MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 33/1983

Darstellungstheorie endlicher Gruppen

24.7. bis 30.7.1983

This conference was organized by Prof. B. Huppert (Mainz) and Prof. G.O. Michler (Essen) and was the first meeting on representation theory of finite groups at the Oberwolfach Institute.

In the talks, methods and results were presented from ordinary, modular and integral representation theory, especially of finite simple groups and groups of Lie type. Special emphasis was given to the recent progress on the long-standing conjectures of R. Brauer, to the block structure of general linear groups, unitary, symplectic and orthogonal groups, and to the applications of cohomological methods in representation theory. Discussions also focused on the extension problem of characters of finite groups and on reduction techniques to the case of quasi-simple groups.

The classification of finite simple groups and the description of characters of the classical groups via the Deligne-Lusztig-theory yield powerful tools in the modern representation theory of finite groups. They play an important role in the present study of several conjectures of Brauer's, e.g. the height zero conjecture.

The conference was attended by 45 participants coming from France, England, Holland, Ireland, Israel, Japan, Switzerland, USA and West Germany. Needless to say that many fruitful discussions evolved during the breaks and in the evenings.

Below, the abstracts of the given talks are listed in alphabetical order.

J. ALPERIN: A proof of the P.A. Smith theorem

P.A. Smith proved in 1937 that if a group G of prime order p acts on a sphere then the fixed point set is a mod p homology sphere. We give a new proof of this result. We use the standard reduction to a finite simplicial complex and then decompose the associated mod p complex into a direct sum of indecomposable complexes from which the result is transparent.

D. BENSON: Diagrams for modular lattices

In modular representation theory, it is very useful to have a notation for writing down the lattice of submodules of a module. I shall describe some recent work by J. Conway and myself, which to each modular lattice satisfying suitable finiteness conditions associates a diagram. This diagram usually contains significantly fewer vertices than the original modular lattice. The theorem stating that the modular lattice may be recovered from the diagram, depends on an identity for modular lattices; namely that if a and c are elements of a modular lattice, and b is minimal with respect to the condition $a \vee b \geq c$, then

$$\underline{b \wedge (a \vee \mathrm{rad}(c)) = \mathrm{rad}(b)\ .}$$

T. BERGER: Extensions of representations on a normal subgroup

Let G be a finite group with normal subgroup N. Suppose that ϕ_1, ϕ_2, $\ldots$, ϕ_n is a G-orbit of irreducible characters of N. There exists a group G^* with normal abelian subgroup N^* so that $G^*/N^* \cong G/N$. Further, there is a G^*-orbit $\lambda_1, \lambda_2, \ldots, \lambda_n$ of irreducible characters of N^* and 1-1 correspondences between $\mathrm{Irr}(H|\phi_1)$ and $\mathrm{Irr}(H^*|\lambda_1)$ where $H/N \cong H^*/N^*$ such that if $H < G$, $K < G$, $H,K < T < G$ and $H/N \cong H^*/N^*$, $K/N \cong K^*/N^*$, $T/N = T^*/N^*$ and if $\theta \in \mathrm{Irr}(H|\phi_1)$, $\psi \in \mathrm{Irr}(K|\phi_1)$ and θ, ψ correspond with θ^* and ψ^* then

$$(\theta\uparrow^T, \psi\uparrow^T)_T = (\theta^*\uparrow^{T^*}, \psi^*\uparrow^{T^*})_{T^*} \quad .$$

(Throughout, if $H/N \cong H^*/N^*$ then we mean by restriction of the isomorphism $G/N \cong G^*/N^*$.)

C. BESSENRODT: Complexity of modules and periodic modules

Let R be a complete discrete valuation ring with quotient field k of characteristic 0, residue class field F of characteristic $p > 0$, $A \in \{R,F\}$ and G a finite group. AG-modules will always mean modules which are f.g. and free over A. First, we will be concerned with some properties of complexity. Especially, an improved version of Green's lower bound for the p-part of the A-rank of a module is given. Then, irreducible RG-lattices are considered. Counterexamples show that the first guesses one might have on the R-forms of an irreducible character are not true. Some bounds for the complexity of an R-form for a given character are stated.

For periodic AG-modules with abelian vertices we get a better lower bound for the p-part of the A-rank than the one above. Furthermore, characters of periodic lattices of odd period are linear combinations of characters of projective lattices, so they are zero on p-elements and $|G|_p$ divides the rank. Because of this, irreducible periodic lattices are always of even period if K is a splitting field for G.

M. BROUÉ: Brauer pairs in the general linear groups

Let ℓ be a prime number and let G be a direct product of general linear groups over finite fields with characteristic $p \neq \ell$. If S is any semi-simple subgroup of G, we set $C_G(S) = \prod_{\sigma \in I_G(S)} GL_{k_\sigma}(v_\sigma)$, and for σ in $I_G(S)$, we set $q_\sigma = |k_\sigma|$, $\phi_{q_\sigma}(\ell)$ = order of q_σ in $(\mathbb{Z}/\ell\mathbb{Z})^\times$.

