MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 41/1983

Kategorientheorie

18.9. bis 25.9.1983

Die Tagung fand unter der Leitung von Herrn Gray (Urbana), Herrn Herrlich (Bremen) und Herrn Pumplün (Hagen) statt und hatte durch die vielen ausländischen Gäste internationalen Charakter. Es wurde u.a. über sehr interessante Entwicklungen auf dem Gebiet der kategoriellen Topologie, der Theorie der Topoi und der Anwendungen der Kategorientheorie auf die Analysis berichtet. Während und nach den Vorträgen gab es einen lebhaften Gedankenaustausch, der für alle Teilnehmer, besonders aber für die jüngeren Kollegen, sehr anregend gewesen sein dürfte.

Vortragsauszüge

J. Adámek: Classification of Structures Using Set Functors

By a result of Kučera and Pultr, each strongly fibre-small concrete category has a full, concrete embedding into some $S(F)$. Here F is a set functor, and $S(F)$ has objects (A,a) where $a \subseteq FA$ and morphisms $f : (A,a) \to (B,b)$ are maps such that $Ff(a) \subseteq b$. Since all usual categories are strongly fibre-small, set functors can be used to "classify" concrete categories. For example, the definition of structure in Bourbaki uses $S(F)$ where F is a composition of P (the power-set functor), hom(M,-) and K_M (hom-functor and its left adjoint); all these categories have *universal final structures* (final structures preserved by pullbacks), and hence, they are quasitopoi in the sense of Penon.

Theorem. $S(F)$ is a quasitopos iff F covers pullbacks.

A category has a full concrete embedding into a quasitopos $S(F)$ iff it is strictly fibre-small in the sense of Koubek and the author. Moreover, these quasitopoi have a simple characterization. Let us call a concrete category *atomic* if in the fibre of any set, the join of atoms is the largest element.

Theorem. An atomic category has universal final structures iff it is concretely isomorphic to $S(F)$ for some set functor F covering pullbacks.

B. Banaschewski: Abelian Groups in a Localic Topos: Indecomposable Injectives

A natural question concerning the category $AbShL$ of abelian groups in the topos ShL of sheaves on a locale L is: How are (1) special objects, or (2) properties, of $AbShL$ determined by L? Regarding indecomposable injectives, any point $\xi : L \to 2$ of L and any indecomposable injective P in Ab, i.e. P is $Z(p^\infty)$ for some prime p or $\mathbb{Q}$, provide the indecomposable $\xi_* P$ in $AbShL$ given by

$$(\xi_* P)U = P \text{ for } \xi(U) = 1 \text{ , } 0 \text{ for } \xi(U) = 0 \text{ .}$$

PROPOSITION 1. *The indecomposable injectives of* $AbShL$ *are exactly these* $\xi_* P$.

As to the plentitude of indecomposable injectives in $AbShL$ we have:

PROPOSITION 2. *The indecomposable injectives cogenerate* $AbShL$ *iff* L *is spatial.*

PROPOSITION 3. *Every injective in* $AbShL$ *is a direct sum of indecomposable injectives iff* L *is covered by the* $U \in L$ *for which the Ascending Chain Condition holds below* U *.*
The subgroups of indecomposable injectives are the *uniform* groups.

PROPOSITION 4. *Every* $A \in AbShL$ *is a sum of uniform groups iff* L *is Boolean atomic.*
This work was done jointly with Kiran Bhutani.

J. Benabou: Cartesian matrices

Let $\langle \delta, \delta' \rangle : \mathbb{X} \to \mathbb{A} \times \mathbb{B}$ be a functor. For each object $B \in \mathbb{B}$ (resp. $A \in \mathbb{A}$) we denote by: $\mathbb{X}(-,B)$ (resp. $\mathbb{X}(A,-)$) the fiber of δ' (resp. δ) over B (resp. A), by $\delta_B : \mathbb{X}(-,B) \to \mathbb{A}$ (resp. $\delta'_A : \mathbb{X}(A,-) \to \mathbb{B}$) the restriction of δ (resp. δ') and $j_B : \mathbb{X}(-,B) \hookrightarrow \mathbb{X}$ (resp. $j_A : \mathbb{X}(A,-) \hookrightarrow \mathbb{X}$) the inclusion considered as a functor over $\mathbb{A}$ (resp. $\mathbb{B}$).
We say that $\mathbb{X}$ (with $\langle \delta, \delta' \rangle$) is a (cartesian) matrix of type $(\mathbb{A}, \mathbb{B})$ if it satisfies:
(i) The functors δ_B (resp. δ'_A) are fibrations (resp. co-fibrations) and the functors j_B (resp. j'_B) are cartesian over $\mathbb{A}$ (resp. cocartesian over $\mathbb{B}$).
(ii) The maps of $\mathbb{X}$ which are cocartesian over $\mathbb{B}$ are stable under inverse images along the maps which are cartesian over $\mathbb{A}$.
We denote by $\mathfrak{Cart}(\mathbb{X}, \mathbb{X}')$ the 2-category of maps between matrices $\mathbb{X}, \mathbb{X}'$ of type $(\mathbb{A}, \mathbb{B})$.

