MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 30/1983

Quasigroups and Loops

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The conference was chaired by H. Pflugfelder (Philadelphia, U.S.A.). The nature of the conference was that of a workshop oriented toward a project of producing a book on Quasigroups, Related Structures and their Applications.

The Theory of Quasigroups is rapidly gaining importance because of its connections with other fields of pure and applied mathematics: Universal Algebra, Topology, Combinatorics, Algebraic and Differential Geometry and others. Although many significant research papers on quasigroups appeared during the last two decades the only existing book devoted to that subject remains the classical book by R.H. Bruck published in 1958. There is definitely an urgent need for an up to date book which could serve as a textbook as well as a reference source for researchers working in this field.

The first volume of such a book is presently being written by H. Pflugfelder and D.A. Robinson. It is written on an introductory level and is intended to serve as a textbook for a course or a seminar on Quasigroups and Loops.

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The topic of this conference was the proposed second volume which should be written by several co-authors presenting complete up to date reports on major developments in their special fields.

Discussion sessions and lectures at the conference were devoted to special parts of the projected volume as well as to the book as a whole. Emphasis was placed on cross-connections between different fields and applications of the theory, but foundations of the subject were reviewed too for the purpose of achieving a uniformity of presentation in the book.

The proposed chapters and their authors are as follows:

I. Universal algebra - T. Evans

II. Methods of construction and examples - O. Chein

III. Centrality - J. Smith

IV. CMLs and related groupoids - L. Beneteau

V. Isotopy-Isomorphy - D.A. Robinson

VI. Combinatorial structures arising from CMLs - M. Deza and N. Homada

VII. Systems with two binary operations - M. Kallaher and H. Pflugfelder

VIII. Geometry of quasigroups - A. Barlotti

IX. Topological quasigroups - K. Hofmann and K. Strambach

X. Locally differentiable quasigroups and webs - V. Goldberg

XI. Cubic hypersurface quasigroups - L. Beneteau

XII. Quasigroups arising from differential equations - D. Gerber

The editors of the volume should be O. Chein, H. Pflugfelder and J. Smith.

ABSTRACTS OF LECTURES

K.H. HOFMANN

A survey of topological quasigroups and loops

In the course of discussing the content of a projected textbook and monograph on quasigroups, related structures and their applications, we proposed a chapter on the state of information on topological and analytical quasigroups and loops. We sketched a general frame for the position of non-associative binary algebra and its applications in geometry within topological algebra in general, and, based on this framework, we proposed a tentative table of contents for the chapter of the monograph to the extent it covers topological quasigroups and loops.

Notably, we sketched the contents of the following four subchapters:

I. The general background of the structure of topological quasigroups and loops (Definitions, morphisms and congruences, separation, connectivity, the translation groups as topological transformation groups, uniformities, universal covering, construction methods, example catalogue)

II. Algebraic hypotheses (power-associativity, di-associativity, idempotency, distributivity)

III. Analytical loops (Hudson's partial solution of Hilbert's Fifth Problem for loops, Moufang Lie loops and Malcev-algebras, disassociative topological loops)

IV. Topological double loops (Generalities, double loops with associative multiplication - addition, distributivity, classification via projectivities and collineatious, characterizations on the multiplications of the classical Hurvitz algebras)

K. STRAMBACH

Connections between loops and their webs

Presented was a classification of loops which is based on a classification of related webs. Furthermore, it was discussed how the properties of loops are reflected in the group of projections and in the group of collineations of the corresponding webs.

V. GOLDBERG

A survey on local differentiable quasigroups and webs

The following topics were reviewed:

1. Foliations on a differentiable manifold.
2. A d-web $W(d,n,r)$, $d \geq n+1$, of codimension r on a differentiable manifold of dimension $n \cdot r$.
3. Local differentiable n-ary quasigroups Q_r connected
4. Canonical expansions of finite equations of Q_r.
5. W-algebras of $W(3,2,r)$.
6. Fundamental tensors of Q_r.

7. Closure conditions on $W(n+1,n,r)$ and corresponding algebraic identities in Q_r (It was stressed that only in multicodimensional cases $r > 1$ there is a perfect correspondence between special webs and special quasigroups while for $r=1$ all special webs coincide although in the corresponding quasigroups different identities hold).
8. 1-parameter subquasigroup and conditions of existence of 1-parameter subloops and subgroups in any direction.
9. A four-web $W(4,2,r)$ on $(2r)$-dimensional manifold and two corresponding orthogonal quasigroups.

