MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 39/1983

Topologie

4.9. bis 10.9.1983

Die Tagung wurde von den Herren L. Siebenmann (Orsay), Ch. Thomas (Cambridge) und F. Waldhausen (Bielefeld) organisiert.
Es wurden Fragen aus unterschiedlichen Bereichen der Topologie erörtert.

Vortragsauszüge:

M. BÖKSTEDT:

Generalized Hochschild Homology and Stable K-theory

Let R be a ring. Recall that Hochschild homology is a simplicial object

$$[n] \longmapsto \underbrace{R \otimes R \otimes \ldots\ldots\ldots \otimes R}_{n+1}$$

with boundary maps

$$d_0(r_0 \otimes \ldots \otimes r_n) = (r_0 r_1 \otimes \ldots \otimes r_n)$$
$$d_i(r_0 \otimes \ldots \otimes r_n) = (r_0 \otimes \ldots \otimes r_i r_{i+1} \otimes \ldots r_n)$$
$$d_n(r_0 \otimes \ldots \otimes r_n) = (r_n r_0 \otimes r_1 \ldots \otimes r_{n-1}).$$

Following Goodwillie we want to define a simplicial object $H(R)_{QS^0}$, which morally should be given by

$$[n] \longmapsto \underbrace{R \underset{QS^0}{\otimes} \qquad \underset{QS^0}{\otimes} R}_{n+1}$$

and the same boundary maps as above.

After suitable technical modification, this idea can be made precise. In particular we define $A \underset{QS^0}{\otimes} B$ as a smash product of spectra.

There is a commutative diagram

$$\begin{tikzcd} K(\mathbb{Z}) \arrow[rr, "a"] \arrow[dr, "b"'] & & K^S(\mathbb{Z}) \arrow[dl, "c"] \\ & H(\mathbb{Z})_Q & \end{tikzcd}$$

where $K^S(\mathbb{Z})$ is stable K-theory in the sense of Waldhausen.

Conjecture 1 (Goodwillie): c is a homotopy equivalence.

Theorem 2:

$$\pi_{2i}(H(\mathbb{Z})_{QS^o}) = 0 \qquad i \geq 1$$

$$\pi_{2i-1}(H(\mathbb{Z})_{QS^o}) = \mathbb{Z}/i \quad i \geq 1 .$$

This theorem was originally also conjectured by Goodwillie.

D. BURGHELEA:

Cyclic (Connes) homology versus algebraic K-theory of spaces (Waldhausen)

Let k be a field of characteristic zero. A k-chain differential graded algebra $(\mathbb{A},d)$ is a k-associative (with unit) graded algebra $\mathbb{A}$ equipped with $d : \mathbb{A}_* \longrightarrow \mathbb{A}_{*-1}$, k-linear and satisfying $d(ab) = da \cdot b + (-1)^{|a|} a\, db$ for $a \in \mathbb{A}$. $(\mathbb{A},d)$ is connected if $\mathbb{A}_o = k$ and $f : (\mathbb{A},d) \longrightarrow (\mathbb{B},d)$ is a quasiisomorphism if f induces an isomorphism for homology. Denote by $k-\mathrm{Ch\ DGA}$ the category of k-chain differential graded algebras and by $k-\mathrm{Ch\ DGA}^o$ the full subcategory of connected ones. It is well known that there exist two functors $\|\ \ \| : Q^{*)}-\mathrm{Ch\ DGA}^o \longrightarrow 1-\mathrm{Top}_*$ and $C_*(\Omega -) : 1-\mathrm{Top}_* \longrightarrow Q-\mathrm{Ch\ DGA}^o$ adjoint to each other which convert quasiisomorphisms to homotopy equivalences, resp. homotopy equivalences to quasiisomorphisms.

Theorem: 1) The Hochschild homology and Connes homology functors defined on k-algebras with values in k-graded vector spaces extend to the functors H_* and HC_* defined on $k-\mathrm{Ch\ DGA}$ with values in k-graded vector spaces so that $H_*(f)$ and $HC_*(f)$ are isomorphisms if f is a quasiisomorphism.

