) the power sets of ω modulo sets of density zero is isomorphic to the power set of ω mod sets of logarithmic density zero.

Remarks: It was shown by Silver (1974) that under (MA) + NON(CH) $\mathcal{P}(\omega_1)$

has a minimal generator. The last result was proved by Just and Krawczyk under (CH).

E. GRZEGOREK

Remarks on some Borel structures

Let I be the unit interval and let I! be the group of all permutations of I. B.V. Rao (Coll. Math. 1970) proved that (a) There are two separable σ-fields $\mathcal{A}_1, \mathcal{A}_2$ on I such that for all $p,q \in I!$ the σ-field $p(\mathcal{A}_1) \cap q(\mathcal{A}_2)$ is not countably generated. (b) There are two separable σ-fields $\mathcal{A}_1, \mathcal{A}_2$ on I such that $\mathcal{A}_1 \cap \mathcal{A}_2$ does not contain any nontrivial countably generated σ-field. Under (CH) this result was strengthened to

Theorem: (CH) There are separable σ-fields $\mathcal{A}_1$ and $\mathcal{A}_2$ on I such that for all $p,q \in I!$ the σ-field $p(\mathcal{A}_1) \cap q(\mathcal{A}_2)$ does not contain any non-trivial countably generated σ-field.

Further, a very short proof to the following recent theorem of R.M. Shortt was given: If A is an analytic non Borel set in $\mathbb{R}$ such that $\mathbb{R} \setminus A$ is totally imperfect then A is not isomorphic with any product $A_1 \times A_2$ of two uncountable analytic spaces A_1 and A_2.

A. JOVANOVIC

Some combinatorial properties of measures

For a measure μ its norm $\|\mu\| = \min\{|X| : \mu(X) > 0\}$ was defined and compared with its additivity add(μ). (Obviously add(μ) $\leq \|\mu\|$). Using methods of Solovay and axioms a little stronger than the existence of measurable cardinals it was shown to be consistent to have a real valued measure μ with $2^\omega \geq \text{add}(\mu) < \|\mu\|$. Starting from that the transition from large cardinals to real valued large cardinals was proposed, changing in the definitions "binary measure" to "real valued measure". The consistency of real valued large cardinals relative to the existence of large cardinals can be proved using essentially Solovay's forcing method. A number of filter combinatorial properties can be translated into the language of measures, so that it makes sense to consider classifications of real-valued measures analogous to the Rudin-Keisler classification of ultrafilters.

Lifting und meßbare Selektionen

R.W. HANSELL

Selectors in nonseparable spaces

Let T be a set with a paving $\mathcal{M} \subseteq \mathcal{P}(T)$ closed to finite intersections. Let (X, ϱ) be a metric space. Suppose $F: T \longrightarrow X$ is a weakly $\mathcal{M}_\sigma$-measurable multimap with values that are nonempty, separable, ϱ-complete and totally bounded with respect to some metric (not necessarily complete) on X. This latter property holds, for example, when the values of F are compact or when X is separable. Let $S(F) = \{ f: T \to X$: f is point valued, $(\mathcal{M}^-)_\sigma$-measurable and $f(t) \in F(t)$ for all $t \in T \}$. ($(\mathcal{M}^-)_\sigma$ = countable unions of differences of sets in $\mathcal{M}$). The following theorem was proved

Theorem: Suppose T is metrizable and $F: T \to X$ is as above. Then S(F) is nonempty whenever

(i) $\mathcal{M}_\sigma$ is the family of all Borel sets of class $\alpha < \omega_1$ (to be precise, F has a Borel selector of class $\omega \cdot \alpha$).

(ii) $\mathcal{M}_\varrho$ is the family of Souslin subsets of T.

(iii) $\mathcal{M}_\sigma$ is a countably generated σ-algebra on T (any sets).

The proof uses a nonseparable analogue of the countable reduction property (shown by Maitra and Rao to be equivalent to the basic selection theorem of Kuratowski and Ryll-Nardzewski).

J.E. JAYNE

Borel measurable selectors and the Radon-Nikodym property

Several applications of the following theorem were discussed:

Theorem: An upper semicontinuous set-valued map from a metric space to a Banach space with its weak topology has a Borel measurable selection, provided the range is everywhere dentable, or equivalently has the Radon-Nikodym property.

V. LOSERT

Some remarks on invariant liftings

The following results were discussed: If G is a non-discrete locally compact group, then there exists no left-invariant Borel lifting. G admits a bi-invariant lifting iff for each $x \in G$ the set $C_G(x) := \{y \in G: xy=yx\}$ is open in G. A connected locally compact group admits a bi-invariant lifting iff it is amenable. For $X = \mathbb{R}^n$, G the group of affine transformations with determinant 1,there is no G-invariant linear lifting on X (with respect to Lebesgue measure).

