MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 1/1984

Mathematische Logik

2.1. bis 7.1.1984

Die Tagung fand unter der Leitung von Herrn W. Felscher (Tübingen) und Herrn H. Schwichtenberg (München) statt. An ihr nahmen 35 Wissenschaftler aus 6 Ländern teil, darunter 21 aus Deutschland.

Obgleich die Tagung nur vier Tage dauerte, wurden 27 Vorträge aus allen wichtigen Teilgebieten der mathematischen Logik gehalten. Einen besonderen Schwerpunkt bildete die Beweistheorie und konstruktive Mathematik mit insgesamt 12 Vorträgen und einer Abendveranstaltung, in der Herr S.G. Simpson (USA, z.Zt. München) über neuere Ergebnisse in Zusammenhang mit einem Satz von Kruskal berichtete. Fragen der mathematischen Logik im Zusammenhang mit der Informatik waren mit vier Vorträgen vertreten. Herr G. Goos (Karlsruhe) stellte sich in einer weiteren Abendveranstaltung für eine längere Diskussion seines Vortrags über logische Probleme in der praktischen Informatik zur Verfügung.

Weitere Schwerpunkte der Tagung bildeten die Modelltheorie mit einem Übersichtsvortrag von Herrn V. Weispfenning (Heidelberg) über Quantorenelimination sowie Mengenlehre, Rekursionstheorie und allgemeine Logik.

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Vortragsauszüge

K. AMBOS-SPIES: On the structure of polynomial time degrees

In the setting of complexity theory polynomial (time) degrees have been defined in analogy to degrees of unsolvability. They provide a classification of the computable but not feasibly computable (= deterministically computable in polynomial time) sets according to their complexity. We sorvey earlier work on the polynomial degrees by Ladner (1975) and Landweber, Lipton & Robertson (1981) and present some new results. We discuss algebraic properties of the upper semilattice of polynomial degrees (like distributivity) and look at sublattices of this structure. E.g. we show that any countable distributive lattice can be embedded in any interval of polynomial degrees by maps which preserve the least respective greatest element.

H. BARENDREGT: Typing lambda terms

In [3] Curry introduced a natural deduction system for assigning tipes to type free elements. Scott [4] gave a natural semantics for this theory by interpreting types as subsets of a λ-model. By extending the type system, as in Coppo et al. [2], it is possible to assign types to all λ-terms. Filters of these extended types form a λ-model. Using this model it is possible to prove completeness for extended type assignment. By a normalization argument extended type assignment is conservative over Curry type assignment. Therefore also for this theory completeness holds. These results will appear in [1].

[1] H. Barendregt, M. Coppo and M. Dezani: A filter lambda model and the completeness of type assignment, JSL, to appear.

[2] M. Coppo, M. Dezani and B. Venneri: Functional characters of solvable terms, Zeitschr. Math. Logik (1981), 45 - 58.

[3] H. Curry and R. Feys: Combinatory Logic I, North Holland, Amsterdam (1958).

[4] D. Scott: Open problem n° II 4 in: λ-calculus and computer science theory, ed. C. Böhm, LN Computer Science 37, Springer-Verlag, Berlin (1975), 369.

N. BRUNNER: Compact spaces in models of ZF - set theory

Variants of Tychonoff's product theorem and criteria, under what weaker conditions a topological space is compact are considered in several models of ZF^{o} (AC and the axiom of foundation are not assumed).

(1) Cohen Halpern Levy Model: There is an infinite, Dedekind-finite subset of $\mathbb{R}$ (Cohen) and a product of compact T_2 spaces is compact (Halpern, Levy). If there is a Dedekind set, then a T_1-space is compact, iff it is Lindelöf. Hence in this model a product of Lindelöf + T_2 spaces is Lindelöf (contradicting the AC-example of Sorgenfrey).

(2) Fraenkel-Halpern model (= least permutation model, Lin): A T_2 space is compact, iff it is metacompact, its wellorderable subsets are relatively compact and there is a multiple choice function on the family of its nonempty closed subsets. Applications: (a) A wellorderable product of compact T_2 spaces is compact. (b) A product of compact T_2 spaces is compact, iff it is metacompact.