2) The natural transformations $H_* \xrightarrow{I} HC_*$, $HC_* \xrightarrow{J} HC_{*-2}$, $HC_{*-2} \xrightarrow{\delta} H_{*-1}$ (as in the case of associative algebras) define a long exact sequence

$$H_n(\mathbb{A},d) \to HC_n(\mathbb{A},d) \to HC_{n-2}(\mathbb{A},d) \to H_{n-1}(\mathbb{A},d).$$

3) If $(\mathbb{A},d)$ is a connected chain differential graded algebra over Q then $HC_*(\mathbb{A},d) = \tilde{K}_*(\|\mathbb{A},d\|) \otimes Q$ where $\tilde{K}_*(X)$ denotes the reduced Waldhausen algebraic K-theory of the space X.

*) Q denotes the field of rational numbers.

The proof extends to the differential graded case by arguments of Connes, Loday, Quillen and combine with results of Burghelea and Dwyer, Hsiang, Staffeldt on the structure of the rational algebraic K-theory of spaces. (In the case $\mathbb{A}$ is a tensor algebra the result follows from the work of Hsiang and Staffeldt, too).

S. DONALDSON:

Gauge Theory and Topology

We discuss some relations between topology of moduli spaces of Yang-Mills connections and infinite dimensional "function spaces" modelled on the well known use of Morse Theory to pass $\Omega M \longleftrightarrow \{\text{geodesics on } M\}$. The Yang-Mills equations are variational equations for the functional on G-connections over a manifold M:

$$\text{connection} \longmapsto \int_M |\text{curvature}|^2 .$$

Dimension 2. M = Surface, M_g. This is the case like that of geodesics. The consequences of Morse Theory apply and are used by Atiyah and Bott to calculate the cohomology of the manifolds:

$$\begin{aligned}\{\text{irreducible Yang-Mills G-connections}\} &\cong \{\text{Flat connections}\}\\ &\cong \text{Reps } (\pi_1(M_g) \to G)\end{aligned}$$

starting from the function space - essentially Maps (M_g, BG). The former spaces are interesting since they can be interpreted as moduli spaces of algebraic bundles over the algebraic curve associated to M_g.

Dimension 4. There are similar spaces $I_k(M,G)$ of instantons $(k \geq 0)$. If M = algebraic surface these are again essentially the moduli spaces of algebraic bundles. The Morse Theory does not work here but we have an "approximate version" for $M = S^4$:

Conjecture (Atiyah-Jones):

$$\lim_{k\to\infty} H_*(K_k(S^4),G) = H_*(\Omega^3(G)).$$

We sketch a fragment from the proof of a weaker form of this:

$$\lim_{\substack{\ell\to\infty\\ k\to\infty}} H_*(I_k(S^4),\ Sp(\ell)) = H_*(\Omega^3(Sp(\ell)))$$

based on Bott periodicity: $\Omega^3(Sp(\ell)) \simeq BO$ and the Lefshetz "hyperplane theorem".

Dimension 3. There is a similar variational problem for so-called "magnetic monopoles" on $\mathbb{R}^3$. The function spaces associated to these are $[\Omega^2(S^2)]_k$ where K = maps of degree k = "charge" of monopole. We have a geometric correspondence

$$(*) \quad \{\text{monopoles of charge } k\} \cong \{\text{rational maps } S^2 \to S^2 \text{ of degree } k\}.$$

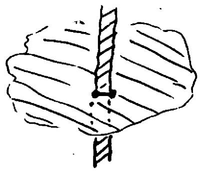
By a theorem of Segal on rational maps these spaces also converge to $\Omega^2(S^2)$ as $k \to \infty$.

Strictly, (*) has only been proved so far for the hyperbolic metric on $\mathbb{R}^3$.

Presumably there will eventually be a way to understand these, and other, results from the point of view of Morse Theory.

J. HOWIE:

Ribbon Discs and Whitehead's Problem



A ribbon knot in S^3 is one which bounds an immersed disc whose singularities consist only of double arcs of the kind in the picture. Pushing this disc slightly into D^4, we get an embedding $k : D^2 \to D^4$, called a ribbon disc.

Question 1. Is $D^4 \setminus k(D^2)$ aspherical?

This would have consequences for higher ribbon knot complements. Question 1 is a special case of Whitehead's problem:

Question 2 (J.H.C. Whitehead). Is every subcomplex of an aspherical 2-complex aspherical?

The decomposition of D^2 by double arcs has a dual tree T.

Theorem 1. $D^4 \setminus k(D^2)$ is aspherical whenever T has diameter ≤ 3.

