

Tagungsbericht 36/1983

Arithmetische Algebraische Geometrie

14.8. bis 20.8.1983

Die Tagung wurde geleitet von den Herren G. Harder (Bonn) und N. Katz (Princeton). Ihr Ziel war es, neue Ergebnisse aus den vielfältigen Zweigen der arithmetischen algebraischen Geometrie vorzustellen. Einen aktuellen Schwerpunkt bekam die Tagung durch drei Vorträge von G. Faltings über seinen kaum vier Monate alten Beweis der Vermutungen von Tate, Šafarevič und Mordell.

Vortragsauszüge

G. FALTINGS:

The Conjectures of Tate and Mordell

The main technical point in the proof of these conjectures is the determination of a height-function on the moduli-space of principally polarized abelian varieties:

If  $p : A \rightarrow \text{Spec}(R)$  ( $R = \text{integers of a number field } K$ ) is a semi-abelian variety, such that the general fibre is proper and principally polarized, then the height of the corresponding point in the moduli-space can be estimated by  $h(A) = \deg(\omega_{A/R})$ , where  $s : \text{Spec}(R) \rightarrow A$  is the zero-section, and  $\omega_{A/R} = s^*(\Omega_{A/R}^g)$  with  $g = \dim A_K$ .

The degree has to be taken (according to Arakelov) in the sense of metricised line-bundles, and the metrics at the infinite places are given by square-integration.

We have to study the behaviour of  $h(A)$  under isogenies: It does

not change very often. E. g., if we divide  $A$  by the levels of an  $\ell$ -divisible subgroup or by an  $\ell$ -group with  $\ell$  big. By well-known arguments, this implies Tate's conjecture on the homomorphisms of abelian varieties over number fields. To derive Šafarevič's conjecture (which was known to imply Mordell by work of Paršin) one needs to know that there are only finitely many abelian varieties (up to isogeny) with prescribed set of bad reduction. This follows from a Čebotarev-type argument.

All the theorems extend to finitely generated extensions of  $\mathbb{Q}$ .

### C. SZPIRO:

#### Intersection Theory on Arithmetic Surfaces

A survey was given of work due to Arakelov and Faltings. The main topics mentioned included:

1. Degree and theorem of Riemann-Roch for a metricised line-bundle over the ring of integers of an algebraic number field.
2. Arakelov's theory:
  - 2.1 Admissible metrics on line-bundles, definition of intersection-pairing.
  - 2.2 Symmetry and rational equivalence.
  - 2.3 Index-theorem (à la Hodge) of Hriljac and Faltings.
  - 2.4 Adjunction-formula and canonical metrics.
  - 2.5 Lemma on section and Conjecture of "small" points.
3. Riemann-Roch d'après Faltings.
4. Faltings' Existence Theorem: If  $(L.L) > 0$  and  $\deg L > 0$ , then  $L^{\otimes n}$  (for  $n \gg 0$ ) has a positive section.

### P. SCHNEIDER:

#### Rigid-analytic Mellin Transforms

Let  $E/\mathbb{Q}_p$  be a finite extension contained in the completion  $\mathbb{C}_p$  of  $\mathbb{Q}_p$ ; let  $\Gamma \leq \mathrm{SL}_2(E)$  be a Schottky group of rank  $r > 1$  and denote by  $\mathcal{L} \subseteq E \cup \{\infty\}$  the set of limit points of  $\Gamma$ .  $\Gamma$  acts discontinuously on the analytic set  $H := \mathbb{C}_p \cup \{\infty\} \setminus \mathcal{L}$  and, according to Mumford,  $C := \Gamma \backslash H$  is a smooth projective curve over  $\mathbb{C}_p$  of genus  $r$ . We always assume  $\infty \in \mathcal{L}$ . A holomorphic function  $f : H \rightarrow \mathbb{C}_p$  is called an automorphic form of weight  $n \in \mathbb{Z}$  for  $\Gamma$ , if  $f\left(\frac{ax+b}{cx+d}\right)^p = (cx+d)^n f(x)$  for