THEOREM: Let $\mathbb{X}, \mathbb{Y}, \mathbb{Z}$ be matrices of types $(\mathbb{A}, \mathbb{B})$, $(\mathbb{B}, \mathbb{C})$ and $(\mathbb{A}, \mathbb{C})$. We define matrices $\mathbb{X} \otimes \mathbb{Y}$, $\mathfrak{Cart}_{\mathbb{A}}(\mathbb{X}, \mathbb{Z})$, $\mathfrak{Cocart}_{\mathbb{B}}(\mathbb{Y}, \mathbb{Z})$ of types $(\mathbb{A}, \mathbb{C})$, $(\mathbb{B}, \mathbb{C})$ and $(\mathbb{A}, \mathbb{B})$ and prove:

$$\mathfrak{Cart}(\mathbb{X}, \mathfrak{Cocart}_{\mathbb{B}}(\mathbb{Y}, \mathbb{Z})) \approx \mathfrak{Cart}(\mathbb{X} \otimes \mathbb{Y}, \mathbb{Z}) \approx \mathfrak{Cart}(\mathbb{Y}, \mathfrak{Cart}_{\mathbb{A}}(\mathbb{X}, \mathbb{Z})).$$

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THEOREM: If $F : \mathbb{F}ib(\mathbb{B}) \to \mathbb{F}ib(\mathbb{A})$ is a 2-functor which has a right 2-ajoint there exists a matrix $\mathbb{X}$ of type $(\mathbb{A}, \mathbb{B})$ such that F is 2-equivalent to $\mathbb{Y} \mapsto \mathbb{X} \otimes \mathbb{Y}$.

F. Borceux: Localic Distributors and the Functoriality of the Spectra Constructions

A spectrum construction for non-commutative rings associates to a ring R some locale Spec(R) . One would expect some morphism of locales Spec(R) $\to$ Spec(S) associated to a ring homomorphism $f : R \to S$. Unfortunately this is generally not the case. Nevertheless there must be some functoriality property in the problem and I intend to exhibit it.
A morphism of locales is a functor Spec(R) $\to$ Spec(S) which preserves finite $\wedge$ and arbitrary $\vee$. If you replace "functor" by "distributor" you obtain the 2-category of locales and localic distributors. The localic distributors between two locales constitute a new locale and the composition on the left or on the right with a localic distributor has a right adjoint. Any localic distributor can be represented by the representable ones. A spectrum construction for rings produces a lax-functor from the category of rings to the 2-category of localic distributors. Many other examples can be produced.

R. Brown: A survey of groupoid methods in mathematics

A *groupoid* is a small category in which every morphism is invertible. Groupoids arise and are used in homotopy theory (the fundamental groupoid, multiple groupoid), category theory (clearly), ring theory (maximal orders), group theory (proofs of subgroup theorems), differential topology (holonomy groupoid, groupoids of germs of homeomorphisms), differential geometry (the theory of connections), ergodic theory (Mackey's Theory of virtual groups), analysis (the C^*-algebra of a measured groupoid), algebraic K-theory (obstruction to excision), algebraic geometry (Picard groupoids).
The aim of the talk is to give concrete examples of groupoids, to attempt to draw together those separate applications and to indicate some overall moral. The conclusion is that the extension

of viewpoint from groups to groupoids, and in particular the applications of *structured groupoids*, is likely to prove increasingly important.

G.C.L. Brümmer: Completion-true Functorial Uniformities

Let Unif_o denote the category of separated uniform spaces, Tych the category of completely regular Hausdorff spaces, $T : \mathrm{Unif}_o \to \mathrm{Tych}$ the forgetfull functor, and $K : \mathrm{Unif}_o \to \mathrm{Unif}_o$ the completion functor. A section F of T is called *completion-true* if $KF \cong FTKF$. An equivalent condition is that F is spanned by a class of complete spaces. If F is completion-true, then TKF is an epireflection in Tych onto a subcategory lying between the compact and the topologically complete spaces. Every epireflective subcategory of Tych can be represented in this way, but in general not uniquely. We discuss results and problems related to the search for examples of sections which are not completion-true.

Y. Diers: An axiomatic description of categories of locally boolean sheaves of simple algebras

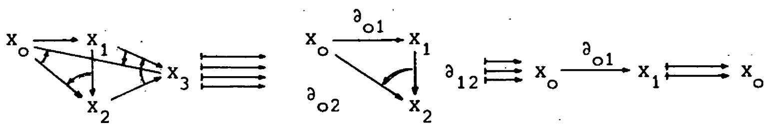
Categories of sheaves of simple algebras on locally boolean spaces can be described axiomatically by universal properties. This description induces the localisation process which associates to any object its maximal spectrum and the family of simple quotient objects, as well as the globalisation process which rebuilds an object from the family of its localisations, by taking the object of continuous sections with boolean support. It provides a unique proof of a lot of representation theorems of algebras by sheaves of simple algebras and allows us to use categorical constructions as fraction categories, monadic categories, comma categories, to get new representation theorems. Unitary objects can be described axiomatically and are shown to be exactly those objects whose maximal spectrum is boolean, so that categories of unitary objects are exactly categories of simple algebras on boolean spaces, and can be described axiomatically. The precise link between those two kinds of categories is stated by an equivalence of 2-categories and we are able to solve, for regular rings and algebras at least, the eternal problem: Good rings must have a unit or not?

