MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Partial Differential Equations in Complex Analysis

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The conference on "Partial Differential Equations in Complex Analysis" organized by K.Diederich (Wuppertal) , I.Lieb (Bonn) and J.J.Kohn (Princeton) has found a very large international interest despite its partial collision with the International Congress of Mathematicians in Warsaw. It was attended by 53 mathematicians from 12 different countries. Because of the strong development in the field since the last such conference in Oberwolfach 3 years ago, there were 2 or 3 longer lectures each day plus shorter reports on new results, such that, all together, 40 participants contributed to the program of the conference. The topics covered belonged to the following areas: Regularity of the Cauchy-Riemann equations and the $\bar{\partial}$-Neumann problem, CR-structures with their corresponding Laplacians and CR-functions, complex Monge-Ampère equations, partial differential operators in the theory of complex vector bundles, harmonic maps,singularities and extendability of positive currents,function algebras,geometry of pseudoconvex boundaries, proper holomorphic maps and correspondences,invariant metrics.

Vortragsauszüge

T.AKAHORI:

On the local embedding theorem of CR-structures

Let $(M, {}^{O}T'')$ be an abstract strongly pseudoconvex CR-structure. Kuranishi solved the local embedding problem in the case $\dim_{\mathbb{R}} M = 2n - 1 \geq 9$. The motivation of our work is to establish the local embedding theorem also in the case $\dim_{\mathbb{R}} M = 7$. Our set-up is as follows. Let $(M, {}^{O}T'')$ be the original CR structure. For this $(M, {}^{O}T'')$, we take a C^{∞}-embedding g satisfying

$$(1)\quad \Phi(g)(p_o) = 0$$

$$(2)\quad \Phi(g) \in \Gamma(M, {}^{O}\overline{T}'' \otimes ({}^{O}T'')^*)$$

where $\Phi(g)$ means the CR-structure induced by g. This is possible because the deformation of the contact structure is trivial. In this situation, the local embedding problem, in Kuranishi's notation D -problem, can be reduced to the following D_b-problem : Find a C^{∞}- embedding f satisfying

(1) $f|C = g$, $D_b f = 0$ along the level set of $t\phi \circ f$

(2) The induced CR-structure $\Phi(f)$ is in $\Gamma(M, {}^{O}\overline{T}'' \otimes ({}^{O}T'')^*)$,

where $t\phi$ is the admissible distance function for the embedding g and $(M, {}^{\Phi(g)}T'')$ and D_b means the operator induced by D along $t_{\phi} \circ f$. Furthermore, the above f satisfying (1) and (2) can be obtained in the case $\dim_{\mathbb{R}} M = 2n - 1 \geq 7$.

E.AMAR :

Extension of Holomorphic Functions in H^{∞}

Let $\Omega = \{\rho < o\}$ be a pseudoconvex domain $\subset\subset \mathbb{C}^n$ with a C^{∞}- smooth boundary and $X \subset \Omega$ be a holomorphic subvariety of the form $X = \{z \in \Omega \,|\, u_1(z) = \ldots = u_k(z) = 0\}$ with $u_j \in A^{\infty}(\overline{\Omega})$. X is called to be transversal if $\partial\rho \wedge \partial u_1 \wedge \ldots \wedge \partial u_k \neq 0$ on $X \cap b\Omega$. The domain Ω is called to be lineally convex at $p \in b\Omega$, iff $T_p^{\mathbb{C}} \cap \overline{\Omega} \cap U = \{p\}$, for a suitable open neighborhood U of p, and pseudo-lineally convex at p, iff it can be made lineally convex by a local holomorphic change of coordinates at p. The following results about extensions of bounded holomorphic functions on X were communicated : Let Ω, X be as above.

Theorem 1: If Ω is lineally convex at each $p \in X \cap b\Omega$, then for any $f \in H^{\infty}(X \cap \Omega)$ there is an $F \in H^{\infty}(\Omega)$ with $F|X = f$.

Theorem 2: If Ω is pseudo-lineally convex at each $p \in X \cap b\Omega$ and X is holomorphic on a neighborhood of $\overline{\Omega}$, then for each $f \in H^{\infty}(X \cap \Omega)$ there is an $F \in H^{\infty}(\Omega)$ such that $F|X = f$.

D.BARRETT :

A smooth bounded domain with irregular Bergman projection

An example was given of a (non-pseudoconvex) smoothly bounded domain D in $\mathbb{C}^2$ whose Bergman projection operator failed to map $C_o^{\infty}(D)$ into $L^p(D)$ for any pre-determined $p > 2$.

P. de BARTOLOMEIS :

Complex Analyticity of Harmonic Maps

(Joint work with M. Derridj)

Let $f: (N,g) \longrightarrow (M,h)$ be a harmonic energy minimizing map between complete non-compact Kähler manifolds. The second variation of the $\bar\partial$ - energy of f induces a semipositive definite quadratic form on $C_o(f^*[TM]_{1,0})$:

$$Q(s) := \int_N |\bar\nabla s|^2 d\mu + \int_N \{ g^{j\bar k} K_{\alpha\bar\beta m\bar\nu} \frac{\partial f^\alpha}{\partial \bar w_j} \overline{\frac{\partial f^\beta}{\partial \bar w_k}} s^m \overline{s^\nu} \} d\mu$$

Assume the holomorphic bisectional curvature of M is positive. If we can find a section $s \not\equiv 0$ of $E := f^*[TM]_{1,0}$ satisfying

$$(P) \begin{cases} \bar\nabla s = 0 \\ Q(s) \geq 0 \end{cases}$$

we obtain $0 \leq \int_N g^{j\bar k} K_{\alpha\bar\beta m\bar\nu} \frac{\partial f^\alpha}{\partial \bar w_j} \overline{\frac{\partial f^\beta}{\partial \bar w_k}} s^m \overline{s^\nu} d\mu \leq 0$,

and so $\bar\partial f = 0$.

In order to solve (P) we study the equation (*) $\bar\nabla u = \alpha$.

Under the assumption

$$(1) \qquad f^*(\omega) = [f(\omega)]^{1,1} ,$$

where ω is the curvature form on M, (*) is reduced to a $\bar\partial$ -problem for a structure of a holomorphic vector bundle on E; we study this $\bar\partial$- problem by means of Hörmander's L^2-theory and, under the condition:

$$(2) \qquad \mathrm{curv}(E) + dd^c\phi + \mathrm{Ric}_N \quad \text{is} \quad \dim_{\mathbb{C}} N - \text{positive}$$

we can construct a $\bar\nabla$ - parallel section $s \not\equiv 0$ of E satisfying

$$\int_N |s|^2 e^{-\phi} d\mu < \infty$$

for a weight factor ϕ. If furthermore, ϕ satisfies

$$(3) \qquad (1+\sigma^2)\, e^{-\phi} \geq k > 0$$

at infinity, where σ is a C^∞- exhaustion on N with bounded gradient, then we obtain $Q(s) \geq 0$, thus (P) is satisfied, and f is holomorphic.