The results presented are due to M.A. Akivis (Moscow, U.S.S.R.) and his students and V. Goldberg (NJIT, U.S.A.)

L. BENETEAU

Cubic hypersurface Quasigroups

In his book ("Cubic forms", North Holland P.C., 1974) Manin generalized the classical construction of an Abelian group in the set of non-singular points of a projective plane curve. Starting from a cubic hypersurface of dimension > 2 the three place relation of collinearity gives rise, in a suitable quotient, to an exponent 6 CML (or equivalently to a totally symmetric quasigroup $(E,\cdot)$) satisfying the laws $x^2 \cdot yz = xy \cdot xz$ and $x^2 \cdot x^2 = x^2$. But it is still an open question whether this loop can be non-associative. The contents of the report are: (1) algebraic presentation of the cubic hypersurface quasigroups, (2) geometrical motivations, (3) the main open problem.

O. CHEIN

A survey of methods of construction of loops and quasigroups

1. Construction of loops rising as extensions of one loop or quasigroup on another (constructions discussed include the direct product, constructions using quasi factor systems, crossed extensions, quasidirect products, twisted direct products, and constructions using normal forms.)

2. Other constructions of new quasigroups from given quasigroups. These include the singular direct product, generalized singular direct product in various forms, generalized semidirect product and generalized twisted singular direct product.

3. Construction by defining new operations on existing algebraic structures. Structures mentioned in this discussion include quasigroups, loops, groups, rings, fields, ternary rings, vector spaces, and exterior algebras.

4. Constructions which define algebraic operations on geometric structures.

5. Constructions based on designs.

6. Constructions which define algebraic operations on unstructured sets.

7. Structures obtained using the right regular representation.

J.D.H. SMITH

Centrality in quasigroups

A brief introduction to the theory of centrality in quasigroups is given, along with a small sample of its applications. Defining the basic concepts requires a more algebraic approach to quasigroups than the "groupoid", "net", or "latin square" approaches commonly used. This algebraic approach is outlined first. The fundamental definitions of central congruence, centre congruence, central series, nilpotence, and Z-quasigroup (the quasigroup analogue of abelian group) are then presented. One of the major specific applications of centrality to quasigroup theory is the notion of central isotopy. This is a relation between quasigroups which is looser than isomorphism but tighter than isotopy. Next follows a look at Z-quasigroups, examining their structure and considering their uses, and concluding with a classification of quasigroups of prime order. Finally, some basic results relating centrality in quasigroups to centrality in their multiplication groups are obtained.

L. BENETEAU

Commutative Moufang loops and related groupoids.

The trimedial quasigroups (TQs) are isotopic to commutative Moufang loop (CMLs). They allow a synthetic study of the distributive quasigroups and the so-called cubic hypersurface quasigroups. If E is a TQ or a CML, and if its rank p (or more generally, its central rank) is finite, then the central nilpotency class k, as well as the orders of the central factors and the classes of the derived congruence are finite: they can be given bounds depending on n only. Several descriptions of free objects are given, including some exterior algebra representations which are faithful if the rank is sufficiently small.

A. ROMANOWSKA

Commutative idempotent entropic (CIE) quasigroups and related groupoids.

The special role of the dyadic numbers D for the variety of CIE-quasigroups was discussed, and the lattice of all varieties described. The integers together with one of the quasigroup operations of D plays a similar role for the variety of entropic symmetric spaces and the dyadic numbers in the interval [0,1] with another of the quasigroup operations of D a similar role for the variety of CIE-groupoids. In both cases the lattices of varieties and equational bases were described.

M. DEZA

p-rank and association schemes of commutative Moufang loops of exponent 3

Using the homomorphism $E_n \to E_n/Z(E_n)$ we construct a new multidimensional association scheme (MD) on the planes of a commutative Moufang loop $E_n = L_3 \times Z_3^{n-4}$ of exponent 3. This is the only one known MD realizing equality in Bosl-Srivastava inequality for MDs. We also calculate the 3-rank of the incidence matrix of the above loop.

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