MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Modelltheorie

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Die Modelltheorie-Tagung 1984 in Oberwolfach wurde von den Herren L. van den Dries (Stanford), U. Felgner (Tübingen) und A. Prestel (Konstanz) geleitet.

Die beiden Schwerpunkte dieser Arbeitstagung waren "Die elementare Theorie der reellen Exponentiation" und "Quantoren-Elimination (QE) für Gruppen und Ringe". In ausführlichen Vorträgen wurden diese Gebiete systematisch behandelt und aktuelle Forschungsergebnisse dargestellt.

Wie es bereits auf früheren Modelltheorie-Tagungen üblich war, wurde darüberhinaus durch mehrere Vorträge der thematische Rahmen in verschiedene Richtungen erweitert, ohne jedoch den Bezug zu den Hauptthemen zu verlieren. So wurde Quantoren-Elimination auch für andere mathematische Strukturen gezeigt ("QE in Discriminator Varieties", "Linear Elimination") und ein Anwendungsbeispiel für QE im Beweis der "Rationality of Poincaré Series" gegeben. Eine Reihe von Vorträgen befaßte sich mit den Problemen des "Reellen" von unterschiedlichen Standpunkten aus: abgesehen vom Schwerpunkt "Reelle Exponentiation" reichten die Themen von den "Rigid Relations on ℝ" über die Theorie der reellen algebraischen Varietäten bis zu den PRC-Körpern, den reell abgeschlossenen und den verallgemeinerten reell abgeschlossenen Körpern, wodurch auch Themen der Modelltheorie-Tagung 1982 fortgeführt wurden.

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Vortragsauszüge

I. Exponential Algebra.

A.J. Wilkie:

Foundations of Exponential Algebra.

Let L be the language $\{0,1,+,\cdot,-,\exp\}$. An exponential ring is an L-structure which is a commutative ring with 1 satisfying the laws $\exp(0) = 1$ and $\exp(x+y) = \exp(x)\cdot\exp(y)$. If k is an exponential field we denote by $k[x_1,\ldots,x_n]^e$ the free exponential ring with generators $x_1,\ldots,x_n$ satisfying the positive diagram of k, that is the elements of $k[x_1,\ldots,x_n]^e$ are terms of L with variables amongst $x_1,\ldots,x_n$ allowing constants from k. Notice that terms of L_k (= L with constants from k) have natural formal partial derivatives (which are also terms of L_k) which induce natural partial derivatives on $k[x_1,\ldots,x_n]^e$ where the set of constants turns out to be just k. We show further that if $P \in k^n$ and $m_P = \{f \in k[x_1,\ldots,x_n]^e : k \models f(P) = 0\}$ then no sub-ideal of m_P is closed under partial differentiation. This implies completeness theorems for suitable k. For example, if $k = \mathbb{R}$ or $\mathbb{C}$ and $k \models \forall\vec{x} f(\vec{x}) = 0$, then f is identically zero in all exponential rings. The method of proof relies heavily on results from differential algebra.

Let L be the language $\{1,+,\cdot,\downarrow\}$ where the intended interpretation is $\mathbb{R}^+$ (the positive reals) and $x \downarrow y = x^y$. Let HSA be the usual "High School Algebra" axioms formulated in L. We produce an example of a law which is true in $\mathbb{R}^+$ but not provable from HSA. However, by increasing the language L suitably (by adding function symbols for certain polynomials not having positive coefficients but nevertheless taking only positive values on $\mathbb{R}^+$) and the axiom system HSA naturally, we show that all laws true in $\mathbb{R}^+$ (in the language L) become provable.

A. Macintyre:

Exponential Fields.

We develop the elements of exponential algebra, via the study of partial E-rings with derivations. The principal object of study is the universal extension of an E-ring R. A notion of E-place is developed, leading to a Completeness Theorem for exponential rational functions over E-fields. The crucial fact is the non-existence (under mild hypotheses) of E-places closed under derivations. In the Completeness Theorem we relate three entities, $K(x)^E$, a field of functions defined sheaf-theoretically, and an algebra of terms. Finally we discussed Dahn's Completeness Theorem and its application to sharpening of Van den Dries' theorem on limits of exponential functions.

L. van den Dries:

On Some Exponentially Definable Functions.