Theorem 2. Suppose the answer to Question 2 is no. Then there is a counter-example of one of the forms

(A) $\quad X \subset Y,\ Y = X \cup e^2$ finite contractible, $\pi_2 X \neq 0$

(B) $\quad X \subset Y,\ X = X_0 \subset X_1, \subset \ldots,\ Y = \cup_i X_i,\ X_i$ finite, $\pi_2 X_i \neq 0,\ X_i \hookrightarrow X_{i+1}$ null homotopic.

Finally, a counterexample of type (A) would (at least modulo the Andrews-Curtis Conjecture) also be a counterexample to Question 1:

Theorem 3. Suppose $X \cup e^2 \underset{3}{\curlywedge} *$ for some finite 2-complex X. Then $X \underset{3}{\curlywedge}$ is a spine of $D^4 \setminus k(D^2)$ for some ribbon disc $k : D^2 \to D^4$.

J.R. HUBBUCK:

Homotopy Types of Compact Lie Groups

It is known (Scheerer 1967) that two compact, connected, simply-connected Lie groups whose underlying spaces are homotopically equivalent must be isomorphic as Lie groups. Developments in the theory of Finite H-spaces subsequent to 1967 enable a more elementary and conceptual proof to be given. A p-local simply connected Finite H-space decomposes uniquely as a cartesian product of indecomposable p-local H-spaces, as a space up to homotopy (Wilkerson 1974). With just two exceptions the 2-localizations of the simple, simply-connected Lie groups are indecomposable; the exceptions are

$$\mathrm{Spin}(7) \simeq_2 G_2 \times S^7 \quad \text{and} \quad \mathrm{Spin}(8) = \mathrm{Sin}(7) \times S^7.$$

The number of Spin(8) factors in a compact 1-connected Lie group G equals

$$\mathrm{Dim\ Cokernel}(P^1 : H^3(G, \mathbb{Z}/3\mathbb{Z}) \to H^7(G, \mathbb{Z}/3\mathbb{Z}))$$

and so Scheerer's result follows almost immediately.

HUỲNH MÙI:

The fundamental groups of the spaces of regular orbits of the affine Weyl groups

This talk is based on a paper with the same title written by Nguyen Viet Dung which will appear in Topology.

Let W be a Weyl group, or an affine Weyl group of rank n. Let W act naturally on $V = \mathbb{R}^n$ and Σ be the set of reflections on V contained in W. For each $\varepsilon \in \Sigma$ denote by H_s the hyperplane of fixed points of s in V. Let

$$Y_W = V_{\mathbb{C}} - D \quad \text{with} \quad D = \bigcup_{s\in\Sigma} (H_s + \sqrt{-1}\, H_s)$$

with $V_{\mathbb{C}} = V + \sqrt{-1}\, V$ $(\approx \mathbb{C}^n)$, the complexification of V. Then W acts properly on Y_W. We prove the following

Theorem: $\pi_1(Y_W/W)$ is the Artin group of the same type as W, i.e. if $W = \langle S;\ s^2 = 1,\ (st)^{m(s,t)} = 1, s,t \in S\rangle$, then

$$\pi_1(Y_W/W) \simeq \langle S;\ \underbrace{ststs\ldots.}_{m(s,t)\ \text{factos}} = \underbrace{tstst}_{m(s,t)\ \text{factos}}\ \ s,t \in S\rangle$$

if W is the Weyl group of type A. The theorem was proved by Fox and Neuwirth [1962]. For every (finite) Weyl group it has been proved by Brieskorn [1971]. For affine Weyl groups, Bannai [1974] proved it in the case of rank 2, and has stated the theorem as a conjecture.

The proof of the theorem is by a cellular decomposition method. For the finite Weyl groups it can be proved by a similar argument of cellular decomposition done by Neuwirth and Fox. However, for the case of affine Weyl groups, one needs to modify this argument. This leads to a notion of CW-semi-complex, a generalization of CW-complex. The above theorem is an application of a method for computing the fundamental groups of a manifold M with CW-semicomplex structure, and that of M/G when G is a group operating properly on M, and M is a G-equivariant CW-semicomplex.