J. Duskin: <u>2-Dimensional-Non-Abelian-Cocycles</u>

The success of the use of simplicial techniques using groupoids (and even categories) as non-abelian coefficients in 1-dimensional cohomology via the definition (for example in a topos)

$$H^1(\mathbb{E};G^{(1)}) \underset{\overline{\overline{\text{def}}}}{} \varinjlim_{C^o\in(\mathrm{Cov}(\mathbb{E}))} \mathrm{Hom}_{\mathrm{Simple}_{\mathbb{E}}}[C^o,\mathrm{Ner}(G^{(1)})]$$

(i.e., homotopy classes of simplicial maps from coverings into the nerve of the category $G^{(1)}$) and its interpretation as isomorphism classes of torsors under $G^{(1)}$ (i.e. as locally representable internal functors, $\mathbb{E}^{G^{op}}$) has led to an investigation of the use of 2 (and higher dimensional categories) as coefficients in the definition of H^2 (and higher dimensional versions) via the same definition using the nerve of a 2-category $G^{(2)}$. This simplicial object has, the form (in the dual)



where the tetrahedra is required to satisfy the equation $\alpha_2 \circ (\alpha_o * f_{o1}) = \alpha_1 \circ (f_{23} * \alpha_3)$, with similar (but more complicated) conditions for higher dimensions, (now calculated through dim 6). Using various choices of $G^{(2)}$ and X_o, the resulting 2-cocycles, $\mathbb{Z}^2(X_o,G^{(2)}) \underset{\overline{\overline{\text{Def}}}}{} \mathrm{Simpl}(X_o,\mathrm{Ner}(G^{(2)}))$, include Schreier factor systems for group; all abelian 2-cocycles; pseudo-functors; the Grothendieck-Giraud H^2 as well as Bauer's natural linear and quadratic systems of groups for his theory of extensions of categories.

A. Frölicher: <u>Differentiable structures</u>

Any set M of maps from $\mathbb{R}$ to $\mathbb{R}$ (or from A to B, where A and B are arbitrary fixed sets) generates a concrete topological category K_M. In the case $M = C^\infty(\mathbb{R},\mathbb{R})$, discussed in the proceedings of the Gummersbach Conference under the title "smooth structures", K_M is cartesian closed and contains not only the category of classical smooth manifolds, but also smooth Banach- and Frêchet-manifolds. We now present a similar approach for the finitely differentiable case. $M = C^k(\mathbb{R}, \mathbb{R})$ turned out to be not useful; but replacing the continuity condition on the k-th derivative by the condition "locally Lipschitz" we obtain the very useful set $M = L^k(\mathbb{R}, \mathbb{R})$. The study of the associated category K_M involves hard analysis; but it yields not only a very elegant and general differentiation theory, but also results which are new even for maps between Banach spaces. Many of the results were obtained in collaboration with Bernard Gisin and Andreas Kriegl.

J. W. Gray: <u>Intrinsic Linear Programming</u>

A linear program is an extremal problem in which cx is maximized subject to the condition $Ax \leq b$. Describe this as a diagram $\mathbb{P}: R \xleftarrow{c} V_1 \xrightarrow{A} V_2 \xleftarrow{b} R$ where V_1 and V_2 are preordered (topological) vector spaces and A, b and c are (continuous) linear transformations. Homomorphisms of programs are suitable translations of diagrams. If $\mathbf{1} : R \xleftarrow{1} R \xrightarrow{1} R \xleftarrow{1} R$ then suitable homomorphisms $\bar{x} : \mathbf{1} \to \mathbb{P}$ are called *points* of $\mathbb{P}$ while suitable homomorphisms $\bar{\omega} : \mathbb{P} \to \mathbf{1}^*$ are called *copoints*. (Here, for any $\mathbb{P}$, the dual is given by $\mathbb{P}^* : R \xleftarrow{-b^*} V_2^* \xrightarrow{-A^*} V_1^* \xleftarrow{-c^*} R$.) Points have values $\mu(\bar{x})$ with $\mu(\mathbb{P}) = \max \mu(\bar{x})$ while copoints have covalues $\nu(\bar{\omega})$ with $\nu(\mathbb{P}) = \max \nu(\bar{\omega})$. It is always the case that $\mu(\mathbb{P}) + \nu(\mathbb{P}) \leq 0$, and $\mathbb{P}$ is said to have a duality gap if this is strictly less than 0 .

THEOREM: Suppose $\mathbb{P}$ is regular or dual regular.

(i) If $\mathbb{P}$ is reflexive then $\mathbb{P}$ does not have a duality gap.

(ii) If $\mathbb{P}$ is not reflexive then the form of the possible duality gap can be precisely described.

(iii) $\mathbb{P}^*$ does not have a duality gap.

