MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 51/1983

Nonstandard-Analysis

4. bis 10. Dezember 1983

Diese fünfte Tagung seit 1970 zur Nichtstandard-Analysis fand unter der Leitung von S. Albeverio (Bochum), D. Laugwitz (Darmstadt) und W.A.J. Luxemburg (Pasadena) statt. Die eindrucksvolle Internationalität des Teilnehmerspektrums (vertreten waren Gäste aus elf Ländern von vier Erdteilen) spiegelte die heute erreichte Fruchtbarkeit infinitesimalanalytischer Techniken in vielfältigen Disziplinen angemessen wider.

Im Mittelpunkt der meisten Vorträge standen die Weiterentwicklungen der Untersuchungen in den schon traditionellen Schwerpunkten Funktionalanalysis und Stochastik, aber auch Algebra und Logik. Besondere Sympathie fanden wiederum die Behandlung von Differentialgleichungen mit infinitesimalen Parametern sowie neuartige infinitesimale Ansätze in der Algebraischen Geometrie. Die der Logik zuzurechnenden Beiträge zielten weniger auf Grundlegung als vielmehr auf Darstellung der methodischen Mittel und Anwendungen bis hin zur Automatentheorie.

Erneut bewährt hat sich die Ergänzung der Vortragsreihen durch organisierte Diskussionsrunden, die diesmal unter den Leitthemen Lehrerfahrungen mit Nichtstandard-Analysis, geschichtliche und philosophische Bedeutung der Nichtstandard-Analysis und Rolle der Nichtstandard-Analysis für das physikalische Weltmodell standen. Die prägnanten, manchmal hitzig vertretenen, oft kontroversen Standpunkte und Meinungen offenbarten relevante Aspekte des Wissenschaftsdiskurses, die erst bei einer in dieser Weise veränderten Kommunikationsstruktur zutage treten.

Vortragsauszüge

L. Arkeryd:

LOEB SOLUTIONS OF THE BOLTZMANN EQUATION

We discuss non-standard and Loeb solutions to the Boltzmann equation, and make a comparison with the standard case. In particular a space-dependent situation is considered, where on the Loeb side there are true solutions.

I.P. van den Berg:

A NONSTANDARD APPROACH TO APPROXIMATIONS BY TAYLOR POLYNOMIALS

A computer drawing, showing the exponential function $x \mapsto e^{-x}$ and its successive Taylor polynomials $x \mapsto \sum_{k=0}^{n}(-1)^k \frac{x^k}{k!}$ permits the following observations.

(1) The graphs of the exponential function and of some Taylor polynomial cannot be distinguished over a certain distance; beyond they separate rapidly.

(2) The picture shows a remarkable regularity.

Our aim is to give a mathematical description of these phenomena. Non-standard analysis appears to supply notions and techniques making it possible to do this in a simple way. Our method applies to convergent and divergent Taylor series $\sum_{k=0}^{\infty} a_k x^k$ provided the sequence of coefficients $(a_k)_{k\in\mathbb{N}}$ has some regularity.

H.-B. Brinkmann:

SOME REMARKS ON THE AXIOMATIC PRESENTATION OF NON-STANDARD EXTENSIONS

We discuss a simple set theoretic formulation of axioms for the teaching of non-standard analysis by extending Keisler's elementary approach from sets of real numbers to all sets. The basic idea of the categorical description of logic is due to Lawvere, while the connection of the type of functors used in the description to non-standard analysis was first noted by Kock and Mikkelsen and formulated in topos theory. In essence the main part of the axioms is a set theoretic formulation of the theorem of Łoś.

P. Cartier

AXIOMATIC FOUNDATIONS OF NON-STANDARD ANALYSIS

In line with the investigations of Lawvere, Nelson and Hrbacek, we present here a finite axiomatization of non-standard analysis. Like Nelson, we introduce only one new primitive concept, namely "standard", but we consider external sets as our framework. The notion of internal set is derived. We insist on the basic operations on sets (Boolean, direct product, and so on) and on the notion of hyperfinite set.

We offer some commentaries on the axiom of idealization and give some applications to a new presentation of topological spaces.

Finally, we mention some interesting philosophical remarks by Benabou.