E.BEDFORD :

Proper holomorphic correspondences of bounded domains in $\mathbb{C}^n$

(Joint work with S.Bell)

Let $\Omega, D \subset\subset \mathbb{C}^n$ be domains. A holomorphic correspondence $f: \Omega \multimap D$ is a set-valued mapping which may be given the algebroid representation $f(z) = \{w_1,\ldots,w_p\}$, where the w's satisfy $w_j^p + a_j^{p-1}(z) w_j^{p-1} + \ldots + a_j^o(z) = 0$, and $a_j^k(z): \Omega \longrightarrow \mathbb{C}^n$ is a bounded function. The following theorem was shown:

Theorem 1: If Ω,D are C^∞ pseudoconvex domains and if Ω satisfies condition (R), then

$$S(z) := u_1 \cdot \ldots \cdot u_p \prod_{i \neq j} (f_i - f_j)(z)$$

belongs to $C^\infty(\bar{\Omega})$, where $f(z) = \{f_1(z),\ldots,f_p(z)\}$ and u_j is the Jacobian determinant of f_j. Furthermore $S(z) f_1^{\alpha_1} \ldots f_p^{\alpha_p}(z)$ is in $C^\infty(\bar{\Omega})$ for all integers $\alpha_1,\ldots,\alpha_p \geq 0$.

One application of this result was to obtain factorization of a proper holomorphic mapping $g: \Omega \longrightarrow Y$. Such a mapping gives a holomorphic correspondence $g^{-1} g : \Omega \multimap \Omega$.

Theorem 2 : If Ω is strongly pseudoconvex and simply connected, $b\Omega \in C^\infty$,then every irreducible component of the graph of $g^{-1}g$ is an automorphism of Ω . Thus there exists a finite subgroup Γ of Aut(Ω) such that g is factored by

$$\Omega \xrightarrow{q} \Omega/\Gamma \xrightarrow{\tilde{f}} Y ,$$

where q is the quotient map and $\tilde{f}$ is a biholomorphism.

M.BEHRENS :

Nonexistence of local plurisubharmonic defining functions

A brief sketch of a proof of the following results was given.

Theorem 1: There exists a bounded domain $\Omega \subset\subset \mathbb{C}^2$ with real analytic smooth boundary $b\Omega$, $0 \in b\Omega$, such that:

a) All points in $b\Omega \setminus \{0\}$ are strongly pseudoconvex, 0 is weakly pseudoconvex

b) There is a local holomorphic supporting function in 0 to Ω

c) Suppose $U = U(0)$ is an open neighborhood of 0 and $\sigma: U \to \mathbb{R}$ is a C^k-defining function for $b\Omega \cap U$, $k \geq 6$, then there are points $p \in b\Omega \cap U$ (arbitrarily close to 0) such that the Levi form of σ has negative eigenvalues at p.

Let ρ be a C^ω-defining function for the above domain Ω .

Theorem 2 : Suppose U is an open neighborhood of 0 and $\psi \in C^\omega(U)$, $\psi \geq 0$, and $\psi | b\Omega \cap U = \tilde{\psi}$ coeff$(\partial\rho\wedge\bar{\partial}\rho\wedge\ \partial\bar{\partial}\rho)$ with some nonnegative $\tilde{\psi} \in C^\omega(U \cap b\Omega)$ (Ω being as in Theorem 1). Then there does not exist a function $u \in C^k(U \cap \bar{\Omega})$, $k \geq 6$, which solves the Dirichlet problem for the Monge-Ampére operator:

$$\begin{cases} \det\left((u_{j\bar{k}})_{1\leq j,k\leq 2}\right) = \psi & \text{on } \bar{\Omega} \cap U \text{ , u plurisubharmonic on } \Omega \cap U \\ u \mid b\Omega \cap U = 0 \end{cases}$$

S.BELL:

Sobolev Estimates for Holomorphic Functions

(Joint work with E.Bedford)

If D is a smooth bounded pseudoconvex domain in $\mathbb{C}^n$ that satisfies condition (R), then $A^{\infty}(D)$ and $A^{-\infty}(D)$ are mutually dual via an extension of the usual L^2 pairing. This fact can be used to study the boundary behavior of proper holomorphic correspondences between smooth bounded pseudoconvex domains.

Theorem: If $f: D_1 \longrightarrow D_2$ is a proper holomorphic correspondence between smooth bounded pseudoconvex domains in $\mathbb{C}^n$, and if D_1 satisfies condition (R), then $\sum_{i=1}^{p} u_i \cdot (h \circ f_i)$ is in $A^{\infty}(D_1)$ whenever h is in $A^{\infty}(D_2)$.

(Here, f_i are the locally defined holomorphic mappings that define f, and $u_i = \det[f_i']$).

A.BONAMI:

Zeros of the Nevalinna class and integrability conditions for closed positive currents

The Blaschke condition $\int_X \delta(z,bD)\,d\sigma_X(z) < \infty$ is necessary and sufficient for X to be the zero of a holomorphic function f in the Nevalinna class of one of the domains

$$D = \{ z \in \mathbb{C}^n \mid |z_1|^{2p_1} + \ldots + |z_n|^{2p_n} < 1 \}.$$

In the simplest case of the domain $D = \{ \rho < 0 \}$, with $\rho(z) = |z_1|^2 + |z_2|^{2p} - 1$, the crucial fact here is that the complex tangent coefficient $\Theta \wedge \partial\rho\wedge\bar{\partial}\rho$ of a closed positive current Θ which satisfies the Blaschke condition $\int \delta(z,bD)\|\Theta\| < \infty$ satisfies the stronger integrability condition

$$\int i\Theta\wedge \partial\rho\wedge\bar{\partial}\rho(1-|z_1|^2)^{-1+\frac{1}{p}} < +\infty$$

This condition generalizes to all domains $D = \{\rho < 0\}$ in $\mathbb{C}^2$ whose all boundary points are of type $\leq m$. Then $i\int S^{-1}\Theta\wedge\partial\rho\wedge\bar{\partial}\rho < \infty$,

where

$$S = \Lambda_2 + \Lambda_3^{2/3}|\rho|^{1-2/3} + \dots + \Lambda_m^{2/m}|\rho|^{1-2/m},$$

where $\Lambda_k = \Sigma\lambda$, the sum being taken over generators of the ideals I_k introduced by J.J.Kohn.

D.BURNS:

Special Exhaustion Functions on Affine Algebraic Manifolds

Let M be an n-dimensional affine algebraic manifold in $\mathbb{C}^N$ such that $\bar{M}$, the projective closure of M in P^N is smooth, and $H_\infty := \mathbb{P}^N \setminus \mathbb{C}^N$ intersects $\bar{M}$ transversally. The proof of the following theorem was sketched

Theorem: There exist strictly plurisubharmonic exhaustions τ on M such that, outside a compact set $K \subset M$, $u := \log\tau$ verifies the homogeneous Monge-Ampère equation: $(dd^c u)^n = 0$.

An estimate for the Ricci curvature of the Kähler metric with potential τ was established, and partial results of R.Foote in the converse direction were discussed, viz., given certain M which admit exhaustions τ as above, one can construct many holomorphic functions which grow polynomially w.r.t. τ .

