

## MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 38/1983

## Special Complex Varieties

28.8. bis 3.9.1983

Die Tagung fand unter der Leitung der Herren W. Barth (Erlangen) und A. Van de Ven (Leiden) statt. In zusammen 25 Vorträgen wurden neue Ergebnisse aus der Algebraischen und Komplex-analytischen Geometrie dargestellt und diskutiert. Ein besonderer Schwerpunkt war die Klassifikationstheorie komplexer Varietäten, wobei wiederum besonderer Wert auf die Theorie der Flächen und dreidimensionalen Varietäten gelegt wurde. Eine Reihe von Beiträgen beschäftigte sich auch mit algebraischen Kurven, Vektorbündeln und Singularitäten. Darüber hinaus wurden Vorträge über holomorphe Zerlegungen, Lefschetzsätze und Deformationstheorie gehalten.

Insgesamt nahmen 40 Mathematiker aus 10 Ländern teil. Insbesondere die Anwesenheit auch vieler ausländischer Gäste bereicherte die Diskussion innerhalb und außerhalb des offiziellen Programms. Durch Freihalten der frühen Nachmittagsstunden und des Mittwoch Nachmittags von Vorträgen blieb trotz eines reichhaltigen Vortragsangebots genügend Zeit für diesen Gedankenaustausch.

Vortragsauszüge

A. BEAUVILLE:

Kähler symplectic manifolds

A compact Kähler manifold  $X$  is symplectic if it admits a holomorphic 2-form  $\varphi$  which is everywhere non-degenerate. Moreover  $X$  is

called irreducible if it is simply connected and  $h^{2,0}(X) = 1$ .

These manifolds give a nice generalization of K3 surfaces. In particular the period map (for 2-forms) looks very much like the one for K3 surfaces: it gives a local isomorphism of the moduli space into a smooth quadric in  $\mathbb{P}(H^2(X, \mathbb{C}))$ .

We present 2 series of examples of such manifolds, called  $S^{[r]}$  and  $K_r$ :

- 1) Let  $S$  be a K3 surface. Then  $S^{[r]}$  is the Douady space of 0-dimensional subspaces  $Z \subset S$  with  $\text{lg}(\mathcal{O}_Z) = r$ .
- 2) Let  $A$  be a 2-dimensional complex torus. Then  $K_r = S^{-1}(0)$ , where  $S: A^{[r+1]} \rightarrow A$  is the morphism "sum".

The local moduli space  $\mathcal{M}$  for  $S^{[r]}$  (resp.  $K_r$ ) is smooth of dimension 21 (resp. 5). The deformations coming from deformations of  $S$  (resp.  $A$ ) form only a smooth hypersurface in  $\mathcal{M}$ . I don't know how to describe a generic deformation of  $S^{[r]}$  or  $K_r$ , except in one example:

Let  $X$  be a smooth cubic hypersurface in  $\mathbb{P}_4$ . Then the Fano variety of lines contained in  $X$  is a (projective) symplectic irreducible fourfold. When  $X$  varies, one gets the complete family of projective deformations of  $S^{[2]}$ , where  $S$  is a K3 surface in  $\mathbb{P}_8$ .

F. CATANESE:

Involutions on rational double points and moduli spaces of surfaces of general type.

Let  $S$  be a minimal surface of general type with given invariants  $K^2, \chi$  and let  $\mathcal{M}$  be the coarse moduli space for surfaces homeomorphic to  $S$ .

I proved recently that the number of irreducible components of  $\mathcal{M}$  can be arbitrarily large, as well as the number of different values for the dimensions of the irreducible components of  $\mathcal{M}$ .

Here I reported on the conjecture that also the number of connected components can be arbitrarily large, and gave a proof of one of the two main steps towards the affirmative answer to the problem.

The irreducible components of  $\mathcal{M}$  under consideration are given as

follows: consider a  $(\mathbb{Z}/2)^2$  Galois cover of  $\mathbb{P}^1 \times \mathbb{P}^1$  of simple type  $z^2 = f(x,y)$ ,  $w^2 = g(x,y)$  with  $f$  of bidegree  $(a,b)$  and  $g$  of bidegree  $(n,m)$ . One gets thus a subvariety  $\mathcal{N}_{(a,b)(n,m)}$ , and the more precise conjecture is that its closure is a connected component of  $\mathcal{M}$  if  $n > 2a$ ,  $b > 2m$ .