Define $E(\mathbb{R}^n)$ - the algebra of n-variable exponential functions - to be the least $\mathbb{R}$-algebra of real analytic functions on $\mathbb{R}^n$ which contains the coordinate fct's $X_1,\dots,X_n$, is closed under $f \mapsto \exp(f)$ $(= e^f)$, and such that each analytic $f : \mathbb{R}^n \to \mathbb{R}$ with $fg = h$ for certain g, h $(g \neq 0)$ in the algebra belongs also to the algebra.

Let $f \in E(\mathbb{R}^2)$ be given and define $g : \mathbb{R} \to \mathbb{R} \cup \{-\infty,\infty\}$ by $g(x) = \lim_{y\to\infty} f(x,y)$. (The limit exists for each x.)

Question (Macintyre): How 'wild' can y be?

Answer: There are real numbers $a_1 < \dots < a_k$ such that on each interval (a_i, a_{i+1}) (with $a_0 = -\infty$, $a_{k+1} = \infty$) the function g is

either $+\infty$

or $-\infty$

or analytic (with finitely many zeros if not identically zero on the interval).

The proof uses nonstandard analysis and actually gives a more general result: if $f \in E(\mathbb{R}^{n+1})$ then each sequence of points (p_k) in $\mathbb{R}$ has a subsequence (q_k) such that the sequence of functions

$(f(x,q_k))_{k\in\mathbb{N}}$ converges pointwise to a function $g : \mathbb{R} \to \mathbb{R} \cup \{-\infty,\infty\}$ satisfying the statement above, and such that moreover on each interval (a_i,a_{i+1}) the convergence is uniform on compacta.

II. QE for Groups and Rings.

U. Felgner:

Quantifier Eliminable Groups.

Call a group $\underset{\sim}{G} = \langle G,\cdot,^{-1},1\rangle$ a QE-group (for 'quantifier eliminable') if for each formula Φ there is a quantifier-free formula Ψ such that $\Phi \longleftrightarrow \Psi$ holds in $\underset{\sim}{G}$. We presented the classification of all solvable QE-groups (modulo the nilpotent QE-groups of exponent 4) and of all finite QE-groups. In particular there are 2 nonabelian finite QE 2-groups (the quaternion group and the Sylow 2-subgroup of $U_3(4)$), four families of finite solvable non-nilpotent QE-groups, the groups $PSL_2(5)$, $PSL_2(7)$, $SL_2(5)$ and direct sums of those. The list of infinite solvable QE-groups is too long to be stated here. This work was done jointly with G. Cherlin.

J. Saffe:

QE-Rings.

The aim of the talk is to give a survey on the classification of rings having quantifier elimination in the lanuage $L = \{+,-,\cdot,0,1\}$. It is easy to see that we can restrict attention to rings of characteristic 0 or a prime power. The results are as follows:

Theorem 1: QE-rings of characteristic zero are exactly algebraically closed commutative fields.

Theorem 2: QE-rings of characteristic p without nilpotent elements are a) algebraically closed or finite fields, b) $\mathbb{F}_{p^n} \times \mathbb{F}_{p^n}$, c) atomless p^n-rings.

Theorem 3: The only semisimple QE-rings beside those of theorem 2 in characteristic p are $M_2(\mathbb{F}_p)$.

Theorem 4: Suppose that R is QE-ring of characteristic p such that the Jacobson-radical $J(R) \neq 0$. Then a) $J^3 = 0$; b) either $R = \mathbb{F}_p[J]$

or $R/J \cong \mathbb{F}_4$, and there are only two possibilities in the second case. But somehow a complete classification is impossible, as is shown by

Theorem 5: There are $2^{\aleph_0}$ pairwise elementarily inequivalent QE-rings of characteristic p.

Finally, we mention the result corresponding to theorem 4 in the case of characteristic p^n.