S. JACKOWSKI:

Families of Subgroups and Completion

The completion conjecture stated for equivariant K-theory in Tagungsbericht (37/1975 p.6) is generalized to arbitrary equivariant cohomology theories. It says that for an equivariant cohomology theory h_G^* and a family $\mathcal{F}$ of subgroups of G the projection $X \times E\mathcal{F} \to X$ induces an isomorphism: $h_G^*(X)^\wedge \to h_G^*(X \times E\mathcal{F})$, where $E\mathcal{F}$ is the classifying space for the family $\mathcal{F}$ and "$\wedge$" denotes the completion with respect to the topology of $h_G^*(pt)$ determined by restriction to subgroups belonging to $\mathcal{F}$. The theorem is first shown for the family of all proper subgroups of the group G. Then for theories satisfying some "finiteness" and "orientability" assumptions (e.g. K_G, KO_G, KR_G) the theorem reducing the general case to the case of the family of all proper subgroups is proven.

Two applications were described. For a normal subgroup $H \leq G$ we obtain a spectral sequence $E_2^* = H^*(B(G/H) : R(H)) \Rightarrow R(G)^\wedge$; where $R(\cdot)$ is the complex representation ring and "$\wedge$" denotes the completion with respect to the $\{\ker(R(G) \to R(H)),\ H \leq G\}$-topology.

The completion theorem applied for K_G-theory and the family of all topologically cyclic subgroups provides the description of $\operatorname{Spec} K_G(X)$ - the prime ideal spectrum of the ring $K_G(X)$.

M. KRECK:

Smooth structures of closed 4-manifolds up to connected sums with $S^2 \times S^2$'s

Two smooth 4-manifolds are called stably diffeomorphic if there exists $r \in \mathbb{N}$ such that $\# r(S^2 \times S^2) \simeq N \# r(S^2 \times S^2)$.

Theorem 1: Let M and N be smooth homeomorphic 4-manifolds, M orientable or $(w_1(M) \neq 0$ and $w_2(\widetilde{M}) = 0)$. Then M and N are stably diffeomorphic.

(The oriented case was proved independently by B. Gompf). Otherwise either M and N are stably diffeomorphic or $M \# K$ and $M \# 11(S^2 \times S^2)$ are stably diffeomorphic, where K is the Kummer surface.

Proposition 2: Let M be a non-orientable 4-manifold. Then $M \# K$ and $M \# 11(S^2 \times S^2)$ are homeomorphic.

Theorem 1 and Proposition 2 imply that stably 4-manifold has at most two differentiable structures which are produced by the Kummer surface: $M \# 11(S^2 \times S^2)$ has a stable exotic structure iff $M \# 11(S^2 \times S^2)$ and $M \# K$ are not stably diffeomorphic. This problem can be translated into homotopy theory but the answer is in general open. The following examples show that there are many closed 4-manifolds with stable exotic structure.

$$\pi \underset{p}{\cong} \langle x_1,\dots,x_n \mid T_1,\dots,T_s \rangle \ , \ \alpha \in H^1(\pi,\mathbb{Z}_2) \ \text{non-trivial},$$

X the two complex corresponding to p. Let $M(p,\alpha)$ be the boundary of a regular neighbourhood of an embedding $X \hookrightarrow \mathbb{R}P^4 \times \mathbb{R}$ homotopic to α.

Theorem 3: $M(p,\alpha) \# K$ is not diffeomorphic to $M(p,\alpha) \# 11(S^2 \times S^2)$. Thus $M(p,\alpha) \# 11(S^2 \times S^2)$ has an exotic differentiable structure.

The proof of Proposition 2 follows from Freedman's work by a simple observation. The proof of Theorem 2 and 3 is based on my surgery theory classifying n-manifolds with prescribed k-skeleton. Theorem 2 follows from computations in stably homotopy and some geometric constructions. Theorem 3 is reduced to the computation of $\pi_4^S(M3H \wedge MSpin)$, $H \longrightarrow \mathbb{R}P^\infty$ the Hopf bundle and the determination of the Kummer surface in this bordism group.

D.D. LONG:

Bounding foliations

A measured foliation $(\mathcal{F},\mu)$ of a closed connected orientable surface F is said to bound in a compression body M for F if there is a sequence of embedded discs $\{D_n\} \subseteq M$, such that $\partial D_n \longrightarrow (\mathcal{F},\mu)$ where convergence is in Thurston's topology on the space of measured foliations.