We show that ℓ-subpairs of G are indexed by triples (P,s,Δ), where P is an ℓ-subpair of G, s a semi-simple ℓ'-element of G commuting with P, and Δ a map from $I_G(s)$ into the set of all Young diagrams such that

(1) $|\Delta(\sigma)| < [v_\sigma{}^P : k_\sigma]$ and $\phi_{q_\sigma}(\ell) \mid [v_\sigma{}^P : k_\sigma] - |\Delta(\sigma)|$

(2) $\Delta(\sigma)$ has no $\phi_{q_\sigma}(\ell)$-hook.

The corresponding ℓ-subpair is denoted by $(P,s,\Delta)_G$. One of the main results is that

$$(P',s',\Delta')_G \subset (P,s,\Delta)_G \iff \begin{array}{ll} (1) & P' \subset P \\ (2) & (\exists g \in C_G(P'))\ (s' = s^g \text{ and } \Delta' = \Delta^g) \end{array}.$$

From that result (and from its proof) follow:

(1) Fong-Srinivasan's classification of blocks of G and of characters in blocks, extended without change to the case $\ell = 2$,

(2) Knowledge of images of blocks through Brauer morphisms,

(3) Structure of the "Brauer-category" of a block, equivalent to an ℓ-Frobenius category of a subgroup of G of the same type.

D.W. BURRY: Lower defect groups with modules, II

Several mathematicians have dealt with the problem of locally determining the lower defect group structure of a block. In his 1969 paper introducing lower defect groups, Brauer gave a method that is sectional. Thus it involves examination of Brauer corresponding blocks of $C(x)$ as x runs through a set of representatives for the conjugate classes of p-elements. The method presented here requires examination of the single local configuration $N(D)$, $C(D)$ to determine the multiplicity of D as a lower defect group of a block. Applications of these results, particularly to the question of the lower defect group multiplictiy of the defect group, will be covered.

J.F. CARLSON: The variety of an indecomposable module is connected

Let G be a finite group and let K be an algebraically closed field of characteristic $p > 0$. The ring $E(K) = \sum_{n \geq 0} \mathrm{Ext}_{KG}^{2n}(K,K)$ is a finitely generated, graded, commutative ring and has an associated affine variety $V(K)$. If M is a KG-module, let $J(M)$ be the annihilator of $\mathrm{Ext}_{KG}^{*}(M,M)$ in $E(K)$, and let $V(M)$ be the corresponding subvariety of $V(K)$. Let $\tilde{V}(M)$ be the associated projective variety. The main theorem is that if M is indecomposable, then $\tilde{V}(M)$ is connected in the sense that it cannot be written as the union of two nonempty, disjoint, closed sets.

The proof is based on the following lemma. Let $\zeta : \Omega^n(K) \to K$ be a non zero homomorphism with kernel L. If $n > 0$ and if $\zeta \in J(M)$ then

$$L \otimes M \cong \Omega^n(M) \oplus \Omega(M) \oplus (\mathrm{proj.}).$$

A.J. CHANTER: Some periodic $SL(2,2^n)$-modules generated by Auslander-Reiten sequences

For a group algebra of infinite representation type, little is known about the indecomposable modules. Work involving the "Auslander-Reiten" quiver of the group, and on the complexity of modules has enabled some kind of classification to be made.

We take advantage of this and construct certain periodic modules for the group $SL(2,2^n)$ using Auslander-Reiten sequences to "generate" new modules from given ones.

E.C. DADE: Extending group modules

If N is a normal subgroup of a finite group G, then an ON-module M can be extended to an OG-module (for any coefficient ring O) if and only if the <u>Clifford extension</u>:

$$X\langle E\rangle : 1 \to U(E_1) \to GrU(E) \to G/N \to 1$$

of the G/N-graded endomorphism ring $E = \mathrm{End}_{OG}(M^G)$ is exact and splits.

When $|G/N|$ is a unit in E and idempotents of $E/J(E)$ can be lifted back to idempotents of E, the splitting of $X\langle E\rangle$ is equivalent to that of the <u>residual Clifford extension</u> $X\langle\overline{E}\rangle$, where $\overline{E}$ is the G/N-graded factor ring $E/J(E_1)E$ of E. Since $\overline{E}$ is much easier to compute this is a great tool for proving extendibility of modules.

R. DIPPER: On the decomposition numbers of the finite general linear groups.

For the symmetric groups there is a well-known theorem that all decomposition matrices in all positive characteristics have lower triangular form with 1 on the diagonal.

Since the Weyl group of the full linear group $GL_n(q)$ is isomorphic to the symmetric groups S_n on n letters, it seems to be natural to ask if a similar statement is true for the general linear groups.