(3) In the Fraenkel-Halpern model there are amorphous sets (infinite subsets are cofinite) and Ramsey's theorem RT holds (Blass). In ZF^{o} + RT X^I is compact, where X is compact, T_2 and a wellorderable $\underline{F} \subseteq C(X)$ separates points from closed sets and $I \subseteq [U]^n$, the n-element subsets of some amorphous set U.

A. CANTINI: A remark on dependent choice and comprehension principles in second order arithmetic

We prove: a) $\Sigma^1_{n+1}\text{-DC}\restriction \equiv (\Pi^1_n\text{-CA})_{<\omega}\omega\restriction$ for suitable classes of sentences;

b) $\Sigma^1_{n+1}\text{-DC}\restriction$ is stronger than $\Sigma^1_{n+1}\text{-AC}\restriction$.

The method of proof is based on a certain asymmetric interpretation of set quantifiers; it can be applied to give elementary proofs of the well-known results relating the subsystems based on choice and comprehension principles, without using the detour through Skolem theories à la Feferman-Sieg.

E. CASARI: Komparationstheorie und Mehrwertigkeit

Es wird eine Logik eingeführt, die eine befriedigende Behandlung der Verknüpfung "α ist höchstens so wahr wie β" $(\alpha \leqslant \beta)$ und der darauf liegenden Begriffsbildungen erlaubt. Aussagenkalkül: Klassischer Aussagenkalkül plus folgende Axiome:
$((\alpha \leqslant \alpha) \leqslant \beta) \leqslant \beta$; $(\alpha \leqslant \alpha) \leqslant (\beta \leqslant \beta)$; $(\alpha \leqslant \beta) \leqslant ((\beta \leqslant \gamma) \leqslant (\alpha \leqslant \gamma))$; $(\alpha \leqslant (\beta \leqslant \gamma)) \leqslant (\beta \leqslant (\alpha \leqslant \gamma))$; $\alpha \leqslant \neg\neg\alpha$; $\neg\neg\alpha \leqslant \alpha$; $(\alpha \leqslant \beta) \leqslant (\neg\beta \leqslant \neg\alpha)$; $\neg(\alpha \leqslant \beta) \leqslant (\beta \leqslant \alpha)$; $(\alpha \leqslant \beta) \to (\alpha \to \beta)$; $((\neg\alpha \leqslant (\beta \leqslant \beta)) \to \alpha) \to ((\neg\alpha \leqslant (\beta \leqslant \beta)) \leqslant \alpha)$; $\alpha \wedge \beta \leqslant \alpha$; $\alpha \wedge \beta \leqslant \beta$; $(\alpha \leqslant \beta) \wedge (\alpha \leqslant \gamma) \leqslant (\alpha \leqslant \beta \wedge \gamma)$; $\alpha \leqslant \alpha \vee \beta$; $\beta \leqslant \alpha \vee \beta$; $(\alpha \leqslant \gamma) \wedge (\beta \leqslant \gamma) \leqslant (\alpha \vee \beta \leqslant \gamma)$. Semantik: Bewertungen über komparative Systeme, d.h. Systeme $\langle G,\sqsubseteq,+,-,\varepsilon \rangle$ wobei: (1) $\langle G,\sqsubseteq,+ \rangle$ eine linear geordnete abelsche Halbgruppe ist; (2) $-$ ist eine Involution $(--x = x$; $x \sqsubseteq \varphi \to -\varphi \sqsubseteq -x)$; (3) $\langle -\varepsilon,\varepsilon \rangle$ ist ein Sprung $(-\varepsilon \sqsubset \varepsilon$; $x \sqsubseteq -\varepsilon \vee \varepsilon \sqsubseteq x)$; (4) $x + -x = \varepsilon$; $x + -\varepsilon = x$; $x \neq -\varepsilon \to x + \varepsilon = x$. Insbesondere ist der Wert $[\![\alpha \leqslant \beta]\!]$ als $-[\![\alpha]\!] + [\![\beta]\!]$ definiert (sonst, z.B. $[\![\alpha \wedge \beta]\!] = \min([\![\alpha]\!], [\![\beta]\!])$) . α gilt gdw $[\![\alpha]\!] \sqsupseteq \varepsilon$. Hierfür ist Vollständigkeit bewiesen. Prädikatenkalkül: neue Axiome: $\forall x\alpha \leqslant \alpha(t)$; $\alpha(t) \leqslant \exists x\alpha$; $\forall x(\alpha \leqslant \beta(x)) \leqslant (\alpha \leqslant \forall \alpha\beta)$; $\forall x(\alpha(x) \leqslant \beta) \leqslant (\exists x\alpha \leqslant \beta)$. Semantik: Realisierungen über Abbildungen von Potenzen einer Menge in ein komparatives System. Einschränkung der Realisierungen auf diejenigen, die überall definiert sind (die erforderten Inf und Sup existieren). Auch hierfür ist Vollständigkeit bewiesen. Bemerkung: $(\alpha \leqslant \exists x\beta) \leqslant \exists x(\alpha \leqslant \beta(x))$ und $(\forall x\alpha \leqslant \beta) \leqslant \exists x(\alpha(x) \leqslant \beta)$ sind nicht beweisbar.