C. Greither: Cancellation for Commutative Algebras

Rings and algebras all commutative.
PROBLEM: Given an R-algebra A s.t. $A[T_1,\ldots,T_m] \cong R[T_1,\ldots,T_{n+m}]$, when does it follow that $A \cong R[T_1,\ldots,T_n]$? (A is called "stably free".)
Two generalizations of stable freeness are introduced. If $n=1$, the rings R with a positive answer can be described (Asanuma). If $n = 2$ and $R = \mathbb{C}$, we have the famous "cancellation theorem". A generalization of this problem (which was recently solved in the affirmative) to arbitrary base fields is discussed. We also treat the generalized cases of invertible and of projective R-algebras.
We overview a K-theory for invertible algebras, due to Connell.

R. Harting: Lattice aspects of radical ideals and T_1-locales

As an approach to the question whether the Boolean Ultrafilter Theorem is strong enough to ensure the existence of prime ideals in arbitrary rings with unit, some lattices of radical ideals are examined. It is shown that the lattices LIdA, JIdA and BIdA of Levitzki, Jacobson and Brown-McCoy radical ideals, respectively, are compact locales, and that BIdA is even a compact T_1 (= subfit (Isbell) = conjunctive (Simmons)) locale. On the other hand, classically (i.e. using the Axiom of Choice), BIdA is the topology of the maximal spectrum Max A, and each compact T_1 locale is actually a BIdA, even for commutative A. Also it is proved that compact T_1 locales are spatial iff the Almost Maximal Ideal Theorem (Johnstone) holds. Thus, at least the latter guarantees the existence of prime ideals in arbitrary rings.

H. Herrlich: Universal Topology

Topological categories are described as full subcategories of (co-)functor-structured categories, definable by means of topological (co-)axioms. Convenience properties for topological categories are introduced providing a natural generalization of such seemingly different concepts as locales and cartesian closed topological categories.

M. Höppner: Deltas of Dimension 1

A delta A, i.e. a small skeletal category with trivial endomorphisms only, is called of (weak) dimension 1 if

(weak) global dim (A,M) = 1 + (weak) global dim M

for every module category M. Based on homological and module-theoretic arguments we give the following characterizations:

1) A delta A is of weak dimension 1 iff A is rigid.

2) A delta A is of dimension 1 iff A is rigid and locally artinian.

(A category C is called rigid if for every commutative square

$$\begin{array}{ccc} A & \rightarrow & B \\ \downarrow & & \downarrow \\ C & \rightarrow & D \end{array}$$

there is a morphism $B \rightarrow C$ or $C \rightarrow B$ in C s.t. both triangles commute.)

The corresponding results for partially ordered sets are already known and due to Brune and Mitchell.

R.-E. Hoffmann: Some Categorical Aspects of Continuous Lattice Theory

Three topics from contiunous lattice theory are discussed to which the ideas of category theory have made a significant contribution:

1) Some monadic functors.

2) Projective sober spaces, projectiveness w.r. to a right adjoint.

3) A topological functor.

ad 1) Locally quasicompact sober spaces such that finite intersections of quasicompact saturated subsets are quasicompact and continuous maps which are "perfect" (inverse images of sat. quasic. subsets are quasic.) have a monadic inclusion into TOP_o (H. Simmons, Top. Appl. 13, 201-23 (1982)) (Saturated = intersection of its neighborhoods). We replace in this result "quasic." by "supercompact" (or "monogenic": every open cover must contain the total set): Observe that the resulting spaces are the continuous lattices (in disguised form), and, thus, the "transferred" result is due to A. Day (Canad. J. 27, 50-9 (1975)). A more recent characterization of the injective hull of a locally supercompact sober space = (continuous poset, Scott topology)

is transferred into an existence theorem for a certain extension of locally quasicompact spaces, the "Fell compactification" (Arbeitspapiere 27, Univ. Bremen 1982).
ad 2) M. Sobral's (Quaest. Math. 1984?) solution to a question raised in Springer LNM 871 (1981), p. 135 is delineated.
ad 3) The topologicity of a functor (2.7 in Manuscripta Math. 27, 401-32 (1979)) is explained.

M. Hušek: Topolocigal motivation for category theory

Suppose that V is a functor on a category $\mathcal{A}$ into a category $\mathcal{B}$, $\mathcal{F}$ is a class of morphisms in $\mathcal{B}$ and $\mathcal{M}$ a class of objects of $\mathcal{B}$ such that for any $B_1 \xrightarrow{f} B_2$ from $\mathcal{F}$ and any $M \in \mathcal{M}$, the map $\mathcal{B}(B_2,M) \xrightarrow{f_M} \mathcal{B}(B_1,M)$ is onto. There is a procedure using the functor V, which allows to enlarge the class $\mathcal{F}$ with preservation property described above with $\mathcal{M}$. As an example one may take the Dugundji extension theorem in Top = $\mathcal{B}$ ($\mathcal{F}$ is the class of closed embeddings of metrizable spaces, $\mathcal{M}$ is the class of Banach spaces), take $\mathcal{A}$ = Unif, and the result is that the Dugundji theorem is valid for the bigger class of embeddings of completely regular spaces $X \hookrightarrow Y$ preserving fine uniformities. This is e.g. the case when one takes for Y any product of metric spaces and for X its subspace such that $\bar{X}$ is regularly closed and X is G_δ-dense in $\bar{X}$ (the recent result due to Hušek and Pelant). Consequences and related results will be mentioned.