D.W.CATLIN:

Subelliptic estimates for pseudoconvex domains of finite type

Let $\Omega = \{r < 0\} \subset \mathbb{C}^n$ be a bounded domain , $b\Omega$ smooth. We say that a point $z^o \in b\Omega$ is of finite type, if there is a number $\tau > o$ such that if V is a 1-dimensional complex analytic variety with $z^o \in V$, then the inequality $|r(z)| \leq c||z - z^o||^\eta$, $z \in V$, z near z^o, does not hold for any $\eta > \tau$. Let $\tau(z^o)$, the type of z^o , denote the smallest such number τ . A subelliptic estimate holds near z^o, iff

$$(SE_\varepsilon) \quad |||u|||^2 \leq C\{\ ||u||^2 + ||\bar\partial^* u||^2 + ||\bar\partial u||^2\ \}$$

holds for all $u \in \mathrm{dom}(\bar\partial) \cap \mathrm{dom}(\bar\partial^*) \cap C^\infty_{o,(p,q)}(U)$,where U is a neighborhood of z^o.

Theorem: Suppose $b\Omega$ is pseudoconvex near z^o, and $\tau(z^o) < \infty$. Then the estimate (SE_ε) holds in some neighborhood U of z^o, with $\varepsilon = (\tau(z^o))^{-N}$, $N = (n-1)^{n-1}$.

By combining this result with another result on necessity of the finite type property, it follows that finite type is necessary and sufficient for the existence of a subelliptic estimate on a pseudoconvex domain.

J.CHAUMAT - A.M. CHOLLET :

Zero sets and interpolation sets for $A^\infty(D)$

Let D be a bounded strictly pseudoconvex domain in $\mathbb{C}^n$ with a C^∞-boundary. Let E be a closed subset of bD. E is said to be an interpolation set for $A^\infty(D)$, if E is totally real, and if for any function $f \in C^\infty(\mathbb{C}^n)$ such that $\bar\partial f$ vanishes to infinite order on E, there exists a function $F \in A^\infty(D)$ such that $D^\alpha F = D^\alpha f$ on E for every multi-index α.

Theorem 1: Let Γ be a C^∞ curve on bD such that $\forall\, p \in \Gamma$ $T_p(\Gamma) \not\subset T_p^c$ (bD) . Let E be a compact subset of Γ .

(1) E is a zero set for $A^\infty(D)$ iff $-\int_\Gamma \log d(\zeta,E)\, dl(\zeta) < \infty$

(2) E is an interpolation set for $A^\infty(D)$ iff there exist $c, c' > 0$ such that for any interval I along Γ :

$$\frac{1}{l(I)} \int_I \log \frac{1}{d(\zeta,E)}\, dl(\zeta) \leq c + c' \log \frac{1}{l(I)}$$

These results extend those obtained by Carleson (1952) and Alexander-Taylor-Williams (1971) in the case of the unit disc in $\mathbb{C}$. The necessity of condition (1) follows from a more general result.

Theorem 2: Let D be a domain in $\mathbb{C}^n$ with a C^∞- boundary. Let Γ be a C^∞ curve on bD such that for any $p \in \Gamma$ $T_p\Gamma \not\subset T_p^c(bD)$. Let F be a function in $A^\infty(D)$ which does not vanish inside of D; then $\log|F|$ is locally integrable along Γ with respect to the length measure dl .

J.P. D'ANGELO :

Intersection theory and the Cauchy Riemann equations

In this talk proofs of the following theorems on the behavior of the order of contact of complex analytic varieties with a real hypersurface were sketched.

Theorem: Let M be a real hypersurface in $\mathbb{C}^n$; let $\Delta(M,p)$ denote the maximal order of contact of 1-dimensional complex analytic varieties with M at p, where $p \in M$.

(1) Suppose for a $p_o \in M$ $\Delta(M,p_o)$ is finite. Then there is a neighborhood of p_o on which $\Delta(M,p) \leq 2\,(\Delta(M,p_o))^{n-1}$

(2) Suppose $(M_\lambda)_\lambda$ is a family of hypersurfaces all containing p. Then $\Delta(M_\lambda,p) \leq 2\,(\Delta(M_o,p))^{n-1}$.

(3) Suppose M is also pseudoconvex. Then the bound can be improved to

$$\Delta(M,p) \leq 2\left(\frac{\Delta(M,p_o)}{2}\right)^{n-1-q},$$

where q is the number of positive eigenvalues of the Leviform of M at p .

(4) Suppose $M = \{r = 0\}$, $p \in M$, where r is a polynomial of degree d. Then either

(a) There is a complex analytic variety $V \subset M$ through p

or (b) $\Delta(M,p) \leq 2\,d(d-1)^{n-1}$.

Let M be a real analytic hypersurface. The following facts were also stated: (i) M is strictly pseudoconvex at p , iff $I(U,p) = m_p$ for all U (family of ideals in $\mathcal{O}_p$, m_p = maximal ideal in $\mathcal{O}_p$)
(ii) M is of finite type at p , iff $\sqrt{I(U,p)} = m_p$,where $\sqrt{}$ denotes holomorphic radical.

J.P.DEMAILLY :

Propagation of singularities of closed positive currents

Let T be a closed positive (p,p) current defined on a Runge open subset $\Omega \subset \mathbb{C}^n$. The following question was discussed:

(Q) Can T be extended to a (p,p) current Θ defined on $\mathbb{C}^n$?

It was shown that the answer is positive if the masses of T are of sufficiently low density, and furthermore, that there is no propagation of singularities, i.e. Θ may be chosen C^∞ outside $\overline{\Omega}$. This gave an extension theorem for analytic subsets in mean value.

Conversely, by means of results of Skoda and El Mir on the extension of currents across pluripolar sets, counterexamples were given to show that the above sufficient conditions are best possible at least in the hypersurface case.

M.DERRIDJ :

The Cauchy problem for $\bar\partial$

In this talk the following problem was treated. Let $S \subset \mathbb{C}^n$ be a hypersurface, containing 0, say smooth. Let U be a neighborhood of 0 which splits into $U = U^+ \cup S \cup U^-$. Then find a neighborhood V of 0 in $\mathbb{C}^n$, such that the following holds:

Given a $\bar\partial$ closed (0,q+1) form, say smooth, with support in $\overline{U}^+$ then there is a (0,q) form u such that

$$\begin{cases} \bar\partial u = f & \text{on } V \\ u|S = 0 & \end{cases} .$$

Some sufficient conditions on S were discussed (essentially when one has degeneracies of the eigenvalues of the Levi form, otherwise the problem was solved by Andreotti-Hill) under which the precedent problem is solvable. Such solvability has applications in extension of $\bar{\partial}_b$-closed forms.

J. GLOBEVNIK :

A holomorphic function in the ball with wild boundary behavior

The following is known.

Theorem 1 (Stout-Globevnik-Alexander): Let D be a bounded domain in $\mathbb{C}^N$, $N \geq 2$, and let f be holomorphic in D. There are a point $p \in bD$, and an arc Λ contained in D except for its end point p, along which f is a constant.