Theorem 6: Suppose that R is a QE-ring of characteristic p^n, $n > 1$. Let R_1 be the annihilator of p^{n-1} in R and R_1^* be the subring generated by the central idempotents of R_1. Then:

a) $R_1 = R_1^* \oplus J$ (in particular $J \subset R_1$)

b) J is nilpotent of order at most 2n+1 and J is a QE-nilring in the language $\{+,-,\cdot,0,p\}$ having a constant for the element p

c) R/R_1 is either $\mathbb{F}_p$, $\mathbb{F}_4$, or $\mathbb{F}_2 \times \mathbb{F}_2$

d) R_1^* is either (0), $\mathbb{F}_{p^k}$ or the subring of continuous functions from a Boolean space without isolated points into $\mathbb{F}_{p^k}$ vanishing at a certain point.

C. Wood:

Finite QE-Rings.

We indicate the current state of joint research with Dan Saracino on the classification of finite QE-rings. The work is complete for characteristic p (Berline and Cherlin, Mons 1978) and for characteristic p^2 when p is an odd prime (Saracino and Wood, preprint). We also sketch the program for handling p^n (the general case) by an analysis of the Jacobson radical as in Berline-Cherlin (J.S.L. 1983).

V. Weispfenning:

QE for Ordered Abelian Groups and Modules.

1. Ordered abelian groups.

A theorem of W.Szmielew (1949) says that any abelian group admits quantifier elimination (q.e.) in the language $L^{\equiv} = \{0,+,-,\{\overset{\equiv}{n}\}_{n<\omega}\}$. We study q.e. ordered abelian groups in the language $L^{\equiv}_{<}(C) = L^{\equiv} \cup \{<,C\}$, where C is an arbitrary set of constant-symbols.

Theorem 1. Let G be an ordered abelian group such that $\langle C\rangle$ is of finite order rank n. Then G admits q.e. in $L_{<}^{\equiv}(C)$ iff

(1) G is dense regular <u>or</u>

(2) Ex. a chain $H_0 \subseteq H_1 \subseteq \ldots \subseteq H_m = G$ of convex subgroups of G ($m \leq n$) such that

(i) H_i/H_{i-1} is a $\mathbb{Z}$-group w. smallest positive element 1_i and for some $k_i < \omega$, $k_i 1_i$ is represented by a constant term

(ii) H_0 is trivial or a dense regular group s.t. for all $k < \omega$ H_0/kH_0 is finite and represented by constant terms.

2. Modules.

All modules are left, unitary over a fixed ring R. We apply the Baur-Monk theorem on q.e. for modules relative to pos. primitive formulas and a reduction of pos. prim. formulas using specific generators for left ideals of R to prove:

Theorem 2. Let R be a Dedekind domain and M an R-module. Then M admits q.e. iff M is divisible <u>or</u> a torsion module with finitely many homocyclic primary components.

Theorem 3. The following are equivalent:

(i) Every R-module admits q.e.

(ii) Every 2-generated R-module admits q.e.

(iii) R is von Neumann regular.

Theorem 4. Let M_R^+ be the class of nontrivial R-modules. Then t.f.a.e.

(i) M_R^+ admits q.e.

(ii) M_R^+ is model-complete

(iii) R is an infinite, simple, v.Neumann regular ring.

Theorem 2 - 4 generalize results of Cherlin-Felgner, Belegradek, Sabbagh, Hodges and Tyukavkin.

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III. Other Topics.

C.N. Delzell:

Application of Hilbertian Function Fields to Exponential Diophantine Equations.

Let K be a field & let w, x, y be variables. Let $f \in K[w,x,y]$ be irreducible in x,y over K(w), & assume y occurs in f. Let $S = \{n \mid f(w,w^n,y) \text{ is irreducible in } y \text{ over } K(w)\}$.

Ex.: If $f(w,x,y) = y^2-x$, then S = {odd integers}. If $f = y^2-xw$, then S = {even integers}.

Ex.: If f = y, then $S = \mathbb{N}$. If $f = y^m-xw^l$, then $S = \{n \mid (n+l,m) = 1\}$.

Ex.: If $f = y^2-x-1$, then $S = \mathbb{N} - \{0\}$.

Question (J. Ohm, 1983): Must S be infinite?

Answer (L. van den Dries): No. (Ex. for char $K \neq 2$: $f = y^4 - 2(w+x)y^2 + (w-x)^2$. Then $S = \emptyset$.)

Main Thm.: If char K = 0, then S is a union of residue classes for some modules, minus a set of density 0. (Cor.: $S \neq \emptyset \Rightarrow$ S has positive density $\Rightarrow$ S infinite.)