P. CLOTE: Applications of the low basis theorem in arithmetic

We report on some applications of recursion theoretic techniques within the context of arithmetic to obtain model theoretic and combinatorial equivalences of Σ_n-collection and Σ_n-induction schemes. One possible application is to prove by model theoretic reasoning that certain fast growing recursive functions are provably total in Σ_n-induction. For instance,

$$I\Sigma_n (- \forall x \exists y \quad [x,y] \xrightarrow[*(k)]{} (n+2)^{n+1}_m \quad \text{for all} \quad m,k \in N$$

where the arrow relation $*^{(k)}$ means that the homogeneous set is of the form

$$X = \{x_0,\ldots,x_{x_0},\ldots,x_{x_{x_{\cdot_{\cdot_{\cdot_{x_0}}}}}}\ldots\} \quad k \text{ times}$$

(actually by the argument used, much more can be proven).

Theorem 1. For $n \geqslant 1$ and M a countable model of bounded induction $I\Sigma_0$, the following are equivalent:

(1) $M \models I\Sigma_n$

(2) M satisfies a type of collection scheme for Σ_n "partial functions":
$M \models \forall a \exists b \forall x \leqslant a(\exists y\varphi \leftrightarrow \exists y < b\varphi)$, where φ is any Π_{n-1} formula

(3) $M \xrightarrow[\Delta_n]{} (2)^1_M$ a version of the Δ_n infinitary pigeonhole scheme stating that for any Δ_n definable partition F of an unbounded Δ_n definable subset of M into M-finitely many pieces, there are two elements going into the same piece.

(4) $(M, \mathrm{Def}_0 M) \models w-\Sigma_n CA_0$ where $\mathrm{Def}_0 M$ is the collection of Σ_0 or boundedly definable subsets of M and $w-\Sigma_n CA_0$ is the system of second order arithmetic as in ACA_0 discussed by S.G. Simpson but with the comprehension scheme replaced by the weak Σ_n comprehension scheme
$\forall m \exists X \forall x \leqslant m(x \in X \leftrightarrow \varphi(x))$, where φ is Σ_n.

Corollary. (also noticed independently by Harrington, Paris and doubtlessly many others)
$I\Sigma_n$ is equivalent over $I\Sigma_0$ to the scheme of induction for the closure under bounded quantification of the collection of Boolean combinations of Σ_n formulas, denoted by $I\mathcal{B}^c_n$.

Corollary. The Σ_n definable points of a model of $I\Sigma_n$ are not cofinal in that model.

Remark. Independently and much earlier, H. Friedman discovered the equivalence of (1) and (2).

Theorem 2. For any (even uncountable model) M of bounded induction, if $n \geqslant 1$

$$M \models \Sigma_{n+1} \text{ collection iff } M \xrightarrow[\Delta_n]{} (M)^n_{<M}.$$

Theorem 3. For any countable model M of bounded induction and $n \geqslant 1$,

$$M \models \Sigma_{n+1} \text{ collection iff } M \xrightarrow[\Delta_n]{} (\text{order type } M)^{n+1}_{<M} .$$

Theorem 4. For any countable model M of bounded induction and $n \geqslant 1$,

$M \models \Sigma_{n+1}$ collection iff there exists a Δ_0-complete ultrafilter on the Δ_0 definable subsets of the Cartesian product M^n.