Theorem 2 (Stout-Globevnik): Let $D \subset\subset \mathbb{C}^n$ be a strongly pseudoconvex domain with a C^2- boundary. There is a holomorphic function f on D such that if $\gamma:[0,1] \to \bar{D}$ is a smooth map, $\gamma([0,1)) \subset D$, $\gamma(1) \in bD$, and $\dot{\gamma}(1)$ is not tangent to bD at $\gamma(1)$, then $\lim_{t\to 1} f(\gamma(t))$ does not exist.

We sketch the proof of the following new result, related to Theorem 1 and 2.

Theorem 3: Let B be the open unit ball in $\mathbb{C}^N$, $N > 1$. There is an $r > o$ and a function f, holomorphic in B with the property that if $x \in bB$ and if Λ is a path with x as one end point such that $\Lambda \setminus \{x\}$ is contained in the open ball with radius r which is contained in B and tangent to bB at x, then $\lim_{\substack{z\to x \\ z\in\Lambda\setminus\{x\}}} f(z)$ does not exist.

R.E.GREENE :

The Automorphism Groups of C^2-Strongly Pseudoconvex Domains

(Joint work with S.Krantz)

By definition, a sequence $\{D_i\}_i$ of C^2- bounded domains converges C^2 to a C^2-bounded domain D_o if there exists for all i sufficiently large, diffeomorphisms $F_i : \overline{D}_o \to \overline{D}_i$ such that $\{F_i\}_i$ converges to the identity on $\overline{D}_o$ in the C^2 topology.

Theorem: If D_o is a C^2 strongly pseudoconvex (bounded) domain and if $\{D_i\}_i$ converges C^2 to D_o, then, for i sufficiently large, $\mathrm{Aut}(D_i)$ ($\underset{\text{def.}}{=}$ { The biholomorphic self-maps of D_i }) is isomorphic to a subgroup of Aut (D_o) .

One notes that because of lack of differentiability no local invariant theory (Chern-Moser-Tanaka) could be relevant here. It can be shown by examples that in the $C^{1-\varepsilon}$, ($\varepsilon>o$ given) , topology on $C^{1-\varepsilon}$ domains no corresponding result holds. The proof of the theorem uses the fact that if $\alpha_i \in \mathrm{Aut}(D_i)$,each i, then there exists an $\alpha_o \in \mathrm{Aut}(D_o)$ such that some subsequence of $\{\alpha_i\}_i$ converges uniformly on compact subsets to α_o. This result is proved by exploiting certain stability properties of the Eisenman and Caratheodory volume forms to establish the nondegeneracy of the limit map. The proof is then completed by using group invariant exhaustion functions to create a situation in which the theory of group actions on compact manifolds can be applied.

G.HERBORT :

Closed Geodesics and Geodesic Spirals for the Bergman Metric

Two theorems were discussed which deal with the existence of geodesics for the Bergman metric with a special behavior. Let Ω be a smooth bounded pseudoconvex domain in $\mathbb{C}^n$ of the form $\Omega = \{r < o\}$, and let B^2_Ω denote the Bergman metric of Ω.

Theorem 1: Suppose the following holds:

(B) There are $c, \eta > o$ such that $B^2_\Omega \geq c\,|r|^{-\eta} ds^2_{eucl}$

Then, given a closed curve c_o in Ω, not homotopic to zero, there exists a closed geodesic γ for B^2_Ω, which is homotopic to c_o.

Theorem 2: Suppose Ω is strongly pseudoconvex, and the 1^{st} fundamental group of Ω is infinite. Then, given a point $z^o \in \Omega$, lying not on a closed geodesic (except for the trivial one), there exists a geodesic spiral emanating from z^o, i.e. a geodesic γ, such that $\gamma(\mathbb{R}^+) \subset K$, where $K \subset \Omega$ is compact.

It is shown in a joint paper with Diederich-Fornaess that if the $\bar\partial$ - Neumann problem for (0,1) - forms on Ω is subelliptic, then (B) holds. Combining this with the new result of Catlin, that implies the subellipticity of the above $\bar\partial$- Neumann problem when Ω is of finite type, one obviously obtains

Theorem 1': If Ω is a smooth bounded pseudoconvex domain of finite type, then the conclusion of Theorem 1 holds.

The above theorems 1 and 2 were illustrated by explicit statements on the existence of closed or spiraling geodesics of the Bergman metric of an annulus in the plane.

P.JAKOBCZAK :

Extension and decomposition theorems in strictly pseudoconvex domains

In the sequel D denotes a strongly pseudoconvex domain in $\mathbb{C}^n$, $\tilde{D}$ an open neighborhood of $\bar{D}$, M' a closed complex subvariety of $\tilde{D}$ which has no singular points on bD and intersects bD transversally. Set $M := M' \cap D$.

Theorem 1: a) If $bD \in C^{k+5}$ $(k \in \mathbb{N})$ then there exist linear and continous extension operators $P: A^k(M) \longrightarrow A^k(D)$, and $R: H^{\infty,k}(M) \to H^{\infty,k}(D)$.

b) If $bD \in C^{k+6}$ $(k \in \mathbb{N})$,and $k< t< k+1$, then there exist a linear and continous extension operator $R: \Lambda_t(M) \to \Lambda_t(D)$.

Theorem 2: Under the same assumptions as in Theorem 1 there exist linear and continous extension operators $P: \tilde{A}(M \times M) \to \tilde{A}(D \times D)$, where $\tilde{A} = A^k, H^{\infty,k}$ or Λ_t .

For any set Y let Δ_Y denote the diagonal in $Y \times Y$.

Theorem 3: Suppose that

a) $bD \in C^{k+5}$ and $\tilde{A} = A^k$ or $H^{\infty,k}$

or b) $bD \in C^{k+6}$ and $\tilde{A} = \Lambda_t$, $k< t< k+1$.

Set $\tilde{A}_0(D \times D) := \{f \in \tilde{A}(D \times D) \mid f|\Delta_D \equiv 0\}$. Then there is a linear mapping $B: \tilde{A}_0 (D \times D) \to [\mathcal{O}(D \times D)]^n$ such that

$$\text{(i)} \quad f(z,\zeta) = \sum_{j=1}^{n} (z_j-\zeta_j)B_j (z,\zeta)$$

(ii) If $Q := \bar{D} \times \bar{D} \setminus \Delta_{bD}$,then for each $K \subset\subset Q$ the mapping $f \longmapsto (B_1 f|K,..,B_n f|K)$ is a continous map from $\tilde{A}_0(D \times D)$ to $[\tilde{A}(K)]^n$

Theorem 4: If U denotes the unit disc in $\mathbb{C}$ and $g_1,...,g_N \in A(U^2)$ satisfy the conditions

a) $\{(z,\zeta)\in \overline{U}^2 \mid g_j(z,\zeta) = 0 , 1 \leq j \leq N \} = \Delta_{\overline{U}}$.

b) The germs $(g_j)_{(z,z)}$, $1\leq j\leq N$, generate the ideal of germs of holomorphic functions at (z,z) which vanish on Δ_U , $z\in U$

c) There exists a neighborhood V of Δ_{bU} in $\overline{U}^2$ such that $g_i|_{V\setminus\Delta_U} \neq 0$, for $1 \leq i \leq N$.

