MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1983

Konstruktive Verfahren der komplexen Analysis

7.8. bis 13.8.1983

Die Tagung fand unter der Leitung der Herren Gaier (Gießen), Henrici (Zürich) und Varga (Kent) statt. Die Vorträge beschäftigten sich vor allem mit Approximation im Komplexen, unter anderem Padéapproximation, Kettenbrüchen und Interpolation, sowie mit Numerischen Verfahren der konformen Abbildung. Das Nebeneinander von praktischen Problemen (einschließlich der Präsentation von numerischen Experimenten) und theoretischen Problemen, von Numerischer Mathematik und Funktionentheorie, wurde als besonders interessant empfunden. Mehrere Vorträge begannen mit der Bemerkung: "Angeregt durch den Beitrag von Herrn X auf der letzten Tagung über konstruktive Verfahren der komplexen Analysis vor drei Jahren, habe ich die Ergebnisse erzielt, die ich heute vorlege". Sicherlich war die diesjährige Tagung ähnlich fruchtvoll.

Bemerkenswert ist die Sondersitzung am Dienstagabend. In ihr wurden Probleme gestellt, von denen einige im weiteren Verlauf der Konferenz gelöst werden konnten. Die Teilnehmer gingen in der Hoffnung auseinander, sich bald wieder in Oberwolfach treffen zu können.

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Vortragsauszüge

E. B. SAFF:

A unified approach to "incomplete polynomials"

Let $w:\mathbb{R} \to [0,+\infty)$ be a weight function which is continuous on its support Σ. Suppose that $Z := \{x \in \Sigma : w(x) = 0\}$ is finite and, in the case when Σ is unbounded, $|x| w(x) \to 0$ as $|x| \to \infty$, $x \in \Sigma$. Our goal is firstly to find a compact set $S \subset \Sigma - Z$ such that for every $p \in \Pi_n$ and every $n = 1, 2, \ldots,$

$$\| [w(x)]^n p(x) \|_\Sigma = \| [w(x)]^n p(x) \|_S ,$$

where $\| \cdot \|$ denotes the sup norm over the indicated set and Π_n is the collection of all polynomials of degree at most n. We also deduce, under certain hypotheses on Σ and w, the asymptotic behavior (as $n \to \infty$) of the errors for the Tchebycheff problem

$$E_n(w) := \inf_{q \in \Pi_{n-1}} \| [w(x)]^n (x^n - q(x)) \|_\Sigma .$$

Our general theorem implies as special cases the θ^2-result of G.G.Lorentz for incomplete polynomials as well as recent results of Mhaskar and Saff concerning exponentially weighted polynomials $e^{-|x|^\alpha} p(x)$, $\alpha \geq 1$, $p \in \Pi_n$, on $\mathbb{R}$. The work presented is joint with H. Mhaskar.

A. SHARMA

A Theorem of J.L.Walsh (Revisited)

The theorem of Walsh to which we refer refers to functions which are analytic in the circle $|z| < \rho$ $(\rho > 1)$. We denote this class by A_ρ. Walsh showed that if $p_{n-1}(z;f)$ denotes the Lagrange Interpolation to $f(z) \in A_\rho$ in the n roots of unity and if $P_{n-1}(z;f)$ denotes its Taylor polynomial of degree n-1 about the origin then $\lim_{n\to\infty} p_{n-1}(z;f) - P_{n-1}(z;f) = 0$ for $|z| < \rho^2$. Extensions of this simple result have appeared in Resultate der Math. 3(1981), 155-191. Here we survey later extensions and present some new unpublished results.

ST. RUSCHEWEYH:

On a descent function for polynomials

For a polynomial P of degree n we define a function $f:\mathbb{C} \to \mathbb{C}$ such that

(I) $|P(f(z))| \leq (1-\frac{1}{4n})|P(z)|, \ z \in \mathbb{C}$,

(II) f is continuous at the zeros $\hat{z}$ of P, $f(\hat{z}) = \hat{z}$.

This implies that the sequence $z_{k+1} = f(z_k)$, $k = 0,1,\ldots,$ is convergent for an arbitrary $z_o \in \mathbb{C}$ to a zero $\hat{z}$ of P. In addition we have

(III) If $\lim_{k\to\infty} z_k$ is a simple zero of P then $\{z_k\}$ converges quadratically.