N. J. Cutland:

NON-STANDARD METHODS IN CONTROL THEORY

We will first discuss deterministic control problems modelled by equations such as $dx_t = f(t,x_t,u_t)dt$. Loeb-measure techniques give a natural proof of the weak-compactness of relaxed controls, using the usual non-standard criterion for compactness. This yields easy proofs of existence of optimal relaxed controls.

We shall then discuss stochastic control systems $dx_t = f(t,x,u(t,x))dt + g(t,x,u(t,x))db_t$ in which the control u depends at time t in a partial observation of the past of x, such as a cummulative digital read-out. Under appropriate conditions on the coefficients f, g we can show that relaxed controls are weakly compact, and obtain optimality results. In the case where the diffusion g is controlled the essential idea is, for a non-standard control U, to solve an internal equation $dX_t = F(t,X,U(t,X))dt + G(t,X,U(t,X))d{*}b_t$ and show that $x = {}^{\circ}X$ is a solution to the foregoing equation for the relaxed control $v = {}^{\circ}U$ and a different Brownian motion.

F. Diener:

LOCALISATION LEMMA

In NSA, there are some kinds of arguments which appear usually and which is worth stating, out of any special context: the localisation lemma is one of these. It asserts that if η is an internal function, defined on $[0,a]$, "taking mincing steps", limited at 0 and satisfying the following condition:

"if for all τ, it suffices that η stays limited on $[0,\tau]$ to assure that η is S-continuous on $[0,\tau]$"

then η is limited (and thus S-continuous) on some standard interval $[0,\tau]$. In this talk, we would like to show, taking for example a theorem of existence for solutions of differential equations with delay, that this lemma is usefull and natural.

M. Dresden:

FRACTALS AND NON-STANDARD CONSIDERATIONS

It has been suggested several times that fractal lattices serve as an interesting interpolation between systems of various dimensions. This has applications in statistical mechanics as well as in dimensional regulations. It will be argued that the only manner to deal with fractal lattices is by non-standard considerations. A number of mathematical questions remain such as the Hausdorff dimension of non-standard fractal structures. It seems likely that some of these mathematical questions have important physical consequences.

E. J. Farkas:

A NON-STANDARD SEMANTICS FOR PARALLEL PROGRAMS

In this talk we show that unbounded parallel standard programs are equivalent to *N-bounded parallel non-standard programs and that non-standard-methods therefore lead to a loop-free analysis of parallel programs. We introduce the concept of a star-finite automaton and prove that every nondeterministic star-finite automaton is equivalent to a "star-deterministic" one. By combining these facts we show that unbounded parallel standard programs are naturally equivalent to non-standard star-deterministic programs.

J. E. Fenstad:

HYPERFINITE QUANTUM FIELDS

In this lecture we first proved the existence of the free field based on a hyperfinite lattice with infinitesimal spacing. We then proved the existence in dimension $d = 2$ of a non-gaussian measure obtained by perturbing the free measure by an exponential interaction.

M. Goze:

NSA AND ALGEBRAIC GEOMETRY. APPLICATIONS OF THE CURVES.

We describe the decomposition of a point infinitely near of the origin of $\mathbb{C}^n$ (n standard). This decomposition gives a sequence of birational transformations of $\mathbb{C}^n$. We applied these transformations for the desingularization and the parametrization of the algebraic plane curves (standard) in the case $n = 2$ and surfaces for $n = 3$.

C. W. Henson:

BANACH SPACE MODEL THEORY

The talk will give an up to date summary of the logic for Banach spaces which the speaker developed to study the properties of the non-standard hulls of Banach spaces. This logic, which is based on a special class of "positive bounded" formulas and a special notion of "approximate truth", behaves very much like first-order logic and is therefore a useful tool for studying the geometry of Banach spaces, constructing such spaces, etc. The logic is now developed to treat Banach spaces equipped with operators (not necessarily linear) and relations (such as an ordering). The results apply to all Banach spaces, not just to the non-standard hulls studied earlier. Speaking generally, one can say that the logic has been developed in very strong analogy to the usual first-order logic, as is presented for example in the book Model Theory by Chang and Keisler, up to the study of stable spaces.