Then there is a linear mapping $B : A_0(U^2) \to [\mathcal{O}(U^2)]^N$ such that

(i) $f = g_1B_1f + \ldots\ldots + g_NB_Nf$

(ii) If $Q := \overline{U} \times \overline{U}\setminus\Delta_{bU}$ then for every $K\subset\subset Q$ the mapping $f \mapsto (B_1f|K,..,B_Nf|K)$ is a continous map from $A_0(U^2)$ to $[A(K)]^N$.

K.O.KISELMAN :

Can the Monge-Ampère operator be applied to any plurisubharmonic function ?

The real Monge-Ampére operator can be applied to any real-valued convex function f (say,on $\mathbb{R}^2$),although the individual terms in the expression

$$f_{xx}f_{yy} - f_{xy}f_{yx}$$

need not have a sense as distributions. In the complex case one may ask analogously if

$$M_f = f_{x\bar{x}}\, f_{y\bar{y}} - f_{x\bar{y}}\, f_{y\bar{x}}$$

can be defined for any plurisubharmonic function f (say on $\mathbb{C}^2$).

Two difficulties which have to be taken care of by any definition were discussed in the lecture. A definition of M_f was proposed which extends the one due to Bedford-Taylor (Acta Math. 149, (1982)) which is partially but not completely successful.

G.KOMATSU :

Variations of domains and of the Bergman kernel

The boundary variation and the variation by cutting a hole of the Bergman kernel associated to a bounded domain in $\mathbb{C}^n$ was discussed. At first Hadamard's variational formula for the Bergman kernel of a strictly pseudoconvex domain with a smooth boundary was given, after showing the smooth dependence of the Bergman kernel on a small perturbation of the domain. A similar formula for the Szegö kernel was also stated.

Next an explicit formula was presented which exhibits how the Bergman kernel varies by cutting a compact hole from a bounded domain in $\mathbb{C}^n$, $n \geq 2$. (This part is a joint work with S.Ozawa). This formula was applied to the question of preservation of boundedness properties of the Bergman projector in various function spaces.

Also the case when the hole is not compact was treated and a refined version of the monotone dependence of the Bergman kernel on the domain was provided. The proof uses the idea of doubly orthogonal systems. A similar idea was used in order to generalize Bell's duality theorem on holomorphic functions.

J.KOREVAAR :

Zero sets and approximation

By the Müntz theorem, distinct positive integral powers s^{p_n} span $L^2((0,1))$ iff $\sum_{n=1}^{\infty} \frac{1}{p_n} = \infty$. Equivalently, if $\{p_n\}_n$ belongs to the zero set of a bounded holomorphic function $f \not\equiv 0$ on the half plane $\{Re(z) > 0\}$, then $\sum_{n=1}^{\infty} \frac{1}{p_n} < \infty$.

Similar connections between spanning properties and zero sets exist in other cases. Question: When are linear combinations of distinct monomials $s^{p_n} t^{q_n}$ dense in $L^2(\{0 < t,s < 1\})$? Positive upper density of the corresponding set of lattice points (p_n, q_n) is a sufficient condition (Korevaar-Hellerstein). (One may conjecture that the Müntz-type condition $\sum_{n=1}^{\infty} \frac{1}{p_n^2 + q_n^2} = +\infty$ suffices (assuming $\delta < \frac{q_n}{p_n} < \frac{1}{\delta}$)). Let $f(z,w)$ be a bounded holomorphic function $\not\equiv 0$ on the octant $U = \{(z,w) \in \mathbb{C}^2 \mid \mathrm{Re}(z), \mathrm{Re}(w) > 0\}$ and let $A_\delta(R)$ denote the area of the zero set $Z(f)$ in the "box" $\{1 < (\mathrm{Re}\, z)^2 + (\mathrm{Re}\, w)^2 < R^2, \delta < \frac{\mathrm{Re}\, w}{\mathrm{Re}\, z} < \frac{1}{\delta}$ and $|\mathrm{Im}\, z|, |\mathrm{Im}\, w| < 1\}$. The Korevaar-Hellerstein result has led to (and is implied by) the area theorem $A_\delta(R) = o(R^2)$ (due to Ronkin and Berndtson). The above Müntz type conjecture corresponds to the area conjecture $\int_1^{\infty} r^{-2}\, dA(r) < \infty$, for which plausibility arguments can be provided.

S.G.KRANTZ :

<u>Characterizations of Certain Weakly Pseudoconvex Domains by their Automorphism Groups</u>

(Joint work with R.E.Greene)

A generalization of Rosay's well-known theorem was presented which applies to certain domains in $\mathbb{C}^2$.

<u>Theorem:</u> Let $0 < m \in \mathbb{Z}$. Let $E_m \subset \mathbb{C}^2$ be given by

$$E_m = \{ |z_1|^2 + |z_2|^{2m} < 1 \}.$$

Let Ω be a domain in $\mathbb{C}^2$ with a smooth boundary. Suppose

$\mathbb{1} = (1,0) \in b\Omega$ and that there is a defining function ρ for Ω such that near $\mathbb{1}$: $\rho(z) = -1 + |z_1|^2 + |z_2|^{2m} + R(z)$ with $|R(z)| \leq c\|\mathbb{1} - z\|^{2m+1}$. If there is a $z^o \in \Omega$ and biholomorphic self-maps ϕ_j of Ω such that $\phi_j(z^o) \to 1$ as $j \to \infty$, then Ω is biholomorphic to E_m.

L.LEMPERT :

Automorphisms of convex domains

Let Ω be a bounded domain in $\mathbb{C}^n$. H. Cartan's theorem asserts that if $p \in \Omega$, then $\Gamma := \{\phi \in \mathrm{Aut}(\Omega) \mid \phi(p) = p\}$ is compact.

The following question was studied:

(Q) Let Γ be a compact subgroup of $\mathrm{Aut}(\Omega)$. Does this imply that Γ admits a fixed point, i.e. does there exist a point $p \in \Omega$ with $\Gamma p = p$?

The answer is in general "no". (Take e.g. $\Omega = \{z \in \mathbb{C} \mid 0 < |z| < 1\}$). However, Greene and Krantz showed that the answer is positive if Ω is C^∞- close to the ball. Their proof used stability properties of the Bergman metric and a theorem of E.Cartan.

A new theorem was presented the proof of which is based upon ideas of E.Cartan.

Theorem: Let $\Omega \subset \mathbb{C}^n$ be a bounded convex domain and let Γ be a compact subgroup of $\mathrm{Aut}(\Omega)$. Then Γ admits a fixed point.

E.LÖW :

Boundary values of holomophic functions

In this lecture the question of inner functions on the unit ball in $\mathbb{C}^n$, $n \geq 2$, was discussed. They were constructed by Alexandrow at the end of 1981 and independently by Löw at the beginning of 1982.

