# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

## T a g u n g s b e r i c h t 44/83

## Parametrische Optimierung und Approximation

16.10. bis 22.10.1983

Die 1. Tagung über "Parametrische Optimierung und Approximation" fand unter der Leitung von B.Brosowski (Frankfurt) und F.Deutsch (University Park) statt. Es kamen 21 Teilnehmer, darunter 10 aus dem Ausland (Brasilien 1, Bulgarien 2, CSSR 1, DDR 1, Großbritannien 1, Israel 1, Niederlande 1, Südafrika 1, USA 1). Ziel der Tagung war es insbesondere, Spezialisten auf dem Gebiet der parametrischen Optimierung und der parametrischen Approximation zusammenzubringen, um die Zusammenarbeit und gegenseitige Anregungen zu fördern.

In insgesamt 19 Vorträgen wurden neben den theoretischen Grundlagen parametrischer Optimierungs- und Approximationsprobleme auch numerische Verfahren für parametrische Probleme und Anwendungen in den Ingenieurs- und Wirtschaftswissenschaften behandelt.

Vortragsauszüge

W.Alt

## Ein Ritz-Verfahren zur Lösung einer Klasse zustandsbeschränkter Kontroll-Approximationsprobleme

Im Zusammenhang mit Wärmeleitungsvorgängen tritt das Problem auf, durch Steuerung der Wärmezufuhr einen Stab, der aus einem homogenen Material besteht, so aufzuheizen, daß ausgehend von einer konstanten Anfangstemperatur innerhalb einer vorgegebenen Zeit eine erwünschte Endtemperatur möglichst gut erreicht wird. Dabei müssen sowohl die Steuerung als auch die Temperatur gewisse Einschränkungen erfüllen. Zur numerischen Lösung des dadurch definierten Kontroll-Approximationsproblems stellen wir ein Ritz-Verfahren vor; dabei wird das kontinuierliche Problem durch eine Folge diskreter Optimierungsprobleme approximiert. Die Konvergenz des Verfahrens wird theoretisch durch Angabe von Fehlerschranken für die Extremalwerte der diskreten Probleme gezeigt und praktisch an einigen numerischen Beispielen illustriert.

D.Amir

## Best simultaneous approximation and projections on maximal subspaces

The B.S.A. problem is to minimize $r(x;A)=\sup_{a\in A}\|x-a\|$, where A is a boundet set in the normed space E. The optimal value of r (x;A) in the constraint set G is denoted by $r_G(A)$. The solution set $Z_G^o(A)$ is called also the "Chebyshev-center" of A in G. We give some applications of BSA theory to the solutions of problems in approximation theory and in Banach space geometry: (i) Using a bounded A with $Z_E^o(A) = \emptyset$, we construct a maximal subspace in a Banach space which is proximinal but admits no minimal projection (answering in the negative a conjecture of Cheney and Price).

(ii) Using a characterization of Hilbert spaces via Chebyshev-centers, it is shown that if every maximal subspace of E admits projections of norm arbitrarily close to 1, then E is Hilbert (Amir-Franchetti).

(iii) In any Banach space E, the constants $H(E) = \sup_{f \in E^*} \inf \{\|P\| ;\ P \text{ projects } E \text{ onto } f^{-1}\, 0\}$ and $R(E) = \sup\{r_A(A);\ r_E(A) = 1,\ A \text{ convex}\}$ satisfy $R(E) \leq H(E) \leq \frac{3}{2} + \frac{R(E)}{4}$. Hence $R(E) < 2 \iff H(E) < 2$. This defines an interesting convexity property for Banach spaces.

H.-P.Blatt

REMARKS TO THE STRONG UNICITY OF SEMI-INFINITE LINEAR OPTIMIZATION PROBLEMS

We consider a semi-infinite linear optimization problem and the generalization of the Remez algorithm where in each step a solution of a "chain of references" is calculated. The notion of "chain of references" leads in a natural way without further calculations to sharp estimates for an optimal solution. Furthermore we have got a new charcterization of a strongly unique optimal solution: An optimal solution is strongly unique iff the extremal point set contains only regular chain of references consisting of one reference.