So let $G = GL_n(q)$, and let $2 \neq r$ be a prime not dividing q. Using the classification of r-blocks given by P. Fong and B. Srinivasan, and the classification of the irreducible characters of G given by J.A. Green, it is shown that the decomposition matrix of an r-block B has lower triangular form with 1 on the diagonal, if the semi-simple part s of B has the following property: r divides $(q^{\deg\Lambda}-1)$ for all elementary divisors Λ of s. In this case parts of the decomposition matrix of B may be described in terms of decomposition matrices of some Weyl groups. In particular, this applies to all r-blocks of G, if r divides $q-1$.

K. ERDMANN: On projective resolutions for simple $SL_2(p^n)$-modules

Let B be a nontrivial p-block of the group $SL_2(p^n)$, and let Γ be the graph whose vertices are the irreducible B-modules and where the number of edges $S \to T$ equals $\dim \mathrm{Ext}^1_G(S,T)$. There is a covering graph $\tilde{\Gamma}$ for Γ which describes certain filtrations of the indecomposable projective modules in B.

For $n = 2$ (and $p > 2$), these filtrations are used to describe minimal projective resolutions of the simple B-modules and to find the dimensions of $\mathrm{Ext}^i(S,T)$ for arbitrary i.

P. FONG: Brauer trees in classical groups

Let G be one of the groups $GL_n(q)$, $U_n(q)$, $SO_{2n+1}(q)$, or $Sp_{2n}(q)$. Let B be a cyclic r-block of G, where r is an odd prime and $r \nmid q$. Suppose B is a unipotent block in the sense that the non-exceptional characters in B are unipotent characters. The non-exceptional characters in B are then labeled by partitions or symbols λ. The Brauer tree of B, which is an open polygon, is completely described by combinatorial properties of the λ's. In the case G is $GL_n(q)$, $U_n(q)$, or $SO_{2n+1}(q)$, this implies a complete description of the Brauer tree of any cyclic r-block of G. This is joint work with B. Srinivasan.

D. GLUCK: Brauer's height-conjecture for p-solvable groups. Part II

This is a continuation of T.R. Wolf's talk on the same topic. In this talk we discuss some technical aspects of the proof. We concentrate on techniques for handling imprimitive modules. We also indicate some of the consequences of the classification of simple groups.

J.A. GREEN: Construction of almost split sequences

For a given indecomposable, non-projective module M over a symmetric, finite-dimensional k-algebra A (k a field), a method is given to construct a short exact sequence

Gefördert durch DFG Deutsche Forschungsgemeinschaft

(1) $$0 \to \Omega^2 M \to E \to M \to 0$$

which is almost split in the sense of Auslander-Reiten. If

$$0 \to \Omega M \to P_0 \to M \to 0$$

is a minimal projective resolution of M in mod A, let $P_0 \cong \coprod_{\nu=1}^{n} Ae_\nu$ (e_ν idempotents in A). Then one has k-isomorphisms

$$(P_o, M) \cong \coprod_\nu e_\nu M \,, \qquad (M, P_o) \cong \coprod_\nu (DM)e_\nu$$

(DM = dual space of M, regarded as right A-module). By means of these isomorphisms, one constructs, for each $\theta \in (M,P_0)$, a linear form T_0 on End M; then if $T_0 \neq 0$, $T_0(\text{rad End } M) = 0$, one constructs (1) as the pull-back sequence from the minimal projective resolution

$$\begin{array}{ccccccccc} 0 & \to & \Omega^2 M & \to & P_1 & \to & \Omega M & \to & 0 \\ & & & & & & \uparrow \theta & & \\ & & & & & & M & & \end{array}$$

R. GOW: Representations of the general linear group

Let $q = p^m$, p a prime. Let $G = GL(n,q^2)$, $U = U(n,q^2)$, $H = GL(n,q)$. We show that there is a one-to-one correspondence between U,U-double cosets in G and conjugacy classes in H, and between H,H-double cosets in G and conjugacy classes in U. The Hecke algebras associated with the induced characters 1_U^G and 1_H^G are commutative and thus the characters are multiplicity free. The constituents of 1_U^G are precisely the characters of G fixed by F (the Frobenius map $x \to x^q$) and those of 1_H^G are those fixed by the twisted Frobenius map F^*. These characters seem to have other interesting properties.

Analogues for the compact unitary group $SU(n,\mathbb{C})$ also exist.

M. HERZOG: Products of conjugacy classes

<u>Definitions</u>. G a finite group; $cn(G)$ (the <u>covering number</u> of G) is the smallest integer m such that $C^m = G$ for every conjugacy class $C \neq 1$ of G, if it exists; $ecn(G)$ (the <u>extended c.n.</u>) is the smallest m such that $C_1 \cdot \ldots \cdot C_m = G$ for every collection of m nontrivial conjugacy classes of G, if it exists.

