MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# T a g u n g s b e r i c h t 2/1984

## Mathematische Theorien der Fluide

9.1. bis 13.1.1984

Die Tagung wurde von den Professoren Wolfgang Bürger (Karlsruhe) und Ingo Müller (Berlin) organisiert.

Es waren 47 Teilnehmer anwesend, die durch interessante Vorträge und Diskussionen zum Gelingen dieser Tagung beigetragen haben.

Im Mittelpunkt des Interesses standen Fragen zur mathematischen Formulierung von Zustandsgleichungen unterschiedlicher Materialien wie Plasmen, polare Flüssigkeiten, Mehrphasenmischungen, Polymernetzwerke, Glas, reale Gase, klassische und relativistische entartete Gase, viskoelastische und plastische Körper sowie Elektrolyte.

Eine größere Gruppe von Wissenschaftlern hat Zustandsgleichungen an konkreten Randwertproblemen überprüft. Dabei wurden besonders Wellenphänomene, die Stabilität von Strömungen und Eindeutigkeitsprobleme behandelt. Der Forschungsgegenstand einer anderen Gruppe war die mathematische Formulierung der neuen erweiterten Thermodynamik. Neue Entwicklungen hinsichtlich der mathematischen Beschreibung der Plastizität und von Phasenübergängen in Memory-Legierungen konnten ebenfalls diskutiert werden.

Abgerundet wurde die Tagung durch mehrere Vorträge über Grundlagenprobleme der irreversiblen Thermodynamik.

Die 46 Vorträge sind in 10 Sektionen diskutiert worden:

i.) Wave Propagation
ii.) Stability
iii.) Elasticity

iv.) Non-Newtonian Fluids
v.) Polymers and Glasses
vi.) Kinetic Theory
vii.) Hyperbolicity and Extended Thermodynamics
viii.) Variational Principles
ix.) Irreversible Thermodynamics
x.) Plasticity and Phase Changes

Besonderen Anklang fand ein dreistündiger Vortrag von Prof. W. Bürger über die Physik von Spielzeugen mit vielen überraschenden Experimenten.

Vortragsauszüge

An after-dinner talk:

W. BÜRGER

On Physical Toys

Historians of science have found out that the human playfulness is one of the main sources of scientific and technological progress, rather than the necessity (which a well-known proverb calls the "mother of invention"). Playing with physical toys may be a key to surprising experiences. Toys often explain a problem of "severe" physics by analogy, or contain the nucleus of an engineering problem. The "tail-waving dragon" which the child draws behind him has a counterpart in the trailer problem for trucks. The "pick-a-back ball" helps to make clear the swingby manoeuvre of a space probe at the planet Jupiter to the Layman. In the lecture, a large number of physical toys was demonstrated, among others the Newton's cradle, the magic windmill, various types of yo-yo, the tippetop, the rattleback, the corruga horn and the "slinky", the walking spring. Their history was briefly mentioned and their well-known or rather surprising dynamical behaviour explained with simple mathematical models.

Section: Wave Propagation

A.M. ANILE & G. RUSSO

## Propagation of Weak Shock Waves

Propagation of shock waves admits an analytical approach only under very special circumstances. For the case of weak shock waves the two main approximation schemes are the shock-fitting method and non-linearization technique (Chandrasekhar, Landau and Whitham).

The theory exposed is applicable in case of arbitrary geometry, initial profile and unperturbed state. It has been developed for general quasi-linear hyperbolic systems in conservation form and gives a first order approximation for the amplitude and position of the shock as a function of time.

The jump in the field at the shock is expanded in terms of a small parameter and Rankine-Hugoniot relations and $1^{st}$ and $2^{nd}$ order compatibility conditions across the discontinuity surface are used to the different order in the small parameters. A couple of transport equations along the rays is obtained for the amplitude of the shock and the amplitude of the jump in the normal gradient of the field.

Applications are shown in the case of propagation in a constant state and in an atmosphere with power law unperturbed state.

Y.A. BEREZIN

## Riemann's Waves in the Media with Anomalous Equations of State

All the processes in continuum media are described by the equations of Navier-Stokes type. Any concrete problem is realized by setting the geometry, initial and boundary conditions, and equation of state (EOS). The EOS relates the thermodynamic functions: pressure, density, temperature etc. Normal gases have the following properties: $(\partial p/\partial V)_T < 0$, $(\partial^2 p/\partial V^2)_T > 0$. There are the EOS of a different kind: 1) $(\partial p/\partial V)_T > 0$ in some domain; 2) $(\partial p/\partial V)_T < 0$ everywhere, and $(\partial^2 p/\partial V^2)_T$ changes the sign in a few points. The dynamical wave processes in the media with such EOS are studied analytically and numerically. Both cases turn out to exist in superdense matter of the neutron stars. Shock waves of pressure and rarefaction are considered.

M. HAYES

Inhoomogenous Plane Waves in Viscous Fluids

Gibbs (1981) called the combination $\underline{a} + i\underline{b}$ where $\underline{a}$ and $\underline{b}$ are real vectors, a "bivector". He pointed out that associated with each bivector there is an ellipse, the ellipse which has $\underline{a}$ and $\underline{b}$ as conjugate semi-diameters . Also if $\underline{A} = \underline{a} + i\underline{b}$, then $e^{i\theta}(\underline{A}) = c + i\underline{d}$ (say) where $\underline{c}$, $\underline{d}$ is another pair of conjugate semi-diameters of the same ellipse.