T. Kamae:

NON-STANDARD ANALYSIS AND COMBINATORIAL ERGODIC THEORY

Non-standard analysis can be used to simplify the proofs of

(1) (Birkhoff's ergodic theorem) For any integrable function f on a probability space $(X,\mathcal{B},\mu)$ with a measure preserving transformation T, $\hat{f}(x) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x)$ exists almost surely and $\int \hat{f} d\mu = \int f d\mu$ holds.

(2) (van der Waerden's theorem) If $\bigcup_{i=1}^{k} S_i = \mathbb{N}$, then there exists i such that S_i contains an arbitrary long arithmetic progression.

C. Keßler:

HYPERFINITE GENERALIZED RANDOM FIELDS

We extend the well-known hyperfinite representation of adapted stochastic processes in two directions:

- arbitrary dimension of parameterspace
- distributions as "paths" of the process.

Taking $\Omega = {}^*\mathbb{R}^\Gamma$ (Γ a "hyperfine" lattice) and defining adaptedness in a natural way we get a

Lifting theorem: Each adapted generalized field has an adapted hyperfinite field as a lifting.

As a by-product we obtain the equivalence on Loeb spaces of the two current definitions of "X is a field of order n":

(1) "Each path $X(\omega)$ is a distribution of order $\leq n$" (Gelfand/Shilov IV)

(2) "There is a continuous field Y such that X is the n-th distributional derivative of Y" ("Polish" version).

D. Laugwitz:

EULER'S ALGEBRAIC FOUNDATION OF INFINITESIMAL ANALYSIS

After having used polynomials of infinite degree successfully in the 1730's Euler proceeds to a more systematic treatment in his Introductio in analysin infinitorum (1748). The elementary functions exp, log, sin, cos, and the binomial expansion are treated avoiding derivatives, integrals and the Taylor expansions, starting from $(1+\frac{x}{\Omega})^\Omega$ as a representation of the exponential function and proceeding in an algebraical way. Series expansions and infinite products are obtained. It appears that two tools suffice, namely Euler's criterion for convergence of series of 1734, and the following Lemma: If $\Sigma\, a_k$, $\Sigma\, b_k$ have real (or complex) sums A, B and if $a_k \approx b_k$ for all finite k, then $A = B$; (a_k), (b_k) are supposed to be internal sequences. This Lemma does not appear explicitly in Euler but must have been looked obvious to him. It is an immediate consequence of Robinson's Lemma on sequences.

T. Lindström:

SINGULAR PERTURBATIONS OF SCHRÖDINGER OPERATORS

We study perturbations of the Laplacian $-\Delta$ in $\mathbb{R}^d$ supported on sets of measure zero. If B is this set of Lebesgue measure zero and ρ is a probability measure on B, a natural standard way of constructing such operators is to look at the quadratic forms

(*) $\quad E(f,g) = \int(-\Delta f)\cdot g\,dm - \int_B \lambda f\cdot g\,d\rho$

where m is the Lebesgue measure on $\mathbb{R}^d$ and λ is a non-negative function on B. The question is whether this form is closable; i.e. whether it generates a self-adjoint operator.

To approach this problem with non-standard methods we first rephrase it in a *-finite setting: Let Γ be a hyperfinite lattice with infinitesimal mesh in $\mathbb{R}^d$, and let μ be the uniform measure on Γ corresponding to Lebesgue measure. Let $\widetilde{B}$ be an internal subset of Γ such that $B = st(\widetilde{B})$, and let $\widetilde{\rho}$ be an internal probability measure on $\widetilde{B}$ such that $\rho = L(\widetilde{\rho})\circ st^{-1}$. The non-standard counterpart of (*) is the internal form

(**) $\quad \mathcal{E}(f,g) = \int(-\Delta_\Gamma f)\cdot g\,dm - \int_{\widetilde{B}} \widetilde{\lambda} f\cdot g\,d\widetilde{\rho}$

where $-\Delta_\Gamma$ is the discrete Laplace operator on Γ.