<u>Results</u>. 1) $ecn(G)$ (and hence $cn(G)$) exists iff G is simple.

2) $ecn(A_5) = cn(A_5)+1 = 4$; $ecn(A_n) = cn(A_n)+1 = [\frac{n}{2}]+1$ for $n > 6$.

3) $ecn(Sz(q)) = cn(Sz(q))+1 = 4$; $ecn(PSL(2,q)) = cn(PSL(2,q))+1 = 4$.

4) $cn(G) = 2$ iff $G \cong J_1$ and possibly ${}^3D_4(q)$, q odd.

5) There exist infinite groups with $cn = 2$.

<u>"Conjectures"</u>. Let $k = \#$ conjugacy classes of G.

I. $ecn(G) = cn(G)+1$;

II. $cn(G) \leq k-1$;

III. $C_1C_2 \neq C_3$ for nontrivial conjugacy classes C_i;

IV. $C \subset C^2$ for every conjugacy class;

V. $G = C^2$ for some conjugacy class.

<u>Remarks</u>. (I) <u>Wrong</u>; $ecn(C_3) = 5$, $cn = 3$; is Conway's group the only exception? (II) Can prove $cn(G) \leq \frac{4}{9}k^2$; (III) Would imply Burnside's Theorem. (IV) <u>Wrong</u>; C_3 and others. (V) Thompson's conjecture; no counterexample found.

I.M. ISAACS: Characters of π-separable groups

If G is a p-solvable group, then in some sense, the characteristic p representation theory of G is contained within the ordinary character theory. For instance, using the Fong-Swan theorem, it is easy to see that the irreducible Brauer characters of G are precisely those restrictions of ordinary characters to p-regular elements which cannot be written as sums of other restrictions. Since one need not consider characteristic p representations, it seems natural to replace p by a set π of primes and consider π-separable groups. One of our results is that in this case, the "irreducible" restrictions of characters to π-regular elements really do behave like π-Brauer characters. In particular, they form a basis for the space of π-regular class functions.

A. JUHASZ: On the restriction and induction of modular representations

Let M be an indecomposable KG-module and let $M_H = L_1 \oplus \ldots \oplus L_r$, L_i indecomposable KH-modules for a subgroup H of the finite group G. We consider the following problem and its natural dual for induction of modules: Assume M belongs to a p-block B. Find the blocks b_i of H which contain the L_i. A typical result for restriction we proved is the following: If B has defect group D and $DC_G(D) \leq H$ then every block b of H having defect group D and satisfying $b^G = B$ contains an L_i. For induction we proved: If L is an indecomposable KH-module in a block b of KH with a vertex V and $C_G(V) \leq H$ then all the components of L^G with vertices not containing $V \cap V^g$, $g \in G\backslash N_G(V)$ belong to b^G. This improves a result of J.A. Green. Our approach leads to a short proof of Brauer's third main theorem and also gives a short module-theoretic proof to a theorem of J. Cassey and W. Gaschütz and generalizes Brauer's third main theorem in several directions.

A. KERBER: Eulerian numbers and characters of S_n

The aim was to point to a series of ordinary and in general reducible characters $\chi^{n,k}$ of S_n, $0 < k < n-1$. The main property of them is that each character $\chi : S_n \to \mathbb{C}$ such that $\chi(\pi)$ depends only on the number of cyclic factors of $\pi \in S_n$, is a $\mathbb{Q}$-linear combination of the $\chi^{n,k}$. For details cf. A. Kerber / K.-J. Thürlings: Symmetrieklassen von Funktionen und ihre Abzählungstheorie II. Bayreuther Math. Schriften (in print).

R. KNÖRR, T.R. BERGER: On the height-0-conjecture

It is shown that the "if"-part of Brauer's height 0 conjecture holds for any finite group provided it holds for quasi-simple groups; here a nontrivial perfect group is called quasi-simple if every proper normal subgroup is central. More precisely, we prove

<u>Theorem</u> Assume that for all quasi-simple finite groups the characters in blocks with abelian defect groups are all of height 0. Then the same is true for all finite groups.

An essential step in the reduction is contained in the following proposition and its corollary which may be of independent interest:

<u>Proposition</u> Let $G = DN$ be finite where D is a p-group and $N \lhd G$.
Then: (1) For any block b of N, there is precisely one block B of G covering b.

(2) If B has defect group D then b is the only block of N covered by B.

(3) If B has defect group D and $\chi \in B$ is an irreducible character such that there are $M =_G D$ for any RG-lattice M affording χ, then $\chi{\downarrow}_N$ is irreducible.

<u>Corollary</u> Let B be a block of G with abelian defect group D and let N be a normal subgroup of G. Then there exists a block b of N covered by B such that every irreducible character of b extends to DN.