Theorem: If  $n > 2a$ ,  $b > 2m$   $\mathcal{N}_{(a,b)(n,m)}$  is an irreducible component of  $\mathcal{M}$ , smooth at the points corresponding to smooth covers of  $\mathbb{P}^1 \times \mathbb{P}^1$ , and the points in  $\mathcal{N}_{(a,b)(n,m)}$  consist of  $(\mathbb{Z}/2)^2$  simple covers of  $\mathbb{F}_{2m}$  of type  $(a,b)(n,m)$ .

The proof relies on the classification of the singularities (#) which occur as quotients of rational double points by  $\mathbb{Z}/2$  or  $(\mathbb{Z}/2)^2$ , and the following:

Theorem: Let  $f: Z \rightarrow \Delta$  be a 1-parameter family, s.t.

- 1)  $f^{-1}(t) = \mathbb{P}^1 \times \mathbb{P}^1$  for  $t \neq 0$ .
- 2)  $Z$  is normal,  $f^{-1}(0)$  is reduced with singularities only of type (#). Then  $f^{-1}(0)$  is either  $\mathbb{F}_{2m}$ , or  $\mathbb{F}_2$ ,  $\mathbb{F}_4$  with the negative section contracted to a point.

To prove the conjecture it suffices to analyse the local deformation when  $K$  is not ample.

G. ELENCAWAJG

Brauer group of fibrations and symmetric products of curves.

Let  $\pi: P \rightarrow X$  be a fibre bundle with fibre  $\mathbb{P}_{s-1}$ . From the exact sequence  $1 \rightarrow \mathcal{O}^* \rightarrow GL(s, \mathcal{O}) \rightarrow PGL(s, \mathcal{O}) \rightarrow 1$  we get a map  $\delta_s: H^1(X, PGL(s, \mathcal{O})) \rightarrow H^2(X, \mathcal{O}^*)$ .

Define  $Br_s(X) = \text{Im } \delta_s$ ,  $Br(X) = \bigcup (\text{Im } \delta_s)$  and  $Br'(X) = H^2(X, \mathcal{O}^*)_{\text{tors}}$ .

Theorem:

- i) If  $\rho: E \rightarrow Y$  is a fibre bundle with fibre  $\mathbb{P}_r$ , then  $Br(Y) = Br'(Y)$  implies  $Br(E) = Br'(E)$ .
- ii)  $Br(C^{(n)}) = Br'(C^{(n)})$  for the  $n$ -th symmetric product of the compact Riemann surface  $C$ .

PH. ELLIA

The normal bundle of space curves.

In this talk we consider the stability of normal bundles of smooth

connected curves in  $\mathbb{P}^3$ . The results are:

Theorem 1: Denote by  $Z^*$  the open set of Hilb  $\mathbb{P}^3$  consisting of smooth curves of genus 3 and degree 6 not lying on a quadric surface. Then: (a) every curve in  $Z^*$  has semi-stable normal bundle. (b) If  $C$  is general in  $Z^*$  then  $N_C$  is stable. (c) There exist curves in  $Z^*$  with  $N_C$  not stable, indecomposable. (d) If  $N_C$  splits then  $N_C = 2T_C(3)$ . There are curves in  $Z^*$  with decomposed normal bundle.

Theorem 2: (joint work with E. Ballico) Let  $E$  be a stable (resp. semi-stable) rank 2 vector bundle on  $\mathbb{P}^3$ . There exists an integer  $b(E)$  such that for  $t \geq b(E)$  a general section of  $E(t)$  has as zero set a smooth connected curve with stable (resp. semi-stable) normal bundle.

#### G. ELLINGSRUD

##### Irreducibility of the moduli space of stable vector bundles on $\mathbb{P}^2$ .

It is well known that the moduli space  $M(r, c_1, c_2)$  of stable vector bundles on  $\mathbb{P}^2$  of rank  $r$  and Chern classes  $c_1$  and  $c_2$  is irreducible if  $r=2$  (Maruyama, Barth, Hulek) or if  $c_1 \equiv 0 \pmod r$  (Hulek). We prove, using the standard-construction that  $M(r, c_1, c_2)$  is irreducible in the resting cases. An essential ingredient in the proof is the following result obtained by Brun and Hirschowitz: Every stable vector bundle on  $\mathbb{P}^2$  of rank  $\geq 3$  and  $c_1 \not\equiv 0 \pmod r$  may be deformed into a uniform bundle with rigid decomposition type if  $c_1 \not\equiv \pm 1 \pmod r$ , and into a bundle with rigid generic decomposition type and only a finite number of jumping lines if  $c_1 \equiv \pm 1 \pmod r$ .