Abstrakte Integration

P. MARITZ

Bilinear integration of multifunctions

The purpose if this talk was to consider some extensions and also an approximation of Lyapunov's theorem in terms of the bilinear m-integral of N. Dinculeanu. The integration is performed successively with respect to a non-atomic measure, a direct sum measure and a Darboux measure. The necessary counterexamples were provided.

S. OKADA

A tensor product integral

Let X and Y be Banach spaces. An integration theory of X-valued functions with respect to a Y-valued measure λ was presented. To achieve the completeness of the space of integrable functions, a space of functions has to be considered which take values in a locally convex Hausdorff space W containing a copy of X. Let α be a cross-norm on $X \otimes Y$. A function f with values in W is said to be λ-integrable if there exists a sequence (c_i) in X and a sequence of measurable sets (E_i) such that $(c_i \otimes \lambda(F_i))$ is unconditionally summable in the completed space $X \otimes_\alpha Y$ for every sequence of measurable sets $F_i \subseteq E_i$ and if $w' \in W'$ implies that $\langle w', f \rangle$ can be expressed as the sum of the absolutely summable sequence $(\langle w', c_i \cdot 1_{E_i} \rangle)$.
This integral can be applied to obtain the Fubini theorem for scalar valued functions with respect to the product of two Banach space valued measures.

T.P. SRINIVASAN

Measure and integral - a new gambit

A procedure to construct the Daniell integral extension and the Baire integral extension of a pre-integral with swiftly converging sequences taking the role traditionally played by Cauchy seuqences, was presented. Unlike Cauchy sequences, swiftly converging sequences converge almost everywhere dominatedly and almost uniformly. If (I,L) is a pre-integral define (f_n) in L to be swiftly convergent if $\sum_n \|f_{n+1} - f_n\| < \infty$, and define N to be a null set if $N \subseteq \{x : \sum_n |f_{n+1}(x) - f_n(x)| = \infty\}$ for some swiftly convergent sequence $(f_n)_n$. Let L^1 be the class of functions f which are a.e. limits of swiftly convergent sequences(f_n) in L and let $I'(f) = \lim I(f_n)$. Then I' is well defined on L^1.
Theorem 1: I' is an integral extension of I and L^1 is norm complete, order complete (pointwise order a.e.) and null complete.

Theorem 2: If L_1 is the subfamily of L-Baire functions in L^1 and I_1 is the restriction of I^1 to L_1 then (I_1, L_1) is an integral extension of (I,L) and it is the smallest integral extension. The space L_1 is norm complete and order complete.

E.G.F. THOMAS

Invariant Daniell integrals

Let X be a Hausdorff space and let L be a sublattice of the vector lattice of real continuous functions on X. Consider a localizable Daniell integral μ on L, i.e. μ is defined by a Radon measure m on X by the formula $\mu(\varphi) = \int \varphi \, dm$. Then if G is a group of homeomorphisms of X leaving L invariant it was shown that, under appropriate hypotheses, the invariance of μ under the action of G implies the quasi-invariance of a certain class of Radon measures on a quotient G-space Y of X. Conversely every quasi-invariant measure class on Y can be obtained in this way from some G-invariant triple (X,L,μ).

Maße und Integrale mit abstraktem Wertebereich

P. MORALES

Boundedness for uniform semigroup valued set functions

Let $X = (X,\mathcal{U})$ be a uniform space. A subset $\mathcal{V} = \{V_n : n \in \mathbb{N}\}$ of $\mathcal{U}$ is called a uniform bounding system if (i) every V_n is symmetric; (ii) $V_n \subseteq V_m$ for $n < m$; (iii) $V_n \circ V_m \subseteq V_{n+m}$. Let $B \subseteq X$. Then B is $\mathcal{V}$-bounded if $B \subseteq V_n[F]$ for some $n \in \mathbb{N}$ and some finite subset F of X. B is said to be bounded if for every symmetric entourage V, B is $\{V^n : n \in \mathbb{N}\}$-bounded.

Let X be a commutative Hausdorff uniform semigroup with neutral element 0, and let $\mathcal{R}$ be a ring of subsets of a fixed set T. The following results were presented:

1) Let $\mu : \mathfrak{R} \to X$ be s-bounded and additive and let $\mathcal{V}$ be a uniform bounding system. Then μ is $\mathcal{V}$-bounded. (This generalizes Results of Musiał and Kats)

2) Let (μ_n) be a sequence of X-xalued s-bounded additive set functions on $\mathfrak{R}$. If for every $E \in \mathfrak{R}$ the sequence $(\mu_n(E))$ converges to 0 then the μ_n are uniformly bounded.