In considering elliptically polarized inhomogenous plane waves

$$\underline{v} = \underline{A} \exp i\omega (\underline{S}\underline{n} - t), \ \underline{A}, \underline{S} \text{ bivectors}$$

use of the Gibbs bivectors leads to a simple way of visualizung the motion. For $\underline{x} = x^{\alpha}$ (say), if the velocity is along some conjugate semi-diamter the $\partial \underline{v}/\partial t|_{x=x^{\alpha}}$ is along its conjugate.

If $\underline{A} \cdot \underline{B} = 0$, then in general the ellipses of $\underline{A}$ and $\underline{B}$ may not lie on planes which are orthogonal. Also, the projection of the ellipse of $\underline{A}$ upon the plane of the ellipse of $\underline{B}$ is an ellipse which when rotated through a quadrant is similar (same aspect ratio) and similarly situated (major axes ||) tc the ellipse of $\underline{B}$. If $\underline{A} \cdot \underline{A} = 0$,the ellipse is a circle.

In this paper the propagation of some waves is considered in linearly viscous incompressible fluids with constitutive equation

$$t_{ij} = - p\delta_{ij} + \mu (v_{i,j} + v_{j,i}), \qquad v_{i,i} = 0.$$

F. MAINARDI

## Dissipative Effects on Waves in Liquid-Filled Elastic Tubes

For wave propagation in liquid-filled elastic tubes, dissipative effects are essentially due to viscosity of the liquid and visco-elasticity of the tube wall. The two effects are here treated separately in different circumstances, based on recent works by the author.

Viscous effects are considered in view of shock-wave formation, neglecting the radial inertia and using a boundary-layer approximation for the flow. The evolution equation turns out to be a non-linear integro-differential equation known in gas-dynamics as Chester's equation, treated recently by J.J. Keller.

Viscoelastic effects are considered in view of linear dispersive waves, taking into account the radial inertia and adopting the Voigt model. The dispersion relationship turns out to be of K-dV-Burgers type and the solution of a pulse-signalling problem appears in good qualitative agreement with some experimental results.

K. SUCHY

## Constitutive Relations for Media Slowly Varying in Space and Time

A system of linear first-order differential equations (balance equations) is used to derive a constitutive relation between a driving force (e.g. an electric field) and a current (e.g. the electrical current density in a conducting medium). The coefficients of the balance equations vary slowly in space and time. Force and current are Fourier represented with amplitudes slowly varying in space and time, while the Fourier kernel $\exp i(\underline{k} \cdot \underline{x} - \omega t)$ varies fast in space and time. The operator of the Fourier analysed balance equations is not merely algebraic but has an additional differential term. It is inverted with a perturbation method. The Fourier synthesis of the inverted Fourier analysed balance equations gives the

following result:
The integral kernel of the constitutive relation depends on a fast varying space-time scale and a slowly varying one. The first appears in the usual convolution, the latter is identical to the slowly varying space-time scale of the current (in contrast to assumptions sometimes made in the literature).

Z. WESOLOWSKI

Dynamics of One-Dimensional Model of Composite Material

Two parallel rods of different elastic moduliinteract with each other with force poroportional to the difference of the (axial) displacements. Basing on the dispersion relation the solution in form of the Fourier series of the initial problem is calculated.For short time the discontinuity curve has the speeds $c_1, c_2$ corresponding to the first and second rod, respectively. For large time the displacements of both rods are approximately equal and the curve profile has the speed $c = (c_1^2+c_2^2)/2$. Analogous result holds for the rods interacting with force proportional to the difference of speeds.

S. ZAHORSKI

Pressure and Surface-Tension Effects on Surface-Type Waves in Viscoelastic Fluids

The conditions of propagation and damping of small-amplitude harmonic waves in the neighbourhood of plane boundaries or interfaces in compressible and incompressible viscoelastic fluids were discussed elsewhere. In the present contribution the main attention is paid to a possible effect of the hydrodynamic pressure as well as the surface or interface tension on the type and speed of the waves considered. Some examples of homogenous fluids with free or rigid surfaces and two-layer immiscible fluids sliding freely at horizontal interfaces are presented in greater detail.