This last result is somewhat surprising, since Mills and Paris have proved that M satisfies Σ_{2n} collection iff the n-fold filter product $\underbrace{\mathcal{F} \times \ldots \times \mathcal{F}}_{k \text{ times}}$ of the Frêchet filter (collection of co-bounded sets) is Δ_0-complete. Theorems 2, 3, 4 make essential use of a formalization of the "low basis theorem" in recursion theory, due to C.G. Jockusch, Jr. and R.I. Soare, the idea being that an object is low iff all its one-quantifier consequences have the same definitional complexity as the object itself.

D. van DALEN: Some Problems in Constructive Group Theory

So for constructive group theory has mainly been practised in recursive mathematics or in the frame work of 'discrete' algebra, i.e. with decidable equality. Extra difficulties arise when one allows more general structures with an arbitrary equality relation, with or without an apartness relation. As a test problem we consider the existence of free abelian groups. Evidently one cannot restrict the attention to products of $\mathbb{Z}$, since that would presuppose distinctness of the generators. We construct the free abelian group over a set S by first introducing the "positive" free monoid part and next extending this part to the full group. The method is completely general. The well-known theorems on free abelian groups, however, fail. In particular (e.g.) $\mathbb{Z}$ contains subgroups which are not free. In general free abelian groups do not carry an apartness relation.

J. DILLER: <u>Zur Schnittelimination im Schütte - Kalkül</u>

Zu Schüttes System der klassischen (KPL) oder der intuitionistischen (IPL) Prädikatenlogik oder der verzweigten Analysis RA* fügen wir folgende Schnittregel hinzu:
(r-cut) $\mathfrak{P}(A)$, $A \to B \vdash C$, wenn $\mathfrak{P}(B) \vdash^{s} C$ und rang $A < r$.
Herleitungen bei Schütte sind i.a. kürzer als bei Tait 1967; dagegen ist Taits (r-cut) stärker als Schüttes. Trotzdem gilt wie bei Tait das
Reduktionslemma. Aus $\vdash^{k}_{r} \mathfrak{P}(A)$ und $\vdash^{1}_{r} A \to B$ folgt $\vdash^{k+1}_{r} \mathfrak{P}(B)$,
falls rang $A \leqslant r$ ist.
Dabei kann man den Rang sparsamer als üblich definieren, nämlich in KPL und RA* rang$(\neg A)$ = rang A und
rang$(A \overset{\to}{\underset{\vee}{\wedge}} B)$ = max(rang $A + 1$, rang B) , falls $B \not\equiv \bot$ ist, und in IPL rang$(A \overset{\to}{\wedge} B)$ wie oben, rang$(A \vee B)$ = max(rang A , rang B) und rang$(\exists x F(x))$ = rang$(F(a))$.

U. FELGNER: <u>Quantifier eliminable FC-groups</u>

A group $\underset{\sim}{G} = \langle G, \cdot, ^{-1}, 1 \rangle$ admits elimination of quantifiers if for each formula Φ there is a quantifier-free formula Ψ such that $\Phi \leftrightarrow \Psi$ holds in $\underset{\sim}{G}$. A group $\underset{\sim}{G}$ is called an FC-group if each element possesses only a finite number of conjugates in $\underset{\sim}{G}$. Clearly, all abelian groups and all finite groups are FC-groups. We have proved the following <u>theorem</u>: Let $\underset{\sim}{G}$ be an FC-group. Then $\underset{\sim}{G}$ admits elimination of quantifiers if and only if $\underset{\sim}{G}$ has the form $\underset{\sim}{A} \oplus \underset{\sim}{F}$ where $\underset{\sim}{A}$ is an abelian group admitting elimination of quantifiers and $\underset{\sim}{F}$ is a finite group admitting elimination of quantifiers. Moreover, if $\underset{\sim}{A}$ is divisible then $\underset{\sim}{F} = \{1\}$, and if $\underset{\sim}{A}$ has finite exponent d then $|F|$ and d are relatively prime.
Thus, together with the previous classification of all quantifier eliminable abelian groups and quantifier eliminable finite groups (joint work with G. Cherlin) the above theorem yields a classification of all quantifier eliminable FC-groups.