Pf. Sketch: Factor $f = (y-\bar{y}_1(w,x))\dots(y-\bar{y}_m(w,x))$, $\bar{y}_i \in L((x^{-1/\infty})) = \bigcup_{d=1}^{\infty} L((x^{-1/d}))$, where $L = \bar{K}((w^{-1/\infty}))$. $\bar{y}_i$ will be "convergent" relative to the w-adic valuation on L (by Hensel). Some factors are in $L[x^{1/d},y]$ - they contribute the residue classes to S; other factors are not, and they subtract the exceptional set of density 0 from S. "Q.E.D.".

Application: Let $\mathcal{P}$ = {all exponential Dioph. equations of form $P(n,w,y) = 0$, where P is a sum of monomials of form $hw^{an+b}y^c$ ($h \in \mathbb{Z}$, $a,b,c \in \mathbb{N}$)}.

Thm.: $\exists$ an algorithm which, given $P \in \mathcal{P}$, decides whether $\forall n \forall w \exists y\ P(n,w,y) = 0$ (Pf. uses main thm.).
$(n,w,y \in \mathbb{N})$

L. Lipshitz:

Identity Problems.

We say that a formal power series $f(x) \in K[[x]]$ is defined by an algebraic differential equation (ADE) if there is a polynomial P in $x, y, y', \dots, y^{(n)}$ and initial conditions $y(0) = a_0, \dots, y^{(m)}(0) = a_m$ such that $P(x, f, f', \dots, f^{(n)}) = 0$, $\frac{\partial P}{\partial y^{(n)}}(x, f, \dots, f^{(n)}) \neq 0$ and $f(x)$ is the unique solution to $P = 0$, satisfying these initial conditions, in $K[[x]]$. (Every power series in $K[[x]]$ which satisfies an ADE can be defined in this way).Let $f_1, \dots, f_n$ be elements of $K[[x]]$ defined by ADE's, and let J be the set of terms built up from variables, elements of K, +, $\cdot$, $-$, $f_1, \dots, f_n$ by composition (where $f_i(g)$ is only allowed when $g(0) = 0$). Assume that the diagram of K in the language $\langle +, \cdot \rangle$ is decidable, or that we have an oracle for this diagram.

Theorem. There is an algorithm to decide for terms $T \in J$ whether T is identically zero.

J. Denef:

Rationality of Poincaré Series.

Let p be a prime number, $\mathbb{Z}_p$ the p-adic integers, $f_1(x), \dots, f_r(x) \in \mathbb{Z}_p[x]$, where $x = (x_1, \dots, x_m)$. Let $f = (f_1, \dots, f_r)$. Let

$$\tilde{N}_n = \mathrm{Card}\{x \bmod p^n \mid x \in \mathbb{Z}_p^m ,\ f(x) \equiv 0 \bmod p^n\}$$

$$N_n = \mathrm{Card}\{x \bmod p^n \mid x \in \mathbb{Z}_p^m ,\ f(x) = 0\}$$

$$\tilde{P}(T) = \sum_{n=0}^{\infty} \tilde{N}_n T^n$$

$$P(T) = \sum_{n=0}^{\infty} N_n T^n .$$

Igusa (1974) proved that $\tilde{P}(T)$ is a rational function. His proof uses Hironaka's embedded resolution of singularities. We gave a different proof which does not use resolution, but uses instead elimination of quantifiers for $\mathbb{Q}_p$. Our approach also answers in the affirmative a question of Serre, whether $P(T)$ is rational.

H. Läuchli:

Rigid Relations on $\mathbb{R}$.

The relation $R \subseteq \mathbb{R}^m$ is said to be rigid on $\mathbb{R}$, if there is no proper permuation $\pi : \mathbb{R} \rightarrowtail\!\!\twoheadrightarrow \mathbb{R}$ such that $\pi(R) = R$. Besides the well known rigid 4-place relation $R_1(x,y,u,v) :\Leftrightarrow u = x+y$ and $v = x \cdot y$, and any well-ordering of the reals, there are many "natural" examples of rigid relations: $R_2(x,y,z) :\Leftrightarrow (z-x-y)\cdot(z-x\cdot y) = 0$ is rigid (proof: + and $\cdot$ are definable from R_2). $R_3(x,y) :\Leftrightarrow (y-x-1)\cdot(y-x^2) = 0$ is rigid (proof: the set $\{a \in \mathbb{R} \mid$ "$x \geq a$" is definable from $R_3\}$ is dense in $\mathbb{R}$). There is even a C^∞ function f of one variable whose graph is rigid. Theorem: The graph of a polynomial of one variable is not rigid; the automorphism group has the power of the continuum.