Section: Stability

U. AKBAY

## Instabilitäten viskosimetrischer Strömungen

An dem Beispiel der einfachen Scherströmung werden zwei verschiedene Instabilitätsmechanismen dargelegt. Es sind:

a) Instabilitäten der einfachen, schleichenden Scherströmung für eine einfache Klasse der Störungen. Entgegengesetzt der gängigen Meinung kann auch die schleichende Scherströmung einer Maxwell-Flüssigkeit instabil werden, falls die Wandgeschwindigkeit $U_w$ bei einer gegebenen Spaltbreite einen kritischen Wert überschreitet (siehe Abb. 1)

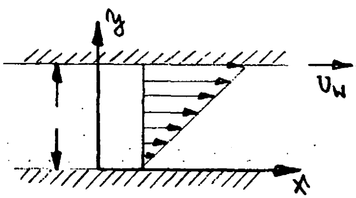


Abb. 1

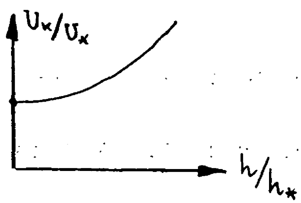


Abb. 2

In dem Modell der Maxwell-Flüssigkeit ist im Gegensatz zur Newton'schen Flüssigkeit ein zusätzlicher Bezugsmaßstab für die Zeit (Relaxationszeit) vorhanden. Mit der kinematischen Viskosität $\nu$ und $\lambda$ kann man zwei Stoffgrößen bilden, die die Dimension einer Geschwindigkeit (U ) und einer Länge (h ) haben. Trägt man die auf $U_*$ bezogene kritische Wandgeschwindigkeit in Abhängigkeit von $h/h_*$ auf, so erhält man den in Abb. 2 qualitativ dargestellten Verlauf und sieht ein, daß auch im Fall Re → 0 eine Instabilität möglich ist.

b) Verschiedene experimentelle Arbeiten berichten von einer Instabilität, die von der Oberfläche der untersuchten Flüssigkeit ausgeht und der Messung Grenzen setzt. Den Mechanismus dieser Instabilität kann man verstehen, wenn man das in Abb. 3 dargestellte Problem zwischen zwei parallelen Wänden und freier Oberfläche studiert. Um die zusätzliche Komplikation durch Oberflächenkrümmung auszuschalten, sei zuerst das Problem der geraden

Oberfläche in ungestörter Form betrachtet. (Die Kanten an dem Flüssigkeitsniveau seien scharf).

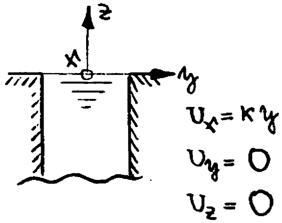


Abb. 3

Untersucht man dieses Problem mit Langwellenansatz und für konstante Stoffdaten, so ist auch die gestörte Strömung viskometrisch, und man erhält für diesen Grenzfall ein recht allgemeines Ergebnis für die kritische Schergeschwindigkeit $\kappa_k$

$$\kappa_k^2 = - \frac{\sigma}{hN_2} \cdot \frac{2}{\pi}$$

wobei h die Spaltbreite, $N_2$ den zweiten Normalspannungskoeffizienten und $\sigma$ die Oberflächenspannung bedeuten.

G.P. GALDI & B. STRAUGHAN

Exchange of Stabilities and the Energy Method for Hydrodynamic Stability

It is shown how the energy method is closely connected with the so-called "exchange of stabilities", and how one might explore in a simple but systematic way the connection between linear and nonlinear stability for a class of convection problems. If time permits, a convection-like process which develops in a suspension of swimming micro-organisms will also be investigated using the energy method.

Section: Elasticity

C. BEEVERS

Materials with a Crust

A. GOLEBIEWSKA-HERRMANN & G. HERRMANN

## Material Balance Laws and Blasius Theorem

One aim of this contribution is to show that a Lagrangian formulation of continuum mechanics leads not only to equations of motion in physical space (or balance of linear physical momentum) but also to conservation laws related to the material symmetries in a perfect continuum. These conservation laws, in the presence of non-homogeneities lead to material balance laws and so-called path-independent integrals. The quantity playing the role of the physical stress tensor in this formulation is the material momentum tensor. - Some of these concepts have been used successfully in fracture mechanics of solids. Here it is shown that extension and application to fluid dynamics is possible. Thus, material forces and moments acting on inclusions, such cylinders or airfoils in an ideal fluid. may be determined within the same mathematical framework of Lagrangian mechanics as defects (e.g. cracks) in elastic solids. - As an example, material forces and moments acting on a rigid inclusion in an ideal fluid (Blasius theorem) are calculated providing new insight into this classical problem.

R.J. KNOPS

## Uniqueness in Some Simple Displacement Boundary Value Problems of Non-Linear Elasticity

Uniqueness of the strong solution is proved to the boundary value problem of non-linear elasticity in which the displacement on the surfaces is affine (of the form $F\mathbf{X} + b$, $F$ a constant 3 x 3 matrix, $b$ a constant vector) and the body force is zero. When the strain-energy function exists and is rank-one-convex, the corresponding homogenous deformation has maximal energy amongst all possible solutions. When, in addition, the stored energy is strictly quasi-convex at F, the homogenous deformation is the unique smooth solution. The method of proof relies upon an identity due to Green, which is related to the normal conservation laws.

Section: Non-Newtonian Fluids

G. CAPRIZ

## The Korteweg Fluid as a Continuum with Latent Microstructure

Some constitutive equations for materials of grade higher than one (in particular the c.e. for the Korteweg fluid, where the stress depends on density and its gradient) are incompatible with a current interpretation of the Clausius-Duhem inequality. To assure at least a partial compatibility J.E. Donn and J. Serrin have proposed a new form of the first principle of thermodynamics where an additional term appears (the interstitial working).