Using resolvent techniques we now find a criterion for when $\mathcal{E}$ induces a standard, closed form on $L^2(\mathbb{R}^d,m)$. It turns out that in some cases we have to chose $\widetilde{\lambda}$ infinitesimal to achieve this, but that the standard part of $\mathcal{E}$ is still a nontrivial perturbation of the form generated by $-\Delta$. This happens for instance when B is a finite set of points in $\mathbb{R}^2$ or $\mathbb{R}^3$, and when B is a Brownian path in $\mathbb{R}^4$ or $\mathbb{R}^5$.

(This is a joint work with S. Albeverio, J.E. Fenstad, R.Höegh-Krohn.)

P. A. Loeb:

APPLICATIONS OF NON-STANDARD ANALYSIS TO POTENTIAL THEORY

The author gives a generalization and easy proof due to Bliedtner and himself of the Fine Limit Theorem of Fatou, Naim and Doob. From this theorem and a non-standard construction one obtains the following generalization of radial limits which also works for Brelot harmonic spaces: Let D be the unit disc $\{z \in \mathbb{C} : |z| \leq 1\}$. For $r \leq 1$ let $C_r = \{z : |z| = r\}$ and let σ_r be normalized Lebesgue measure on C_r. We write C and σ if $r=1$. Let $P(z,y)$ denote the Poisson kernel. Let ν be a probability function on D given by $h(y) = \int_C P(z,y)\nu(dz)$. For each $z \in C$ and $r<1$ let $f_r(z) = \int_{C_r} h(y)P(z,y)\sigma_r(dy)$. Then f_r is continuous on C and $\lim_{r \to 1^-} f_r(z) = \frac{d\nu}{d\sigma}(z)$ for σ a.e. z in C.

W. A. J. Luxemburg:

SPECTRAL AND ASYMPTOTIC SPECTRAL MEASURES OF INTERNAL MEASURES

Let (Ω,Σ,σ) be an internal measure space, where the measure σ is internally finitely additive and say $\sigma(\Omega) = 1$. Let $f \in L^1(\Omega,\Sigma,\sigma)$ with $\int_\Omega |f| d\sigma$ is finite, f real. Then there exists a unique Stieltjes measure σ_f on $\mathbb{R}$ such that for all left-continuous g defined on $\mathbb{R}$ we have $st\left(\int_\Omega {}^*g(f)d\sigma\right) = \int_{-\infty}^{+\infty} g(t)d\sigma_f(t)$. In particular, $\int_\Omega f d\sigma =_1 \int_{-\infty}^{+\infty} t d\sigma_f(t)$. Furthermore, the function $\Phi(a) =_1 \int_{-\infty}^{+\infty} \exp(iaf(\omega))d\sigma(\omega)$ is S-continuous on ${}^*\mathbb{R}$ and positive definite, and so by Bochner's theorem $\Phi(x) =_1 \int_{-\infty}^{+\infty} e^{ixt} d\sigma_f(t)$ for all $x \in \mathbb{R}$.

The measure σ_f is called the spectral measure of f w.r.t. σ.
Let (X,Λ) be a standard measurable space and let $\mathcal{F}$ be a filter of probability measures on Λ. Let f be a bounded real Λ-measurable function defined on X such that for all $t \in \mathbb{R}$ we have $\mu({}^*\{x: f(x) \leq t\}) = \nu({}^*\{x: f(x) \leq t\})$ for all $\mu, \nu \in$ monad of $\mathcal{F}$. Then all the spectral measures μ_{*f}, $\mu \in \text{monad}(\mathcal{F})$ are equal and is called the asymptotic spectral measure of f w.r.t. $\mathcal{F}$. In this case we have that $\lim_{\mathcal{F}} \int {}^*g({}^*f)d\mu = \int_{-\infty}^{+\infty} g(t)dm_f(t)$, where m_f is the asymptotic spectral measure of f w.r.t. $\mathcal{F}$.
Applications are made to the case where $\mathcal{F}$ consists of the sequence $\{\frac{1}{n} \sum_1^n \delta_{x_k}\}_{n=1}^\infty$, $\{x_k\}_1^\infty \subset [0,1]$.