J. Isbell: Polynomials in modules

Polynomial functions of degree at most n between finite-dimensional vector spaces are characterized by functional equations. They apply as well in infinite-dimensional spaces, once one settles what the polynomial functions are; for that see N. Roby, Ann. Sci. Ecole Norm. Sup. (3) 80 (1963), 215-348. However, beyond fields I diverge from Roby, who excludes $x \mapsto \frac{1}{2}(x^2 - x)$ on Z. Over any commutative ring R, polynomial functions are defined here by functional equations (a recursive set of them if

R is thoroughly known). The justification is of the following form: The polynomial functions of degree at most n between modules form the smallest class containing the values of polynomial expressions and mapping bijectively by (1) preceding with a module epimorphism, (2) following with a module monomorphism, and (3) change of base by a surjective ring morphism. The polynomial expressions are defined in a set of variables $\{x_\alpha\}$, with coefficients in any module A : They are expressions $\Sigma\, a_\lambda m_\lambda$, with $a_\lambda \in A$ and the m_λ monomials in the x_α of bounded degree.

G. Jarzembski: <u>Spectral Algebraic Theories and their algebras</u>

The notion of "partially multiadjoint functor" is the extension of the concepts "right adjoint functor" and "functor having a left multiadjoint". Let $U : \mathcal{A} \to \mathcal{B}$ be a functor, $\mathcal{D}$ - a subcat. of $\mathcal{A}$ with $\mathrm{Ob}\mathcal{D} = \mathrm{Ob}\mathcal{A}$. For every $\mathcal{B}$ - object B instead of looking for one (or a set) universal morphism we look for "universal ordered family SB " such that every U-morphism with domain B factors through a morphism from SB followed by a $\mathcal{D}$-morphism and this factorization is "greatest" with respect to the order on SB. Partially multiadjoint functors generate spectral algebraic theories /SAT/ and categories of algebras for such theories are constructed. Classical results by using "part. mult. functors" instead of "adjoint functors" are extended. Theorem: "For every finitary SAT in Set its category of algebras is isomorphic with "weak variety" of partial algebras of suitable finitary type" is proved.

P. T. Johnstone: <u>Open locales and exponentiation</u>

The following theorems were inspired by recent work of Gaunce Lewis on exponentiation in the category of weakly Hausdorff k-space over a fixed base space B.

THEOREM 1: Let A be a locally compact, open locale. Then the exponential functor $(-)^A : \mathit{Loc} \to \mathit{Loc}$ preserve regularity and strong Hausdorffness.

THEOREM 2: Let A be a locally compact, regular locale such that the functor $(-)^A$ sends regular locales to strongly Hausdorff locales. Then A is open.

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Both theorem are proved constructively, using a couple of lemmas which exploit the interaction between the notion of positivity (for elements of a locale) and the way-below relation. When interpreted in the topos of sheaves on B , the theorems yield results similar to, but not identical with, those of Lewis.

A. Kock: The Comohology of Eilenberg-MacLane toposes, according to Joyal and Wraith

Joyal and Wraith have recently proved a longstanding conjecture of Wraith: $H^*(K(\pi,n)) \cong \mathcal{H}^*(\mathcal{K}(\pi,n))$. Here, the left hand side denotes singular cohomology H^* of the *Eilenberg-MacLane space* $K(\pi,n)$; $\mathcal{H}^*$ denotes topos cohomology (derived functor of global-sections functor), and $\mathcal{K}(\pi,n)$ denotes "the" *Eilenberg-MacLane topos*, defined as a topos which classifies topos cohomology. The existence of such a topos was observed by Wraith many years ago, using the 'logical' theory of classifying toposes. The proof of Joyal and Wraith uses Dold-Kan-Puppe Theorem, results of Quillen and Illusie on topos cohomology for simplicial sets, and Gabriel-Zisman's theory of anodyne extensions. The crux is that the set-valued functors on FinAc/$K(\pi,n)$ form a $\mathcal{K}(\pi,n)$ (FinAc is the category of finite acyclic simplicial sets).

Category theoretic problems and results arising in Synthetic Differential Geometry

In many models of synthetic differential geometry, the infinitesimal objects D. are (internal) atoms: $(-)^{D.}$ preserves colimits, or has a right adjoint $(-)_{D.}$. We prove for any class $\mathcal{D}$ of pointed atoms in $\mathcal{E}$:

THEOREM: For fixed M , the category Ét/M of $\mathcal{D}$-étale maps to M is a topos; it comes with an essential geometric surjection $\mathcal{E}$/M → Ét/M . (This generalized recent results of Lawvere and Freyd.)

We also discuss some properties on an object M , analogous to infinitesimal linearity, and use this to improve work of Bunge and Sawyer, constructing a bijective correspondence between sprays on M and symmetric connections on M .