3) A generalization of the Nikodym uniform boundedness theorem.

4) A generaöization of the following result of Dieudonné: Let $\mathcal{M}$ be a family of regular Borel measures on a compact Hausdorff space T such that $\sup \{ |\mu(U)| : \mu \in \mathcal{M} \} < \infty$. Then $\sup \{ \|\mu\|(T) : \mu \in \mathcal{M} \} < \infty$.

D. SENTILLES

Some measure theoretic applications to the Pettis integral

A bounded weakly measurable function f on a probability space (Ω, Σ, μ) into a Banach space X naturally induces a "Stonian transform" $\hat{f} : S \to X^{**}$ which is continuous with respect to the X^*-topology on X^{**}, where S is the Stone space of the measure algebra. Because f is strongly measurable iff $\hat{f} \in X$ a.e., on S the measure $\mu \hat{f}^{-1}$ may be used to prove a decomposition $f = g + h$, where g is Bochner integrable and $\hat{h} \in X^{**} \setminus X$ a.e.. Consequently, the Pettis integrability of f depends only on the extreme case: h. Such a function h is Pettis integrable iff the intersection of X with the weak*-closed convex hull of $\hat{h}(O)$ is nonempty for each nonempty open-closed subset O of S. This result leads to an integral free characterization of Pettis integrability: f is Pettis integrable, iff whenever $\|x_\alpha^*\| \leq 1$ and $x_\alpha^* \to x^*$ in the X-topology, one has that $x_\alpha^* f = 0$ a.e. implies $x^* f = 0$ a.e.

T. TRAYNOR

Frechet-Nikodym topologies on rings and lattices

Modular functions $(m(a \vee b) + m(a \wedge b) = m(a) + m(b))$ on an abstract lattice with values in a topological group were considered. The analogue of the Frechet-Nikodym distance ($d(A,B) = \mu(A \Delta B)$) in this setting is $d_m(a,b) = \sup\{|m(v)-m(u)|: a \wedge b \leq u \leq v \leq a \vee b\}$, (in case the group has a quasinorm, which was assumed). This defines a (generalized) pseudometric on L for which the translations $a \mapsto a \wedge x$ and $a \mapsto a \vee x$ are contractions and $d(a \wedge b,b)=d(a,a \vee b)$. If M is a family of such modular functions for which ma_n increases uniformly for m in M, then the M-topology (generated by $\{d_m: m \in M\}$) coincides with the equi-M-topology generated by the distance $d= \sup d_m$. A consequence is a local Rybakov-type theorem for Banach space valued modular functions on a distributive lattice. Several related problems remain open, even in the real case.

H. WEBER

Gruppen- und vektorwertige s-beschränkte Inhalte

Es wurde eine Methode zur Behandlung von gruppen- und vektorwertigen Inhalten vorgestellt, mit der sich zahlreiche Sätze einheitlich und mit einem Minimum an technischem Aufwand beweisen lassen. Eine wesentliche Rolle spielen dabei die FN-Topologien.

Sei G eine topologische Gruppe, die vollständig und Hausdorffsch ist, R ein Boolescher Ring, u_s die feinste s-beschränkte FN-Topologie auf R und $(\tilde{R}, \tilde{u}_s)$ die vollständige Hülle von (R, u_s). Dann läßt sich jeder s-beschränkte Inhalt $\mu: R \to G$ in eindeutiger Weise stetig zu einem Inhalt $\tilde{\mu}: \tilde{R} \to G$ fortsetzen. Zur Untersuchung von μ betrachtet man zunächst die Fortsetzung $\tilde{\mu}$ und überträgt dann die Ergebnisse auf $\mu = \tilde{\mu}|R$. Die Untersuchung von $\tilde{\mu}$ ist deshalb einfacher als die von μ, weil $\tilde{R}$ eine (als Verband) vollständige Boolesche Algebra ist, auf der $\tilde{\mu}$ τ-stetig ist (d.h. für jedes nach unten gerichtete System (x_γ) in $\tilde{R}$ mit $x_\gamma \downarrow 0$ gilt $\tilde{\mu}(x_\gamma) \to 0$).