B. KÜLSHAMMER: Lower defect groups

Let F be a field, char $F = p \neq 0$, G a finite group, FG the group algebra and ZFG its center. Decompose $ZFG = \Delta ZFG \oplus JZFG$ where ΔZFG is semisimple and $JZFG$ is the radical and denote by $\delta : ZFG \to \Delta ZFG$ the corresponding projection. Let $g \in G$ with p-factor g_p, conjugacy class K in G and p'-section S in $C_G(g_p)$. Decompose $FG = FC_G(g_p) \oplus F[G \backslash C_G(g_p)]$ and $FG = F(C_G(g_p))_{p'} g_p \oplus F[(G\backslash C_G(g_p))_{p'} g_p]$ and denote by $\sigma_g : FG \to FC_G(g_p)$ and $\pi_g : FG \to F(C_G(g_p))_{p'} g_p$ the corresponding projections. Then

$$\pi_g(\delta(z)K^+) = \pi_g(\sigma_g(z)S^+) .$$

For conjugacy classes K, L of G, $\ell \in L$ and $P \in \mathrm{Syl}_p(C_G(\ell))$,

$$\delta(K^+)L^+ = \sum_{C \in \mathrm{Cl}(G)} \frac{|L|}{|C|} \left|\{(c,k) \in (C \cap (C_G(\ell_p))_{p'} \ell_p) \times (K \cap C_G(\ell_p)) : c^{-1}k\ell \in P\}\right| C^+ .$$

This implies that for p-subgroups D,P of G and a p-section T of G the sum of multiplicities of P as a lower defect group of blocks having a defect group contained in D in the p-section T can be described as the rank of a suitably defined $\mathbb{F}_p$ -matrix.

O. MANZ: Representation theory of finite groups

A finite group G is called a $PPC(p_1,\ldots,p_n)$-group, if all $\chi \in Irr_{\mathbb{C}}(G)$ have prime power degree and if the primes which occur are $p_1,\ldots, p_n$. Then G is solvable if and only if $n \leq 2$.

Let G be a PPC(p,q)-group. Then the following assertions hold:

(1) $3 \leq dl(G) \leq 5$ and $2 \leq n(G) \leq 4$.

(2) If $dl(G) = 5$, then $\{p,q\} = \{2,3\}$ and G is nearly the semidirect product of GL(2,3) with its standard module.

(3) $dl(G) = 4$ implies that $p = 2$ and q is a Fermat prime.

T. OKUYAMA: Blocks of finite groups with radical cube zero

I shall talk about blocks of finite groups with radical cube zero. Let G be a finite group and k an algebraically closed field of characteristic $p > 0$. Let B be a block algebra of kG with defect group D and let J(B) denote the Jacobson radical of B. Then we obtain the following:

<u>Theorem</u> If $J(B)^3 = 0$ (and $J(B)^2 \neq 0$), then a defect group D of B is a four group for p = 2 and is of order p for an odd prime p.

Furthermore we will determine structures of such blocks.

J. OLSSON: On a conjecture of Brauer's for linear groups

Let $k(B)$ $(k_0(B))$ be the number of complex irreducible characters (of height 0) in an r-block B of a finite group G. We consider the following conjectures

$$\text{(I)} \quad \text{(Brauer)} \quad k(B) \leq |D|$$

$$\text{(II)} \quad k_0(B) \leq |D:D'|$$

where D is a defect group of B and D' its commutator subgroup.

We discuss how to prove these conjectures if G is a symmetric group $S(n)$ (r arbitrary) or $GL(n,q)$ or $U(n,q)$ $(r \nmid q\ ,\ r \neq 2)$ and consider the possibility of extending the result to the classical groups $Sp(2n,q)$ or $SO(2n+1,q)$. When the numbers $k(B)$ and $k_0(B)$ have been computed for all blocks in the groups above, then (I) and (II) follow easily from a property of the partition generating function.

W. PLESKEN: Permutation groups with uniserial modules

H.M. Neumann raised the question whether there are transitive p-subgroups of the symmetric group S_{p^n} of degree p^n which have an exponent less than p^n but nevertheless act uniserially on their natural permutation module over a field of characteristic p. It is proved that the uniserial permutation groups just defined coincide with the p-groups acting p-uniserially in the sense of C.R. Leedham-Green and M.F. Newman on a free abelian group. Independently a quick proof for the criterion for uniseriality by the latter two authors is given for both cases. In particular, the uniserial p-subgroups of S_{p^n} of exponent smaller than p^n exist iff $n > p$.

G. ROBINSON: Blocks, isometries, and sets of primes

In this talk, we will be concerned with the following situation: G is a finite group, π is a set of primes, L is a subgroup of G, A is a union of π-sections of L. Also, for each π-element $a \in A$, we have $C_G(a) = C_L(a)\, O_{\pi'}(C_G(a))$. Finally, whenever two π-elements of A are conjugate in G, they are conjugate in L.