Restricting to uniquely ergodic foliations with all half-leaves dense, it can be shown that:

Theorem: $(\mathcal{F},\mu)$ bounds iff there is a regular planar covering $p:\widetilde{F} \longrightarrow F$ in which a lift of $\mathcal{F}$ has a strongly homoclinic leaf.

Theorem: $(\mathcal{F},\mu)$ bounds in finitely many minimal compression bodies.

This has the corollary, that some power of a pseudo-Anosov map compresses iff its invariant foliations bound.

W. LÜCK:

Equivariant obstructions to finiteness as universal functorial additive invariants

We give a geometric definition of a so-called Wall group $Wa^G(Z)$ for an arbitrary G-complex Z, in which elements are represented by G-maps $f : X \to Z$ for a finitely dominated arbitrary G-complex X. For a finitely dominated G-complex X $[id_X] \in Wa^G(X)$ is its Wall obstruction $e^G(X)$. X is finite up to G-homotopy if and only if $e^G(X)$ vanishes. $(Wa^G(Z), e^G(Z))$ can be computed by an algebraically defined group $K_o^G(Z)$, which splits as a direct sum of $\tilde{K}_o(\mathbb{Z}[E])$ for certain groups E. (Wa^G, e^G) is the universal functorial additive invariant for the category $\mathcal{K}G$ of finitely dominated G-complexes, where a functorial additive invariant (A,a) consists of a functor $A : \mathcal{K}^G \to ABEL$ and an assignment $a : X \mapsto a(X) \in A(X)$, which satisfies the usual sum formula. This leads to the construction of a transfer for K_o and fibrations.

H.R. MORTON:

Fibred knots in S^4

The best-known examples of fibred knots of S^2 in S^4 are constructed by spinning a fibred knot in S^3 or by r-twist spinning any knot in S^3. By means of a satellite knot construction, described in the talk, many different examples can be constructed which are not of these two known types.

Further fibred knots can be constructed by plumbing the fibres of two fibred knots, using an analogue of Murasugi plumbing.

The satellite construction is made by choosing

(i) $K \simeq S^2 \subset S^4$ and

(ii) $A \cup C \simeq S^2 \cup S^1 \subset S^4$.

The complement h of a neighbourhood of C is homeomorphic to $D^2 \times S^2$; giving a homeomorphism $h : V \longrightarrow$ neighbourhood of K. The sphere $h(A) \subset S^2$ is defined to be a satellite of K.

If (i) K is fibred, (ii) A is fibred and (iii) C covers S^1 regularly n times in the fibre projection of S^4-A then $h(A)$ is fibred. The group $\pi_1(h(A)-S^4)$ alone is enough to distinguish many different such knots.

B. OLIVER:

The Whitehead transfer for oriented circle bundles can be non-zero

Munkholm and Pedersen have shown that for any oriented circle bundle $S^1 \to E \to B$, where B and E are finite CW-complexes, there is a well defined transfer homomorphism $T : Wh(\pi_1 B) \to Wh(\pi_1 E)$ which sends the torsion of any homotopy equivalence $f : B' \to B$ to the torsion of $\tilde{f} : f^*E \to E$. Furthermore, T depends only on the exact sequence $\mathbb{Z} = \pi_1(S^1) \to \pi_1(E) \to \pi_1(B) \to 1$; and it has a simple algebraic description.

Fix an odd prime p, set

$$\tilde{\pi} = \langle a_1, a_2, a_3, a_4, z \mid a_i^p = [a_i, [a_j, a_k]] = [z, a_i] = 1,\ z^p = [a_1, a_2][a_3, a_4] \rangle,$$

and let $\pi = \tilde{\pi}/\langle z \rangle$. Then $|\tilde{\pi}| = p^{11}$, $|\pi| = p^9$, and the transfer map in Whitehead groups for the sequence $\mathbb{Z} \to \tilde{\pi} \to \pi \to 1$ has image of order p. That it is so difficult to construct examples of central extensions $\mathbb{Z} \to \tilde{\pi} \to \pi \to 1$ for which $T \neq 0$ is due partially to the fact that (when $|\tilde{\pi}| < \infty$) $\mathrm{Im}(T) \subseteq Cl_1(\mathbb{Z}\pi)$; which is the part of $Wh(\tilde{\pi})$ where detection of elements is least understood.