Our function is related to similar functions defined by Kellenberger and M. Kneser.

T. J. RIVLIN:

Optimal Recovery among the Bounded Analytic Functions

The notion of optimal recovery (= optimal estimation of some aspect of a function from limited information about it) is described, and some new and old results concerning optimal recovery among the bounded analytic functions are presented.

V. I. BELYǏ:

On some Applications of Quasiconformal Mappings to the Approximation Theory

The purpose of this lecture is to make a brief survey of some methods, which are connected with the theory of quasiconformal mappings. More specifically, the following subjects will be discussed.

1^o. The "strongly" local theory of a distortion under conformal mappings.

2^o. Integral representations for analytic functions in the quasiconformal regions, their generalizations.

3°. Polynomial approximation in the regions with a quasiconformal and a piece-quasiconformal boundary.

4°. Approximation of functions, which have a quasiconformal extension.

5°. The method of conformal invariants and some problems of numerical analysis.

R. S. VARGA:

On the Bernstein Conjecture for $|x|$ on $[-1,+1]$

S. Bernstein proved in 1914 that there exists a positive β for which

$$\lim_{n\to\infty} (2n\, E_{2n}(|x|)) = \beta \quad (\text{where } E_{2n}(|x|) := \inf_{g\in\Pi_{2n}} \| |x|-g(x) \|_{L_\infty[-1,+1]}).$$

He also obtained the following numerical bounds for β:

$$0.278 < \beta < 0.286,$$

and, as the average of these bounds is 0.282, Bernstein noted "as a curious coincidence" that $\frac{1}{2\sqrt{\pi}} = 0.282...$, and then became known as the

Bernstein Conjecture: Is $\beta = \frac{1}{2\sqrt{\pi}}$?

This problem has remained open since 1914. We show, via computer calculations, that the conjecture is false and that, numerically, there exists an asymptotic development

$$2n\, E_{2n}(|x|) = \beta - \frac{K_1}{n^2} + \frac{K_2}{n^4} - \frac{K_3}{n^6} + \ldots$$

where all $K_i > 0$. (This expansion is truly asymptotic as the K_i increase very rapidly.)

J. WALDVOGEL:

The Whittaker Constant

The Whittaker (1935) constant W is the least exponential type of an entire function $\not\equiv 0$, each of whose derivatives has a zero in the closed unit disk. An upper bound for W is obtained from

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the function

$$\mu_a(z) := \sum_{k=o}^{\infty} a^{\binom{k}{2}} \frac{z^k}{k!}, \quad a = e^{i\alpha} \tag{1}$$

considered by Macintyre (1947):

$$W \leq W_o := \inf_{\alpha} |z_o(a)|, \tag{2}$$

where $z_o(a)$ is the smallest zero in modulus of $\mu_a(z)$. If a is a root of unity $\mu_a(z)$ may be written in terms of elementary functions, in particular,

$\mu_{-1}(z) = \cos z + \sin z$, $z_o(-1) = \zeta := -\pi/4$. The behaviour of $z_o(a)$ near $a = -1$ is given by the expansion

$$z_o(-1-u) = \zeta + \zeta^3\left(\frac{u^2}{2} + \frac{u^3}{6}\right) - \zeta^5\left(\frac{7}{24}u^4 + \frac{27}{20}u^5\right) + \dots \,. \tag{3}$$

Applying numerical procedures (Newton, secant method) to Equ.(2) yields

$$W_o = .73775\,07574\,78224 \dots , \tag{4}$$

a value that has been calculated to 60 D. According to Levinson (1944) lower bounds for W may be obtained from the maximum moduli of the Gončarov polynomials $G_n(z;z_o,\dots,z_{n-1})$ on the unit circle. By using the recurrence relations for G_n and for its gradient as well as the MINPACK routine HYBRDI these maxima where easily obtained to 12D for $n \leq 28$. Since the corresponding lower bounds are extremely close to W_o the new conjecture $W = W_o$ is made. This conjecture is corroborated by a theorem stating an explicit relation between Gončarov polynomials and Macintyre's function.