B.Brosowski

## Application of Parametric Programming to the Optimal Design of Stiffened Plates

The optimal design of an axially loaded stiffened plate is minimizing the weight and the costs and maximizing the collapsing load. The design variables are the number, the depth, and the thickness of the stiffeners and the thickness of the plate. This multiobjective design problem is formulated by using the concept of efficient points, which permits a reduction to a parametric optimization problem. This reduction leads to a necessary and a sufficient condition for the efficiency of a point. A refinement of these conditions leads to a reduction of a vector minimization problem with $\ell$ objective functions to a problem with $k \leq \ell - 1$ objective functions. As an application we design an algorithm for the calculation of efficient points in the cases of 2 and 3 objective functions.

A.L.Brown

## Best Approximation by Smooth Functions and Related Problems

Let K be a kernel defining a compact operator $T_K : L^\infty \longrightarrow C$, let M and N be finite dimensional spaces of continuous functions and let

$$W = \{k + T_K f : k \in M,\ f \in L^\infty \cap N^\perp,\ \|f\|_\infty \leq 1\} .$$

The initial problem is that of characterising best uniform approximations to a function $\varphi \in C$ from W. This general situation contains one considered by Saltes ($W = W_{\infty r}$) and one considered independently by Glashoff and Pinkus (K a totally positive kernel, $M = N = \{o\}$ ). A development of Glashoff's duality argument leads to a consideration of the zeros of functions of the form

$$\mu(t) = g(t) + \int K(s,t)d\,\lambda(s)$$

where $g \in N$ and $\lambda$ is a measure orthogonal to M. The case in which $K(s,t) = (s-t)_+^{r-1}/(r-1)!$ yields a new proof of Saltes result on best approximation by smooth functions.

R.Colgen

## On parametric separably-infinite optimization problems

We consider separably-infinite optimization problems. This class of optimization problems introduced by Charnes, Gribik, and Kortanek has some applications in economics. Moreover, it contains semi-infinite optimization problems as well their duals, and has the important property that duals of separably-infinite programs belong to this class, too. We investigate on stability of such optimization problems: We give a sufficient and necessary condition for upper semicontinuous dependence of the set of optimal solutions on parameters.

F.Deutsch

## The Alternating Method of Von Neumann: A Brief Survey

Von Neumann proved the following result in 1933: If A,B are closed subspaces of a Hilbert space X and $P_A$, $P_B$ the corresponding orthogonal projections, then

$$\lim_{n\to\infty} (P_A P_B)^n x = P_{A\cap B}x \quad \text{for all } x \in X.$$

This result has been rediscovered, reinterpreted, extended, and applied in a variety of ways. Some of the areas of application are solving linear systems of equations, linear prediction theory of multivariate stochastic processes, computer tomography, approximating functions of several variables by sums of functions of a single variable. Some of the names associated with these results are: Wiener, Masoni, Kacmarz, K.T.Smith, Hornsfield, and Klee.

A.L.Dontchev

## DIFFERENTIAL INCLUSIONS WITH SMALL PARAMETER IN THE DERIVATIVE

It is known that if the function $f(\cdot)$ is continuous and the spectrum $\sigma(A)$ of the matrix A satisfies $\mathrm{Re}\,\sigma(A) < 0$ then for $t < 0$ the solution of the differential equation

$$\beta\dot{y} = Ay + f(t) \ , \ y(0) = y^0 \ , \ t \geq 0 \ ,$$

converges pointwise to the solution of the algebraic equation

$$0 = Ay + f(t) \ ,$$

corresponding to $\beta = 0$. If we replace $f(\cdot)$ by a multivalued mapping, i.e. a linear differential inclusion is considered

$$\beta\dot{y} \in Ay + F(t), \quad y(0) = y^0 \ ,$$

then the following two phenomena can be observed:
(i) For every $t > 0$ the set of solutions values is convergent (in suitable sense) to a set which may be larger than the set corresponding to $\beta = 0$;
(ii) The condition $\mathrm{Re}\,\sigma(A) < 0$ may be not essential for such a convergence.
In the talk we give a more detailed discussion and some extensions of these two observations.