The same result can be obtained by: (i) taking as starting axioms those proposed in [2] for a theory of continuum with microstructure, (ii) assuming that microinertia and external force on the microstructure are absent, (iii) introducing external constraints relating the microstructural variables to gradients of the deformation gradient, (iv) considering the special case when there is only one scalar order parameter and it is constrained to coincide with the density.

[1] J.E. Dunn & J. Serrin, Inst. Math. Appl. Reprint 24, Univ. Minnesota (1983)
[2] G. Capriz & P. Podio Gaidugli, Ann. Mat. Pure Appl., 135 (1984), 1-25

J. DUNWOODY

## Stability of the Rest State of Viscoelastic Fluids to Retarded Flow Perturbations

Through thermodynamic arguments, it is shown that the rest state of a viscoelastic fluid with fading memory is stable to the class of retarded flow perturbations, for which the second-order approximation to the stress is $\underline{S} = -p\underline{I} + \beta_o\underline{A}_1^2 + \beta_2\underline{A}_1^2 + \beta_1\underline{A}_2$, even though $\beta_1 < 0$. For asymptotic stability a set of sufficient conditions is $\beta_o > 0$, $\beta_1 + \beta_2 = 0$, $\beta_1 \geq 0$. In the absence of the latter two conditions, asymptotic stability is not assured.

F. M. LESLIE

## Transverse Effects in Shear Flow of Certain Isotropic Liquids

This paper discusses shear flow for a theory of isotropic liquids which differs from that for a Newtonian liquid solely by allowing angular velocity to be an independent variable. With the particular constitutive theory employed, one finds that transverse secondary flows occur despite the symmetry of the problem. These effects should prove more readily detectable in practice than existing predictions for such theory of variations in viscosity at small gap widths.

D. LHUILLIER

## Phenomenology of Inertial Effects in a Liquid-Solid or Liquid-Gas Mixture

The inertial forces occurring in the motion of a solid particle relative to a surrounding fluid are now rather well-known. Our aim is to generalize these single particle results with methods of irreversible thermodynamics, to deduce the form of inertial forces in a non-dilute fluid-solid mixture. Besides the relative translation motion, we also consider the case of dilatation-contraction motion and the resulting equations then apply to a bubbly mixture with spherical bubbles of variable radius. The problem of inertial effects associated with more general bubble deformations will be (tentatively) dealt with.

G.A. MAUGIN & MRS. R. DROUOT

## Continuum Modelling of Polyelectrolytes

Polyelectrolytes are macromolecules (such as proteins, DNA, polyvinyl sulfonic acid) having many ionizable groups which, in solution, are dissociated

into polyvalent macroions and a large number of small ions of opposite charges. The macroions are long flexible chains which may present a spherical conformation for a low charge of the solution and a rather rodlike or cylindrical conformation of high charge. The charge or p-H of the solution can be changed by adding a proper simple electrolyte (alkali acid). Still in its infancy, a continuum model involving the conformation of macroions (via a so-called tensorial internal variable) and electrodynamics is presented for polyelectrolytes with a view to modelling, essentially, couplings between conformation and electric state (polarization in particular). The phenomenological thermodynamical modelling which involves so-called "gyroscopic" or entropy non-producting terms couples in an obvious manner the phenomena of viscosity, relaxation of the conformation towards its equilibrium shape and electrical properties. Simple problems of equilibrium conformations in an electric field are presented as well as the problem of the combined influence of a shear flow and an electric field applied longitudinally or orthogonally. This last problem is solved exactly as also for small Deborah numbers. This yields orientation of the macromolecules at an angle not far from (but not equal to) $\pi/4$ of the flow lines and strangely enough an electric polarization may be induced via the conformation and relaxation in a direction orthogonal to the common direction of the flow and the longitudinal electric field. This can be detected since this gives rise to a surface charge density on the top of the fluid layer. Optical properties will be shortly developed along the same lines.

A.I. MURDOCH

Structured Fluids: The Relationship Between Continuum Theories and Microstructural Behaviour

Macroscopic quantities, and the balance relations they satisfy, are motivated from atomistic considerations involving simple molecular models. Continuum fields are defined as averages, both in space and time, of expressions involving motions of, and/or interactions between, point masses. The particular case of large, somewhat inflexible, molecules is discussed, and shown to result in the balance relations appropriate to Cosserat continua with deformable directors, provided molecular deformations are locally co-operative. Conservation of micro-inertia is shown to hold when this co-operative behaviour involves similar homogeneous deformations for neighbouring molecules (a special case being liquid crystals), but not when the co-operative behaviour only involves spatial deformation gradient rates for neighbouring molecules.

J.M. RUBÍ

Hydrodynamics and Brownian Motion of Fluids with Internal Angular Momentum

Fluids with internal angular momentum (spin) serve as a model to study fluids of polyatomic molecules. From the hydrodynamical point of view, such fluids are described by means of a new balance equation for the internal angular momentum which involves a new hydrodynamical variable: the spin. We employ a method of induced forces and torques to study the translational and rotational motion of a sphere in a fluid with spin. When the sphere is a brownian particle we get fluctuation-dissipation theorems for the force and torque on the sphere. Viscoelastic effects are also considered and the velocity correlation function at short and long time is studied.