all  $\begin{pmatrix} ab \\ cd \end{pmatrix} \in \Gamma$  and  $x \in H$ . It was shown that one can associate in a natural way, to an automorphic form  $f$  a  $\mathbb{C}_p$ -valued distribution  $\mu_f$  on the locally compact set  $\mathcal{L} \setminus \{\infty\}$ , which should be considered an analogue of the Mellin transform of an elliptic modular form. As an application (using results of Ribet and Čerednik) one gets a  $p$ -adic  $L$ -series for a certain class of Weil curves with multiplicative reduction at  $p$ .

N. KOBLITZ:

Possibility of a Theory of  $p$ -adic Modular Forms of Half-integral Weight

Let  $g_i \in M_{k_i-1}(N)$  ( $i=1,2$ ) be normalized Hecke eigenforms with (say) rational  $q$ -expansion, and suppose  $N$  is squarefree,  $k_i \equiv 0 \pmod{2}$ ,  $k_1 \equiv k_2 \pmod{2(p-1)}$ .

Generalized Hida-Question: Can the Shimura-Kohnen isomorphisms  $\varphi_i : M_{k_i/2}^+(4N) \simeq M_{k_i-1}(N)$  be normalized in such a way that the  $\varphi_i^{-1}(g_i)$  have rational  $p$ -integral  $q$ -expansions, not  $\equiv 0 \pmod{p}$ , and satisfying  $f_1 \equiv f_2 \pmod{p^N \mathbb{Z}[[q]]}$  ?

An affirmative answer to this question would entail (by a theorem of Zagier and Kohnen) a  $p$ -adic interpolation of the square-roots of  $L$ -values interpolated by Katz's 2-variable  $p$ -adic  $L$ -functions for imaginary quadratic fields.

M. NYGAARD:

K 3 Surfaces of Finite Height

Let  $k$  be a perfect field of characteristic  $p > 0$ , and  $X_0/k$  a K3 surface. We assume that the formal Brauer group  $\widehat{\text{Br}}(X_0)$  is of finite height. The enlarged Brauer group  $\psi(X_0)$  sits in an exact sequence

$$(*) \quad 0 \rightarrow \widehat{\text{Br}}(X_0) \rightarrow \psi(X_0) \rightarrow D \rightarrow 0,$$

$D$  being the étale  $p$ -divisible group associated to  $H_{\text{fl}}^2(\overline{X}_0, \mu_p^\infty)_{\text{div}}$ .

Let  $X/R$  be a lifting of  $X_0$  to some local artinian ring  $R$  with residue field  $k$ .

**Theorem:** There is an exact sequence of filtered F-crystals on Spec R:

$$(**) \quad 0 \rightarrow \mathbb{D}(v^*(X)) \rightarrow H_{\text{crys}}^2(X) \rightarrow \mathbb{D}(\widehat{\text{Br}}(X))(-1) \rightarrow 0.$$

Using the existence of certain liftings  $X$  of  $X_0$  over  $W(k)$ , giving split exact sequences (\*) for  $X$  and (\*\*), one can deduce Tate's conjecture for  $X_0$ , provided  $k$  is finite, and  $p > 3$ .

S. LICHTENBAUM:

Values of Zeta-functions at Non-negative Integers

Let  $X$  be a smooth, projective, geometrically connected variety over a finite field  $k$ . Write  $d = \dim X$  and  $p^f = q = \#k$ . Let  $\zeta(X, s) = Z(X, t) [t = q^{-s}]$  be the zeta-function of  $X$ , and define  $a_n = -\text{ord}_{t=q^{-n}} Z(X, t)$ . Generalizing the known formulas for  $n=0, 1$

(assuming  $H^2(X, G_m)$  finite in the latter case), we conjecture that a complex  $\Gamma(n)$  of étale sheaves exists such that  $\Gamma(0) = \mathbb{Z}$ ,  $\Gamma(1) = G_m[-1]$  and