#### I. ENOKI

##### Complex structures on $S^3 \times S^3$ (after Hajime Tsuji)

Consider over an elliptic curve  $C$  a flat vector bundle  $E$  of the form  $E = \mathbb{C}^n \times \mathbb{C}^* / \langle A, \alpha \rangle$  where  $A \in GL(n, \mathbb{C})$  is a contraction and  $0 < |\alpha| < 1$ . Then there are complex analytic modifications (not bimeromorphic) which replace the zero-section of  $E$  by a Hopf manifold  $D$  of codimension 1, e.g. the result of this modification is  $\mathbb{C}^n - \{0\} \times \mathbb{C} / \langle A, \alpha \rangle$

and  $D = \mathbb{C}^n - \{0\} \times \{0\} / \langle A, \alpha^{-1} \rangle$ . Now we apply this modification to a diagonal Hopf 3-fold  $H = \mathbb{C}^3 - \{0\} / \langle \alpha_1, \alpha_2, \alpha_3 \rangle$  around the elliptic curve defined by  $z_1 = z_2 = 0$  to obtain a family of compact 3-folds which are homeomorphic to  $S^3$ -bundles over lens spaces. These manifolds can be characterized under certain generic conditions; Tsuji reduces the proof to the characterization of Hopf manifolds by showing that by means of the inverse of the above modification we can obtain a Hopf 3-fold from the manifold in question.

G. VAN DER GEER

Projective geometry of moduli spaces of Abelian varieties.

This is a report on joint work with van Geemen. If  $X$  is a principally polarized Abelian variety of dimension  $g$  then the theta functions of second order define a basis of  $\Gamma(X, \mathcal{L}^{\otimes 2})$  where  $\mathcal{L}$  gives the polarization. They give rise to two maps:

- 1)  $\text{Th}: A_g(2,4) \rightarrow \mathbb{P}^{2g-1}$  with  $A_g(2,4)$  the moduli space of (certain) Abelian varieties
- 2)  $\text{Th}_X: X \rightarrow \mathbb{P}^{2g-1}$  the Kummer map.

We look at the intersection of the tangent space to  $\text{Th}(A_g(2,4))$  at  $\text{Th}([X])$  with the image  $\text{Th}_X(X)$  of  $X$ . I formulated precise conjectures on these intersections. This gives a conjectural answer to the Schottky problem. Also conjectures about the intersection of  $\text{Th}_X(X)$  and  $\text{Th}(A_g(2,4))$  were given. Their relation with other approaches (irreducibility of the Schottky locus, Novikov conjecture...) were discussed.

H. GRAUERT

Complex decompositions.

Assume always that  $X, Y$  etc. are reduced complex spaces. An equivalence relation  $E \subset X \times X$  is a complex decomposition, if  $E$  is an analytic set. The quotient space  $Q = X/E$  is equipped with natural topology and  $\mathbb{C}$ -ringed structure. Then the quotient map is a morphism. If  $F: X \rightarrow Y$  is a holomorphic map,  $E = E_F := X \times_F X$  as a set is a complex decomposition. If  $E$  is given, by the B.-Kaup-construction the "simple decomposition"  $\hat{E}$  to  $E$  is obtained.  $\hat{E}$  is the finest complex decomposition whose fibers are locally the same as those



of  $E$ . A holomorphic map  $F: X \rightarrow Y$  is called analytically dependent on  $E$  if  $F$  is locally constant on the fibers. These are just those maps which come from  $Q \rightarrow Y$ . So  $Q$  is a complex base in the sense of Stein.

Assume now that  $X$  is normal. The question is: when is  $Q$  a complex space? By B. Kaup this is true if locally the dimensions of all fibers are the same. If this is not the case there are two necessary and sufficient conditions ( $E = \text{holomorphic} + \text{semiproper}$ ). These should be the weakest conditions which can be obtained. Finally it was shown that  $m$ -bases in the sense of Stein always exist.