D. May Latch: Using Adjoints to Design FP Consistency Checkers, Interpreters and Type Checkers

Each context-free grammer (CFG) G generates two algebras: $\mathcal{D}(G) \equiv$ Free category of derivations in G, $\mathcal{A}\ell g(G) \equiv$ Algebra of derivable expression sets; and a natural homomorphism $F : (\mathcal{D}(G))^{op} \to \mathcal{A}\ell g(G)$ which assigns to each derivation $(d : A \to x)$ the lanuage $F(x)$ of expressions derivation from x.
Except for the set operation of union, it is generally undecidable whether there exist representations in $\mathcal{D}(G)$ of the set operations in $\mathcal{A}\ell g(G)$ of derivable set intersection $(F(x) \cap F(x'))$ and derivable set difference $(F(x) \cdot F(x'))$.
In the talk, a class of CFG's is developed which have:
(i) finite set of generators for both derivable set intersection and set difference;
(ii) each generating concatenation operation in $\mathcal{A}\ell g(G)$,Cartesian.
By using the M-Adjoint Construction (developed by [ADJ-1980]) which naturally attaches typed variables to G to form a new CFG SCH(G), it will be shown how to use:
(i) Intersection representatives to check whether: (a) primitive subroutine definitions actually represent *functions* (FP Consistency Checker); (b) complex programs are consistent (FP Type Checker);
(ii) Set differences to add error messages (FP Semantic Error Generator);
(iii) Both intersection and difference sets to build FP Interpreters.

F.E.J. Linton: A paradoxical distributivity

Unlike the familiar distributivity from ring theory of multiplication through addition, which leads to what Jon Beck has called a distributive law $\mathbb{M} \cdot \mathbb{A} \to \mathbb{A} \cdot \mathbb{M}$ between the monads $\mathbb{M}$ for monoids and $\mathbb{A}$ for abelian groups, the distributivity from lattice theory, of meet through joins $\left(\bigwedge_{i \in I} \left(\bigvee_{j \in J_i} a_j^i \right) = \bigvee_{t \in \prod_{i \in I} J_i} \left(\bigwedge_{i \in I} a_{t(i)}^i \right) \right)$ leads to maps

$$\lambda_X : K^{op}(P(X)) \to P(K^{op}(X))$$

that so badly fail to provide a Beck distributive law that they are not even natural in X . So: can a distributive law possibly fail to be a distributive law? -You bet your booty.

Free locales and Sierpinskian Cantor spaces

THEOREM: The monadic reflection $\mathbb{E}^{\mathbf{T}} \to \mathbb{E}$ of the "object-of-open-sets" functor $\mathcal{O}$: $(\text{Spaces in } \mathbb{E})^{op} \to \mathbb{E}$ assigning to each object $(X,\tau) \in |\text{Spaces in } \mathbb{E}|$ (which consists of an object X of the elementary topos $\mathbb{E}$ and a subframe τ of Ω^X) the object τ ... is exactly the category $\mathcal{Frm}(\mathbb{E})$ of frames in $\mathbb{E}$ (and frame homomorphisms). [REMARK: This assures us that in the attempt to use frames to capture the algebraic essence of topology, we have omitted no nonobvious operations on the lattices τ of "open sets" nor any nonobvious equations on those operations.] PROOF SKETCH: (i) $\mathcal{O}$ has a left adjoint as soon as it is representable. (ii) $\mathcal{O}$ is representable, by the "Sierpinski space" $\mathbb{S} = (\Omega, \leq_\Omega)$. (iii) The adjointness monad arising from $\mathcal{O}$, which assigns to X in $|\mathbb{E}|$ the product topology on the Cantorian power $\mathbb{S}^X$ in (Spaces in $\mathbb{E}$) , is the *same* as the "free locale" monad arising from the (already known-to-be) monadic category $\mathcal{Frm}(\mathbb{E})$: The topology on $\mathbb{S}^X$ is the free locale on X .

R.B. Lüschow: A concept of compactness for sets

The well-known concept of compactness for sets in a topological space will be extended to pairs $(C,\mathfrak{a})$ (i.e. C is $\mathfrak{a}$-compact for a system $\mathfrak{a}$ of sets covering C). By means of this concept one characterizes: injective map, convergence and continuity (and some conditions), and finite-dimensional vector-spaces.
Then this concept is studied for its own interest. There are two functors $\mathcal{Set} \underset{m}{\overset{c}{\rightrightarrows}} \mathcal{Complat}$ involved ($\mathcal{Complat}$: category of complete lattices). All this is done hoping to get a "tool" for the question: Which prebornologies are topological?

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S. MacLane: The Health of Mathematics

This will be an attempt to examine the present state of mathematical research, with comparisons betwenn the merits of different fields and proposals as to the relative importance of such fields - together with comments on current constraints. Too much publications, to little (ideas not published, but told privately), to much attention to past problems with too little attention to direction of research.