J.Flachs

## SENSITIVITY ANALYSIS IN GENERALIZED RATIONAL APPROXIMATION WITH RESTRICTED DENOMINATOR

Generalized rational approximation with restricted denominator is viewed as a parametric program, in which all the given functions which are involved, as well as the compact space T on which they are defined, play the role of parameters. Supposing the Slater condition on the constraints, we establish a local Lipschitz property of the optimal value and the Lipschitz behaviour in a certain sense of the solution set. In particular, these properties are shown for the case where a sequence $T^k \subset T$

converging to T is considered together with a convergent sequence of given functions. The approach encompasses the behaviour of both the optimal value and the solution set under noise and under various discretizations, these aspects being encountered in practice.

T.Gal

## PARAMETRIE PROGRAMMING - A BRIEF HISTORICAL SURVEY

A brief survey of the origins and historical development of parametric programming is given. Various viewpoints of what can be understood by sensivity analysis and parametric programming are formulated. After a mathematical formulation of the parametric optimization problem in general and of the linear parametric optimization problem in particular is introduced, the mainstreams of development of the field are presented.
In the talk also problem formulation, theory and methods of linear parametric optimization are briefly discussed. This part of the talk, however, is to appear in Mathematical Programming Study edited by A.Fiacco (Washington,D.C.) and therefore not included here.

B.Gollan

## PARAMETRIC OPTIMIZATION AND EIGENVALUE PERTURBATIONS

Parameterized finite-dimensional optimization problems are studied. Recent results about their optimal value function are used in order to treat the generalized eigenvalue problem $A(p)x = \lambda B(p)x$; here $A(\cdot)$ and $B(\cdot)$ are real symmetric matrices which depend on a parameter $p \in \mathbb{R}^m$. In the classical theory $m = 1$ and A and B are analytic or (continuously) differentiable functions of p. Here we assume only differentiability at $\overline{p}$ or Lipschitz continuity and we derive similar properties about the eigenvalues as functions of p. When $m > 1$ this leads to results which seem to be new in case of continuously differentiable matrices $A(\cdot)$ and $B(\cdot)$.

J.Guddat

## ON THE POINTWISE APPROXIMATION OF SELECTIONS IN PARAMETRIC OPTIMIZATION

The following parametric optimization problem is considered: $P(t):\{\min h_o(x,t)\,|\,h_j(x,t) \leq 0,\ j = 1,\dots,m\},\ t \in T := [0,1]$ where $h_i : R^n \times T \longrightarrow R,\ i = 0,1,\dots,m$. The paper presents an algorithm to find a pointwise approximation of a continuous selection function $x(t) \in \Psi_{loc}(t)$ on T where $\Psi_{loc}(t)$ is the set of all local minimizer of P(t). More precisely we wish to find a partitioning of T : $0 = t_o < \dots < t_N = 1$ and points $\tilde{x}(t_i)\ (i = 1,\dots,N)$ such that $\tilde{x}(t_i)$ is a sufficiently exact approximation of $x(t_i)$. This approach can be used to globalize locally convergent algorithms for solving nonlinear optimization problems and to compute Pareto-optimal points for vector optimization in an efficient way. This work has been done in cooperation with H.J.Wacker, H.Gfrener and W.Zulehner (Kepler-Universität Linz,Austria).