M. GUTKNECHT:

The computation of the conjugate trigonometric rational function

We show that every continuous real trigonometric rational function (of t, with period 2π) can be written as the real part of a rational function of $z = e^{it}$ that is analytic for $|z| \leq 1$. This result leads immediately to a method for computing the conjugate function, which is again a real trigonometric rational function. We discuss a fast version of this method and possible applications in conformal mapping.

W. SEEWALD:

Fast and accurate Poisson Solvers on the Disk

The Poisson equation, $-\Delta u = f$, can be solved on a rectangle by interpolating the given function f by a two-dimensional trigonometric polynomial. (The same holds for the Helmholtz equation, $-\Delta u + k^2 u = f$.) Due to the slow convergence of the trigonometric polynomial, the error is $O(h^2)$. Methods can be constructed with better convergence.
For the disk, u can be developed into a trigonometric polynomial of the argument only, the coefficients being functions of the modulus which satisfy ordinary differential equations. These can be solved efficiently with errors smaller than $O(h^2)$.

J. HERSCH:

On the reflection principle and the ratio of some conformal radii

Let G and $\tilde{G}$ be two simply connected domains and z_o be a point in their intersection. Then the ratio $R_{z_o}(\tilde{G})/R_{z_o}(G)$ is a conformal invariant. This simple remark yields sometimes the exact ratio of conformal radii, especially if $\tilde{G}$ is obtained from G by reflection of the whole domain G, or of parts of it, with respect to boundary segments or circular arcs. - Now let G_n be a regular polygon with n sides and center O, and $\tilde{G}_n$ the domain obtained from G_n by reflecting across each side s_i of G_n the triangle of base s_i and opposite vertex O. Then $R_o(\tilde{G}_n)/R_o(G_n) = 2^{2/n}$. (This is immediate if $n = 2$ or 4.) The proof uses the notion of the "angular conformal radius" of a trilateral at one of its vertices. - Other elementary ratios of conformal radii are found in some analogous cases.

P. HENRICI:

The Schwarz-Christoffel formula for the conformal mapping of doubly connected polygonal regions

The problem of mapping the disk onto a simply connected (bounded or unbounded) region bounded by a polygon is solved by the well-known Schwarz-Christoffel formula. Koppenfels und Stallmann, Praxis der konformen Abbildung, Grundlehren Bd. 100, give a formula involving elliptic functions for mapping an annulus of modulus μ^{-1} onto a given conformally equivalent doubly connected region bounded by polygons. We represent the same mapping function in terms of the function $\Theta(w) = \prod_d (1-\mu^d w)(1-\mu^d w^{-1})$ (product with respect to odd integers $d > 0$) and, following Ahlfors' Complex Analysis (2nd ed.), we give an elementary proof of our representation. Our formula makes evident that the methods due to Trefethen (SIAM J. Stat.Sci.Comput. 1) for solving the parameter problem in the simply connected case are applicable here also. The function $\Theta(w)$ is rapidly evaluated by means of the series $\sum_{k\in\mathbb{Z}} \mu^{k^2} w^k$.

N. PAPAMICHAEL:

Two expansion methods for the numerical conformal mapping of doubly-connected domains

Let f be the function which maps conformally a given doubly-connected domain Ω onto a circular annulus. We consider the use of two closely related methods for determining approximations to f of the form

$$f_n(z) = z \exp\{\sum_{j=1}^{n} a_j u_j(z)\} ,$$

where $\{u_j\}$ is a set of basis functions. The two methods are respectively a variational method, based on an extremum property of the function

$$H(z) = f'(z)/f(z) - 1/z,$$

and an orthonormalization method, based on approximating the function H by a finite Fourier series sum. Our main purpose is to show that the computational efficiency of the two methods can be improved considerably by using a basis set which contains terms that reflect the main singularities of H on $\partial\Omega$ and in compl $(\bar{\Omega})$.