L. GRUSON

Complexes of singular jumping lines.

Let  $E$  be a reflexive  $\mathcal{O}_{\mathbb{P}^3}$ -module of rank two, let  $\mathcal{O} \rightarrow E$  be a section of  $E$  whose set of zeroes is a curve. Let  $i$  be an integer such that  $2i \leq c_1 = c_1(E)$  and  $H^0(E(i-c_1-1)) = 0$ . Let  $G$  be the grassmannian of lines in  $\mathbb{P}^3$  and  $F \subset G \times \mathbb{P}^3$  be the incidence correspondence. Then one defines a subvariety  $S_i$  of pure codimension one in  $F$ . A pair  $(L, P)$  of  $F$  lies in  $S_i$  if and only if the following condition holds: form the cartesian square

$$\begin{array}{ccc} \mathcal{O}_L & \longrightarrow & E_L \\ \downarrow & & \downarrow \\ \mathcal{O}_L((c_1-2i)P) & \longrightarrow & E_1 \end{array} \quad \text{of } \mathcal{O}_L\text{-modules;}$$

then the expected decomposition  $E_1 = \mathcal{O}_L^2(c_1-i)$  does not hold. There is a natural scheme structure on this set of points, this is  $S_i$ . One proves that each non-reduced irreducible component of  $S_i$  is (set-theoretically) the inverse image in  $F$  of some "special" complex in  $G$ , i.e. consists of points  $(L, P)$  in  $F$  such that  $L$  meets some fixed irreducible curve  $\Gamma$ . Moreover there is a map  $E \rightarrow \mathcal{J}_\Gamma^m(n)$ , with  $n-m \leq i-1$ , where  $\mathcal{J}_\Gamma^m$  denotes the idealdefining  $\Gamma$ .

K. HULEK

Complete intersection curves and the splitting of the normal bundle.

This talk was a report on recent joint work with J. Harris. An out-

line of a proof of the following result was given.

Theorem: Let  $S \subseteq \mathbb{P}_n$  be a smooth complete intersection surface, and let  $C \subseteq S$  be a smooth curve. Then the normal bundle sequence

$$0 \rightarrow N_{C/S} \rightarrow N_{C/\mathbb{P}_n} \rightarrow N_{S/\mathbb{P}_n} \Big|_C \rightarrow 0$$

splits if and only if  $C$  is a complete intersection with  $S$ , i.e.  $C = S \cap F$ .

By means of a counterexample (an elliptic curve  $C$  of degree 6 on the Veronese surface  $S \subseteq \mathbb{P}_5$ ) it was shown that the above statement does not hold for arbitrary smooth surfaces  $S \subseteq \mathbb{P}_n$ .

### S. IITAKA

#### Small size birational geometry.

Let  $(D, X)$  be a pair of a non-singular curve  $D$  and a complete non-singular rational surface  $X$  defined over the field of complex numbers such that  $D$  lies on  $X$ . Two such pairs  $(D, X)$  and  $(D', X')$  are called birational if there exists a birational map  $\varphi: X' \rightarrow X$  such that  $\varphi[D'] = D$ .

Definition.  $(D, X)$  is said to be relatively minimal, if  $D$  is not an exceptional curve of the 1-st kind and any exceptional curve  $E$  satisfies  $D \cdot E \geq 2$ .

Let  $\kappa[D] = \kappa(K + D, X)$ , that is the Kodaira dimension for  $(D, X)$ .

Theorem 1. Let  $(D, X)$  be a relatively minimal pair such that  $\kappa[D] \geq 0$ . Then it is minimal or birationally equivalent to a hyperelliptic pair.

Note. This is an analog of a fundamental theorem in the minimal model theory by Enriques.

### H. KNÖRRER

#### Torsion free sheaves and simple curve singularities.

The purpose of the talk is to describe and comment the following result, which was obtained in collaboration with G.-M. Greuel:

Theorem: Let  $R$  be the local ring of a reduced plane curve singularity. Then the following statements are equivalent:

- 1) Up to isomorphism there are only finitely many torsion free modules of rank one over  $R$

- ii) Up to isomorphism there are only finitely many indecomposable torsion free modules over  $R$
- iii)  $(D, p)$  is isomorphic to a simple curve singularity.

H. MAEDA

Classification of logarithmic Fano 3-folds.