Theorem: For any positive bounded lower semicontinous function ϕ on the boundary S of B_n there is a bounded holomorphic function f on B_n with boundary limits ϕ almost everywhere. If σ is the area measure on S, there is also a function $f \in A(B_n)$ whose boundary values satisfy $|f| = \phi$ on a set $M \subset S$ with $\sigma(M)$ close to $\sigma(S)$.

J.MICHEL :

The generalized Koppelman formula and its application to strictly q-convex domains

The following result on integral representations of $(0,r)$ - forms on strictly q-convex domains $,r \geq q-1$, was presented.

Theorem: Let $G \subset\subset \mathbb{C}^n$ be a strictly q-convex domain with a C^∞-smooth boundary. For $r= q-1,\dots,n$ there exist linear integral operators T_r, such that for all $\gamma \in C^1_{o,r}(G)$,with γ and $\bar\partial\gamma$ bounded the following holds :

$$\text{(i)}\quad \gamma = \bar\partial T_r\gamma + T_{r+1}\bar\partial\gamma \quad , \text{ if } r \geq q$$

$$\gamma = T_{q-1}\gamma + T_q\bar\partial\gamma \quad , \text{if } r = q-1$$

(ii) $\sup_G |T_r\gamma| \leq c\{\sup_G|\gamma| + \sup_G|\bar\partial\gamma|\}$, for $r \geq q-1$.

The proof of this theorem is an application of the generalized Koppelman formula and some special coordinated barrier forms.

T.OHSAWA :

Global realization of strongly pseudoconvex CR manifolds

Let S be a real hypersurface in a complex manifold M. Then, $T_M^{1,0}$, the holomorphic tangent bundle of M, determines an integrable subbundle $T_S' := T_M^{1,0}|S \cap (T_S \otimes \mathbb{C}) \subset T_S \otimes \mathbb{C}$. Modelled on (S,T_S'), a CR manifold is defined as a pair (X,T_X') consisting of a differentiable manifold X and a subbundle $T_X' \subset T_X \otimes \mathbb{C}$ of corank 1 satisfying the following two conditions: (i) $T_X' \cap \overline{T_X'} = 0$, (ii) T_X' is closed under the Poisson bracket. Then Boutet de Monvel showed

Theorem 1: Any strictly pseudoconvex manifold is holomorphically imbeddable into some $\mathbb{C}^N$, provided that dim X $\geq$ 5.

Our result is as follows:

Theorem 2: Let (X,T_X') be a compact strictly pseudoconvex CR manifold of dimension $\geq$ 5. Then (X,T_X') is realizable as a hypersurface of a complex manifold.

The proof is based upon Boutet de Monvel's imbedding theorem which assures the existence of a realization $X \subset \mathbb{C}^N$ for sufficiently large N. We apply then Tanaka's stability theorem to perform a finite number of bumps on X and obtain a strictly pseudoconvex manifold $\hat{X}$ which is the boundary of a complex manifold containing X as a hypersurface.

J.C.POLKING :

Holomorphic extension of CR functions

Let M^{2n-d} be a generic CR manifold in $\mathbb{C}^n$ of real codimension d. Two theorems were discussed concerning the problem of describing quantitatively the open set $\Omega \subset \mathbb{C}^n$ to which all CR functions on M can be extended holomorphically.

Theorem 1: (due to Bogess-Polking and independently to Baouendi,Chang, and Treves) Let $p_o \in M$ be a given point and Γ_{p_o} be the convex closed hull of the image of the Levi form of M at p_o. Assume $\Gamma_{p_o} \neq \emptyset$. Then given a neighborhood ω of p_o there exist open sets $\omega' \subset M$, $\Omega \subset \mathbb{C}^n$ such that

a) $p_o \in \omega' \subset \overline{\Omega} \cap M \subset \omega$

b) For every cone $\Gamma \subsetneq \Gamma_{p_o}$ there is an open set $\omega_\Gamma \subset M$ and an $\varepsilon_\Gamma > o$ such that $\omega_\Gamma + (\Gamma \cap B_{\varepsilon_\Gamma}) \subset \Omega$

c) Every continous holomorphic function f on ω has an extension $F \in \mathcal{O}(\Omega) \cap C(\overline{\Omega})$.

Theorem 2: (due independently to Bogess-Pitts, Baouendi-Treves,and to Rea) If $M^{2n-1} \subset \mathbb{C}^n$ is a hypersurface of type k at a point $p_o \in M$, then the following holds:

a) If k is odd, every CR function on M is locally the restriction of a holomorphic function near p_o.

b) If k is even, every CR function extends to a holomorphic function on at least one side of M.

R.M.RANGE :

Integral Representations on Hermitian Manifolds: The $\bar\partial$ - Neumann Solution to the Cauchy-Riemann Equations

Let X be a Hermitian complex manifold and $D \subset X$ a smoothly bounded strictly pseudoconvex domain. If N is the $\bar\partial$ - Neumann operator for D, the operator $\bar\partial^* N$ solves the $\bar\partial$ equation : given $f \in L^2_{o,q+1}(D)$, $\bar\partial f = 0$, $f \perp \ker \Box$, then $u = \bar\partial^* Nf$ is the unique solution $u \in L^2_{o,q}$ of $\bar\partial u = f$, with $u \perp \ker \bar\partial$. In this lecture results were discussed which deal with finding explicit representations for $\bar\partial^* N$ in terms of integral kernels. The main objective was to establish the basic representation formula

$$(1) \qquad \phi = T_q \bar\partial\phi + T^*_{q-1} \bar\partial^*\phi + E_q\phi \quad , \text{ for } \phi \in C^1_{o,q}(\bar D) \cap \operatorname{dom} \bar\partial^* .$$

where all operators are explicit and smoothing of some order.

The main result was as follows:

Theorem: (Lieb-Range) If the metric on X is a Levi metric for D, then (1) holds for all $q \geq 1$, with T_q, T^*_{q-1} and E_q smoothing of order 1/2 , (i.e. they send L^∞- forms to Lipschitz - 1/2 - forms) .

Corollary: $\bar\partial^* N = T_q$ + more smoothing terms on range$(\bar\partial)$.

Less complete results hold in case of arbitrary Hermitian metrics; on the other extreme, the formulas become exact in the case of the ball.

H.L.ROYDEN :

Dual Extremal Problems and the Kobayashi Metric for Convex Domains

(Joint work with P.M.Wong)

Let D be a bounded convex domain in $\mathbb{C}^n$. The Kobayashi problem for a point $q \in D$ and a direction $v \in \mathbb{C}^n$ is to minimize $\frac{1}{\lambda}$ for all holomorphic maps

$f: \Delta \to D$ with $f(0) = q$ and $f'(0) = \lambda v$, $\lambda > o$. Here Δ denotes the unit disc in $\mathbb{C}$. By means of the method of dual extremal problems a direct proof of the following result was given.