We describe conditions under which certain generalized characters of L which vanish outside A can be "lifted" to generalized characters of G of the same norm. In particular, we show that any generalized character of L which is constant on π-sections and vanishes outside A can be extended to a generalized character of G of the same norm. This particular result gives a positive answer to a conjecture of Reynolds and yields as special cases isometries of Dade and Reynolds. Block theory and methods similar to those used by Puig play an important role in our work.

K.W. ROGGENKAMP, I. REITEN: Inner products on some Green rings

Let T be a Brauer tree with e edges and multiplicity m at the exceptional vertex. Let A be a $\bar{k}$-algebra to T, $\bar{k}$ a field, and

$$\begin{tikzcd}[column sep=small]
\Omega : 0 \arrow[r] & \Omega_0 \arrow[r] & P_{2e-1} \arrow[r] \arrow[dr] & \cdots & \cdots & P_1 \arrow[rr] \arrow[dr] & & P_0 \arrow[r] & \Omega_0 \arrow[r] & 0 \\
& & & \Omega_{2e-1} \arrow[ur] & & & \Omega_1 \arrow[ur] & & &
\end{tikzcd}$$

be a minimal projective resolution of a simple A-module corresponding to a non-exceptional endpoint of T. Let $Q_1,\dots,Q_e$ be the indecomposable projectives and $Q_i^{\perp}$ the duals to Q_i with respect to the Cartan matrix in the rational Grothendieck group. Let $\overline{a(\Omega)}$ $(a(\Omega))$ be the rational Grothendieck group generated by $\{\Omega_i\}$ $(\{\Omega_i, Q_j\})$ with respect to direct sums. For $X,Y \in \{\Omega_i, Q_j\}$ we put $[X,Y] = P(X,Y)$, the A-homomorphisms that factor via projectives, and consider the bilinear form

$$\langle X,Y\rangle = [X,Y] = \sum_{i=1}^{e} [P_i,Y][X,P_i^{\perp}].$$

<u>Proposition:</u> Assume $T \neq \overset{1}{\bullet}\!\!-\!\!\!-\!\!\!-\!\!\overset{1}{\bullet}$.

1.) $[\ ,\ ]$ is non degenerate on $a(\Omega)$.

2.) $\langle\ ,\ \rangle$ is non degenerate on $\overline{a(\Omega)}$.

3.) $\langle M,N\rangle = -\delta_{M,N} + \frac{m}{me+1}\,\mathrm{sgn}\,M\ \mathrm{sgn}\,N$, when $\mathrm{sgn}\,\Omega_i = (-1)^i$. Hence $\langle\ ,\ \rangle$ is invariant under stable equivalence.

Applications to blocks with cyclic defect are given modularly and p-adically. Moreover, a Bachshai order of T is constructed.

P. SCHMID: Invariant characters and invariant lattices

Let $0 \to H \to G \to S \to 0$ be a finite group extension and let ζ be an (absolutely) irreducible character of H which is G-invariant. Let p be a prime (dividing $|H|$) and let K be a finite extension of the p-adics $\mathbb{Q}_p$ such that ζ is realizable over K. (A smallest field of realization over $\mathbb{Q}_p$ is the unramified extension K_ζ of $\mathbb{Q}_p(\zeta)$ of degree $m_{\mathbb{Q}_p}(\zeta)$.) Let R be the ring of integers in K and let $\mathbb{M} = \mathbb{M}(R,\zeta)$ be the set of isomorphism types of RH-lattices affording ζ. By the Jordan-Zassenhaus theorem, $\mathbb{M}$ is a finite set. G (or S) acts on $\mathbb{M}$ via conjugation, and we are interested in finding fixed points, i.e., invariant lattices.

In general no invariant lattices do exist. Examples are provided by the exceptional characters in blocks with cyclic defect groups, taking $K = K_\zeta$ (which here equals $\mathbb{Q}_p(\zeta,\phi)$ for any irreducible Brauer character ϕ in the block). However, enlarging the field of scalars produces invariant lattices:

<u>Theorem.</u> There exists a G-invariant RH-lattice affording ζ if R contains the p-th roots of unity when p is odd and the 4-th roots of unity in case $p = 2$.

The proof uses techniques of Clifford theory. As a consequence one obtains that there is always an invariant lattice if the greatest common divisor of $|S/S'|$ and $\zeta(1)$ is relatively prime to $p-1$ for odd p and to $p = 2$ otherwise.

L. SCOTT: Some Mackey theorems for algebraic groups

A general Mackey imprimitivity theory is presented (joint work with E. Cline and B. Parshall) and applied to give Mackey decomposition theorems in special cases for algebraic groups. Included is a historical and conceptual introduction to the subject, from the perspective of finite group theory and Mackey's original work on continuous groups.