A. PAPADOPOULOS:

A Composition Theorem for Pseudo-Anosov Classes of Surface Diffeomorphisms

Let S be a closed orientable surface of genus ≥ 2. We prove the following theorem:

Theorem: If f is a pseudo-Anosov homeomorphism of S, with unstable and stable measured foliations (F^u, μ^u) and (F^s, μ^s) respectively, and if φ is any diffeomorphism of S such that $[\varphi]([F^u, \mu^u]) \neq [F^s, \mu^s]$, then there exists an integer k such that for all $n \geq k$, $f^n \circ \varphi$ is isotopic to a pseudo-Anosov map.

As a corollary, we prove the following (using Thurston's construction of pseudo-Anosov maps inducing the identity on $H_1(S, \mathbb{Z})$).

Corollary: For every symplectic automorphism of $H_1(S, \mathbb{Z})$ there exists a pseudo-Anosov map which induces it.

J. PRZYTYCKI:

Incompressible surfaces in complements of closed 3-braids

We analyse incompressible surfaces in the complement of a hyperbolic 3-strand braid. In particular we solve the problem whether the complement of a 3-braid contains a closed, incompressible, Γ_1-injective, non-peripheral surface.

Example: Let be a hyperbolic 3-braid.

$\gamma \sim \Delta^{2n}\bar{\sigma}_1^{p_1}\sigma_2^{q_1}\ldots\bar{\sigma}_1^{p_k}\sigma_2^{q_k}$ then the associated closed 3-braid $\bar{\gamma}$ has a unique (up to isotopy) incompressible surface. The genus of this surface is equal to $\frac{1}{2}(|\sum_{i=1}^{k} q_i + 3n| + |\sum_{i=1}^{k} p_i - 3n|) - 1$.

We assume that $\bar{\gamma}$ is a knot.

Problem: Find all 3-braids γ such that the chosen $\bar{\gamma}$ is a 2-bridge link. We have 5 families of "candidates" and we conjecture that that is all.

A. RANICKI:

Infinite cyclic coverings

The talk considered CW complexes X with a finitely dominated infinite cyclic cover $\bar{X}$, a very popular topic in the 1960's. The mapping torus trick of M. Mather shows that X is homotopy equivalent to a finite CW complex, indeed has a preferred finite homotopy type. If X is already a finite CW complex the difference of the two finite homotopy types is an invariant $\phi(X) \in Wh(\pi_1(X))$, which was first defined by Siebenmann. If X is a compact manifold $\phi(X)$ is the Farrell-Siebenmann obstruction to fibering X over S^1. The Bass-Heller-Swan-Farrell-Hsiang map $B: Wh(\pi_1(X)) \longrightarrow \tilde{K}_0(\mathbb{Z}[\pi_1(\bar{X})])$ sends $\phi(X)$ to the Novikov-Siebenmann splitting obstruction $[\bar{X}^+]$, which is the Wall finiteness ostruction of the positive end $\bar{X}^+$ of $\bar{X}$. The motivation for the talk came from the problem of the geometric realization of the duality involution

$$*: \tilde{K}_0(\mathbb{Z}[\pi]) \to \tilde{K}_0(\mathbb{Z}[\pi]) ; [P] \to [P^*] = [\mathrm{Hom}_{\mathbb{Z}[\pi]}(P, \mathbb{Z}[\pi])].$$

For a finitely presented group π every element $x \in \tilde{K}_0(\mathbb{Z}[\pi])$ can be realized as $x = [\bar{X}^+]$ for a compact n-manifold X with finitely dominated infinite cyclic cover $\bar{X}$, $\pi_1(\bar{X}) = \pi$, and $x^* = (-)^{n-1}[\bar{X}^-]$.In fact, X can be taken to be the boundary of a regular neighbourhood in S^{n+1} (n large) of a finite CW complex $Y \subset S^{n+1}$ in the preferred finite homotopy type of $Z \times S^1$, with Z any finitely dominated CW complex such that $\pi_1(Z) = \pi_1[Z] = x$.

D. REPOVŠ:

Resolving acyclic images of 3-manifolds

It is well-known that a finite dimensional quotient M/G of an n-manifold M by a cell-like decomposition G must be a generalized n-manifold. The converse is (in general) false, e.g. shrinking out a spine of a PL n-manifold yields even an n-manifold. However, in this example G is almost cell-like. Next, Bing's "figure eight" decomposition of S^3 shows there should be some restriction on dim G. We can prove:

Theorem 1: Suppose G is a 0-dimensional usc decomposition of a 3-manifold M such that M/G is a generalized 3-manifold. Then G is almost cell-like, i.e. $C_f = \{x \in M/G \mid f^{-1}(x)$ is not cell-like$\}$ is a locally finite set, where $f : M \to M/G$ is the quotient map.