- (1) the cohomology sheaves  $H^q(\Gamma(n))$  are essentially equal to  $\text{Gr}_Y^n K_{2n-q}^{\text{ét}}(X)$ , where  $\text{Gr}_Y^n$  denotes the  $n$ -th graded piece of Soulé's  $\gamma$ -filtration on  $K$ -theory.
- (2)  $\Gamma(n)$  is acyclic outside of  $[1, n]$ .
- (3) There are natural pairings  $H^i(X, \Gamma(r)) \times H^j(X, \Gamma(s)) \rightarrow H^{i+j}(X, \Gamma(r+s))$ .
- (4)  $H^i(X, \Gamma(r))$  is zero for  $i$  large, and finite for  $i \neq 2r, 2r + 2$ .  $H^{2r}(X, \Gamma(r))$  is finitely-generated,  $H^{2d+2}(X, \Gamma(d)) \cong \mathbb{Q}/\mathbb{Z}$ , and the natural pairing from  $H^i(X, \Gamma(r)) \times H^{2d+2-i}(X, \Gamma(d-r))$  to  $H^{2d+2}(X, \Gamma(d))$  gives a duality. This implies that  $H^{2d}(X, \Gamma(d))$  should be the group of zero-cycles on  $X$  modulo rational equivalence and so have a degree map to  $\mathbb{Z}$ .
- (5)  $\lim_{t \rightarrow q^{-n}} (1-qt)^{a_n} Z(X, t) = \pm \chi(X, \mathcal{O}_X, n) \chi(X, \Gamma(n))$ , where

$$\chi(X, \Gamma(n)) = \frac{\#H^0(X, \Gamma(n)) \dots \#H^{2n}(X, \Gamma(n))}{\#H^1(X, \Gamma(n)) \dots \#H^{2n+1}(X, \Gamma(n)) \dots R_n(X)} \text{tor} \#H^{2n+2}(X, \Gamma(n)) \text{cotor} \dots$$

and  $R_n(X)$  is a "regulator term" involving the above pairing between  $H^{2n}(X, \Gamma(n))$  and  $H^{2d-2n}(X, \Gamma(d-n))$  into  $H^{2d}(X, \Gamma(d))$ , and  $\chi(X, \mathcal{O}_X, n)$  is a  $p$ -torsion-term whose explicit form has been given by Milne.

Y. IHARA:

Infinite Unramified Galois Extensions of Global Fields and Related Geometric Questions

Let  $M/k$  be an infinite unramified Galois extension of a global field  $k$ . By investigating an analogue of  $d \log \zeta(s)$  ( $\zeta$ : the zeta-function) for the infinite extension field  $M$  and its analytic continuation especially towards  $s = \frac{1}{2}$ , we obtain an upper bound for some "weighted cardinality" of the set  $S$  of non-archimedean prime divisors of  $k$  that decompose "almost completely" in  $M$ . To describe this, call  $f(P)$  ( $P \in S$ ) the residue degree of  $P$  in  $M/k$ ,  $S_\infty$  the set of all archimedean places of  $k$  and put

$$\alpha_P = \begin{cases} \frac{\log N(P)}{\frac{1}{2}f(P) - 1} & \dots P \in S, \\ \frac{1}{2}(\log 8\pi + \gamma + \frac{\pi}{2}) & \dots P \in S_\infty, \text{ real} \\ \log 8\pi + \gamma & \dots P \in S_\infty, \text{ imaginary} \end{cases} \quad (\gamma = \text{Euler's constant})$$

Theorem 1:

$$\sum_{P \in S \cup S_\infty} \alpha_P \leq \begin{cases} (g-1) \log q & \text{if } k \text{ function-field} \\ \frac{1}{2} \log |d_k| & \text{if } k \text{ number field} \end{cases}$$