A. SZANIAWSKI

Non-Equilibrium Flows of a One-Component Three-Phase Mixture

The one-component three-phase mixture has such a peculiar property that in thermodynamic equilibrium it may exist in so-called triple point state, in isochoric and isothermal condition only. In consequence the equilibrium model of the triple point mixture is easy to analyze, but it allows to obtain some results, which seem to be paradoxical. As the pressure does not change with the density, the speed of sound of the triple point mixture should vanish. Also its static equilibrium in a gravitation field is im-possible outside a unique horizontal surface, where the pressure equals its triple point value. The motion along possible stream lines according to the free fall rule and with variable phase computation is here admitted only.

Another model of weakly perturbed thermodynamic equilibrium has been introduced, in which the small spheres of condensed phases are dilutely dispersed in a continuous gaseous phase. The equations with a larger number of constants and variables for such model have been presented and qualitatively analyzed. Also the acoustic prospects of t he considered model of the mixture have been determined, giving very strong increase of the speed of sound and of the attenuation coefficient with the increase of small frequency of acoustic wave.

K. WALTERS

Numerical Simulation of Non-Newtonian Flow

One of the main research areas in non-Newtonian fluid mechanism at the present time concerns the prediction of the behaviour of fluids with memory (elastic liquids) in complex geometries involving abrupt changes in geometry. Experiments have shown that the interaction between fluid memory and abrupt changes in geometry can lead to qualitatively different behaviour to that found in Newtonian liquids.

Setting up the governing equations for various simple fluid models is not difficult or controversial and it is generally acknowledged that analytic structures are not feasible. This means that numerical methods are indispensible and both finite-difference and finite-element techniques have been applied with a certain degree of success. However, all existing algorithms break down at a critical value of the relevant non-dimensional elastic parameter. This critical value is frustratingly small!

Various possible explanations for the breakdown may be obtained, including bifurcation, change of type of the governing equation, failure of the iterative schemes. At the present time, it appears that a judicious choice of fluid model and/or the use of filtering techniques may provide a partial answer to this pressing problem.

Section: Polymers and Glasses

T. ALTS

## On the Statistical Description of Hysteresis and Pseudo-Plasticity in Natural Rubber

A new statistical theory of deformation-induced crystallization of weakly cured rubbers in the so-called "rubber-elastic" domain is presented. For this Kuhn's model of the molecular chains is modified by consideration of crystalline contributions and the configurational and crystallization entropies and internal energies are calculated as function of deformation, temperature and crystallinity. With these a stress-strain-crystallinity relation is derived, with which stress relaxation, hysteresis and thermoplastic residual deformations can be calculated and can be traced back to the time-delayed equilibrium adjustment of crystallinity. The results are compared with measurements on sulphur-cured natural caoutchouc and are in good agreement. Changes of the microstructure under strain are ascertained by X-ray experiments and electron microscopical photographs and are correlated to changes of crystallinity.

E. KRÖNER & R. TAKSERMAN-KROZER

## On Flow of Temporary Polymer Networks

These networks are distinguished from permanent networks by the occurrence of decay and formation of junctions. By this a new viscoelastic component is added to the mobility of the network. Equilibrium thermodynamics and statistics are developed. The latter lead to the equilibrium distribution function and (grand-canonical) partition function in explicit form. The distribution of chain contour lengths z in equilibrium follows in the form

$$z^3 \exp(-Cz).$$

With the distribution function in equilibrium known, the non-equilibrium function is calculated in the relaxation time approach. To this end the transition probabilities for decay and formation of junctions have been calculated. After this, the second moments (of the spring-bead model) are calculated. They lead to the stress tensor, given the velocity gradient, i.e. to the non-Newtonian law of flow. Also other physical properties can be calculated in this frame.

The theory contains many simplifications which, however, leave the physical picture intact. Most of the simplifications can be removed in a more refined theory.

P. STREHLOW

## On Statistical Thermodynamics of Glass

In phenomenological thermodynamics we consider glass as a thermoelastic body with well defined internal variables $\xi_\alpha$, whereby $\xi_\alpha$ is proportional to the number of $\alpha$-type rings within a network.The entropy principle yields restrictions for the constitutive functions: The second Piola-Kirchhoff-tensor is given by the derivative of the Helmholtz free energy with respect to the right Cauchy-Green-tensor and $\xi_\alpha$ is proportional to the derivative of free energy with respect to the internal variables. The determination of Helmholtz free energy is a problem of statistical thermodynamics.

By means of a two-dimensional network we have determined the free energy as a function of deformation gradient, temperature and internal variables.

The only parameter which does occur has been determined by vibrational experiments.

This procedure enables us

1) to calculate the stress-strain relation for a brittle-elastic glass;
2) to determine the ring-structure of the glass-network;
3) to describe the viscoelastic behaviour.