A. Möbus: Relational algebras of the filter monad

Let $\mathbb{F} = (F, \varphi, \kappa)$ be the filter monad on $\mathcal{Set}$ [2], and let $\mathcal{Set}^R(\mathbb{F})$ be the category of $\mathbb{F}$-relational algebras [1]. We can consider $\mathcal{Clos}$ [2] and $\mathcal{Top}$ as full reflective subcategories of $\mathcal{Set}^R(\mathbb{F})$ by the adherence resp. convergence relation. We construct the reflective hull $\mathcal{LI}$ of $\mathcal{Clos} \cup \mathcal{Top}$ in $\mathcal{Set}^R(\mathbb{F})$. Objects of $\mathcal{LI}$ are considered as sets equipped with a closure cl and a topology, such that $\mathrm{cl}(F \cup G) \subset \mathrm{cl}(F) \cup \bar{G}$. Morphisms are maps continuous w.r.t. both structures. Let $\mathcal{B}$ be the category of bitopological spaces $(X, \mathfrak{S}, \mathfrak{T})$ with $\mathfrak{S}$ finer than $\mathfrak{T}$, and let $\mathcal{TP}$ be the category of sets X equipped with a topology $\mathfrak{S}$ and a preorder, such that $\downarrow x$ is $\mathfrak{S}$-closed for all $x \in X$, with the obvious morphisms. Then we get full embeddings:

$$\begin{array}{ccccccc} \mathcal{TP} & \xrightarrow{U} & \mathcal{B} & \xrightarrow{V} & \mathcal{LI} & \xrightarrow{H} & \mathcal{Set}^R(\mathbb{F}) \\ & & & & \uparrow J & & \\ & & & & T_3\text{-}\mathcal{Set}^R(\mathbb{F}) & & \end{array}$$

with H,J reflective, U,V coreflective. HVU assigns to each object of $\mathcal{TP}$ a relation, which generalizes the limit inferior relation of the reals.

[1] M. Barr, Relational Algebras, in: Reports of the Midwest Category Seminar IV, LN 137 (1970), 39-55

[2] A. Day, Filter Monads, Continuous Lattices and Closure Systems, Canad. Journ. Math. 27 (1975), 50-59.

T. Müller: Chain homotopy pullbacks

In any category $\mathcal{C}$ with homotopy system fulfilling the KAN-conditions E(2), E(3) one has the structure of a groupoid enriched category and can define the notion of a homotopy pullback (pushout). We further assume, that the cylinder functor of the homotopy system has a right adjoint and that pullbacks of $\cdot \xrightarrow{k} \cdot \leftarrow \cdot$ with k a fibration, pushouts of $\cdot \xleftarrow{i} \cdot \rightarrow \cdot$ with i a cofibration exist. Supposing $\mathcal{C}$ has zero morphisms, we can prove the following

THEOREM: a) If the cofibrations are normal and (cofibrations, fibrations) are pushout-stable, then every homotopy pushout is a homotopy pullback. b) Dually, if the fibrations are conormal and (cofibrations, fibrations) are pullback-stable, then every homotopy pullback is a homotopy pushout.

EXAMPLE: If $\mathcal{A}$ is an abelian category and $\mathcal{C} := \partial\mathcal{A}$ is the category of chain complexes over $\mathcal{A}$ provided with the usual homotopy system, then, by the theorem, every homotopy pushout is a homotopy pullback and vice versa.

COROLLARY: Mather's Cube Theorems and their duals hold and therefore homotopy pullbacks commute with homotopy pushouts, if, in the corresponding diagram, any pair of composable squares are homotopy pullbacks (or pushouts).

C. Mulvey: <u>And</u>

In considering the representation of non-commutative C^*-algebras introduced by Giles and Kummer, one meets the lattice Lid A of closed left ideals of a C^*-algebra A . The Gelfand representation of A is determined by mappings Lid $A^{**} \rightleftarrows$ Lid A given by specialisation of and extension to ideals of the double dual algebra A^{**} . The lattices Lid A^{**} and Lid A are generalisations respectively of those of arbitrary and of open subsets of the maximal spectrum of a commutative C^*-algebra A . To obtain a Gelfand duality in the non-commutative case, one needs to characterise the lattices involved, demonstrating the way in which a C^*-algebra may be recovered from its non-commutative maximal spectrum. Rephrased in terms of a question in the foundations of quantum mechanics, this problem becomes that of finding a logic within which propositions $a \in (r,s)$, expressing the extent to which states admit the observable a in the rational interval (r,s) , may be manipulated to yield the quantised spectrum of a system of non-commuting observables. It is conjectured that the analogue of a locale in this non-commutative situation is that of a *quantale*, obtained by considering a complete lattice together with a binary operation $\cdot$, which is associative and distributive over arbitrary joins on both sides. The logical significance of this product is that of a non-commutative operation & which interprets the connective 'and later'. The lattice of closed left ideals of a C^*-algebra is a quantale with respect to multiplication of left ideals. In particular, any quantum logic canonically determines a quantale.

L.D. Nel: Enriched duality for smooth maps

Continuous maps $f: X \to Y$ correspond via natural bijection to multiplicative linear maps $C(f): C(Y) \to C(X)$ (classical Gelfand-Naimark). E. Binz (1975 monograph) made the correspondence $f \leftrightarrow C(f)$ continuous in both directions. Now we make it smooth in both directions. To do this, we reduce (for abstract categories $\mathcal{X}$) existence of a Binz-like duality to three rather simple properties of the reals $\mathbb{R}$ as object of $\mathcal{X}$; at concrete level we introduce a new category *Liss* for smooth analysis; *Liss* is a "topological universe" (≈ well-fibred concrete quasi-topos).