Geometrische Maßtheorie

P. MATTILA

Hausdorff dimension of intersections of sets in n-space

The following problem was considered: Let A and B be Borel sets in R^n with Hausdorff dimensions dim A = s and dim B = t. What can be said about the Hausdorff dimensions of the intersections $A \cap fB$, where f runs through the isometry group of R^n ? Some examples indicate that in general there is very little to say. But if t is assumed integral and B sufficiently nice, e.g. a C^1 manifold or t-rectifiable, then dim $A \cap fB$ = s+t-n holds for many isometries f provided $s+t-n \geq 0$. For general Borelsets A and B a larger family of transformations has to be used; e.g. dim $A \cap fB \geq$ s+t-n holds for many similarities , i.e. maps composed of translations, rotations and homotheties. Equality does not hold in general, but it does under some extra conditions on B. For example, it suffices to assume that B has positive t-dimensional lower density at each of its points.

A. VOLČIČ

On the reconstruction of convex bodies from a finite number of Steiner symmetrals

The following problem was considered: Given a convex body K in the plane, is it possible to find a finite number of directions such that K is determined by its Steiner symmetrals with respect to these directions ? Giering (1962) showed that three directions are enough to distinguish a given K from all other convex bodies; Gardner and McMullen (1980) proved that a set of directions, D, distinguishes each convex body from each other iff it is not the linear image of the diagonals of a regular n-gon. As a counterpart to this result it was shown that if there are two convex bodies not distinguished by D then there are already continuously many.

From the result of Gardner and McMullen it follows in particular that there is a distinguishing set of four directions, hence the map $\mathcal{S}$ sending K to its Steiner symmetrals with respect to these directions is injective (defined on $\mathcal{K}_*$, the set of all convex bodies and taking values in $\mathcal{K}_*^4$). It was shown that $\mathcal{S}$ as well as $\mathcal{S}^{-1}: \mathcal{S}(\mathcal{K}_*) \to \mathcal{K}_*$ are continuous (with respect to the Hausdorff metric), hence the problem of reconstructing K from $\mathcal{S}(K)$ is well posed. For sets which are unions of inscribed parallelograms with sides parallel to two given directions, a recunstruction procedure was presented.

Extremalprobleme

H.G. KELLERER

Duality theorems for marginal problems

Given Hausdorff spaces X_i, tight Borel probability measures μ_i on X_i and a (bounded) function g: $X = \prod_{1 \leq i \leq n} X_i \longrightarrow R$, the following two problems were investigated:

(MP) maximize $\mu^*(g)$ over all tight Borel probabilities on X with marginals $\pi_i(\mu) = \mu_i$, for $1 \leq i \leq n$,

(DP) minimize $\sum_{1 \leq i \leq n} \mu_i(f_i)$ over all $(f_1, \ldots, f_n)$, $f_i: X_i \longrightarrow R$ μ_i-integrable and $\sum_{1 \leq i \leq n} f_i \circ \pi_i \geq g$.

Let S(g) be the supremunm corresponding to (MP) and I(g) be the infimum corresponding to (DP). Then, using the theorems of Hahn-Banach and Riesz, it is not hard to show the "duality theorem": S(g) = I(g), provided that the spaces X_i are assumed to be compact metrizable and the function g continuous. A thorough examination of the functionals S and I then shows that they have the properties of a C^u capacity, where C^u is the lattice of upper semicontinuous functions on X. Therefore, by first proving S(g)=I(g) for all g in C^u one obtains the duality theorem for all C^u-analytic functions. This result holds without special topological assumptions and can be carried over to all $\bigotimes_{1 \leq i \leq n} \mathcal{B}(X_i)$ -measurable functions g.

V.N. SUDAKOV

Two problems connected with Kantorovic distance

1. Let $\{X_t : t \in T\}$, $X_t : (\Omega, \mathcal{O}\!\mathcal{l}, P) \to E$ be a family of E-valued random variables where (E, ρ) is a complete separable metric space, and let $\{\mu_t : t \in T\}$ be the family of their distributions. Such a family is called a Kantorovic set if for any t , t' in T the equality $E_P\, \rho(X_t , X_{t'}) = \kappa(\mu_t, \mu_{t'})$ holds, where κ denotes the Kantorovic distance.

A class of spaces (E, ρ) was described, such that for every family $\{\mu_t : t \in T\}$ of probability distributions on E there exists a corresponding Kantorovic set of E-valued random variables.

2. Let μ and ν be Borel probability measures on $\mathbb{R}^n$, absolutely continuous with respect to Lebesgue measure and such that $\iint \|x-y\|\, d\mu\, d\nu < \infty$. Then there is a one-to-one optimal plan to transport μ to ν , i.e. a Borel measure m_o on $\mathbb{R}^n \times \mathbb{R}^n$, concentrated on the graph of a 1-1 measure preserving transformation of $\mathbb{R}^n$, whose marginal distributions are μ and ν such that $\int \|x-y\|\, dm_o = \kappa(\mu, \nu)$.

H. von WEIZSÄCKER

Extremal families of probability measures

This talk gave a survey on some questions about the extremal structure of convex sets of probability measures.