W. L. WENDLAND:

On the Spline Approximation of Singular Integral Equations

The lecture gives a short survey on recent asymptotic error results for the Galerkin and the collocation methods with splines applied to singular integral equations with Cauchy kernel on a closed smooth curve Γ having principal part $A_o I + C_o H$ with I identity, H Hilbert transform. Let $\mathcal{S}_d(\Delta)$ be the smoothest splines of degree d associated with the family of grids Δ with maximal meshwidth h. Let u be the actual solution and u_Δ the approximate sln. obtained by one of the above methods, $e = u - u_\Delta$. Then $\|e\|_{H^\tau(\Gamma)} \leq ch^{\alpha-\tau}\|u\|_{H^\alpha}$ with $\tau < d + \frac{1}{2}$ and $-d-1 \leq \tau \leq \alpha \leq d+1$, $0 \leq \alpha$ for Galerkin's method [Hsiao-Wendland J.Int.Equ. 3 (81)], and with $0 \leq \tau \leq \alpha \leq d+1$ for collocation at the break points for d odd or collocation at the midpoints between breakpoints for d even if and only if the singular integral equations are strongly elliptic, i.e. $\det (A_o + \lambda C_o) \neq 0$ on Γ and for all $\lambda \in [-1,1]$. [Arnold-Wendland Math.Comp. 41 (83)] [Saranen-Wendland (83)][Prössdorf-Schmidt Math.Nachr. 100 (81)] [Schmidt 83]. This result has been extended by G. Schmidt by choosing the collocation points as $s_j = h(j+\varepsilon)$, $0 \leq \varepsilon < 1$ and introducing a generalized principal symbol to a much wider class of equations. All these results hold also for pseudodifferential operators of arbitrary order m on Γ (including Symms integral equation where $m = -1$ which is strongly elliptic).

H.-P. HOIDN:

The collocation method applied to Symm's integral equation

The conformal mapping of a simply connected, bounded Jordan region G onto the unit disk may be described by a boundary correspondence function θ. For θ' a relationship is given by Symm's integral equation. If G has a boundary curve in C^1, Symm's integral operator defines an isomorphism between the Sobolevspaces H^0 and H^1. In the space of spline functions of degree p, θ' is approximated by θ_n', whose coefficients are determined by collocation. We obtain optimal rates of convergence and the super approximation property, e.g. for $t = -1, 0$

$$\|\theta' - \theta_n'\|_{H^t} = \mathcal{O}(h^{s-t}), \quad \theta' \in H^s, \quad t \leq s \leq p+1.$$

We notice that the asymptotic error estimates are in good agreement with numerical results.

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R. WEGMANN:

An iterative method for conformal mapping

An iterative method is described for the determination of the conformal mapping of the unit disc onto a simply connected region G with smooth boundary. It requires in each step the solution of a Riemann-Hilbert-problem. It is shown that an explicit solution of this problem can be obtained in closed form in terms of the operator K of conjugation. For boundary curves with Lipschitz continuous second derivative the method converges quadratically in the Sobolev-space $W^{1,2}$. If K is approximated by Wittich's method, then the numerical realization of the method on N grid points requires in each step four real FFTs of length N. The Sobolev-norm of the numerical error can be estimated for boundaries in Hölder-spaces $C^{1+\mu}$ as well as for analytic boundaries. Test calculations show that compared with the methods of Theodorsen or Fornberg this method is the most efficient one to obtain results with high accuracy.

O. HÜBNER:

The Newton-method for solving the Theodorsen integral equation

Each step of the Newton-method for the nonlinear and singular Theodorsen integral equation (T) requires to solve a linear integral equation. The exact solution of this linear equation is given by using the solution of a certain Riemann-Hilbert-problem for the unit disc. Computation of the discretized solution of the linear equation is much less work than computing one step of the Newton-method for the discretized Theodorsen equation (T_N). Convergence conditions for the Newton-method for (T) are given. With help of the machinery used, it can also be shown that - under certain assumptions - (T_N) has always exactly one solution in the neighborhood of the solution of (T).