H. Osswald:

ON STOCHASTIC DIFFERENTIAL EQUATIONS WITH RESPECT TO SQUARE INTEGRABLE CONTINUOUS MARTINGALES

By means of results of Lindström and techniques used by Keisler for solving stochastic differential equations it was shown that the equation

$$x(\omega,t) = x_0(\omega) + \int_0^t f(\omega,s,x(\omega,s))d\lambda(s) + \int_0^t g(\omega,s,x(\omega,s))dm(\omega,s)$$

has an adapted continuous solution $x\colon \Omega\times[0,1]\to\mathbb{R}$ in an adapted Loeb space $(\Omega,L(\Omega),\hat{\mu},(b_{\hat{t}})_{\hat{t}\in[0,1]})$, if

(1) x_0 is a b_0-measurable initial condition

(2) $f, g : \Omega\times[0,1]\times\mathbb{R}\to\mathbb{R}$ are predictable in the first two arguments and continuous in the third argument.

(3) m is a square integrable continuous martingale with respect to $(b_{\hat{t}})_{\hat{t}\in[0,1]}$

(4) $\sup_{a\in\mathbb{R}}(f(\cdot,\cdot,a))$ is integrable with respect to $\hat{\mu}\times\lambda$ (where $\hat{\mu}$ is the Loeb measure and λ the Lebesgue measure on $[0,1]$).

(5) $\sup_{a\in\mathbb{R}}(g(\cdot,\cdot,a))^2$ is integrable w.r.t. the Doleans measure defined by m.

K. Potthoff:

APPLICATIONS OF NON-STANDARD METHODS TO GROUP THEORY

We show how non-standard methods can help to prove theorems about profinite groups and groups with finiteness conditions. For profinite groups we use that a profinite group as a projektive limit of finite groups is a homomorphic image of some star-finite group. Hence it is possible for example to transfer results from Finite Group Theory to Profinite Group Theory.
As an example a result of Mel'nikov concerning automorphism group of profinite group is improved from the abelian to the general case.
Some indications are made to show how star-finite sets and groups can be used to prove theorems for locally normal groups, FC-groups, etc..

M. Rašković:

MODEL THEORY FOR L_M LOGIC

In "Hyperfinite model theory" (1976) H.J. Keisler introduced several probability logics (L_P, $L(\ )_1$, etc.) and developed model theory for them together with D. Hoover.
We introduced L_M which instead of probability measure has a star-finite one and gave a method to transfer results from L_P to our logic.

J. R. Reveilles:

ALEXANDER SPANIER COHOMOLOGY AND NON-STANDARD ANALYSIS

Using internal set theory we give external definition of Alexander Spanier theory and verify Eilenberg Steenrod axioms. We examine duality and relation with differential forms.

T. Sari:

THE STROBOSCOPIC METHOD AND ITS APPLICATIONS IN PERTURBATION THEORY

L'ideé de base de la technique de stroboscopie est la suivante. Soit $f(t)$ une fonction de $\mathbb{R}$ dans $\mathbb{R}^n$. On suppose que pour chaque t il existe un nombre réel infiniment petit ε tel que la pente $\frac{f(t+\varepsilon)-f(t)}{\varepsilon}$ soit infiniment proche de $\Phi(f(t))$ ou $\Phi(x)$ est une fonction standard de $\mathbb{R}^n$ dans $\mathbb{R}^n$. Dans ces conditions $f(t)$ est infiniment proche d'une solution de l'équation differentielle $\frac{dx}{dt} = \Phi(x)$.
Comme conséquences de ce résultat, on obtient de nombreuse théorèmes classiques (dépendance continue des solutions en fonction des paramètres, théorèmes de moyennisation, invariants adiabatiques, ...). Cette technique de strobo copie permet aussi d'aborder l'étude de systémes differentiels pouvant réveler deux compartements differents, et qui echappent aux méthodes classique. Comme illustration on propose l'étude de l'équation differentielle

$$\frac{dx}{dt} = a + \sin\frac{tx}{\varepsilon} .$$

A. Stoll:

LEVY BROWNIAN MOTION WITH SEVERAL TIME PARAMETERS

P. Lévy's generalization of Brownian motion to the case of a multidimensional time set $\mathbb{R}^d$ is characterised by $E(L_x - L_y)^2 = \|x-y\|$. A non-standard construction of this Lévy Brownian motion is presented, which extends the well known non-standard representation of Brownian motion due to R.M. Anderson. Since it involves a non-standard construction of white noise, one obtains as a classical corollary a further white noise integral representation of Lévy Brownian motion, namely $L_x - L_y = c_d\int((x-z)|x-z|^{-(d+1)/2}-(y-z)|y-z|^{-(d+1)/2})Xdz$, where X is a d-dimensional white noise on $\mathbb{R}^d$. In the same manner as Donker's invariance principle follows from Anderson's construction, a new invariance principle can be deduced.