Section: Kinetic Theory

C. CERCIGNANI

## Kinetic Theory of Gases and Fluid Dynamics

The kinetic theory of gases has been frequently considered as a guideline for establishing constitutive equations or discussing basic principles of continuum mechanics. While the first application has a long story, the second one has been considered only in recent times, particularly by I. Müller. In the present talk other aspects of kinetic theory are discussed:
1) boundary conditions for continuum mechanics as derived from kinetic theory, with particular concern for the non-negligible jump of the temperature at the surface of an evaporating body. 2) the speed of propagation of disturbances in a relativistic fluid. 3) the kinetic theory of polyatomic gases.

W. FISZDON & T. PLATKOWSKI

## The Discrete Velocity Models of the Kinetic Theory and Transition to Continuum Fluid Dynamics

Basic discrete velocity models are reviewed showing their advantages for a microscopic description of large systems of particles. The problem of transition from a kinetic to hydrodynamic description of gases is considered Results of asymptotic Hilbert and Chapman-Enskog procedures leading to the fluid dynamic approximation for the Carleman and Broadwell models are discussed. Assumptions sufficient to obtain corresponding limiting theorems are compared. For the Carleman model, the approach based on the nonlinear semigroups of the corresponding accretive operators in the positive cone of the Banach space $L_1(R) \times L_1(R)$ is presented. Results concerned the fluid dynamic limit of the semigroups for the Carleman equation, when the Knudsen number $Kn \to 0$, are discussed. Some directions of possible further investigations and some open problems are suggested.

Section: Hyperbolicity and Extended Thermodynamics

G. BOILLAT

Symmetrization of Systems of Partial Differential Equations with Constraints

If an additional conservation law with a convex energy density exists it is shown that the system can be written in a symmetric conservative form.

W. DREYER & I. MÜLLER

On Relativistic Thermodynamics of Degenerate Gases

Extended thermodynamics deals with fourteen independent basic fields rather than with five fields as it is done in the older thermodynamic theories, namely particle density, four velocity and the energy momentum tensor instead of particle number, four velocity and temperature.

The objectives of this lecture are:

i.) To give a short oversight on the general scheme of Relativistic Extended Thermodynamics.

ii.) It is shown that the influence of relativity to nondegenerate gases and to Bose gases with nonzero rest mass is very weak.

iii.) One of the most striking results is, that a completely degenerate Fermi gas, for which the relativistic effects are very important, cannot be at rest in equilibrium if a gravitational field is applied.

Such a gas is forced to make a so-called solid body rotation in a spherical gravitational field.

D. JOU

Generalized Equations of State in Extended Irreversible Thermodynamics

The inclusion of dissipative fluxes as independent thermodynamic variables leads to many modifications in a thermodynamic formalismn.Some of these modifications have been extensively studied in recent years, as for instance relaxational constitutive equations (Nettleton, Müller, Lebon ...) or hydrodynamical fluctuations (Jou, Rubi, Casas-Vazquez ...). Here we point out the modifications which arise in the equations of state. First of all we provide a physical justification for a generalized Gibbs equation. Our arguments, proposed in a wider and more abstract form by other authors (Muschik), confirm the results obtained by the methods of irreversible thermodynamics, of kinetic theory and of fluctuation theory. Secondly, starting from such a generalized Gibbs equation, we obtain the generalized equations of state for internal energy and for thermodynamic pressure. Such equations exhibit a dependence on dissipative fluxes. Finally, we apply, as an illustration, such equations to the case of a Van der Waals fluid and calculate the modifications in the critical point. Our development shows the need of a non-analytical generalization of EIT to be able to fit the experimental results.

G.M. KREMER

Extended Thermodynamics of Dense Gases

The objective of extended thermodynamics is the determination of the 13 fields of density, velocity, stress-tensor and heat flux. It was first formulated by Müller, who wanted to avoid the so-called paradox of heat conduction in the Navier-Stokes-Fourier fluid, which predicts an infinite speed of propagation for disturbances of temperature. The theory was reformulated recently by Liu & Müller within the framework of a rational thermodynamics with Lagrange multipliers. Among other results it leads to a complete description of classical and degenerate ideal gases. This theory has pressure and internal energy density related by the equation $3\ p = 2\rho\varepsilon$ which, as we know, is valid only for one-atomic ideal gases.

Our purpose is the formulation of a theory of dense gases.

G. LEBON

An Extended Thermodynamic Description of Viscous Fluids

A model of viscous fluids is formulated within the framework of extended irreversible thermodynamics. In contrast with the classical description, the heat flux and the viscous stress tensor are elevated to the status of independent variables, on the same level as the density velocity and temperature fields. Extra evolution equations governing the behaviour of these extra variables are proposed. Restriction on the form of the constitutive and response functions are placed by the second principle and the principle of objectivity. Explicit expressions for these constitutive equations and the evolution equations are derived in the vicinity of equilibrium (linearized term). The results are shown to be in agreement with the 13-moment kinetic theory of Grad and with a more recent kinetic model proposed by Schmidt-Köhler and Hess.

I. MÜLLER

Extended Thermodynamics of Dilute and Degenerate Gases

Extended Thermodynamics is a fields theory with the principal objective of determining the 13 fields of density, velocity, energy, stress and heat flux. The formulation of field equations requires constitutive equations which are restricted by the principle of material objectivity and by the entropy principle so that for gases only the transport coefficients and the thermal equation of state remain unknown. The thermal equation of state

may be calculated from statistical mechanics of Bose and Fermi particles. Thus appears a non-equilibrium thermodynamic theory of degenerate gases which can also be applied to superfluid Helium.