O. WIDLUND:

An analysis of a numerical conformal mapping method due to Fornberg

This method was introduced in Fornberg, SIAM J.Sci.Stat.Comput., 1, 1980, pp.386-400. The purpose of this contribution is to provide a theoretical analysis. It is established, that i) the $N \times N$ linear system of equations with the positive semidefinite, symmetric matrix G, introduced in Fornberg's paper is always solvable; ii) in the vicinity of the solution of the nonlinear problem and for N large enough, $N-1$ eigenvalues of G are bounded away from zero; iii) the eigenvalues of $\hat{G}$, obtained from G by deleting the first row and first column of G are bounded from below by constant/N; iv) the eigenvalues of $\hat{G}$ cluster at 1. These results make it possible to establish the rapid convergence of the conjugate gradient method, used in an inner iteration, which has been observed in numerical experiments by Fornberg and others.

K. MENKE:

On Tsuji Points in a Continuum

Let $D: = \{z : |z| < 1\}$ and E be a compact subset of D. Tsuji introduced a point system on E by which the hyperbolic capacity $\rho(E)$ of E can be approximated. If E is a continuum, the point system can also be used to approximate the conformal mapping of $D \setminus E$ onto $\{z : \rho(E) < |z| < 1\}$. Here some sharper estimates for the approximations are given under additional assumptions on E.

W. J. THRON:

Basic Lemmas in the Theory of Modifications of Continued Fractions

In the theory of modifications of continued fractions certain estimates play a key role. After a brief historical introduction to the subject as a whole we shall trace the development of these fundamental lemmas from the work of H. Waadeland and the author to the recent results of L. Jacobsen.

L. JACOBSEN:

On solutions of three-term recurrence relations

The solutions $\{y_n\}$ of the linear, homogenous three-term recurrence relations

(*) $$Y_{n+1} + b_n Y_n = a_n Y_{n-1} \quad \text{for } n = 1,2,3,\ldots$$

$$a_n, b_n, Y_n \in F \text{ - normed field, and } a_n \neq 0 \text{ for all } n,$$

are uniquely defined, up to a constant factor, by the solutions of the two-term recurrence relations

(**) $$g_n + b_n = a_n / g_{n-1} \quad \text{for } n = 1,2,3,\ldots \quad (g_n = Y_{n+1}/Y_n.)$$

By Pincherles theorem, (*) has a non-trivial minimal solution iff $K(a_n/b_n)$ converges, and in that case $g_n = f^{(n)} = \underset{m=n+1}{\overset{\infty}{K}} (a_m/b_m)$ for all $n \geq 0$. We shall prove that if $K(a_n/b_n)$ converges, then any two dominant solutions $\{y_n\}$, $\{y'_n\}$ of (*) satisfy

$$y_{n+1}/y_n - y'_{n+1}/y'_n = g_n - g'_n \to 0.$$

W. B. JONES:

Digital Filters and Schur Continued Fractions

Connections between continued fractions and digital filters are discussed; particularly, digital filters that are implemented by ladder directed graphs. Such filters are used in the analysis and synthesis of speech and in signal detection. It is shown that the transfer function of ladder filters can be represented by modified Schur fractions and hence the filters are stable. Applications of the qd-algorithm to compute the poles of the transfer function are also discussed. More generally, it is shown that a necessary and sufficient condition for a polynomial to have all of its zeros inside the interior of the unit circle is that a certain rational (test) function be represented by a modified Schur fraction. This result is the analogue of a theorem of Wall and Frank on stabel (Hurwitz) polynomials.

W. NIETHAMMER:

A remark on the acceleration of limit periodic continued fractions

A continued fraction (c.f.) $K(a_n/1)$ is called limit periodic if $\lim_{n\to\infty} a_n = a$. For $a \in \mathbb{C}$ with $a \notin (-\infty,-1/4)$, $a \neq 0$, Thron-Waadeland (1980) developed a modification of a limit periodic c.f. which yields faster convergence. There is a strong acceleration if the convergence of the sequence $\{a_k\}_{k\geq 1}$ is superlinear or linear, whereas the acceleration remains modest, if the $\{a_k\}_{k\geq 1}$ converge sublinearly. On the other side, many c.f., e.g., those belonging to hypergeometric series, show this behavior. It is proposed to apply an appropriate Euler summability method to the series equivalent to the limit periodic c.f.. Some theoretical and numerical results show the effectivity of this method especially in the case of sublinear convergence mentioned above.