Theorem : A map $f : \Delta \to D$ is minimal for the Kobayashi problem $K(q,v)$ for the pair $(q,v) \in D \times \mathbb{C}^n$ if and only if $f(e^{i\theta}) \in bD$ for almost all $e^{i\theta} \in \partial\Delta$ and there is a map $h : \Delta \to \mathbb{C}^n$ of Hardy class $H^1(\Delta)$ such that for almost all $e^{i\theta} \in \partial\Delta$ the hyperplane

$$\{ \operatorname{Re} \sum_{k=1}^{n} e^{-i\theta} h_k(e^{i\theta}) (z_k - f_k(e^{i\theta})) = 0 \}$$

is a supporting hyperplane to D at $f(e^{i\theta})$. If D is strictly convex, then f is unique.

This result was obtained by Lempert using indirect methods, when the boundary of D is sufficiently smooth.

Corollary : The Kobayashi and Caratheodory metrics for D coincide. This result has also been obtained by Lempert.

G.SCHUMACHER :

An application of the Calabi-Yau theorem to the construction of a coarse moduli space

In algebraic geometry coarse moduli spaces are known for polarized abelian varieties (Mumford) and for canonically polarized compact varieties (Popp) . Interpreting a polarization as an integer-valued Kähler class, we carry over this notion to compact Kähler manifolds by the assignment of a fixed Kähler class. The existence of a cooresponding embedding into a projective space has to be replaced by transcendental methods which are based upon S.T. Yau's solution of the Calabi problem. For polarized manifolds with $c_1 = 0$ we construct a coarse moduli space. This is carried out in three steps. First we show the existence of universal Kähler deformations. In order to get out of this a local patch of the desired moduli space, one has to identify those points of the base which correspond to isomorphic fibres. By means of the Barlet space one can show that the graph of the induced equivalence relation is constructible. Under the assumption $c_1 = 0$ it is closed by the isomorphism properties for such families. Then the graph can actually be computed as an orbit of a finite group acting holomorphically on the base. Finally the coarse moduli space can the patched together from the quotients as a Hausdorff complex space.

With the same methods and the global Torelli theorem of Burns-Rapopot one gets a fine moduli space for marked polarized K 3 surfaces.

N.SIBONY :

Piuripositive currents.

Let T be a positive current on an open subset $\Omega \subset \mathbb{C}^n$. The current T is said to be pluripositive iff T is a normal current satisfying $dd^cT \geq 0$.

Proposition: Let T be a pluripositive current of bidimension (p,p) in Ω. Then

(1) If T has a compact support, then $T = 0$

(2) If $A \subset \Omega$ such that $\Lambda_{2p}(A) = 0$, then $\chi_A T = 0$.

(Here for any set $A \subset \mathbb{C}^n$, χ_A denotes the characteristic function of A and Λ_{2p} denotes the 2p-dimensional Hausdorff measure).

Theorem : Given a pluripositive current T of bidimension (p,p) in Ω , and a closed complete pluripolar set $A \subset \Omega$, then

$$(1) \quad d\,(\chi_A T) = \chi_A dT$$

$$(2) \quad dd^c(\chi_A T) \geq 0$$

The result was shown by Skoda - El Mir in the case that the current is closed.

Corollary: If T is supported on an irreducible variety V of dimension p then $T = \phi[V]$, where ϕ is plurisubharmonic, and positive on V.

Y.T. SIU :

Vanishing Theorems for Semipositive Bundles over Non-Kähler Manifolds

Let M be a compact complex manifold and L be a Hermitian holomorphic complex line bundle over M so that the curvature of L is semipositive everywhere on M and is strictly positive on an open subset G of M. Grauert and Riemenschneider conjectured that M is Moišezon if G is dense in M (and as a consequence $H^\nu(M,L^kK_M)$ vanishes for $k,\nu \geq 1$, where K_M is the canonical line bundle of M). By following the method of Poincaré-Siegel of using the Schwarz lemma, we prove that the Grauert-Riemenschneider conjecture is true if $M \setminus G$ has measure zero.

Moreover the more general conjecture with the weaker assumption that G is nonempty is true if one has the following statement concerning the first eigenvalues : $\inf_{k>0} \lambda(M,L^k) > 0$, where $\lambda(M,L^k)$ is the first eigenvalue of the Laplacian $\square = \bar\partial\,\bar\partial^*$ on the space of sections of L^k over M.

M.SKWARCZYNSKI :

Alternating projections in complex analysis

Let D be an arbitrary domain in $\mathbb{C}^n$. A constructive description of the Bergman kernel function (and equivalently the Bergman projection P_D) was given.

Theorem : Let $D \subset \mathbb{C}^n$ be the union of domains D_i, $1 \leq i \leq m$ (with known Bergman kernel functions K_{D_i}) . In the Hilbert space $H = L^2(D)$ we consider of each i the closed linear subspace F_i of all elements which are holomorphic in D_i. Let $P_i : H \longrightarrow F_i$ be the corresponding orthogonal projection. Let $t \in D$ ($t \in D_1$ with no loss of generality). Consider $f \in H$ defined by

$$f(z) := \begin{cases} K_{D_1}(z,t) & ; z \in D_1 \\ 0 & ; z \in D \setminus D_1 \end{cases} .$$

Then the sequence of alternating projections :

$$f_1 = P_1 f \ , \ f_2 = P_2 f_1 \ , \ \ldots . \ , \ f_m = P_m f_{m-1} \ , \ f_{m+1} = P_1 f_m \ , \ldots$$

converges in H to $K_D(\,\cdot\,, t)$.

The proof uses a theorem of Halperin (Acta Sci. Szeged 23, 96-99 (1962)). A general case $D \subset \mathbb{C}^n$ follows by a theorem of Ramadanov.

Ch.M.STANTON :

Intrinsec connections for Levi metrics

Let (M,g) be an integrable nondegenerate CR manifold with Levi metric g. Let $H(M)$ be the maximal codimension 1 subbundle of TM.

Let X denote a unit vector field orthogonal to H(M). Then an idea of a proof of the following result was given :

Theorem : On M one can construct an intrinsically defined metric connection in such a way that X becomes a parallel vector field.

The construction of this connection used E.Cartan's method of equivalence. The normal coordinates for this connection are useful for studying some problems of analysis on CR manifolds.

N.K. STANTON :

The heat equation for $\Box_b$

Let M be a compact strictly pseudoconvex CR manifold , dim M = 2n+1 $\geq$ 5 equipped with a Levi metric g. Let p be the fundamental solution of $\frac{\partial}{\partial t} + \Box_b$ on (0,1)-forms , $p \in C^\infty(\Lambda^{0,1} \otimes \Lambda^{0,1}(M \times M \times \mathbb{R}^+))$. Let r be the kernel on M modelled on the fundamental solution of $\frac{\partial}{\partial t} + \mathcal{L}_{n-2}$ on the Heisenberg group, set $q := (\frac{\partial}{\partial t} + \Box_b)\, r$.

Theorem 1: (Stanton-Tartakoff) The fundamental solution p is given by

$$p = r + \sum_{k=1}^{\infty} (-1)^k\, r \# q^k$$

where $q^1 := q$ and $q^{k+1} = q^k \# q$, for $k \in \mathbb{N}$, and # is an operation on kernels corresponding to composition of operators on $M \times \mathbb{R}^+$.

Theorem 2: (Beals - Greiner - Stanton)

Trace $p(x,x,t) \sim t^{-n-1} \sum_{j=0}^{\infty} K_j(x)\, t^j\, dV(x)$ as $t \to 0$.