F. Point:

QE in Discriminator Varieties.

We classify those classes of elements of a discriminator variety V, which admit quantifier elimination (q.e.) in terms of classes of simple elements of V.

From now on, we suppose that the language L of V contains at least one constant denoted by O. Let A be a non trivial element of V. Our starting point is a representation theorem due to Werner. Namely that A is isomorphic to the structure $\Gamma_c(X, \bigcup_{x \in X} A_x)$ of all sections with clopen compact supports of a locally boolean sheaf of simple elements of V.

We show that quantifier elimination in such structures implies certain properties both of the base space and of the class of the stalks. For proving that the conditions that we derive are indeed sufficient we use the Feferman-Vaught theorem on finite direct products, the construction, due to Boffa and Cherlin, of quantifier eliminable boolean valued structures starting with certain quantifier eliminable classes of structures, and a generalization of this later result to a non-compact situation. Then we analyze the effectiveness of the transfer of q.e. from the class of stalks to the structure of sections and vice versa.

M. Boffa:

Linear Elimination.

This is some kind of Quantifier Elimination (QE) related to solvability of systems of linear equations. The context is that of rings (with a unity; commutativity is not assumed). Let S_{mn} denote the linear system $\sum_{j=1}^{n} a_{ij}x_j = b_i$ $(i = 1,\dots,m)$ and let $\psi_{mn}(a_{ij},b_i)$ be the existential formula expressing that S_{mn} has a solution.

By definition:

Linear Elimination (LE) means QE (in the language $L = \{+,-,\cdot,0,1\}$) for all the ψ_{mn}'s.

Weaker notions can be introduced, for example:

1-LE = QE for the formula $\exists x\ (ax = b)$,

2-LE = QE for the formula $\exists x\ (ax = b \ \&\ cx = d)$.

The 1-LE rings without nilpotent elements and the 1-LE semi-simple rings are (cf. F.Point, Thesis, Mons, 1983): the division rings, the rings of characteristic p (prime) satisfying an identity $x^q = x$ $(q = p^n)$, the matrix rings $M_2(\mathbb{F}_q)$ $(q = p^n)$, and the finite direct products of these rings provided the factors have distinct prime characteristics.

Among them, all the commutative one have LE. Among the matrix rings, only those of the form $M_2(\mathbb{F}_p)$ (and perhaps also $M_2(\mathbb{F}_4)$) have 2-LE. $M_2(\mathbb{F}_p)$ has in fact QE.

Moreover, it can be shown that a 2-LE division ring satisfies a polynomial identity and so is finitedimensional over the centre. The division rings have LE for $L \cup \{^{-1}\}$, but for L the following questio[n] remain open:

- are there non commutative division rings having 2-LE?
- are there non commutative division rings having LE?
- what about $\mathbb{H}$ (the quaternions)?

M. Droste:

On the Lattice of Normal Subgroups of Transitive Automorphism Groups of Linearly Ordered Sets.

An infinite linearly ordered set ("chain") $(\Omega,\leq)$ is called doubly-homogeneous if its automorphism group $A(\Omega) = \mathrm{Aut}((\Omega,\leq))$ acts 2-transitively on it. It is known that $A(\Omega)$ has precisely 3 non-trivial proper normal subgroups, if both the cofinality $(\mathrm{cof}(\Omega))$ and the coinitiality of Ω are countable. Using model-theoretic means, in particular reduced products of partially ordered sets, we show that $A(\Omega)$ has at least 2^{2^κ} normal (or even maximal normal) subgroups if $\kappa = \mathrm{cof}(\Omega) \neq \aleph_0$. We establish several other properties of the lattice $N(A(\Omega))$ of all normal subgroups of $A(\Omega)$, and we obtain (this in joint work with S. Shelah) a complete classification and construction of the class of all such lattices $N(A(\Omega))$ (Ω a doubly homogeneous chain).