K. D. Stroyan:

BOUNDED WEAK-STAR CONTINUITY

The bounded weak-star topology arises as a kind of inductive limit on bounded subsets of a topological vector space. In holomorphic function spaces it is closely related to bounded sequential approximation, although the topology is never metrizable. On sequence spaces modeling economies, this topology characterizes impatient consumers. It has many applications, both pure and applied, and it has a rich family of continuous nonlinear functions (unlike the weak-star topology).
Using syntactical methods on predicates similar to the ones arising in topological model theory, we can give simple nonlinear continuity criteria for generalized inductive limits including bounded weak-star topologies, mixed topologies and ordinary linear inductive limits. The criteria use a kind of (non-topological) bounded infinitesimal (in Robinson's sense). They can be verified by computations in many concrete examples.
(Joint work with B. Benninghofen)

M. E. Szabo:

NON-STANDARD COMPUTATION THEORY

In this joint paper with M. Richter we examine a variety of infinitary aspects of computation theory (absence of a finite base for regular expressions; undecidability of the termination problem for computable functions; running times of loops; incompleteness of first order logics of programs; limit requirements for the theory of recursive procedures and higher order functionals, etc) and describe the relevance of the theory of hyperfinite sets and relations to the study of these problems. We exhibit a variety of monads which arise naturally in this context (for regular sets; finite state modicums; sequential programs; parallel programs; procedure terms; computable functionals; etc.) and show that they can be used to "finitize" the infinitary situations mentioned above. In this way we achieve a smooth theory which leads, among others, to a limit-free construction of a model of the lambda calculus and a plausible semantics for parallel programs (cf. the separate abstract by E.J. Farkas). Finally we discuss the use of ultraproducts in the proofs of hierarchy theorems and equivalence results in dynamic logic. We print out that many of the incomplete logics of programs (Floyd-Hoare, Burstall, Pomeli, Manna-Cooper, et.al.) become complete w.r.t. non-standard models and that these completeness theorems yield the only known methods for the construction of counter-examples to the provability of formulas in these logics.

F. Wattenberg:

APPLICATIONS OF NON-STANDARD ANALYSIS TO SOME FAMILIAR PDEs

The use of non-standard analysis allows us to rigorously derive partial differential equations from physical models. These lead to infinitesimal finite difference equations. Techniques of numerical analysis and non-standard analysis combine to give illuminating proofs of existence and uniqueness.

M. Wolff:

A REMARK ON FINITE-DIMENSIONAL REPRESENTATIONS OF BANACH RADICAL ALGEBRAS

Let A be a Banach radical algebra and let the Banach space X be a left A-modul. In this talk we present a necessary and sufficient condition for X to be finite-dimensional.

As applications of this result we obtain a characterization of finite-dimensional Banach algebras and in addition a characterization of Riesz points in the spectrum of a representation of a regular commutative Banach algebra.

R. T. Živaljević:

INTERNAL HOMOLOGY AND COHOMOLOGY THEORY

M.C. McCord in his paper "Non-standard analysis and homology", Fund. Math. 74/1, 21-28, established a homology theory based on hyperfinite chains of infinitesimal simplices. More precisely, McCord proved that this theory satisfies all axioms of Eilenberg and Steenrod including the exactness axiom. The last feature is interesting because the Čech theory, which is a direct motivation for this theory, does not satisfy this axiom in full generality, for example if the coefficient group is Z , the group of integers. Because of that McCord at the end of his paper asks:

(1) What is the relation of (so called) internal homology to Čech homology theory?

(2) What sort of invers limit continuity does the internal enjoy?

The author shall give an answer to these questions and try to convince the audience that this theory is worth further consideration.

Berichterstatter: D. Spalt (Darmstadt)

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