H. WAADELAND:

Thoughts and pre-thoughts about value regions and pre value regions

A. Let $\overset{\infty}{\underset{n=1}{K}} \frac{a_n}{1}$ be a continued fraction where all $a_n \in E$ (the element region), Let $V \neq \emptyset$ be a set such that 1) $E \subseteq V$, 2) $\frac{E}{1+V} \subseteq V$. Then V is called a value region, and contains all the approximants $\frac{a_1}{1+}\cdots\frac{a_n}{+1}$. The set cl V contains also all possible values of converging continued fractions with all $a_n \in E$. Knowledge of V is useful in estimating truncation errors for continued fractions, but V is sometimes an awkward set. A pre value region is a set V for which 2), but not necessarily 1) is satisfied. If in particular it contains all possible values of continued fractions, it is called a limit region. The limit regions can be used to estimate modified truncation errors. Examples are given to illustrate that this approach in many cases and in many ways is superior to the traditional one.

B. Corresponding element- and value regions are often determined by a backward procedure, i.e. by choosing V first. A forward procedure, calculating in turn E, $\frac{E}{1+E}$, $\frac{E}{1+\frac{E}{1+E}}$, ... is in most cases unrealistic. However, in several of the known results the value region (or limit region) is completely determined by a very sparse set of continued fractions from E. These observations may be useful in numerical and computer graphics experiments, some of which are being carried out in Trondheim presently.

G. G. LORENTZ:

Bernstein Inequalities and Functions of a Hardy Class

An inequality of Bernstein type is proved, which is based on the "$\prec$" order relation of Hardy, Littlewood and Pólya. This is precisely the correct lemma to derive certain theorems about sequences $T_n \to f$ of trigonometric polynomials, which converge in the L_1 norm. Previously, v.Golitschek and G.G.Lorentz have proved a similar theorem for bounded converges. These theorems, in turn, imply results about convergent sequences $P_n(z) \to f(z)$ of polynomials in the Hardy space H_1. It follows that if f is not identically zero, the polynomials P_n can have only $o(n)$ zeros on the circle $|z| = 1$.

K. ZELLER:

Der Approximationssatz von Carleman

Der Vortrag erörtert verschiedene Zugänge zu diesem Satz: Komplexe Approximation, Operatoren, unendliche Interpolation (vergleiche Gaier, Hoischen, Kaplan, Niethammer, Sinclair). Dabei wird der Nutzen funktionalanalytischer Methoden hervorgehoben. Im Mittelpunkt steht ein kurzer Beweis (Zerlegung der Funktion in endkonstante Summanden, Anwendung von Operatoren, Konvergenz im Sinne von Halbnormen). Zum Schluß folgen Hinweise auf Verallgemeinerungen und Erweiterungen.

C. CHAFFY:

Polynomial and rational interpolation in $\mathbb{C}^2$; integral forms of the errors

In the case of two complex variables, the problem of interpolation formulas for holomorphic functions depends on the set of interpolation points. Here we present two constructive methods.
First, the "tensor product" process - repeated application of formulas with one variable - gives results for sets of points, that satisfy some "rectangular" conditions. Bidimensional divided differences naturally appear in polynomials and remainders, that we express in an integral form.
Second, the "homogeneous" process, concerning Taylor conditions, provide an other expression of the remainders in the case of triangular polynomials. Moreover, this process produces rational functions that already exist: Abstract Padé approximants.

W. SCHEMPP:

Kurvenintegraldarstellungen kardinaler Splinefunktionen

Die exponentiellen und die logarithmischen Splinefunktionen zur Schrittweite $h > 0$ sind in ihrem Konvergenzverhalten beim Grenzübergang $h \to 0+$ vollständig verschieden. Mit Hilfe der inversen Laplace- bzw. der inversen Mellin-Transformation werden für die exponentiellen und die logarithmischen kardinalen Splinefunktionen komplexe Kurvenintegraldarstellungen mit nicht kompakten Integrationswegen angegeben und damit deren Konvergenzverhalten auf der reellen Zahlengeraden $\mathbb{R}$ bzw. auf der offenen positiven reellen Halbgeraden $\mathbb{R}_+^\times$ studiert. Als Nebenergebnis erhält man eine komplexe Kurvenintegraldarstellung für die Euler-Frobenius-Polynome, der man alle ihre wesentlichen Eigenschaften entnehmen kann.