G.L. Cherlin:

Finite Homogeneous Structures.

We consider finite structures which are homogeneous for a finite relational language L. Lachlan has systematically investigated the classification of such structures, and in joint work we completed the proof of his conjecture:

Given L (finite, relational), the finite, homogeneous L-structures fall into finitely many families in such a way that within a given family the isomorphism types of the structures are determined by simple numerical invariants, called dimensions; the notion of dimension arises from an explicit classification of the large, primitive L-structures.

A necessary technical lemma of independent interest says:

Lemma. Given r, s one can find n so that in all finite structures M satisfying

$$|M| > n$$
$$|M^5/\mathrm{Aut}\ M| < s$$

there is a subset I on which Aut M induces Sym(I), with $|I| = r$.

L. Bröcker:

Hovanskii's Theorem and Betti Numbers of Real Algebraic Varieties.

For $x = x_1,\dots,x_n$ the functions $u_1(x),\dots,u_k(x) : \mathbb{R}^n \to \mathbb{R}$ form a Pfaffian chain (by def.), if $\frac{\partial}{\partial x_i} u_j = p_{ij}(x,u_1,\dots,u_j)$ for a polynomial p_{ij}, $i = 1,\dots,n$; $j = 1,\dots,k$. Consider equations $Q_i(x,u(x)) = 0$ where $Q_i(x,u)$ is a polynomial, $i = 1,\dots,s$. The complexity of such a system is the collection of the numbers n, k, $\deg(p_{ij})$, $\deg(Q_i)$.

Hovanskii's Theorem. The number of the non singular solutions of a system $Q_1 = 0,\dots,Q_n = 0$ is bounded by a function of the complexity of that system.

Corollary. The number of non singular solutions of a system $P_1(x) = 0,\dots,P_n(x) = 0$ of polynomials is bounded by a function which depends only on n and the number k of monomials occuring in the P_i.

Theorem. For V = real solutions of $P_1(x) = 0,\dots,P_r(x) = 0$ the sum of the Betti numbers of V is bounded by a function, which depends only on n, k.

Hovanskii's theorem was proved following the original paper and the second theorem with Morse-theory and the Hovanskii-estimation following ideas of Milnor.

The explicit bound seems to be bad. For instance, easy calculation shows, that for a curve defined by a threenomial the sum of the Betti numbers is ≤ 9 whereas the general bounds give 2^{16}.

Y. Ershov:

Involutory Groups and RRC-(PRC)-Fields.

An involutory group $\mathbb{G}$ is a triple $\langle G,G_0,I_G\rangle$: G is a profinite group, G_0 is an open normal subgroup of G, $[G : G_0] = 2$, $I_G \subseteq G \setminus G_0$ is a non empty set of involutions closed in G-topology and under conjugations in G. If $\mathbb{G} = \langle G,G_0,I_G\rangle$ and $\mathbb{H} = \langle H,H_0,I_H\rangle$ are the involutory groups then a morphism $\varphi : \mathbb{G} \to \mathbb{H}$ is a homomorphism (continuous) from G to H such that $\varphi(G_0) \leq H_0$ and $(I_G) \subseteq I_H$. Morphism φ is an epimorphism if $\varphi(G) = H$ and $\varphi(I_G) = I_H$; φ is a monomorphism if φ is a monomorphism on G.

Preordered field $\langle F,P\rangle$ is regularly real closed (RRC or PRC) if for every regular totally real extension $F_o \sqsupseteq \langle F,P\rangle$ (the last means: every linear order $L \supseteq P$ of F has an extention on F_o) then for every $\alpha_1,\ldots,\alpha_n \in F$ there is a F-homomorphism $\varphi : F[\vec{\alpha}] \to F$.

CDM-presentation $\Phi(G)$ is defined in Cherlin-van den Dries-Macintyre paper on RC fields for every profinite group G.