G. GOOS: <u>Problems in Mathematical Logic as viewed by a practical Informatician</u>

Computer scientists working in the area of software construction are faced with many problems belonging into the area of Mathematical

Logic. This talk reports about such problems in the areas of program verification, logic programming and program specification, abstract data types and the practical significance of results in the areas of dynamic and temporal logic, unification theory etc. It will be stressed that very often the theoretically best methods are practically not feasible because the required amount of formalism is too high. Also incompleteness and undecidability results are often less important than it may seen on the first view.

L. GORDEEW: Collapsing functions

We show an alternative way of defining various systems of ordinal notations, particularly for those before Howard ordinal $|ID_1|$. This approach is based on appropriate axiomatization of the notion of collapsing function (known in proof theory). In contrast to standard methods, no normal-function terminology in the sense of Veblen - Bachmann hierarchies is involved in our formalisms. Instead, we show how those functions are definable in terms of collapsing functions. A minimal model of our theory for $|ID_1|$ is shown to be given by a short and simple cut-free calculus, which provides a simple elementary (in the rec.-theor. sense) system of ordinal notations for $|ID_1|$. By iterating this method one can produce simple theories and models for ordinals corresponding to iterated inductive definition theories as well.

H.R. JERVELL: Π^1_n - Completeness

A completeness theorem for Π^1_n - logic is given.
We discuss the kind of objects needed for such a theorem.

H. KOTLARSKI: False Truth and Bounded Induction

Let Δ_o-PA(S) denote the theory PA + S is a full satisfaction class + Δ_o - induction in $L_{PA} \cup \{S\}$. See [2] for the notion of a full satisfaction class.

THEOREM 1. Let $M \models PA$, let S be a full satisfaction class on M . Suppose $(M,S) \models \forall\varphi[PA \vdash \varphi \rightarrow S(\varphi,\langle\rangle)]$. Then $(M,S) \models \Delta_o - PA(S)$.

COROLLARY 2. $\Delta_o - PA(S)$ is finitely axiomatisable.

COROLLARY 3. If $N \subseteq M$ is such that $S \cap N$ is a full satisfaction class on N, then if $(M,S) \models \Delta_o - PA(S)$, then $(N,S) \models \Delta_o - PA(S)$.

Observe that if we assume that $N \subset_{end} M$ then the assumption that S is full on both models is not needed.

DEFINITION 4. $F(0)$ = the Gödel number of the formula $v_2 = v_1 + 1$
$F(i+1) = \min y : \forall \varphi \in \Sigma_i \ \forall u \leq F(i) [\varphi \leq F(i) \ \& \ \exists z S(\varphi, u^\frown z) \rightarrow$
$\rightarrow \exists z \leq y \ S(\varphi, u^\frown z)]$. This definition written explicitely is Σ in the language of $\Delta_o - PA(S)$, nevertheless

LEMMA 5. $\Delta_o - PA(S) \vdash \forall i \in F(i)$.

THEOREM 6. If G is a Δ_o formula of $L_{PA} \cup \{S\}$ and $\Delta_o - PA(S) \vdash \forall x \exists y \ G(x,y)$, then there is $k \in \omega$ such that $\Delta_o - PA(S) \vdash \forall x \exists y < F^k(x) \ G(x,y)$. Here F^k denotes F composed k times with itself.

THEOREM 7. $\{\varphi \in L_{PA} : \Delta_o - PA(S) \vdash \varphi\}$ is axiomatised by PA + full reflection principle.

This result follows from Ratajczyk's work [3] and theorem 1. See also [1] for truth notions which are more false than those considered here.

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[3] Ratajczyk, Satisfaction classes and sentences independent from PA , Zeitschr. Math. Log. 28(2), 1982.

H. LUCKHARDT: <u>On the complexity of TAUT - decisions</u>

TAUT - algorithms and their complexity analysis are given for the following boolean subclasses to which every boolean formula can be easily reduced:

(i) generalized disjunctive normal forms GDNF: classes are built up from literals by $\wedge$ and $\vee$, $\wedge$ outside;

(ii) restricted disjunctive normal forms RDNF where a variable occurs at most once positively or negatively.