Let  $V$  be a non-singular projective variety and  $D = D_1 + \dots + D_s$  be a divisor with simple normal crossings on  $V$ . A logarithmic Fano variety is defined to be a pair  $(V, D)$  such that  $-K_V - D$  is an ample divisor on  $V$ . This is an extension of the classical Fano variety.

The purpose of this talk is to state the structure of logarithmic Fano varieties of dimension 3. The proof is based on the idea of the theory of open algebraic varieties due to S. Iitaka and on the theory of extremal rational curves due to S. Mori.

Roughly speaking, the logarithmic Fano 3-folds are (1)  $\mathbb{P}^3, Q_2, V_1, V_2, V_3, V_4, V_5$  in the notations of Iskovskih, (2)  $\mathbb{P}^1$ -bundles over non-singular surfaces which are del Pezzo surfaces or Hirzebruch surfaces, (3)  $\mathbb{P}^2$ -bundles over  $\mathbb{P}^1$ , (4) quadric fiberings over  $\mathbb{P}^1$ , or (5) some points blowing up of  $\mathbb{P}^3, Q_2$  or  $\mathbb{P}^2$ -bundles over  $\mathbb{P}^1$ .

H.H. MARTENS

Mappings of closed Riemann surfaces.

Holomorphic mappings of closed Riemann surfaces induce homomorphisms between their homology groups. Conditions for a given homomorphism to be induced by a holomorphic map are discussed. The problem of characterizing such homomorphisms is, in general open and interesting.

Y. MIYAOKA

Maximal number of rational double points on surfaces.

Let  $X$  be a normal projective surface with only rational double points (R.D.P.'s) and put

$$v(P) = \# \text{ (irreducible components of the minimal resolution) } + 1/2,$$



for each rational double point P.

Theorem:  $\sum_{P:R.D.P.} \nu(P) \leq \frac{4}{3} (9X(\mathcal{O}_X) - \omega_X^2)$  if the (invertible)

dualizing sheaf is nef. In particular,

$$+(\text{rational double points}) \leq \frac{8}{9} (9X(\mathcal{O}_X) - \omega_X^2).$$

This is an easy application of the following

Theorem: Let  $\mathcal{E}$  be a rank 2 vector bundle on a smooth projective surface S such that (i)  $\mathcal{E} \subset \Omega_X^1(\log D)$  (ii)  $c_1(\mathcal{E})$  is nef.

Then  $3c_2(\mathcal{E}) - c_1^2(\mathcal{E}) \geq 0$ .

Y. NAMIKAWA

Automorphisms of Enriques surfaces.

For an Enriques surface S let G be a subgroup of the group of isometries  $O(H^2(S, \mathbb{Z})_{\text{free}})$  consisting of isometries which have extensions of isometries of the covering K3 surface preserving periods. Also let W be the group generated by symmetries associated with irreducible curves C on S with  $C^2 = -2$ . Set  $A = \text{Im}(\text{Aut}(S) \rightarrow O(H^2(S, \mathbb{Z})_{\text{free}}))$  and  $D = \pm 1$ . Then the fundamental result is

Theorem: i) W is a normal subgroup of G; ii)  $G = (D \times W) \rtimes A$  (semi-direct product), iii)  $G \supset O(2) = \{g; g \equiv 1 \pmod{2}\}$  (hence  $[O(H^2); G] < \infty$ ). As corollaries we get: 1) (Dolgachev)  $\text{Aut}(S)$  is finite if and only if  $[O; W] < \infty$ , 2) (Barth - Peters) for generic S one has  $\text{Aut}(S) = O(2)$ . The kernel of  $\text{Aut}(S) \rightarrow O(H^2_{\text{free}})$  is non-trivial in exactly three cases which can be exhibited concretely. One can propose a problem to classify all S with finite  $\text{Aut}(S)$ , which is now not so far from the complete answer, though only one example is known so far.

C. PESKINE

Complete series for space curves.

Theorem (Castelnuovo): If C is a smooth connected curve of  $\mathbb{P}_{\mathbb{C}}^3$ , then surfaces of degree n cut out a complete linear system on C for  $n \geq d^0(C) - 2$ , and for  $n \geq d^0(C) - 3$  if C is not rational. The proof suggested by Castelnuovo for the second assertion is not complete if C lies on a surface of 4-secants.