In the function-field case, the equality holds for those types of  $M/k$  which correspond to torsion-free cocompact irreducible discrete subgroups  $\Gamma$  of  $\text{PSL}_2(\mathbb{R}) \times \text{PSL}_2(F_\mathfrak{p})$  ( $F_\mathfrak{p}$ : a  $\mathfrak{p}$ -adic field), and in this case the Galois group of  $M/k$  is isomorphic to the profinite completion of  $\Gamma$ . In the number field case, we must assume the generalized Riemann hypothesis, but comparison with the function field case seems suggestive. Yamamura found an example of  $M/k$  in the NF-case for which in theorem 1:  $(\text{LHS})/(\text{RHS}) \geq 0.9059$ .

Open questions

- (i) Are there any other cases where the equality holds?
- (ii) Is the set  $S$  always finite? In the function field case, this is related to the question of Zariski closedness of "super-singular loci".

B. BIRCH:

Heegner Points on Weil Curves

A short account was given of recent results of Gross and Zagier. Let  $\varphi : X_0(N) \rightarrow E$  be a parametrized Weil curve, corresponding to a newform  $f(\tau) = \sum_{n \geq 1} a_n q^n$  on  $\Gamma_0(N)$ , so that  $L(E, s) = \sum a_n n^{-s}$ .

Suppose  $D$  is the discriminant of a complex quadratic order  $\sigma$  in which 1) all prime factors of  $D$  split or ramify, 2) only simple factors of  $N$  ramify, and 3)  $N$  is prime to the index of  $\sigma$  in  $K = \mathbb{Q}(\sqrt{D})$ . Writing  $(N) = \pi \bar{\pi}$ , with  $\pi$  having no non-trivial integer factor, there is a natural construction of a "Heegner point"  $\varphi(\sigma, \pi)$  in  $E(H)$ ,  $H$  being the ring class field of  $\sigma$ . Further, to any factorization  $D = d_1 \cdot d_2$  into discriminants of quadratic fields one can define a conveniently weighted sum of conjugates of  $\varphi(\sigma, \pi)$ :  $P(d_1, d_2) \in E^{(d_1)}(\mathbb{Q})$ , for a certain choice of  $d_1 \in \{d_1, d_2\}$ . Then the theorem of Gross-Zagier asserts:

$$\text{Canonical height}(P(d_1, d_2)) = 2^{\pi(N, D)+2} \frac{L'(E^{(d_1)}, 1)}{\int_E (d_1)^\omega(\mathbb{R})} \frac{L'(E^{(d_2)}, 1)}{\int_E (d_2)^\omega(\mathbb{R})}$$

where  $\pi(N, D)$  is the number of common prime factors of  $N$  and  $D$ .

So far, the proof of this theorem has been spelled out completely only in the case  $(d_1, d_2) = 1 = (d_1 d_2, N)$ . - The theorem provides explicit examples of L-functions with triple zeroes, so Goldfeld's method for proving effective bounds for class numbers of complex quadratic fields really works. - As further consequence, the conjectural interpretation of a number (with an intrinsic sign!) as  $\sqrt{\# \text{III}(E^{(d_1)})^7}$  was also pointed out.

H. P. F. SWINNERTON-DYER:

The Châtelet Problem and the Intersection of Two Quadrics

It was explained how the following theorem (the proof of which was also sketched) generalizes results concerning Châtelet's 4-parameter families of solutions to (a non singular cubic surface birational to)

$$(X-a_1)(X-a_2)(X-a_3) = Y^2 - cZ^2,$$

where  $c$  is not a square.

**Theorem:** Let  $V \subset \mathbb{P}^N$  with  $N \geq 4$  be the intersection of two quadrics defined over  $\mathbb{Q}$ . Assume:

- (i)  $V$  is absolutely irreducible and has no conical points;
- (ii)  $V$  contains nonsingular  $p$ -adic points, for each  $p$  (including  $\infty$ );
- (iii)  $V$  contains a rational pair of skew lines which contain no singular points of  $V$ .