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Our algorithms work in polynomial space $S = O(|i|^2)$ and time $T(i) = P(|i|)\cdot 2^{h\cdot n}$ (i input, P a low degree polynomial, n the number of variables). h is a finite measure for the extent of the reduction employed. It is shown how to compute h . - For formulae having clauses with $p \geqslant 3$ variables our result is as follows: over p-GDNF h(p) starts with 0,6943 and tends to 1 as $p \to \infty$; on p-RDNF h(p) behaves quite different: it goes from 0,5286 to 0 .

A.R.D. MATHIAS: Unsound Ordinals

For Θ an ordinal, $A = A_0, A_1, A_2, \ldots$ a sequence of subsets of Θ, and $x \subseteq \omega$, define $\tau_A(a)$ = the ordinal that is the order type of $\bigcup\{A(n) \mid n \in a\}$.

Θ is called sound if for every A , $\{\tau_A(a) \mid a \subseteq \omega\}$ is countable, and unsound if for some A , $\{\tau_A(a) \mid a \subseteq \omega\}$ is uncountable.

THEOREM Assume that ω_1 is regular. Then

(i) every ordinal less than $\omega_1^{\omega+2}$ is sound

(ii) if $\aleph_1 < 2^{\aleph_0}$, $\omega_1^{\omega+2}$ is unsound

(iii) if $\aleph_1 \nless 2^{\aleph_0}$, every ordinal less than $\omega_1^{\omega+\omega+1}$ is sound.

PROBLEMS: Is it consistent with ZF that every ordinal is sound?

If ω_1 is singular, is it unsound?

If there is an unsound ordinal, what is the least such?

If $\aleph_1 \nless 2^{\aleph_0}$ and ω_1 is regular, are $\omega_1^{\omega+\omega+1}$ and $\omega_1^{\omega_1}$ sound?

Using a lemma of Kechris and a reflection argument, Woodin has shown that under the Axiom of Determinacy there is an unsound ordinal less than ω_2 .

I. MOERDYK: Heine - Borel does not imply the Fan Theorem

We consider four formal spaces, or locales, namely formal Cantor space C , formal Baire space B , the formal real line R , and the formal functionspace R^R . Classically, these locales all have enough points of course, but intuitionistically or in (Grothendieck) toposes this may fail in each case. In fact, C has enough points iff the Fan Theorem (FT) holds, i.e. the Cantor space $2^{\mathbb{N}}$ is compact; B has enough points iff Bar Induction (BI) holds; R has enough points iff the Heine - Borel theorem (HB) holds, i.e. the space of Dedekind reals is locally compact. "R^R has enough points" will be abbreviated by (EF). We show that in Grothendieck toposes (in HAH, or in intuitionistic ZF) the following implications are the only ones that hold:

$$\begin{array}{ccc} (BI) & \Rightarrow & (FT) \\ \Downarrow & & \Downarrow \\ (EF) & \Rightarrow & (HB) \end{array}$$

A. OBERSCHELP: Zur sogenannten Biberfunktion

Ein "fleißiger Biber" ist eine Turing-Maschine, die auf das leere Band angesetzt zum Stoppen kommt. Er ist um so fleißiger, je mehr Marken er auf dem Stop-Band hinterläßt. B(n,m) ist diese Zahl für einen fleißigsten Biber mit n Zuständen und m Symbolen. Es ist leicht zu sehen (Rado 1962), daß B nicht berechenbar ist.

Diese Funktion ist bisher für $m = 1$ und für Turing-Maschinen betrachtet worden, die in einem Rechenschritt zugleich drucken und bewegen können (Quintupelmaschinen) und für die Stoppen ein zusätzlicher Zustand ist. Hier betrachten wir Maschinen, die in einem Rechenschritt nur eine dieser Tätigkeiten durchführen (Quadrupelmaschinen) und im Stopschritt keine davon. Zusätzlich lassen wir auch mehr als ein Symbol zu.

Satz: $\langle m \mapsto B(3,m)\rangle$ ist nicht berechenbar.

Satz: $\langle m \mapsto B(2,m)\rangle$ ist berechenbar und wächst quadratisch mit m.

Einige Funktionswerte: $B(1,m) = 1$, $B(2,1) = 2$, $B(3,1) = 3$, $B(4,1) = 8$, $B(5,1) = 15$, $B(2,2) = 4$, $B(2,3) = 5$, $B(3,2) \geq 12$.