Let X be a Polish space, P(X) the space of Borel probability measures on X and H be a convex subset of P(X).

Q1: Is H a Choquet set, i.e. (a) is H equal to $\{r(\pi) : \pi \in P(\mathrm{ex}H)\}$ where $r(\pi)(B) = \int \nu(B)\, \pi(d\nu)$. (b) in part (a) π is uniquely determined by $r(\pi)$ iff $\mathbb{R}_+ H$ is a lattice cone. A simple sufficient condition is that H is of the form $H = \bigcap_{n=1}^{\infty} \{\mu : \int f_n\, d\mu \leq a_n\}$ where (f_n) is a sequence of Borel functions and (a_n) a sequence in R. (v. Weizsäcker-Winkler, 1979)

Q2: Characterize exH. Here, following the ideas of P. Martin-Löf and

Dynkin, the Martin boundary for Brownian motion was explained. It was pointed out that for other (possibly infinite dimensional) diffusion processes the analogous questions are open and interesting.

Q3: Is there a measurable map $\varphi : X \to \mathrm{ex}H$ such that $\nu\{x: \varphi(x)=\nu\} = 1$ for all ν in exH ? The answer is "yes" if H is a Choquet set, $\mathbb{R}_+H - \mathbb{R}_+H$ is a sublattice of the lattice of signed Borel measures on X and exH is σ-compact with respect to $\sigma(M(X),C_b(X))$. An example by D. Preiss (contained in a paper by Mauldin, v. Weizsäcker and Preiss, Ann. of Prob.) shows that the answer becomes "no, in general" if the last condition is omitted.

Q4: Find a computational algorithm for π based on statistical data on $r(\pi)$. Here results of W. Krüger were reported on.

Maßtheorie und Funktionalanalysis

M.A. AKCOGLU

Sub-Banach lattices of L_p spaces

Let $1 < p < \infty$, $(X, \mathfrak{F}, \mu)$ be a Lebesgue measure space and $L_p = L_p(X, \mathfrak{F}, \mu)$. For f in L_p let f^* denote the unique vector in $L_p^* = L_q$ such that $\|f\|_p^p = \|f^*\|_q^q = (f,f^*)$. Then it was observed that arguments of T. Ando (Pacific J. Math. 17(1966)) yield the following theorem.

Theorem: Let $p \neq 2$ and let $M \subseteq L_p$. Then the following statements are equivalent: (i) M and $M^* = \{f^* : f \in M\}$ are closed linear manifolds. (ii) There is an f in M and a sub-σ-algebra $\mathfrak{G}$ of $\mathfrak{F}$ such that $M = \{gf: g$ is $\mathfrak{G}$-measurable and $gf \in L_p\}$. (iii) M is isometrically isomorphic to the L_p space of another measure space. - The implication (i) (ii) (which is the only non-trivial part of the theorem) illustrates the fact that for $p \neq 2$ some pointwise properties of the elements of L_p can be formulated in terms of Banach space conditions on L_p. Note that if $T: L_p \to L_p$ is a contraction then $M = \{f: f=Tf\}$ satisfies the conditions of the theorem ((i) for example). This gives Ando's theorem.

R. BECKER

Quelques aspects de la théorie des mesures coniques

Soit E un espace localement convexe séparée de dual E'; une mesure conique positive sur E est une forme lineaire positive sur le treillis de fonctions sur E, engendré par E'. Il y a plusieurs types de problèmes:

a) quand μ est elle donnée par une mesure de Radon ?

b) si $X \subseteq E$ est une cône, quand toute μ portée par X est elle donnée par une mesure de Radon ?

c) que se passe-t-il si on affaibli la topologie de E ?

G.A. EDGAR

Realcompactness and measure-compactness of the unit Ball in a Banach space

The realcompactness and measure-compactness of a Banach space in its weak toplogy have been of interest in connection with the theory of integration in the Banach space. Similar properties can be investigated for the unit ball of the Banach space. In the talk several examples were worked out to illustrate these properties. It can be conjectured that the ball is realcompact (or measure-compact) if and only if the whole space is. The speaker expected that this is false, but did not know of a counterexample.