S.D. SMITH: Homology representations and local subgroups

Construction of representations from homology of a coefficient system (defined at local subgroups) was first used by Lusztig in his "discrete series" work; and later by Alvis, Curtis, Deigne-Lusztig in studying "duality". Ronan first observed that these techniques, applied in characteristic p, give a description of modular representations in terms of the geometry of p-local subgroups (including "local geometries of many sporadic simple groups, as well as Chevalley-group buildings). Some basic theoretical results will be described, with a discussion of how they may be applied - for example, construction of modular irreducibles, extension problems, "local recognition" of modules. Applications in "revisionism" of simple-group classification will be emphasized.

J. THÉVENAZ: Burnside ring, combinatorics and topology

The primitive idempotents of the Burnside ring $\Omega(G)$ of a finite group G can be expressed as linear combinations of the canonical basis of $\Omega(G)$ with coefficients involving the Möbius function $\mu(S,T)$ of the lattice L of all subgroups of G. Moreover, $\mu(1,G)$ is equal to the reduced Euler characteristic of the simplicial complex $S(G)$ associated to L. Then:

1) If G is soluble, $\mu(1,G)$ can be computed using a chief series of G.
2) The Möbius function has nice divisibility properties (e.g. $|G|$ divides $|G:G'|\mu(1,G)$).
3) If G is soluble, the simplicial complex $S(G)$ has the homotopy type of a bouquet of spheres of dimension $n-2$, where n is the length of a chief series of G.
4) There exist easy group-theoretic conditions for the contractibility of $S(G)$.

Y. TSUSHIMA: Ext_G^1 for irreducible modules

I'd like to talk about the following theorem which has been established jointly by T. Okuyama and myself.

<u>Theorem</u>. Let p be a prime number and G a finite p-solvable group with a Sylow p-subgroup of order p^n. Let k be an algebraically closed field of characteristic p and S,T simple kG-modules. If $\dim_k S = p^s g$ and $\dim_k T = p^t h$ with $(p,g) = (p,h) = 1$, then we have

$$\dim_k \mathrm{Ext}_G^1(S,T) \leq \min\{(n-s)\dim_k S/\dim_k T \;,\; (n-t)\dim_k T/\dim_k S\}.$$

In particular, we have $\dim_k \mathrm{Ext}_G^1(S,T) \leq 1$, provided $\dim_k S$ is sufficiently smaller than $\dim_k T$. In that case, there is at most one non-split extension of S by T.

R.W. VAN DER WAALL: M-groups and symplectic modules

In this talk we give a survey about recent developments (1980-1983) in the theory of: n-isoclinism, M-groups, symplectic modules. In particular, we shall speak about

1) the results in I.M. Isaacs' paper Math. Z. 182 (1983), 205-221:

   Ex.A. There exists an M-group of order $2^{15}\cdot7^2$ in which the center, which is of order 2, is the unique maximal abelian normal subgroup.

   Th.B. Let G be an M-group of odd order in which every abelian normal subgroup is cyclic. Then G is supersolvable.

   Ex.C. Let p and q be odd primes with $q \mid p^2+1$. Then there exists an M-group of order $p^{10}q$ in which the center, being of order p^2, is the unique maximal abelian normal subgroup.

2) a theorem like Schur and Clifford, for symplectic modules.

3) some results on n-isoclinism, obtained by N.S. Hekster (Spring 1983).

P.J. WEBB: Local formulae for cohomology

Any cohomology group (or more generally Ext group) for a finite group may be expressed in terms of the cohomology of normalizers of elementary abelian subgroups in a rather easy way. On the one hand this allows the cohomology to be determined in many specific cases, and also the algebraic approach adopted connects with equivariant cohomology as used by some authors. Results on Quillen's complex of elementary abelian p-subgroups are given.

T.R. WOLF: Brauer's height conjecture for p-solvable groups, I.

Brauer's height conjecture states that the defect group of a p-block of a finite group is abelian if and only if each irreducible character in that block has height 0. Fong proved one direction for p-solvable groups and we show for p-solvable groups that if the character heights are all zero, then the defect group is abelian. In a minimal counterexample, a group G acts on a vector space V in such a way that each element of V is centralized by a Sylow p-subgroup of G. We determine all such actions for solvable G and prove the height conjecture for solvable groups. We use the classification of finite simple groups to pass from the solvable case to the p-solvable case.