T. RUGGERI

On the Structure of Partial Differential Systems Compatible with an "Entropy Principle"

We consider a system of partial differential system of balance laws (first-order, quasi-linear and hyperbolic) and we suppose that all solutions satisfy a supplementary "conservation" law expressed as an inequality; in continuum thermomechanics these represent "entropy principle" conditions. In this situation it is possible to have a theorem that guarantees the existence of a privileged field such that all the equations are "generated" by 4 "potentials" expressed in terms of this "main field" $\underline{u}'$. Therefore for new mathematical models of some physical realities such as "extended thermodynamics" it is more convenient to commence by considering $\underline{u}'$ as a field, because in this manner it is necessary to identify only these 4 potentials to have the field equation.

The main advantage of this procedure is the possibility to control by convexity arguments the hyperbolicity of the system and the well posedness of Cauchy problems.

Section: Variational Principles

K.-H. ANTHONY

## Thermodynamics of Irreversible Processes in the Framework of Lagrange-Formalism

It has been shown that Lagrange-Formalism allows for thermodynamics of irreversible processes. The total information of a process is contained in a Lagrange-Funtional, which depends on the fundamental field variables of the system. Together with Hamilton's Principle and a set of universal invariance principles, which apply to all physics, the total structure of thermodynamics can be deduced, i.e. all balance equations and all constitutive equations are a straightforward outcome of the t heory. Especially the entropy concept is deduced, which within Lagrange-Formalism is closely related with stability theory in Ljapunov's sense. The theory has been realized for the case of heat conduction and, for the first time, for the case of the combined process of heat conduction and material diffusion. An essential point is the use of complex valued fields, namely a "thermal excitation field" and a "matter field". As in the case of pure heat conduction Onsager's theory is again completely reproduced for the combined process. This means, that using Lagrange-Formalism,the proposed unification of different phenomenological theories of matter, including thermodynamics, seems to be a realistic aim.

A. MORRO

## Variational Principles in Fluiddynamics

A theorem due to Volterra, and re-visited by Vainberg, yields a necessary and sufficient condition for an operator to be the derivative of a functional. On appealing to such a theorem, the possibility of setting up variational

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formulations in fluiddynamics is investigated. The motion of the fluid is supposed to be adiabatic and is described in terms of Lagrangian coordinates. Then, upon a standard procedure, a Lagrangian density is found explicitly for the equation of motion and the continuity equation. By the same token it is found a Lagrangian density for fluiddynamics when the continuity equation is viewed as a constraint; incidentally the Lagrangian multiplier turns out to be the pressure as it happens in the case of incompressible fluids. Furthermore, a hierarchy between different variational principles is exhibited. Finally, variational principles for outstanding equations in fluiddynamics (e.g. KdV and BBM eqs.) are given.

Section: Irreversible Thermodynamics

W.G. DIXON

Generalized Thermodynamics of Fluids

The most fundamental variables of continuum mechanics are the densities and fluxes of mass, momentum and energy, which are governed by appropriate balance equations. From a mathematical viewpoint the physical significance of these variables is contained in their transformation properties under a Galilean transformation. An approach to thermodynamics is presented which relies on mathematical development based on these transformation properties, rather than on imprecise verbal physical reasoning. The power of their approach is seen most clearly in complex situations or for generalized materials, where the physical concepts of stress, heat flux, etc. can become ambiguous. The method is illustrated by considering a fluid in which the thermodynamic variables are allowed to depend on the gradient of the mass density. The results are found to be equivalent ot those of a theory involving hyperstress.

W. MUSCHIK

## Quantumstatistical Foundation of Non-Equilibrium Contact Quantities

Intensitive quantities as temperature, chemical potentials and generalized forces are defined for discrete non-equilibrium systems. This is achieved by the zeros of the conjugate exchange quantities as heat exchange, mass exchange and work exchange. The quantumstatistical expectation values of these quantities are evaluated by using the generalized canonical statistical operator according to Fick and Schwegler for the time of contacting an equilibrium system with a non-equilibrium one. The zeros of these expectation values are unique and determine the intensitive quantities mentioned above.

J. SCHRÖTER

## Thermodynamics of Systems with Long-Range Interactions

Two-point distribution functions $f_{\alpha\beta} : R^{12} \to R$ are defined which describe correlation effects for a system of M kinds of changed particle. The particles are supported to interact by a Coulomb force together with a hard core. In addition they interact with neutral particles. Then $f_{\alpha\beta}$ satisfies the following transport equation:

$$\partial_t f_{\alpha\beta} + v_1 \cdot \nabla_{x_1} f_{\alpha\beta} + v_2 \quad \nabla_{x_2} f_{\alpha\beta} + (k_\alpha + k_{\alpha\beta}) \nabla_{v_1} f_{\alpha\beta} +$$