Let  $e = e(C) = \max \{ n; h^1(O_C(n)) \neq 0 \}$  be the index of speciality of the embedding of C. We prove the following result:

Theorem: Surfaces of degree  $n$  cut out a complete linear system on  $C$  for  $n \geq d^0(C) - e - 3$ .

Sketch of the proof: A non zero section of  $\omega_C(-e)$  induces an extension  $0 \rightarrow \mathcal{O}_{\mathbb{P}^3}(-e-4) \rightarrow \mathcal{M}(-e-4) \rightarrow \mathcal{O}_{\mathbb{P}^3} \rightarrow \mathcal{O}_C \rightarrow 0$ , where  $\mathcal{M}$  is a reflexive rank-2 sheaf on  $\mathbb{P}^3$ .

The result is first proved when the variety of  $[(e+2)/2]$ -jumping lines for  $\mathcal{M}$  does not contain any non-reduced complex of lines. If there is such a complex, one proves that  $C$  lies on a rational ruled surface  $S$  of  $(e+4)$ -secants to  $C$  having for singular locus the  $(d^0(S)-2)^{th}$  infinitesimal neighbourhood of a line and concludes by using the classification of curves on such a surface.

M. PÉTERNELL

A Lefschetz theorem on sections in projective manifolds.

The following theorem is proved: Let  $X$  be a submanifold of  $\mathbb{P}_n$  and  $Y$  be an algebraic set with not too large codimension in  $\mathbb{P}_n$ . Then certain homotopy groups of  $X$  and  $X \cap Y$  are isomorphic. This theorem is a generalization of the Lefschetz theorem on hyperplane sections.

The proof is based on Morse theory. One considers the behaviour of the homotopy type. The used Morse function is not differentiable but the minimum of differentiable functions.

C. PETERS

$\zeta(3)$  and a family of K3-surfaces.

In Apéry's irrationality proof for  $\zeta(3)$  the numbers  $a_n = \sum_{k=0}^n \binom{n}{k}^2 \binom{n+k}{k}^2$  play a central role. The generating function  $A(t) = \sum_{n \geq 0} a_n t^n$  satisfies the differential equation

$$LA(t) = 0, L = (t^4 - 34t^3 + t^2) \left( \frac{d^3}{dt^3} \right) + (6t^3 - 153t^2 + 3t) \left( \frac{d^2}{dt^2} \right) + (7t^2 - 12t + 1) \frac{d}{dt} + t - 5$$

There has been a strong suspicion that  $L$  comes from algebraic geometry. The main result, obtained in collaboration with F. Beukers is:

Theorem: If  $t \notin \{0, 1, (\sqrt{2} \pm 1)^4, \infty\}$  then  $A(t) = \int_Y \omega_t$ , where  $\omega_t$  is a holomorphic 2-form on a K3-surface  $X_t$ . For  $t$  generic the Picard number of  $X_t$  is 19, so that the transcendental lattice  $T_t$  is of rank 3. Since  $T_t$  is horizontal with respect to the Gauß-Manin connection one obtains a system of three differential equations of order 1, whose associated third order differential equation (the "Picard-Fuchs equation") is  $LA(t) = 0$  (in a suitable basis for  $T_t$ ).

M. REID

Classification of threefolds.

Assume  $k = \mathbb{C}$  and let  $X$  be a projective 3-fold with canonical singularities. It is hoped to classify  $X$  by the numerical properties of the canonical class:

Conjecture 1: There exists a model  $X'$  birational to  $X$  for which either  $K_{X'}$  is nef (that is,  $K_{X'} \cdot C \geq 0$  for all  $C \subset X'$ )

or there exists a fibre space  $\phi: X' \rightarrow Z$  with  $\dim Z = 0, 1$  or  $2$  s.t.  $-K_{X'}$  is relatively ample.

About  $1/2$  of this conjecture has been proved thanks to work of S. Mori and Y. Kawamata.

Conjecture 2: Assume  $K_X$  is nef; then for  $m \gg 0$ ,  $|mK_X|$  is free, defining  $\phi: X \rightarrow Z$  which contracts precisely the curves  $C \subset X$  with  $K_X \cdot C = 0$ .

Substantial cases of this conjecture are also known.

E. SERNESI

Counting moduli in families of projective curves.