W.A.J. LUXEMBURG

The Radon-Nikodym theorem for positive operators

The main purpose of the talk was to discuss the following Radon-Nikodym type factorization theorem for positive operators defined on vector lattices. Let L and M be Dedekind complete vector lattices and let $\mathcal{L}_n(L,M)$ denote the Dedekind vector lattice of order continuous and order bounded linear transformations of L into M. A local operator on a vector lattice is a positive linear transformation that leaves invariant all the bands of the underlying space. The family of all such densely defined operators is denoted by $\mathrm{Orth}^{\infty}(L)$. By a Radon-Nikodym type factorization theorem we mean a factorization of the form $S=T\circ\pi$, where S,T are positive order continuous and π is a local operator (In the case of measures the Radon-Nikodym derivative plays the role of the local operator). For positive operators we have the following result. Let $S,T \in \mathcal{L}_n(L,M)$ be positive. If T has the Maharam property, i.e. maps intervals onto intervals, then the following conditions are equivalent: (i) S is contained in the band generated by T; (ii) S is absolutely continuous w.r.t. T, i.e. for all $0 \leq u \in L$, Su is contained in the band generated by Tu; (iii) there exists a local operator $\pi \in \mathrm{Orth}^{\infty}(L)$ such that $S = T\circ\pi$.

A dual form of this result leads to a factorization theorem for linear lattice homomorphisms generalizing a result of Kutateladze. For spaces of measurable functions the result is related to earlier results of D. Maharam-Stone. Since every order bounded operator from L to M may be uniquely extended to a larger space containing L, the extension having the Maharam property and the order continuity property, the above Radon-Nikodym type factorization theorem has a wide application range analogous to the classical Radon.Nikodym theorem for measures.

G. PISIER

Tensor products of Banach spaces

The talk reported on some recent results concerning the injective and projective tensor products (denoted by $X\check{\otimes}Y$ and $X\hat{\otimes}Y$ respectively) of two Banach spaces X and Y.

Recent examples show that X and Y can have the RNP (Radon-Nikodym property) and be weakly sequentially complete while $X\hat{\otimes}Y$ contains c_o and hence fails to have these properties. Further the following theorem was presented (to appear in Acta Math.) which answers a conjecture of Grothendieck: Every Banach space E of cotype 2 (every separable Banach space E) can be isometrically embedded into a space X (into a separable space X) such that

$X \check{\otimes} X = X \hat{\otimes} X$ and such that both X and X^* are of cotype 2, and X/E has the RNP and the Schur property. Related results were discussed concerning the possibility of embedding an arbitrary Banach space in a suitable way into an $\mathcal{L}^\infty$-space (cf. a joint paper with J. Bourgain, in preparation).

C. STEGALL

Gateaux differentiability and a class of topological spaces

Let $\mathcal{C}$ be the category of topological spaces K defined by the following property: K belongs to $\mathcal{C}$ if and only if for all topological spaces C, S, T, V where V is a Baire space and $C \subseteq K \times S$, and for all perfect maps $\varphi: C \to T$ and all continuous maps $\lambda: V \to T$ there exists a selection $(\psi(v), \xi(v))$ and a dense G_δ set G so that ψ is continuous at each point of G (ξ does not matter).
This category has nice permanence properties and contains, for example, the duals of Asplund spaces (in the weak* topology), Eberlein compacts, compact metric spaces and RNP sets. The important property of $\mathcal{C}$ is that if X^* (in the weak* topology) is in $\mathcal{C}$ then X is a weak Asplund space. This is the first theorem that gives permanence properties of a large class of weak Asplund spaces.

Ergodentheorie

S.J. EIGEN

Ergodicity of Cartesian products via triangle sets

Let S,T be non-singular, invertible ergodic transformations of the unit interval I. When is $S \times T$ ergodic and when is $T \times S^{n(x)}$ ergodic ? Here, $T \times S^{n(x)}$ denotes the power-skew product $(x,y) \mapsto (Tx, S^{n(x)}y)$ where n is a map from I to the integers. Using the fact that T is ergodic iff for all measurable sets A, B of positive measure there is $n > 0$ such that $T^n A \cap B$ has positive measure as a definition, one would like to study $T \times S$ on rectangles $A \times C$, $B \times D$. However, this does not seem sufficient: But no example of ergodic transformations S, T to show this insufficiency was known to the speaker.

Definition: A measurable set of positive measure $F \subseteq I \times I$ is a triangle set if (1) $F \subseteq A \times I$ for some measurable subset A of I; (2) for all $0 < \varepsilon < 1$ one has $\mu(Q_\varepsilon) > 0$ where $Q_\varepsilon = \{y: \mu(F \cap (A \times \{y\})) \geq (1-\varepsilon)\,\mu(A)\}$.

Results such as the following theorems can then be obtained:

Theorem 1: Every measurable subset of the unit square having positive measure is a disjoint union of triangle sets.

Theorem 2: $T \times S$ is ergodic if T and S have the following property :for all $\varepsilon > 0$ and all sets A, B of positive measure the sets $N_S(A,B) = \{n > 0: T^n A \cap B$ has positive measure$\}$ and $N_T = \{n > 0: \mu(T^n A \cap B) \geq (1-\varepsilon)\,\mu(A)\,\mu(B)\}$ have nonempty intersection.