$$+(k_\beta + k_{\beta\alpha}) \nabla_{v_2} f_{\alpha\beta} + \sum_\gamma \int (k_{\alpha\gamma} \nabla_{v_1} + k_{\beta\gamma} \cdot \nabla_{v_2}) f_{\alpha\beta\gamma} dx_3 dv_3 = \sum_\gamma (I_{\alpha\gamma}(f_{\alpha\beta}, f_\gamma) +$$

$$+ I_{\beta\gamma}(f_{\alpha\beta}, f_\gamma)) + S_{\alpha_1} f_{\alpha\beta} + S_{\beta_2} f_{\alpha\beta}$$

where $I_{\alpha\gamma}$ are Boltzmann collision operators and $S_\alpha$ are Fokker-Planck operators. Balance equations for the correlation densities, the correlation momentum densities and the internal energy are derived. In the hydrodynamic approximation of $f_{\alpha\beta}$ constitutive relations are given which close the system of balance equations thus forming a thermodynamical theory for particles with long range interactions. As an example the electric conductivity in an electrolyte is discussed.

M. ŠILHAVÝ

On the Clausius-Duhem inequality, Non-Simple Materials, and Phase Transitions

The paper describes thermostatics of materials whose stress and free energy in equilibrium depend on the higher gradients of deformation. In contrast to the existing theories of such materials, the present approach employs the conventional forms of the balance equations and of the Clausius-Duhem inequality. Within this framework, the restrictions on the static response functions at points of stability are studied. It is shown that the equilibrium stress relation and the Legendre Hadamard condition hold at points of stability. These results, as well as certain more general consequences of stability, are then employed to analyse the continuum approach to phase transitions.

K. WILMANSKI

Multicomponent Thermodynamic Models of Plasma

The aim of this work is to construct such thermodynamic models of cold plasma, i.e. with temperature $14 \times 10^{3}$ °K, that the equations of heat transfer, following from those models would account for the experimental data. It has been found that the heat conductivity, measured in experiments, is app. 10 times higher than the values, following from the kinetic theory. By use of the modern thermodynamics of mixtures, we derive the governing set of equations for such plasmas under the following assumptions.

1. All ions have the positive charge. 2. One of the components (electrons) has the molecular mass much smaller than all remaining components. 3. Mixture is electrically neutral. 4. The motion of charged components is ambipolar, i.e. all those components have the same velocity field (ambipolar diffusion). 5. The electric polarization is negligible.

The above assumptions, obviously, eliminate from the description the plasma with negative ions (molecular gases), but still the model is sufficiently rich to eliminate the above mentioned discrepancy in the values of heat conductivity.

Section: Plasticity and Phase Changes

J. BAUMGART

Memory Alloys and Torsion Theory

F. FALK

Domain Walls and nuclei in Shape Memory Alloys as Solitary Waves

The shape memory effect occuring in certain alloys is due to a martensitic first order structural phase transition which is connected with a permanent shear strain. The transition may be induced either by changing temperature or by applying an external load. A one-dimensional continuum model with couple stress is presented. In order to get a phase transition the shear stress-shear strain relation is non-linear. The non-linear equations of motion allow for solitary wave solutions of kink as well as of soliton type. The kink solutions represent domain walls between different phases whereas the soliton solutions can be interpreted as nuclei of one phase in a matrix of another one.

P. HAUPT & CH. TSAKAMKIS

## Stress Rates and Incremental Constitutive Equations

Incremental constitutive equations involve time derivatives of tensor-valued variables, which should have certain invariance properties. However, invariance requirements are not sufficient to ensure a representation of material behaviours, which is consistent in every respect. In particular, the application of the conventional Jaumann derivative to hypoelasticity and plasticity leads to difficulties, which result from the difference between the vorticity tensor and the true spin of the material line elements. The properties of three different types of tensorial time rates are outlined:

1. the material derivative
2. the derivative relative to the material
3. various derivatives relative to rotating frames

For each particular kind of derivative, applications are discussed.

J. KRATOCHVIL

## Microscopic vs. Macroscopic Approaches to Plasticity

The most distinguished feature of plastic materials is a strong dependence of their mechanical response on the deformation history. At the microscale the plastic memory is governed by interactions among dislocations and other crystal defects. However, the attempts to build the theory of plasticity starting at the microscale dislocation and encounters serious difficulties The models of plastic behaviour of the structural (mesoscopic) type are more successful at present. One can look at these models as the examples of a convenient discretization of hypothetical constitutive equation of plasticity one likes to discover.

H. LIPPMANN

## Rolling Theory of a Rheological Material

An "elementary" theory is developed for the strip rolling process of a Bingham material, based on a generalization of the Siedel/v. Karman "elementary" rolling theory for a rigid ideal plastic material conforming with Sommerfelds lubrication theory. In the rolling gap the results agree fairly well with those obtained from Büche!'s rolling theory for plastics, though even then additional information is found regarding the pressure distribution. Moreover the transition zones are investigated, between the rolling gap and the rigid strip before and behind it. It is shown that for a Newtonian fluid no stationary state exists. Rather the material must be expected to pile up increasingly (and, as it is known from experiments, finally to form the so-called moulding vortex). Otherwise the theory yields monotonically shaped regimes of transition so that the bulge observed in practice, to form in front of the rolling gap must have an elastic origin.

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