Then  $V$  contains nonsingular rational points, except perhaps when  $N = 5$  and there are two distinct quadratic forms of rank 4 which vanish on  $V$ .

The proof is by induction on  $N$ , since the case  $N = 4$  is almost known; the induction is by intersection with a carefully chosen hyperplane. - The exception is genuine as is shown by an example due to Colliot-Thélène, Coray and Sansuc.

J.-L. COLLIOT-THÉLÈNE:

Torseurs under Tori and Rational Surfaces

Let  $k$  be a field,  $\bar{k}$  a separable closure,  $G = \text{Gal}(\bar{k}/k)$ , and let  $X/k$  be a rational surface (i. e.,  $X_{X, \bar{k}}$  is birational to  $\mathbb{P}^2$ ). Let  $S_0/k$  be the torus dual to the  $\mathbb{Z}$ -free finitely generated  $G$ -module  $\text{Pic } \bar{X}$ .

Definition (Sansuc & C-T): A torseur  $\mathcal{T}$  over  $X$  under  $S_0$  is universal, if  $[\mathcal{T}] \in H^1(X, S_0)$  is sent to 1 by the natural map  $H^1(X, S_0) \rightarrow \text{Hom}_G(\hat{S}_0, \text{Pic } \bar{X})$ .

Basic questions:

- A. Is any universal torseur with a  $k$ -rational point  $k$ -rational (i. e.,  $k$ -birational to  $\mathbb{P}^N$ )?
- B. ( $k$  a number field:) Do the universal torseurs satisfy the Hasse principle?

For a number field  $k$ , A. implies the finiteness of  $R$ -equivalence on  $X(k)$ , and B. implies, e. g., if  $\text{Br}(X) = \text{Br}(k)$ , then the Hasse principle holds for  $X$ .

In the case of generalized Châtelet surfaces (i. e.,  $Y^2 - cZ^2 = P(X)$ ,  $c$  not a square in  $k$ , and  $P$  separable of degree 3 or 4) the universal torseurs are essentially  $k$ -birational to intersections of

two quadrics. So the theorem of Swinnerton-Dyer's talk applies, and A. and B. are answered in the affirmative in these cases. Various consequences of this fact were discussed.

G. LAUMON:

Mellin Transform, Difference Equations and Boyarski Principle

Using the classical fact that the Mellin transform  $f \rightarrow \int_0^{+\infty} f(x)x^p \frac{dx}{x} = \Gamma_f(x)$  exchanges differential equations and difference equations, a sheaf analogue of the Mellin transform over  $\mathbb{C}$  was defined. This transformation induces an equivalence of categories between the modules over  $\mathcal{D}_{\mathbb{G}_m, \mathbb{C}}$  (the sheaf of differential operators on  $\mathbb{G}_m, \mathbb{C}$ ) and the modules over  $\Delta_{\mathbb{G}_a, \mathbb{C}}$  (the sheaf of difference operators on  $\mathbb{G}_a, \mathbb{C}$ ). The Mellin transform of a holonomic  $\mathcal{D}_{\mathbb{G}_m, \mathbb{C}}$ -module has the following property:  $\exists (\alpha_i)_{i \in I}$ , a finite family of complex numbers, such that this Mellin transform restricted to  $\mathbb{G}_a - \bigcup_{i \in I} (\alpha_i + \mathbb{Z})$  is free of finite type over  $\mathcal{O}_{\mathbb{G}_a} \subset \Delta_{\mathbb{G}_a}$ . In some sense, the Boyarski principle of Dwork is a p-adic formulation of this result.

T. EKEDAHL:

The de Rham-Witt Complex and F-gauge Structures

Let  $k$  be a perfect field of characteristic  $p > 0$ , and  $R$  the Raynaud ring over  $k$ . A bounded complex  $X$  of graded  $R$ -modules is called coherent, if

- (i)  $X \simeq R \varprojlim \{R_n \otimes_R^L X\}$ , for  $R_n = R/V^n R + dV^n$ .
- (ii)  $R_1 \otimes_R^L X$  is a complex of  $k$ -vector spaces with finite dimensional cohomology.