The functions K_j may be calculated by evaluating a polynomial (depending only on n,j) in the components of the curvature and torsion of the Webster-C.Stanton connection and their covariant derivatives, calculated in normal coordinates.

Theorem 1 is clearly formally true. To make the formal arguments work, we prove appropriate estimates. Theorem 2 is proved by developing an appropriate pseudodifferential operator calculus. Stanton and Tartakoff have obtained a second proof by analyzing the terms in the series of Theorem 1. This proof is in the spirit of the Mc Kean - Singer work on the heat equation in Riemannian geometry. The original proof is in the spirit of the Seeley and Greiner proofs in Riemannian geometry.

W. STOLL :

A defect relation for moving targets

Nevalinna theory is extended to meromorphic maps $f: M \to \mathbb{P}^n$ where M is an m-dimensional parabolic manifold. Let $\mathcal{G}$ be a set of meromorphic maps $g: M \to (\mathbb{P}^n)^*$,where $(\mathbb{P}^n)^*$ is the dual projective space. Then each $g \in \mathcal{G}$ is a "moving hyperplane" .

We assume that $n+1 < \#\mathcal{G} < +\infty$. Also $\mathcal{G}$ is in general position at at least one point $z_o \in M$. The incidence $f(z) \in g(z)$ is studied. A First and a Second Main Theorem is obtained.

If $M = \mathbb{C}^m$ these results are applied to obtain a defect relation over a function field F of rank m-1 on a proper covering of $\mathbb{C}^m$.

E.L.STOUT :

Differentiable interpolation on the polydisc

(Joint work with R.Saerens)

Denote by U^N the unit polydisc in $\mathbb{C}^N$ and by T^N its distinguished boundary. The main result was this :

Theorem : Let $E \subset T^N$ be a closed set and $p \in \mathbb{Z}$, $p \geq 1$.

a) If E is locally a peak set for $A^p(U^N)$, then given $f \in C^p(T^N)$, there is $F \in \bigcap_{0<\alpha<1} A^{p-1,\alpha}(U^N)$ with $F|E = f|E$.

b) If E is locally a peak set for $A^{p-1,\alpha}(U^N)$, then given $f \in C^p(T^N)$, there is $F \in A^{p-1,\alpha}(U^N)$ with $F|E = f|E$.

c) If E is locally a peak set for $A^{p-1,\alpha}(U^N)$, then given $f \in C^{p-1,\alpha}(T^N)$ there is $F \in A_N^{p-1,\alpha}(U^N)$ with $F|E = f|E$.

(In (c) $A_N^{p-1,\alpha}$ denotes the space of functions on U^N whose $(p-1)^{st}$ derivatives admit const$\cdot\, \delta^\alpha(\log \frac{1}{\delta})^{N-1}$ as modulus of continuity).

It was shown, in addition, that the peak set of an $f \in A^p(U^N)$ is contained in a locally closed submanifold M of T^N of class C^p that is an interpolation manifold in the sense that for each $x \in M$ the tangent space T_xM meets the cone in $T_x T^N$ generated by the vectors $\frac{\partial}{\partial\theta_1}\big|_x, \dots, \frac{\partial}{\partial\theta_N}\big|_x$ only at the origin.

An example was given that shows that "locally closed" cannot be replaced by "closed" .

G. TRAUTMANN :

The Atiyah-Ward theorem for arbitrary SU(r)

Let $\mathbb{P}_3(\mathbb{C}) \xrightarrow{\sigma} \mathbb{P}_3(\mathbb{C})$ be the involution

$$(z_o:z_1:z_2:z_3) \longmapsto (-\bar{z}_1:\bar{z}_o:-\bar{z}_3:\bar{z}_2)$$

Then the mapping $z \underset{\pi}{\mapsto} \overline{z,\sigma z}$ (:= line in $\mathbb{P}_3(\mathbb{C})$ passing through z and σz) from $\mathbb{P}_3(\mathbb{C})$ into the Grassmannian $\mathbb{G}_{1,3}$ has values in S^4, where S^4 is identified with the real submanifold of $\mathbb{G}_{1,3}$ defined by the real structure of $\mathbb{G}_{1,3} \subset \mathbb{P}_5(\mathbb{C})$. It turns out that π is a fibre bundle with fibre $\mathbb{P}_1(\mathbb{C})$.

The theorem of Atiyah-Ward (Comm. math. phys. 55 (1977)) gives a correspondence between self-dual su(2) - connections on S^4 and holomorphic rank 2 vector bundles E on $\mathbb{P}_3(\mathbb{C})$ modulo isomorphisms , having a holomorphic isomorphism $E \xrightarrow[\tau]{} \sigma^*\bar{E}$, $\tau^2 = -1$.

This correspondence generalizes to su(r) - connections which then correspond to a holomorphic $*$- operator $\Lambda^p E \xrightarrow[*]{} \Lambda^{r-p}\, \sigma^*\bar{E}$ satisfying $\sigma^* \bar{*} \circ * = (-1)^{p(r-p)}$.

S.M.WEBSTER :

Real n-manifolds in $\mathbb{C}^n$

We consider a smooth compact orientable real n-manifold M^n imbedded in $\mathbb{C}^n$; it is assumed that all complex tangents are of dimension one, and are suitably non-exceptional. They are then of elliptic, hyperbolic or parabolic character according to an invariant of E.Bishop. Elliptic points contribute to the hull of holomorphy of M.

The main theorem says that (under suitable conditions on M) the Euler number of M

$$\chi(M) = e - h + p ,$$

where $e = \chi(N_e)$, $h = \chi(N_h)$ and p is the parabolic index. Here N_e and N_h denote the manifolds of elliptic and hyperbolic points, respectively.

P.M.WONG :

A PDE approach of constructing holomorphic vector bundles

Let M be a compact Kähler surface and E an SU(k)- bundle over M. Using perturbation methods analogous to that of Taubes' construction of self-dual Yang-Mills fields, we obtain the result that such bundles admit holomorphic structures provided that $c_2 \geq 2\, h^{0,2}$, where $h^{0,2} := \dim_{\mathbb{C}}\{$holomorphic 2-forms on M$\}$. The proof uses the fact that any SU(k)-bundle over M is the pull-back of an SU(k) - bundle over S^4 via a mapping of degree 1, and if $c_2 \geq 0$ there exists a connection on the SU(k)-bundle over S^4 whose curvature on $\mathbb{C}^2 \cong \mathbb{R}^4 \hookrightarrow S^4$ is of type (1,1). Furthermore on $\mathbb{C}^2$ we can scale the curvature by automorphisms of $\mathbb{C}^2$ so that the pull-back curvature even though not of type (1,1) is, however, very close to it (in some appropriate L_p-space). If we use $\mathbb{CP}^2$ as a model then in some very special cases M may be mapped into $\mathbb{CP}^2$ in such a way that the pull-back curvature is still close to type (1,1). This approach can be repeated in higher dimensional manifolds under stringent conditions. We hope to investigate this problem more extensively in the future.

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