S.Helbig

## PARAMETRISCHE OPTIMIERUNG AUF EXTREMALEN ALGEBREN: CHARAKTERISIERUNGSSÄTZE UND STETIGKEITSAUSSAGEN BEI VARIABLER RECHTER SEITE

Eine Algebra F heißt extremal, wenn (1) $x \oplus y = x$ oder $y$ für alle $x,y \in F$ und (2) $x \leq y \Leftrightarrow x \oplus y = y$. Außerdem werden zwei Axiome benötigt: (3) $(x,y,z \in F$ mit $x < y \leq z) \Rightarrow (\exists \alpha: \bar{0} < \alpha < \bar{1}$ mit $x < \alpha \circ z < y)$ und (4) $(x \in f,\ x \neq \bar{0}) \Rightarrow (\exists x^{-1} : x \circ x^{-1} = \bar{1})$.

$\bar{0}$ und $\bar{1}$ sind dabei das Null- bzw. Einselement von F. Durch Umgebungen der Art $U_{ab} := \{z \in F \mid a < z < b\}$ mit $a < b \in F$ wird eine Topologie $\mathfrak{F}$ auf F erklärt; auf $F^n$ wird die Produkttopologie eingeführt. Eine Menge $A \subset F^n$ heißt extremalkonvex, wenn $\forall x,y \in F^n$ und $\alpha,\beta \in F$ mit $\alpha \oplus \beta = \bar{1}$ gilt: $\alpha \circ x \oplus \beta \circ y \in A$. Sei T kompakt; dann betrachte (LMP): Min $(p,x); (B(t),x) \geq b(t)$ $\forall t \in T$; dabei sind $p \in F^n$ und $b(t)$, $(B(t),x)$ stetige Abbildungen. Charakterisierungssätze:
(1) Sei $v_o \in F^n$ so, daß $\forall v \in F^n$ $\exists t \in M_\sigma, v_o$: $(B(t),v_o) \geq (B(t),v)$ (*); dann ist $v_o$ Minimalpunkt von (LMP).
(2) Wenn $v_o$ Minimalpunkt von (LMP) ist, gilt Eigenschaft (*).
Bei variabler rechter Seite konnten folgende Stetigkeitsaussagen gemacht werden: (1) $Z : L_{B,P} \longrightarrow POT(F^n)$ ist abgeschlossen und uhs; (2) $E : L_{B,p} \longrightarrow F$ ist stetig und (3) $P : L_{B,P} \longrightarrow POT(F^n)$ ohs, wobei $L_{B,P} := \{b : T \longrightarrow F \mid b$ stetig und (LMP) besitzt Lösung$\}$.

H.Th.Jongen

## CRITICAL SETS IN PARAMETRIC OPTIMIZATION

We consider finite-dimensional optimization problems depending on one parameter. The constraints are of both equality and inequality type. Under generic regularity conditions we classify the local structure of the set of (generalized) critical points. In particular, the change of linear and quadratic indices will be studied in detail. Finally, we discuss the subset of critical points of Kuhn-Tucker type.

P.S.Kenderov

## MOST OF THE OPTIMIZATION PROBLEMS HAVE UNIQUE SOLUTION

Let X be a compact metric space and C(X) be the space of all continuous real-valued functions in X. Every pair (A,f), where A belongs to the set $2^X$ of all closed subsets of X and f is from C(X), determines a (constrained) optimizationproblem: find $x \in A$ at which f attains its minimum over A. Suppose $2^X$ is endowed with the Hausdorff metric and C(X) is topologized by the usual uniform norm. We prove that there exists a dense $G_\delta$ - subset of $2^X \times C(X)$ such that every minimization problem (A,f) from this set has unique solution. I.e. the set $\{x \in A : f(x) = \min\{f(y) : y \in A\}\}$ consists of only one point for each (A,f) outside some first Baire category subset of $2^X \times C(X)$. The result is also valid for some non-metrizable compact spaces X. This is so, for instance, when X is a weakly compact subset of a normed space or X is homeomorph to a weak*-compact subset of a dual Banach space with the Radon-Nikodym property.