An  $F$ -gauge structure of level  $n$  is a  $\mathbb{Z}/n\mathbb{Z}$ -graded  $W$ -module  $M$  with additive mappings  $\tilde{F}, \tilde{V}$  of degree 1, resp. -1, which are  $W$ -linear except between  $M^0$  and  $M^{n-1}$  where they are  $\sigma$ -linear, resp.  $\sigma^{-1}$ -linear.

The main result is an equivalence of categories between, on the one hand, coherent complexes containing only  $R$ -modules  $M$  with  $M^i = 0$  unless  $0 \leq i \leq n$ , and  $F$  bijective on  $M^n$ , and, on the other hand, bounded complexes of  $F$ -gauge structures of level  $n$  with cohomology of finite type over  $W$ . As an application, the complex



$R\Gamma(Y, W\Omega_Y^i)$ , for a smooth proper variety  $Y/k$  can be recovered explicitly from the crystalline cohomology of  $Y$ , provided the sum of the Hodge numbers of  $Y$  equals the sum of its crystalline Betti numbers.

S. SPERBER:

Newton Polygons of Exponential Sums

The following analogues of results of Dwork and Katz, resp., were proved:

- 1) If  $f_D \in \mathbb{F}_q[X_1, \dots, X_n]$  is a form of degree  $D$  defining a non-singular hypersurface in  $\mathbb{P}^{n-1}$ , and  $f = f_D + g$ , with  $g \in \mathbb{F}_q[X_1, \dots, X_n]$  of degree  $< D$ , then the Newton polygon of  $L(f, T)^{(-1)^{n+1}}$  ( $L(f, T)$  being the L-function attached to the exponential sums  $S_m = \sum_{x \in (\mathbb{F}_q^m)^n} \psi \cdot \text{tr}_{\mathbb{F}_q^m / \mathbb{F}_q}(f(x))$ ) lies over the

$$\text{Newton polygon of } \prod_{i=0}^{n(D-1)} (1 - q^i T)^{\gamma_i}, \text{ where } \gamma_i = \text{coeff. of } z^i \text{ in } (z(1-z^{D-1})/(1-z)).$$

2. If  $f \in \mathbb{F}_q[X_1, \dots, X_n]$  has degree  $D$ , and  $X$  is an affine variety over  $\mathbb{F}_q$ , given by polynomials  $g_1, \dots, g_r \in \mathbb{F}_q[X_1, \dots, X_n]$  of respective degrees  $d_1, \dots, d_r$ , then

$$\text{ord}_q \sum_{x \in X(\mathbb{F}_q)} \psi(f(x)) \geq \inf \left\{ \alpha \in \frac{1}{D} \mathbb{Z} \mid \alpha \geq 0; \alpha \geq \frac{n - \sum_i d_i}{\sup_i (D, d_i)} \right\}.$$

Various examples were discussed.

W. E. LANG

Classical Enriques Surfaces in Characteristic Two

Let  $X$  be a classical Enriques surface over an algebraically closed field  $k$  of characteristic 2. Then  $X$  has a  $\mu_2$  double cover  $Y$  which in general has 12  $A_1$  rational double points. Let  $\tilde{Y}$  be a minimal resolution of  $Y$ . Then  $\tilde{Y}$  is a supersingular K3 surface. Answering (generically) a question of Illusie, it was proved that if  $X$  is a generic classical Enriques surface, then the Artin invariant

$G_0(\tilde{Y}) = 10$  (maximum possible).

The proof makes use of the fact that  $X$  is an elliptic surface to unscrew the Néron-Severi group.