V.S. PRASAD

Nonsingular ergodic transformations

Let $(X, \mathfrak{B}, \mu)$ be a Lebesgue probability space and G(X) be the of nonsingular transformations of X onto itself. On G(X) put the coarse topology, i.e. $T_n \to T$ coarsely if $\|U_{T_n} f - U_T f\|_1 \to 0$ for all $f \in L^1(X)$, where $U_T : L^1 \to L^1$ is the L^1 isometry associated to T, $(U_T f)(x) = f(Tx)\,(d\mu T/d\mu)(x)$ for $f \in L^1$. With this topology G(X) is a complete metrizable space.

Theorem: The transformations T in G(X) such that the skew product extension $T^* : X \times \mathbb{R} \to X \times \mathbb{R}$, $(x,t) \longmapsto (Tx,\ t + \log(d\mu T/d\mu)(x))$ is ergodic on $X \times \mathbb{R}$ with the product measure $d\mu \times e^{-t}dt$, form a dense G_δ subset of G(X) with the coarse topology.

L. SUCHESTON

(reporting on joint work with M. AKCOGLU)

Ergodic theory and truncated limits

Let E be a Banach lattice such that (A) There is a weak unit u, i.e. $u \in E_+$ and $u \wedge |f| = 0$ implies $f=0$, and (B) Every norm bounded increasing sequence (φ_k) converges (strongly). For a sequence (f_n) in E_+, $\mathrm{WTL} f_n = f$ (weak truncated limit of f_n is f) means that for all $k > 0$, $f_n \wedge ku \to \varphi_k$ (weakly) and

φ_k increases to f. $WTLf_n = WTLf_n^+ - WTLf_n^-$. TL (strong truncated limit) is defined analogously. If $f_n \in E_+$ then $(WTLf_n=0) \Rightarrow (TLf_n=0) \Rightarrow$ (f_n has a subsequence $f_{n_i}=g_i+h_i$, $g_i,h_i \in E_+$, $\|g_i\| \to 0$ and $h_i \wedge h_j=0$ for $i\neq j$). The interest of WTL stems from the fact that every norm bounded sequence has a subsequence with a weak truncated limit. WTL is unique and $g_i \to g$ weakly, $WTLg_i=0$ imply $g=0$. In L_1, $(W)TLg_i = 0$ iff $g_i \to 0$ in measure on sets of finite measure. Let T be a positive linear operator on E; if $WTLf_n = \varphi$ and $WTL\, Tf_n = \psi$ then $T\varphi \leq \psi$. Hence if $\|f_n - Tf_n\| \to 0$ then $T\varphi \leq \varphi$. Let $A_n = (1/n)\sum_0^{n-1} T^i$, $\sup\|A_n\| < \infty$. Theorem 1: The following statements are equivalent: (i) there is a weak unit $\bar{u}$ such that $T\bar{u}=\bar{u}$; (ii) for every band projection $P\neq 0$, $PA_n u$ has no TL-null subsequence; (iii) for every $0\neq H \in E_+^*$ one has $\lim H(A_n u) > 0$. If $\sup \|T_n\| < \infty$, then A_n can be replaced by T_n. Theorem 2: If $0 \leq \varsigma \leq T\varsigma$, P_ς is the band projection on ς, $f \in E_+$ then TL $P_\varsigma A_n f$ exists.

Assume also (C): For every $h \in E_+$ and $\alpha > 0$ there exists $\beta = \beta(h,\alpha)$ such that $0 \leq f \leq h$, $\|f\| > 0$, $g \in E_+$, $\|g\| \leq 1$ implies $\|f+g\| \geq \|g\| + \beta$. Then one has

Theorem 3: If $\|T\| \leq 1$ and $f \in E_+$ then $A_n f = g_n + h_n$ with $g_n, h_n \in E_+$, $TLh_n=0$ and g_n converges strongly to some φ with $T\varphi = \varphi$.

Wahrscheinlichkeitstheorie

S.D. CHATTERJI

Measure theory and "amarts"

Let $\mathcal{A}_n$ be an increasing sequence of algebras of subsets of a space Ω and $\mathcal{A} = \bigcup \mathcal{A}_n$. Let E be a Banach space and $\Theta_n : \mathcal{A}_n \to E$ be a sequence of additive set functions of bounded variation such that (i) $\lim \Theta_n(A) = \Theta(A)$ exists for all $A \in \mathcal{A}$; (i') $\Theta : \mathcal{A} \to E$ is of bounded variation; (ii) there exists a sequence $\nu_n : \mathcal{A}_n \to [0,\infty[$ of additive set functions with $\nu_n(\Omega) \to 0$ and $\nu_{n+1}|\mathcal{A}_n \leq \nu_n$ for all n.