G. SCHMEIßER:

A cardinal series and its applications

In communication, control theory, and data processing the following problem arises: Is it possible to reconstruct a signal function f from its values (samples $f(x_n)$) taken at equally spaced points x_n $(n = 0, \pm 1, \pm 2, \ldots)$ on the real axis? The following fundamental answer, independently found by Whittaker, Kotel'nikov, and Shannon, leads us to constructive complex analysis: If f is the restriction of an entire function of exponential type at most τ and $f \in L^2(\mathbb{R})$ - in this case the engineers call the signal bandlimited to $[-\tau, \tau]$ - then

$$f(x) = \sum_{n=-\infty}^{\infty} f(n\pi/\tau) \frac{\sin(\tau x - n\pi)}{\tau x - n\pi} =: C_\tau[f](x) .$$

Unfortunately this interpolation operator shows several bad properties which we try to overcome by constructing a modified interpolation operator $\mathfrak{C}_\tau$ such that

(a) $\mathfrak{C}_\tau$ reproduces a larger class of functions.

(b) The evaluation of $\mathfrak{C}_\tau$ is numerically more efficient.

(c) Applied to uniformly continuous and bounded functions $\mathfrak{C}_\tau$ attains the order of best approximation by entire functions of exponential type at most τ.

W. B. GRAGG:

Superfast Padé fractions

Brent has given an elegant and concise derivation of a fast, $O(n \log_2^2 n)$, algorithm for computing Padé fractions on downward sloping "stair-cases" in the Padé table. Use of the Gohberg-Semenecul formula, a matrix interpretation of the Christoffel-Darboux formula, then permits the fast solution of related Hankel or Toeplitz systems of linear equations. Unfortunately, Brent's "proof of correctness" of his algorithm is not complete. In this note we give a similar, and complete, result which leads to an algorithm of more general applicability. There remain open questions regarding the numerical stability of such algorithms.

N. TREFETHEN:

Complex Rational Chebyshev and Padé Approximation

We outline a number of new results with the common thread that they involve degenerate or near-degenerate rational functions. ($r = \frac{p_m}{q_m} \in R_{mn}$ is degenerate if $\deg p < m$ and $\deg q < n$.)

(1) Complex rational best approximations (BA's) on $|z| \leq 1$ are in general nonunique.

(2) Complex local BA's are in general not global BA's.

(3) Complex BA's to real functions on $[-1,1]$ are in general arbitrarily much better than real ones.

(4) Real or complete BA's on $[-\varepsilon,\varepsilon]$, $[0,\varepsilon]$, or $|z| \leq \varepsilon$ do not in general approach the Padé approximant r^P as $\varepsilon \to 0$.

(5) Identical entries in the "Walsh table" for complex Chebyshev approximation do not in general fall into square blocks.

M. EIERMANN:

On the Convergence of Padé-type Approximants to Analytic Functions

For a given summability method, the Okada Theorem describes a domain into which an arbitrary power series f can be analytically continued, if such a domain is known for the geometric series. Okada's result is extended to more general methods of analytic continuation. Especially, sequences $\{(m-1/m)_f\}_m$ of Padé-type approximants to f are considered. ($(m-1/m)_f$ is a rational function of the type $(m-1,m)$ satisfying the condition $f(z)-(m-1/m)_f(z) = O(z^m)$.)

The convergence theorem is applied to sequences of Padé-type approximants having special denominator polynomials.