Theorem: For all  $g, r, n$  satisfying the following inequalities  $n - r \leq g \leq \frac{r(n-r)-1}{r-1}$ ,  $n \geq r+1 \geq 4$ , there exists a smooth irreducible curve  $C \subset \mathbb{P}^r$  of genus  $g$  and degree  $n$  such that  $h^0(D) = r+1$ ,  $h^1(N_C) = 0$  and the natural map  $\mu_0: H^0(D) \otimes H^0(K-D) \rightarrow H^0(K)$  has maximal rank ( $D$  is a hyperplane section of  $C$ ,  $N_C$  is the normal bundle of  $C$  in  $\mathbb{P}^r$ ,  $K$  is a canonical divisor).

This theorem is proved inductively studying certain reducible curves in  $\mathbb{P}^r$  and then using deformation theory to smooth them. As a corollary we obtain for all  $g, r, n$  as above, generically smooth components of the Hilbert scheme of  $\mathbb{P}^r$  generically parametrizing smooth curves of degree  $n$  and genus  $g$  which correspond to a locus in the moduli space  $\mathcal{M}_g$  having dimension equal to  $\min(3g-3, 3g-3+\rho(g, r, n))$ , when  $\rho(g, r, n) = g - (r+1)(g-n+r)$  is the Brill-Noether number.

J. SHAH

Stability of surface singularities.

The notion of stability of a local ring is defined in such a way that if the local ring of a point on a projective scheme  $X$  is unstable, then  $X$  itself is unstable in the sense of the geometric invariant theory. The results obtained in dimension 2 are as follows: Let  $R$  be a two-dimensional, semistable, Cohen-Macaulay, local ring of multiplicity  $e$  and embedding dimension  $\rho$ . Then  $e \leq 6$  and  $e = \rho$  or  $\rho - 1$ . If  $e = \rho$ , then  $\text{Gr}_m R$  must be Cohen-Macaulay and  $\text{Proj Gr}_m R$  must be either an elliptic curve or a cycle of rational curves. If  $e = 2$  and  $\rho = 3$ , then  $R$  must be a rational, simple elliptic or cusp singularity or a non-normal limit of such singularities. Partial results for the case  $e \geq 3$  and  $\rho = e + 1$  suggest, in this case,  $R$  must be a quotient singularity, a non-normal limit of such singularities or a quotient of a simple elliptic singularity.

K. UENO

On compact analytic threefolds with non-trivial Albanese tori.

Let  $\alpha: M \rightarrow A(M)$  be the Albanese mapping of a compact complex manifold of dimension 3. The structure of the Albanese mapping is well-known, if  $M$  is bimeromorphic to a Kähler manifold. The main purpose of the talk is to show that if  $M$  is not bimeromorphic to a Kähler manifold, the structure of the Albanese mapping may be quite different from that of a Kähler threefold. Such examples are constructed by means of strange non-Kähler degenerations of surfaces.

J. WEHLER

Deformation of global complete intersections.

In 1958 Kodaira and Spencer proved in their fundamental work on deformations of complex analytic structures that every smooth hypersurface  $X$  of the projective space remains a smooth hypersurface under arbitrary small deformations if  $X$  is not a Riemann surface or a K3-surface. In this talk we sketch the proof of the following generalisation:

If  $X$  is a complete intersection, not necessarily smooth, in a compact homogeneous Kähler manifold  $Z$  with  $b_2(Z) = 1$ , then every small deformation of  $X$  is again a complete intersection in  $Z$  if  $\dim X \geq 2$  and  $X$  is not a K3-surface.

For  $X$  smooth this result has been obtained first by C. Borcea in 1983. The assumptions made on  $Z$  are fulfilled by all Grassmann manifolds, for example. The proof of the theorem uses the Kodaira-Spencer completeness criterium and the vanishing theorem of Bott for homogeneous vector bundles on  $Z$ .

S.-T. YAU

Equivalence relations among holomorphic functions and exotic differentiable structures on singular varieties.

There are three well-known equivalence relations among holomorphic functions. They are right equivalence, right-left equivalence and contact equivalence. Among these three equivalence relations, perhaps the contact equivalence is the most interesting one to complex geometers. We introduce two more equivalence relations:

Milnor equivalence and moduli equivalence. These last two equivalence relations are easy to understand. We study the relationship between these equivalence relations. As a result of our study, we find that a general singular variety has more than one embedding differentiable structure (with underlying embedding topological structure being fixed).

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