This result follows from a much stronger statement concerning almost acyclic maps on 3-manifolds (and generalizes a result of J.L. Bryant and R.C. Lacher, Math. Proc. Camb. Phil. Soc. 88 (1980), 311-319):

Theorem 2: Suppose $f : M^3 \to X$ is a closed, monotone, almost 1-acyclic $(\mathbb{Z}_2)$ map (i.e. f is 1-acyclic $(\mathbb{Z}_2)$ over the complement of a possibly non-empty 0-dimensional set in X) from a 3-manifold M onto a locally simply connected $\mathbb{Z}_2$-homology 3-manifold X. Then C_f is locally finite. Moreover, (M,f) can be repaired to become a (conservative) resolution of X. Thus X is a generalized 3-manifold and all of its singularities are "hard", i.e. not of the Wilder type.

R.J. Daverman and J.J. Walsh have examples which show Theorem 2 to be, to a certain extent, a best possible statement. However, from another point of view, one may want to ask if M^3 could be replaced by a generalized 3-manifold.

Question 1: Is a locally contractible proper $\mathbb{Z}_2$-acyclic image of a generalized 3-manifold always a generalized 3-manifold?

Question 2: Does there exist an ENR which is a $\mathbb{Z}_2$-homology 3-manifold but not a $\mathbb{Z}$-homology 3-manifold? (Note that there are examples of ENR $\mathbb{Z}_p$-homology 3-manifolds, p an odd prime, which fail to be $\mathbb{Z}$-homology 3-manifolds, e.g. ΣP^2, the suspension of the projective plane.)

This research was done jointly with R.C. Lacher.

R. RUBINSZTEIN:

Transfer and Equivariantly Parallelizable Manifolds

If G is a compact Lie group and M is a compact G-manifold, one has the transfer map $\tau(M) : BG_+ \to \Omega^\infty S^\infty((EG \times_G M)_+)$ associated to the fibration $EG \times_G M \to BG$. We prove

Proposition: If M is G-parallelizable and $m = \dim M$, then $\tau(M)$ is contractible over the $(m-1)^{st}$ skeleton of BG_+.

We also study under what conditions a "G-strongly stably parallelizable" manifold M is actually G-parallelizable. For such a "G-strongly stably parallelizable" manifold M we introduce an abelian group $C(M)$ and an obstruction element $\gamma(M) \in C(M)$.

Theorem: If G is a finite group and M is a G-strongly stably parallelizable manifold such that

(i) $\dim M^H$ is even for every subgroup $H \subset G$,

(ii) if $H \subset G$ is such that $\dim M^H > 0$, then $\chi(M^H) = 0$ and $N_G(H)/H$ acts on M^H preserving orientation,

(iii) $\gamma(M) = 0$ in $C(M)$,

then M is G-parallelizable.

Example: Let $G_k = \mathbb{Z}_2 \times \ldots \times \mathbb{Z}_2$ (k factors) and let V_k be the real regular representation of G_k. Denote by $S(V_k)$ the unit sphere in V_k. The product $S(V_k) \times S(V_k)$ is G_k-specially stably parallelizable (since $S(V_k)$ is such). We show that for $k > 3$ the obstruction element $\gamma(S(V_k) \times S(V_k)) \neq 0$ and, consequently, $S(V_k) \times S(V_k)$ is not G_k-parallelizable. For $k = 1,2,3$ both $S(V_k)$ and $S(V_k) \times S(V_k)$ are G_k-parallelizable.

R. SCHWÄNZL:

Reidemeister and Picard invariant of abelian homotopy representations

There is a question of Rothenberg in what way the Picard invariant of tom Dieck/Petrie is related to his (oriented) Reidemeister torsion invariant.

Theorem: G abelian, X,Y even homotopy representations of the same dimension function, then $\mathrm{Pic}(X,Y) = \bar{\tau}X - \bar{\tau}Y$, where $\bar{\tau}$ is the oriented torsion mod "simple equivalence".