F.Nožička

## VERALLGEMEINERUNG DER KONVEXITÄTSBEGRIFFE AUFGRUND BESTIMMTER PARAMETRISCHER OPTIMIERUNGSAUFGABEN

Ist F(x) eine über den ganzen Raum $E_n$ definierte Funktion, die gewissen Bedingungen unterworfen ist, so kann man aufgrund dieser einzigen Funktion eine spezielle Kurvenklasse in $E_n$ einführen. Durch Punkt und Richtung, bzw. durch zwei verschiedene Punkte aus $E_n$ (wobei keiner dieser Punkte einen Optimalpunkt des freien Minimums von F(x) über $E_n$ darstellt) wird eine Kurve der fraglichen Klasse in ihrer Globalität eindeutig bestimmt. Führt man die Mannigfaltigkeiten

$$V_n := \{x \in E_n \mid F(x) = \mu\}, \quad \mu > \hat{\mu} := \inf_{x \in E_m} \{F(x)\} ,$$

ein und stellt man die folgenden parametrischen Optimierungsaufgabe auf

a) $\max \{(v,x) \mid x \in V_\mu\}$ mit $\mu > \hat{\mu}$ und $v \neq 0$ festgewählt,

b) $\max \{(v_1 \cos\varphi + v_2 \sin\varphi, x) \mid x \in V_\mu\}$ mit $\mu > \hat{\mu}$ festgewählt, $\varphi \in \langle 0, 2\pi)$,

c) $\max \{(v_1 \cos \mu + v_2 \sin , x) \mid x \in V_\mu\}, \; \mu > \hat{\mu}$ , mit festgewählten orthonormalen Vektoren $v_1$ und $v_2$, so ergibt sich mit die entsprechende Kurvenklasse als die Menge der Lösungskurven der obigen parametrischen Optimierungsprobleme. Mit Hilfe der auf diese Weise gewonnenen Kurven kann der Begriff der verallgemeinerten Konvexität von Mengen und Funktionen bezüglich der vorgegebenen Funktion $F(x)$ eingeführt werden.

G.Nürnberger

## Strong unicity in parametric semi-infinite optimization

Semi-infinite optimization problems of the following type are considered. Minimize $\langle p,x\rangle = \sum_{i=1}^{N} p_i x_i$ subject to $\langle B(t),x\rangle \leq b(t)$ for all $t \in T$. We denote by SU the set of parameters $\sigma = (p,B,b)$ for which the corresponding optimization problem has a strongly unique solution. Since in practice the parameter $\sigma$ may be only known approximatively, it is natural to ask which $\sigma$ in SU are "stable under small perturbations", i.e. which $\sigma$ are contained in the interior of SU. We give a complete characterization of parameters $\sigma$ with this property. Moreover, the relationship between unique and strongly unique solutions is investigated. The results are applied to varions types of minimization problems.

A.R. da Silva

## On parametric infinite optimization

We consider infinite parametric problems. We show that the best approximation problems in real normed linear spaces, the parametric semi-infinite problems and certain types of infinite programming problems can be reformulated as parametric infinite problems. Further we give sufficient conditions to the lower semicontinuity of the optimal set mapping in the linear case.

M.Sommer

## Continuous Selections and Convergence of Best $L_p$-Approximations in Subspaces of Spline Functions

The purpose of this talk is to study the problem of existence of continuous selections for the metric projection and of convergence of best $L_p$ - approximations in subspaces of polynomial spline functions defined on a real compact interval I. Recently, G.Nürnberger and the author have shown that exists a continuous selection s if and only if the number of knots k is less than or equal to the order m of the splines. Using their construction we have proved that the sequence of best $L_p$ - approximations of f converges to s(f) as $p \to \infty$ for every f in C(I). We are furthermore able to show that also in the case when $k > m$ the above stated results remain valid provided that our approximation problem is considered on certain subsets of I.

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