If $\lambda : \mathcal{A} \to [0,\infty[$ is countably additive then $\Theta_n(A) = \int_A f_n d\lambda + \Theta_n'(A)$ where $\Theta_n' \perp \lambda$ and $f_n \in L_E^1$, provided that E has RNP. One can prove that f_n converges

almost everywhere (λ) to f where $\Theta(A) = \int_A f\, d\lambda + \theta'(A)$, $\theta' \perp \lambda$.
The relationship of this theorem (proven in Manuscripta Math. 4(1971) and Lect. Notes in Math. 541(1976)) to certain other convergence theorems including those concerning "amarts" was discussed.

M. TALAGRAND

Characterization of Glivenko classes and Banach space valued maps satisfying the Law of Large Numbers

Let (Ω, Σ, μ) be a complete probability space, E be a Banach space and $f: \Omega \to E$ be a map (no measurability assumed). For $n \in \mathbb{N}$ let $g_n: \Omega^{\mathbb{N}} \to E$, $(t_i) \mapsto \sum_{i \leq n} f(t_i)$. The following results were proved.
Theorem: The following statements are equivalent: (a) $\mu^{\mathbb{N}}$ almost everywhere lim $g_n(t)$ exists in norm; (b) f is Pettis integrable and $\mu^{\mathbb{N}}$ almost everywhere lim $g_n(t)$ = P- $\int f d\mu$; (c) f is Pettis integrable and $\int \| g_n(t) - P\text{-}\int f d\mu \| d\mu^{\mathbb{N}}(t)$ converges to 0; (d) $\int^* \|f\| d\mu < \infty$ and the set $Z = \{x^* \circ f:\ x^* \in E_s^*\}$ is stable. (For the definition of stability see the abstract of the speaker's first talk, p.7).
Corollary: A sequence (C_n) in Σ is not a Glivenko-Cantelli class iff there is a measurable set A of positive measure and a natural number n such that for almost all choices $t_1,\dots,t_n \in A$, each subset of $\{t_1,\dots,t_n\}$ is the trace of a set C_p on $\{t_1,\dots,t_n\}$.

W.A. WOYCZYNSKI

On multiple random measures and integrals

The aim of the talk was to study integrals of the form
$I_n(f) = \int_0^1 \dots \int_0^1 f(x_1,\dots,x_n)dM(x_1)\dots dM(x_n)$, where M(x) is the homogeneous process with independent increments determined by Lévy measure.
The basic questions are (1) For what class of functions f does $I_n(f)$ exist ? (2) What is the distribution of $I_n(f)$? For n=1 there is an answer of Urbanik and Woyczynski to (1), for n=2 Ito's formula can be used to answer (2) and for general n a result of Cameron and Martin. Applications are, e.g.

in quantum field theory and statistics.

For the second order case the following result of Rosinski-Szulga and Engel was mentioned: let the product measure $M_2(A)=M(A_1)\cdot M(A_2)$, $A=A_1\times A_2$ be in L_1 and $F(A)= EM_2(A)$. then μ defined by $\mu(B)= F(\pi(B\cap D) + F\otimes F(B\setminus D)$ where π is the projection and D the diagonal is a control measure for M_2. Thus M_2 extends to a countably additive measure. Furthermore a condition for a function f to be M_2 integrable can be given.

The stable case was treated by Szulga and Woyczynski: Let (Φ_k) be the Haar system normalized in L_p $(1<p<2)$ and $f(s,t) = \sum c_{k,j}\,\Phi_k(s)\,\Phi_j(t)$.

Then if $\sum_{k,j} |c_{k,j}|^{p/2} < \infty$ then $I_2(f) = \sum c_{k,j} \int \Phi_k dM(s) \cdot \int \Phi_j dM(t)$ converges almost surely.

Finally the following result on iterated integrals (Cambanis and Woyczynski) was presented: Let (discrete version) $Q_n = \sum_{j=1}^{n} \sum_{k=1}^{j-1} f(k,j)\, M_k\, M_j$ where (M_i) is independent identically distributed and stable. Then Q_n converges in probability iff

$$\sum_k \sum_{j=1}^{k-1} |f(k,j)|^p (1+ \log(1/ \sum_{j=1}^{k-1} f(k,j)) < \infty .$$

The proof uses a lemma characterizing p-stable-radonifying operators $T= f(k,j)): l^q \to l^p$. The necessity of the above condition was noticed by Pisier. For multiple integrals one gets higher powers of the logarithm.

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