Berichterstatter: Gerhard Schneider

List of participants

Prof. J. Alperin	Dept. Math. University of Chicago	1118 E 58th St. Chicago, Ill. 60637 USA
Dr. D. Benson	Dept. Math. Yale University	New Haven, CT 06520 USA
Prof. T. Berger	Dept. Math. University of Minnesota	Minneapolis, MN 55455 USA
Dr. C. Bessenrodt	Dept. Math. Univ. of Ill. at Urbana	Urbana, Ill. 61801 USA
Prof. M Broué		9, Rue Brézin 75014 Paris FRANCE
Prof. Burry	Dept. Math. University of Hartford	200 Bloomfield Ave. West Hartford, CT 06117 USA
Prof. J. Carlson	Dept. Math. University of Georgia	Athens, Georgia 30602 USA
Mr. A. Chanter	Mathematics Institute University of Warwick	Coventry CV4 7AL ENGLAND
Prof. M.J. Collins	Mathematics Institute University College	Oxford OX1 4BH ENGLAND
Prof. E.C. Dade	Dept. Math. Univ. of Ill. at Urbana	Urbana, Ill. 61801 USA
Dr. R. Dipper	Fachbereich Mathematik Universität Essen	Universitätsstr. 4300 Essen 1 BRD
Dr. K. Erdmann- -Hannabuss	Mathematics Institute Oxford University	24-29 St. Giles Oxford OX1 3LB ENGLAND
A. Feyzioglu	Fachbereich Mathematik Universität Mainz	Saarstr. 21 6500 Mainz BRD
Prof. P. Fong	Dept. Math. University of Illinois	Box 4348 Chicago, Ill. 60680 USA
Prof. W. Gaschütz	Mathem. Seminar Universität Kiel	Olshausenstr. 40-60 2300 Kiel 1 BRD
Prof. D. Gluck	Dept. Math. University of Wisconsin	Madison, WI 53706 USA

Prof. R. Gow	Mathematics Institute University College Dublin	Belfield Dublin 4 IRELAND
Prof. J.A. Green	Mathematics Institute University of Warwick	Coventry CV4 7AL ENGLAND
Prof. M. Herzog	Mathematics Institute Tel-Aviv University	Tel Aviv ISRAEL
Prof. B. Huppert	Fachbereich Mathematik Universität Mainz	Saarstr. 21 6500 Mainz BRD
Prof. I.M. Isaacs	Dept. Math. University of Wisconsin	Madison, WI 53706 USA
Prof. Juhasz	Institute of Mathematics Hebrew University	Jerusalem ISRAEL
Prof. A. Kerber	Fachbereich Mathematik Universität Bayreuth	Postfach 3008 8580 Bayreuth BRD
Dr. W. Knapp	Mathematisches Institut Universität Tübingen	Auf der Morgenstelle 10 7400 Tübingen 1 BRD
Dr. R. Knörr	Fachbereich Mathematik Universität Essen	Universitätsstr. 4300 Essen 1 BRD
Dr. B. Külshammer	Fachbereich Mathematik Universität Dortmund	Postfach 500 500 4600 Dortmund 50 BRD
O. Manz	Fachbereich Mathematik Universität Mainz	Saarstr. 21 6500 Mainz BRD
Prof. G. Michler	Fachbereich Mathematik Universität Essen	Universitätsstr. 3 4300 Essen 1 BRD
Prof. Okuyama	Department of Mathematics Osaka City University	558, Osaka JAPAN
Dr. B. Olsson	Fachbereich Mathematik Universität Dortmund	Postfach 500 500 4600 Dortmund 50 BRD
Prof. H. Pahlings	Fachbereich Mathematik Universität Aachen	Templergraben 64 5100 Aachen BRD
Dr. W. Plesken	Fachbereich Mathematik Universität Aachen	Templergraben 64 5100 Aachen BRD
Prof. C.M. Ringel	Fachbereich Mathematik Universität Bielefeld	Universitätsstr. 4800 Bielefeld BRD

Prof. G. Robinson	Dept. Math. University of Chicago	1118 E 58th St. Chicago, Ill. 60637 USA
Prof. K.W. Roggenkamp	Mathematisches Institut B Universität Stuttgart	Pfaffenwaldring 57 7000 Stuttgart 80 BRD
Prof. P. Schmid	Mathematisches Institut Universität Tübingen	Auf der Morgenstelle 10 7400 Tübingen 1 BRD
Dr. G. Schneider	Fachbereich Mathematik Universität Essen	Universitätsstr. 3 4300 Essen 1 BRD
Prof. L. Scott	Dept. Math. University of Virginia	Charlottesville, Va. 22903 USA
Prof. St. Smith	Dept. Math. University of Illinois	at Chicago Circle Chicago, Ill. 60680 USA
M. J. Thévenaz	Dépt. de Mathématiques Université de Lausanne	CH-1015 Lausanne-Ecublens SUISSE
Prof. Y. Tsushima	Department of Mathematics Osaka City University	558, Osaka JAPAN
Dr. R.W. van der Waall	Mathematisch Instituut Universiteit v. Amsterdam	Roitenstraat 15 1018 WB Amsterdam NEDERLANDE
Dr. P. Webb	Department of Mathematics Manchester University	Manchester M13 9PL ENGLAND
Dr. W. Willems	Fachbereich Mathematik Universität Mainz	Saarstr. 21 6500 Mainz BRD
Prof. T.R. Wolf	Dept. Math. Ohio University	Athens, Ohio 45701 USA