As a consequence one obtains of course that torsion (mod simple equivalence) classifies up to (stable) G-homotopy-equivalence. For the proof one redefines torsion using special projective modules over the orbit ring and then derives a Mayer-Vietoris sequence to relate the groups the invariants live in. To prove the equality one uses Swan modifications. The K-theory invariants are defined strictly, opposite to Rothenberg, LNM 763, 573-590.

P. SCOTT:

Homotopic homeomorphisms of Seifert fibre spaces

In 1968, Waldhausen proved that homotopic homeomorphisms of a closed, Haken manifold must be isotopic. It seems reasonable to conjecture that the same result holds for any closed, P^2-irreducible 3-manifold with infinite fundamental group. In this lecture, we considered the special case of Seifert fibre spaces. Most Seifert fibre spaces which are closed, P^2-irreducible and have infinite fundamental group are Haken, so that Waldhausen's result applies. If M is a non-Haken Seifert fibre space of the above type, then there is an exact sequence

$$1 \longrightarrow \mathbb{Z} \longrightarrow \pi_1(M) \longrightarrow \Delta(p,q,r) \longrightarrow 1,$$

where $\Delta(p,q,r)$ is a hyperbolic or Euclidean triangle group. We will say that M is <u>of type $\Delta(p,q,r)$.</u> For each such triangle group, there are infinitely many distinct Seifert fibre spaces.

<u>Theorem:</u> If $p,q \geq 4$ and M is a Seifert fibre space of type $\Delta(p,q,r)$, then homotopic homeomorphisms of M are isotopic.

The proof involves consideration of a singular, incompressible torus X in M, and the simplification of the intersection of X with its image under a homeomorphism of M. The condition that p and q exceed 4 is required for purely technical reasons and it seems likely that it can be removed.

V. SNAITH:

<u>Algebraic K-theory and topological K-theory</u>

Let ℓ be a prime, X a scheme over a ring, A, containing $1/\ell$ and ξ_{ℓ^∞}, all ℓ-ary roots of unity. Choose $\beta \in K_2(A; \mathbb{Z}_{\ell^\nu})$ whose Bockstein is $\xi_{\ell^\nu} \in K_1(A)$. If $K_*(X; \mathbb{Z}_{\ell^\nu})$ is the mod ℓ^ν algebraic K-theory of X, define $\mathcal{K}_*(X; \mathbb{Z}_{\ell^\nu}) = K_*(X; \mathbb{Z}_{\ell^\nu})\,[1/\beta]$. We have natural localization maps

$$\rho : K_i(X; \mathbb{Z}_{\ell^\nu}) \longrightarrow \mathcal{K}_i(X; \mathbb{Z}_{\ell^\nu})$$

and an ℓ-adic analogue

$$\hat{\rho} : K_i(X) \longrightarrow \varprojlim_{\nu} \mathcal{K}_i(X; \mathbb{Z}_{\ell^\nu}).$$

The study of $\operatorname{im}(\rho)$ and $\operatorname{im}(\hat{\rho})$ is central to the Lichtenbaum-Quillen conjecture and to the problem of Lefschetz concerning the determination of the subgroup of $H^*(X; Q)$ generated by Chern classes of analytic vector bundles on a smooth, projective variety over $\mathbb{C}$. In this survey, I described a number of results concerning the kernel of ρ and $\hat{\rho}$ and circumstances when ρ is onto.

R. VOGT:

Homotopy ring spaces

There are two important types of homotopy ring spaces: E_∞ ring spaces which have coherently h-unital, h-associative and h-commutative multiplication and addition satisfying distributivity up to homotopy (h stands for "homotopy"), and A_∞ ring spaces having the same structure with h-commutativity of the multiplication dropped.

1978 Waldhausen made the proposal to take an A_∞ ring space, give its n-square matrices $M_n X$ and A_∞ ring structure, take the h-invertible ones, $Gl_n R$, stabilize and apply the classifying space construction B and define $KR = \mathbf{Z} \times (BGl_\infty R)^+$.

For $R = Q((\Omega X)_+)$, KR contains information about the pseudo-isotopy space. This proposal was followed up in parts only. Generalizing the existing definitions to more flexible ones we can show (joint work with R. Schwänzl):

(1) A_∞ and E_∞ ring structures are h-invariants; there is a satisfactory theory of h-homomorphisms.

(2) Waldhausen's program can be realized completely and KR is an E_∞ ring space if R is.

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