(Untersuchung gemeinsam mit Karsten Schmidt und Günther Todt. Bestimmung von B(5,1) durch Klaus Muus und Holger Sönnichsen)

P. PÄPPINGHAUS: Categories of Terms for Functors over the Ordinals

Sei $PT^O = ON$ die Kategorie der Ordinalzahlen mit den ordnungserhaltenden Abbildungen als Morphismen, und sei $PT^{\sigma\to\tau}$ die Kategorie der Funktoren von PT^σ in PT^τ, die direkte Limiten und pull-backs erhalten, mit den natürlichen Transformationen als

Morphismen. Girard hat gezeigt, daß die PT^{σ} ein Modell einer Variante von Gödels T sind. Es werden Kategorien von unendlichen Termen definiert, die ebenfalls die PT^{σ} als Modell haben. Ferner wird ein Längenbegriff für diese Terme definiert, wobei die Längen Funktoren aus $PT^{o\to o}$ sind. Gödels T wird in die Term-Kategorien eingebettet, die unendlichen Terme werden reduziert und normalisiert. Es gelten dafür die folgenden Abschätzungen:

$$|t[x/s]| \leq |s|\cdot|t|$$

$$|RED(t)| \leq \underline{2}^{\underline{2}^{|t|}}$$

$$|NF(t)| \leq \left.\underline{2}^{\cdot^{\cdot^{\cdot^{\underline{2}^{|t|}}}}}\right\} 2.\mathrm{Rang}(t) \text{ oft.}$$

Für Übersetzungen von Termen aus $T \cup \{\omega\}$ gilt: $|t| \leq \underline{\omega}^{\underline{1}+Id+\underline{1}}$. Der Wert von abgeschlossenen Termen vom Typ o in Normalform läßt sich mit Hilfe eines Funktors $\Lambda \in PT^{(o\to o)\to(o\to o)}$ wie folgt abschätzen:

$$\mathrm{Val}(t) \leq \Lambda|t|0 .$$

Damit ergibt sich für abgeschlossene Terme aus T vom Typ $o \to o$ die folgende Abschätzung:

$$\mathrm{Val}(t\omega) \leq \Lambda \underline{2}^{\cdot^{\cdot^{\underline{2}^{\underline{\omega}^{\underline{1}+Id+\underline{1}}}}}} 0 .$$

H. PFEIFFER: <u>Ein Bezeichnungssystem für Ordinalzahlen</u>

In Verallgemeinerung des Systems der $\overline{\Theta}$-Funktionen von W. Buchholz ("Normalfunktionen und konstruktive Systeme von Ordinalzahlen", Springer Lecture Notes in Math., 500) und des Systems $\theta(I)$, das K. Schütte in einer unveröffentlichten Arbeit mit dem Titel "Das Ordinalzahlensystem $\overline{\theta}(I)$" 1980 aufgestellt hat, wird ein abstraktes Bezeichnungssystem $\underset{\sim}{\Theta}$ für Ordinalzahlen angegeben. Es enthält vermutlich Bezeichnungen für alle Mahloschen π_{ν}-Zahlen kleiner als die ρ_{o}-Zahl von Mahlo bzw. deren rekursive Analoga. Der Wohlordnungsbeweis für das System $\underset{\sim}{\Theta}$ wird unter geeigneter Erweiterung des Begriffsapparates geführt, den Buchholz in dem Beweis für das oben erwähnte System benutzt.

M.M. RICHTER: Program Verification

We use the area of program verification in order to discuss the interplay between logic, mathematics and computer science. Program verification is embedded in the bigger area of producing reliable software and has therefore many non-logical aspects. The logic enters the picture first on the level of principles (e.g. dynamic logic, complete systems for first order predicate calculus) but appears also at various much more concrete levels. Examples are equational theories, systems of reductions; in particular a special joint project for the gradual development and verification of microprograms (with W. Damm, H. Langmaack, V. Penner) is mentioned. In this context some recent work of W. Zadrozny on the axiomatization of floating point arithmetic is finally discussed.

G. SAMBIN: Some proof theory for the modal logic of provability GL

GL is defined as the logic of modal propositional formulae which, when $\Box$ is interpreted as Pr_{PA} , give rise to theorems of PA for any assignment of formulae of PA to propositional variables. A short survey of GL is given, including completeness w.r.t. Kripke semantics, decidability, fixed point theorem etc. Particular attention is given to cut elimination for a sequent calculus for GL .