The following extends one principal result of CDM:

Theorem. If $F_o \leq F_1$ are RRC-fields ($P_i = P(F_i)$ the minimal cone on F_i), embedding is regular then

$$F_o \preccurlyeq F_1 \Leftrightarrow \Phi G(F_o) \preccurlyeq \Phi G(F_1) .$$

If $\langle F,P\rangle$ is a preordered field, $F_o \geq F$ an i-extension of F (= F_o is Galois-extension of F and $i \in F_o$, $i^2 = -1$), then $\mathbb{G}(F_o,F,P) = \langle G,G_o,I_G\rangle$ is the involutory group defined as follows: $G = \mathrm{Aut}_F(F_o)$, $G_o \leq G$ and $F_o^{G_o} = F(i)$; $I_G = \{G \mid \sigma \neq e,\ \sigma^2 = e$, there is a linear order L_σ on F_o^σ such that $L_\sigma \supseteq P\}$; $\mathbb{G}(F) := \mathbb{G}(\tilde{F},F,P(F))$.

Theorem. An involutory group $\mathbb{G}$ is isomorphic to an i-group $\mathbb{G}(F)$ for some RRC-field F iff $\mathbb{G}$ is finitely projective.

There is a reasonable definition of the admissible classes of involutory groups such that the known theory of admissible classes for RC-fields works for RRC-fields as well.

A. Prestel:
On the Polynomials $X^4 + nX^2 + 1$.

We gave a talk on Becker's problem which asks for a characterization of polynomials $f \in \mathbb{R}[X_1,\ldots,X_n] = \mathbb{R}[\vec{X}]$ belonging to the set $\sum \mathbb{R}(\vec{X})^{2m}$ of sums of 2m-th powers of rational functions in $\vec{X}$. It is no restriction to consider only homogeneous polynomials.

Theorem. Let $f \in \mathbb{R}[X_1,\ldots,X_n]$ be pos. semidefinite, homogeneous of degree 2m. Then $f \in \sum \mathbb{R}(\vec{X})^{2m}$ iff $2m \mid \mathrm{ord}_o f(p_1(t),\ldots,p_n(t))$ for all $p_1,\ldots,p_n \in \mathbb{R}[t]$.

This theorem suggests that the property of being a sum of 2m-th powers should be elementary over the theory T of real closed fields. But the following theorem shows that there is no formula $\varphi(y_o,\ldots,y_d)$ such that $a_o + \ldots + a_d x^d \in \sum R(X)^{2m}$ iff $\varphi(a_o,\ldots,a_d)$ holds in R, for all real closed fields R.

Theorem 2. $X^{2m} + nX^2 + 1 = \sum_{i=1}^{s(m)} \frac{g_i^{(n)}(X)^{2m}}{g^{(n)}(X)^{2m}}$ and $n \to \infty$ implies $\deg g^{(n)} \to \infty$ for $m \geq 2$.

(By a theorem of Becker, $\sum R(X)^m = \sum^{s(m)} R(X)^m$ for all real closed fields R.)

E. Becker:

Rational Points on Varieties over Generalized Real Closed Fields.

This is a report on a joint work with B. Jacob which is based on his modeltheoretic investigation of these fields. Let (K,P) be a real closed field of level n, i.e. P is an ordering of level n ($P + P \subset P$, $-1 \notin P$, $K^{2n} \subset P$, $K^\times/P^\times$ cyclic of order 2n) and (K,P) does not admit any proper algebraic extension. Let A be an affine K-algebra, $\mathfrak{a}$ an ideal of A, then $X = \mathrm{Hom}_K(A,K)$ is the set of K-rational points of the affine varieties attached to A, moreover set $V(\mathfrak{a}) = \{x \in X \mid x(f) = 0 \text{ for all } f \in \mathfrak{a}\}$ and $IV(\mathfrak{a}) = \{f \in A \mid x(f) = 0 \text{ for all } x \in V(\mathfrak{a})\}$. It is proved

$$IV(\mathfrak{a}) = \mathrm{rad}_S(\mathfrak{a}) := \{f \in A \mid f^{2nk} + q \in \mathfrak{a} \text{ for some } k \in \mathbb{N},\ q \in S\}$$

where S is a certain semiring in A. This result mainly depends on a precise description of the relation "K is existentially closed in an extension field". Moreover, using this explicit description it is possible to solve the 17^{th} problem of Hilbert, adapted to this situation. The definite functions, i.e. $f(x) \in P$ whenever $f(x)$ is defined, are "explicitly" described.

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