A. Obtułowicz: Categories of Partial Algebras

The categories of partial algebras axiomatizable in p-equational logic (briefly p-equational categories) are considered. The p-equational logic has common features with equational logic due to A. Tarski but differs from it in forming equations. A p-equational category is a category whose class of objects is the class of partial algebras being models of some theory formulated in p-equational logic. The arrows of a p-equational category are weak homomorphisms of partial algebras. The category of applicative systems (partial combinatory algebras) and weak homomorphisms is a p-equational category. The category of small toposes with canonical subobjects and logical morphisms preserving these subobjects is a p-equational category.
After giving an outline of main ideas of p-equational logic the necessary and sufficient conditions are formulated for a category to be equivalent to a p-equational category.

H. E. Porst: T-regular functors

Regular functors as introduced by H. Herrlich are the appropriate notion for semantical axiomatization of algebra as long as the base category is $\mathcal{S}et$ or a cartesian closed topological category $T : X \to \mathcal{S}et$. However, the underlying space functors of categories of universal algebra $A \to X$ over arbitrary (mono-) topological categories $T : X \to \mathcal{S}et$ in general will fail to be regular. Hence, given a mono-topological functor $T : X \to \mathcal{S}et$ we introduce the concept of a T-regular functor s.t. $1_{\mathcal{S}et}$-regular = regular. T-regular functors then turn out to be in a sense precisely the underlying space functors of (general) topological algebra. Besides some useful properties of these functors we prove a representation theorem, which - when specialized to $T = 1_{\mathcal{S}et}$ - gives a new proof of the well known representation theorem for regular functors.

D. Pumplün: Convex spaces and the Hahn-Banach theorem

Rodé proved an abstract version of the Hahn-Banach theorem (Arch. Math. 31, 1978), from which all known versions of this theorem follow. The frame within which he proves his result is a (single) universal algebra with commuting operations. The Eilenberg-Moore algebras of Banach spaces, called totally convex spaces (cp. Pumplün-Röhrl, Banach spaces and totally convex spaces I, to appear in Comm. Alg.) furnish a whole category of algebras with commuting operations. Modifying Rodé's idea in a non trivial way it is possible to prove a Hahn-Banach theorem for totally convex spaces. The result generalizes Rodé's substantially, because the operations on totally convex spaces are infinitary and the convex resp. concave bounding functionals are bounded. It is shown that all of Rodé's examples are in fact Hahn-Banach theorems for categories of universal algebras.

J. Reitermann: On locally small based equational theories

Locally small based equational theories, which include varietal theories = those corresponding to triples and some non-varietal ones, such as complete lattice theory, are shown to correspond precisely to varieties of functor algebras. Thus, the category of T-algebras for T l.s.b. is always legitimate, in fact, small fibred. We present several examples showing that the basic structure-sematics relations viz that T StrSem T , Sem T SemStrSem T are iso are fulfilled for some non-varietal l.s.b. theories, but for some they fail to be true in a very strong way.

G. Richter: Locally noetherian categories of functors

We try to characterize small categories X for which the following generalized version of the *Hilbert Basis Theorem* holds:
(+) C-*Mod* locally noetherian $\Rightarrow$ C-*Mod*^{X} locally noetherian,
where $C \neq 0$ is a "ring with several objects" in the sense of B. Mitchell. The special case, where X is a group and C a commutative field, is an unsolved classical problem. But there are characterizations for other important classes of small categories X , namely posets (M. Höppner) and path categories of quivers (D. Höinghaus, -). As a common generalization of these two and other known cases we have the following
THEOREM: Let X be well ordered (i.e. every morphism functor $X(X,-)$ factorizes over the category of well ordered sets and strictly increasing maps). Then the following are equivalent:
(i) (+) holds for X (and some $C \neq 0$)
(ii) every morphism functor $X(X,-)$ fulfils the a.c.c. on subfunctors.

J. Rosický: Categories of models

Let Mod(T) be the (concrete) category of models of a first order theory T with homomorphisms as morphisms. T is finitary, one-sorted and of a finitary type τ , empty models and 0-ary relation symbols are admitted. The determination of the theory by its category of models was considered by Makkai and Lascar. We show that the concrete case is connected with the category invariance of ultraproducts. It enables us to find a large class φ of theories (including all Horn theories) with the property that $S,T \in \varphi$ are isomorphic (via $\vee\exists\wedge$-interpretations) iff Mod(S) , Mod(T) are concretely isomorphic. The characterization of (concrete) categories Mod(T) is unclear even among locally finitely presentable categories. The reason for the last fact is that limits in Mod($\mathcal{T}$) need not be canonical, i.e. computed in Mod(τ) . The canonical case is covered by the Coste's result characterising (concrete) categories of models of limit theories. We will characterize concrete categories of models of more general regular theories (in the sense of Makkai). Weakly initial objects are used here and they also make possible to prove that the dual of the category of compact Hausdorff topological spaces is not isomorphic to Mod($\mathcal{T}$) for any $\mathcal{T}$ with canonical products (it solves the problem of Bankston).

G. Strecker: <u>Cartesian closed topological hulls</u>

A method is given for the construction of cartesian closed topological hulls of concrete categories $K \xrightarrow{U} X$ over suitable X . In the case that $X = \mathcal{Set}$ and K has constant morphisms, it specializes to an earlier result of Adâmek and the speaker. In the case that X is a terminal category, it provides a "locale hull" for meet semi-lattices.

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