G. WÜSTHOLZ

Algebraic Points on analytic subgroups of algebraic groups

Let  $G/\bar{\mathbb{Q}}$  be a commutative algebraic group of dimension  $n$  with tangent space  $T(G)$  in  $O$ , and  $A \hookrightarrow G$  an analytic subgroup of  $G$ ,  $0 < \dim A \leq n$ .  $A$  is said to be defined over  $\bar{\mathbb{Q}}$ , if the subspace  $T(A) \subset T(G)$  is defined over  $\bar{\mathbb{Q}}$ .

Main Theorem: Suppose  $O \neq \exp_A^{-1}(G(\bar{\mathbb{Q}}))$ . Then there exists an algebraic subgroup  $H \subset G$  with  $\dim H > 0$  and  $H \subset A$ .

This result implies the solutions to a number of longstanding problems in transcendence theory. But the most beautiful application is probably the following:

Let  $U$  be a smooth quasiprojective irreducible variety over  $\bar{\mathbb{Q}}$ ,  $\omega \in \Gamma(U, \Omega_U^1)$  with  $d\omega = 0$ , and  $\gamma \in H_1(U, \mathbb{Z})$ . Then the period  $\int_{\gamma} \omega$  is either zero or transcendental.

N. SCHAPPACHER

Motives for Hecke L-functions

Let  $K$  be a number field. Denote by  $(CM)_K$  the category of motives constructed (via absolute Hodge cycles) from algebraic varieties of dimension 0 and abelian varieties over  $K$ , of CM-type over  $\bar{K}$ . From Deligne's study of this category in Springer Lecture Notes 900, chapter IV, one easily deduces a 1-1 correspondence between algebraic Hecke characters  $\chi$  of  $K$  with values in a given number field  $E$  and rank 1 motives  $M(\chi)$  in  $(CM)_K$  with coefficients in  $E$  whose  $\lambda$ -adic realizations, for  $\lambda$  a place of  $E$ , are abelian 1-dimensional given by  $\chi$ .

Using various geometric constructions of motives in  $(CM)_K$  - in particular, G. Anderson's motives for Jacobi-sum Hecke characters found in the cohomology of certain twisted forms of Fermat hypersurfaces - the uniqueness of  $M(\chi)$  implies many period relations, e. g., for  $K$  imaginary quadratic, a relation up to  $K^*$ , whose norm

$(K/\mathbb{Q})$  is the so-called Chowla-Selberg formula, taken up to  $\mathbb{Q}^*$ . Likewise, Shimura's monomial relations between periods of CM-abelian varieties follow from the uniqueness of  $M(\chi)$ .

This fact is also used in Don Blasius' recent proof of Deligne's rationality conjecture for "critical" values of Hecke L-functions of CM-fields  $K$ .

G. HARDER:

Eisenstein Cohomology for  $SU(2,1)$

Let  $F = \mathbb{Q}(\sqrt{-d}) \subset \mathbb{C}$  an imaginary quadratic field. Consider on  $F^3$  the hermitian form

$$h_0 = z_1 \bar{w}_3 + z_2 \bar{w}_2 + z_3 \bar{w}_1,$$

and the associated group  $G/\mathbb{Q}$  of similitudes. The associated Shimura variety

$$S_K = G(\mathbb{Q}) \backslash G(\mathbb{A}) / K_\infty K_f$$

(for  $K_f$  = open compact subgroup of  $G(\mathbb{A}_f)$ ) can be compactified (Baily-Borel) by adding a finite number of cusps. The resulting projective variety  $\overline{S}_K$  has singularities at the cusps. The desingularization  $\tilde{S}_K \xrightarrow{\pi} \overline{S}_K$  has elliptic curves with complex multiplication by  $F$  as fibres over the cusps.

The main result implies in particular that the image of

$$\text{Pic}^\circ(\tilde{S}_K) \rightarrow \text{Pic}^\circ(\pi^{-1}(\text{cusps}))$$

is nontrivial in general. The precise statement requires the theory of representations of  $G(\mathbb{Q}_p)$ , for all  $p$ . Possible arithmetic applications were discussed.

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