B. SCARPELLINI: Diagonalisierung und Längen von Formeln

U. SCHMERL: Diophantische Gleichungen in schwachen Systemen der Arithmetik

Es werden drei (relativ schwache) formale Systeme der Arithmetik vorgestellt und jeweils Charakterisierungen angegeben, welche diophantischen Gleichungen in diesen Systemen entschieden werden können.

S.G. SIMPSON: Subsystems of second order arithmetic

Given a theorem τ of ordinary mathematics, it is possible to ask: what is the weakest natural subsystem $S(\tau)$ of second-order arithmetic in which τ is provable? We investigate this question for specific theorems τ . It turns out that very often $S(\tau)$ is one of the five systems RCA_0 (recursive comprehension), WKL_0 (weak König's lemma), ACA_0 (arithmetic comprehension), ATR_0 (arithmetic transfinite recursion), Π^1_1-CA_0 (Π^1_1 comprehension). Furthermore, these results often turn out to be best possible in the sense that τ is provably equivalent to the principal axiom of $S(\tau)$ over the weak base system RCA_0 . For example, the Hahn-Banach theorem for separable Banach spaces is equivalent to weak König's lemma, provably over RCA_0 ; the theorem that every countable commutative ring has a maximal ideal is equivalent to arithmetic comprehension, provably over RCA_0 ; open determinacy is equivalent to arithmetic transfinite recursion, provably over RCA_0 . We develop also the philosophical implications of such results.

G. TAKEUTI: Globalization in set theory

In order to treat global notions in the same framwork with local notions, we introduce a new logical connective globalization. We discuss logical axioms on globalization and a formulation of set theory with globalization.

W. THOMAS: An application of the Ehrenfeucht-Fraïssé game to formal language theory

The class of star-free regular word-sets over a given alphabet $\mathbb{A}$ is the smallest class which contains the finite subsets of $\mathbb{A}^*$ and is closed under boolean operations and concatenation product. A well known classification of these word-sets is the so-called dot-depth hierarchy, in which word-sets are distinguished by the "levels of concatenation" used in their definition. Using a characterization of this hierarchy in terms of quantifier complexity of first-order sentences, we present a proof that the hierarchy is infinite, based on a suitable version of the Ehrenfeucht-Fraïssé game. This rather simple application of a model-theoretic method eliminates the heavy use of semigroup theory that appears in the original hierarchy proof (given by Brzozowski and Knast in 1978).

V. WEISPFENNING: A Survey on Quantifier Elimination

We give a survey on the method of quantifier elimination in model theory and its applications and connections to algebra, following roughly the historical development of ideas and results.

Prehistory: Uniform explicit methods in algebra: Linear and algebraic elimination theory.

Period 1 ($\geqslant$ 1919) is characterized by explicit, primitive recursive quantifier elimination procedures with applications to elementary invariants and decidability. We review the results up to 1950 and emphasize: The combinatorial nature of these procedures, the close connections in tools to algebraic elimination theory, the problem of finding a language appropriate for quantifier elimination.

Period 2 ($\geqslant$ 1950): Introduction of indirect methods for quantifier elimination by A. Robinson associated with the concepts of model-completeness and substructure-completeness, using compactness and diagrams. This opens the way for an application of structural algebra to quantifier elimination and leads to many deep theorems, in particular in field theory. We sketch the method of quantifier elimination relative to a set Ψ of formulas with weak closure conditions and give an interpolation-style persistence theorem.

Period 3 ($\geqslant$ 1970) We distinguish three new developments:

(i) quantifier elimination for generalized languages,

(ii) characterization of algebraic q.e. structures in their basic language and natural extensions of this language. We discuss the results and some problems for fields, rings, differential fields, ordered rings, abelian groups, modules, ordered ab. groups, lattices, boolean algebras and Heyting algebras,

(iii) feasability: quantifier elimination obtained by indirect means is usually general recursive by the completeness theorem. Explicit procedures for natural algebraic theories have turned out to be primitive recursive. Since polynomial time procedures cannot be expected, the interest is in Kalmar-elementary procedures; they require as a rule new mathematical ideas (e.g. integer and real addition, real and valued fields).

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