G. OPFER:

T-Approximations on Segments in $\mathbb{C}$

Bei der Richardson Iteration $x_{j+1} = x_j - \alpha_j(Ax_j - a), j=1,2,\ldots,n$ zur Lösung eines nichtsingulären $N \times N$-Gleichungssystems $Ax = a$ führt die optimale Wahl der Parameter $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{C}$ auf die

Lösung des Tschebyscheff-Approximationsproblems

(*) $\min\{\|p\|_\infty : p \in \Pi_n, p(0) = 1, \ \|p\|_\infty = \max_{z \in E} |p(z)|\}$

mit $\sigma(A) \subset E$ ($\sigma(A)$ = Spektrum von A). Nach einem Satz von Bendixson (vgl. z.B. Stoer-Bulirsch) kann ein Rechteck angegeben werden, in dem die Eigenwerte von A liegen. Dieses Rechteck kann auch auf ein (achsenparalleles) Segment zusammenschrumpfen. Es ist daher naheliegend, auch komplexe T-Approximationen auf Segmenten zu betrachten. Es werden weitere Eigenschaften des Problems (*) erörtert und einige numerische Beispiele dazu angegeben.

S. ELLACOTT:

Complex Polynomial and Rational Approximation

Faber-Padé and Faber-CF approximations are constructed for Faber domains; the rational functions being handled by a theorem that the Faber transform of a rational function is rational. Some theoretical and numerical results on these approximations are presented.
An elementary proof of Pommerenke's representation of the Faber polynomials is also given.

L. REICHEL:

On complex rational approximation by interpolation and the discrete least squares method

Approximation of analytic functions on regions in the complex plane by polynomials and rational functions is discussed. We consider approximation by interpolation and by the discrete least squares method using nodes on the boundary of the region distributed with given distribution functions. First we consider polynomial approximation. Approximation by interpolation does generally not converge, however, we show that the discrete least squares method gives good results if the degree of the polynomial is chosen as a suitable function of the number of least squares nodes. We then turn to approximation by rational functions (generalized polynomials) and show how for a given distribution of nodes we can construct a space of generalized polynomials so that in many cases convergence is obtained even for approximation by interpolation. Applications to conformal mapping and to the boundary collocation method will be presented if time permits.

A. RUTTAN:

Best Uniform Complex Rational Approximants

Let f be a continuous function on a compact set K of the complex plane, and let R = q/p be a rational approximant of f from the set of rational function $\Pi_{mn} := \{q/p : \deg q \leq m,\ \deg p \leq n\}$. Assume that the set C of critical points of $f - q/p$ contains exactly m+n+2 points. Then there is an $(m+n+2) \times (m+n+2)$ Hermitian matrix H defined in terms of f,q,p and the points of C such that R is a best uniform approximant of f on K from Π_{mn} if and only if H is positive semi-definite.

R. FREUND:

Über ein spezielles (Tschebyscheff-)Approximationsproblem

Wir (mit St. Ruscheweyh) betrachten Approximationsprobleme der Form

$$(P) \qquad \min_{\substack{p \in \Pi_k \\ p(a)=1}} \ \max_{z \in [-1,1]} |p(z)| \ ,$$

wobei $a \in \mathbb{C}$, $\Pi_k = \{p(z) \equiv \sigma_0 + \sigma_1 z + \ldots + \sigma_k z^k \mid \sigma_j \in \mathbb{C}\}$. Für den Fall, daß a auf der imaginären Achse liegt, wird die Lösung von (P) explizit angegeben. Ferner wird auf einen Zusammenhang zwischen (P) und dem Problem, die scharfe Konstante $s_k(R)$ in der Ungleichung

$$\max_{z \in E_R} |p(z)| \leq s_k(R) \max_{z \in [-1,1]} |p(z)| \qquad \text{für alle } p \in \Pi_k$$

(E_R, $R > 1$, bezeichnet die Ellipse mit Brennpunkten ±1 und großer Halbachse $\frac{1}{2}(R + \frac{1}{R})$) zu bestimmen, eingegangen.

H.-P. BLATT:

Zeros of best linear Tschebyscheff-Approximations

Let E be a compact subset of $\mathbb{C}$, such that the complement K of E is connected and regular. We study the zeros of the best polynomial approximations $p_n \in \Pi_n$ of f on E. Characterizing the asymptotic behaviour of $|p_n(z)|^{1/n}$, we conclude that under weak assumptions every point of the boundary ∂E is a limit point of zeros of p_n. If ∂E is an analytic Jordan curve the zeros of a subsequence of p_n are in some sense equidistributed. The results generalize theorems of Rosenbloom, Walsh, Gončar, Borwein. The work presented is joint with E.B.Saff.

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