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Combinatorics, Invariant Theory and Representation Theory of Symmetric Groups

22.1. bis 28.1.1984

This conference was organized by Prof. G.-C. Rota (MIT) and Prof. A. Kerber (Bayreuth). The lectures presented methods and results concerning combinatorics, invariant theory and representation theory. They showed the broad range of applications of the representation theory of symmetric groups in mathematics, physics and chemistry together with related structures as there are symmetric functions, tableaux, diagrams, dominance order, cores, hooks, q-quotients, Specht modules, plethysms and so on.

The conference was attended by 45 participants coming from Austria, Bulgaria, Canada, Britain, France, Netherlands, Italy, Jugoslavia, Poland, Switzerland, U.S.A., West-Germany. The evenings were filled with fruitful and intensive discussions.

G. ANDREWS: The decorated hard hexagon model

This talk concerns joint work with R.J. Baxter (A.N.U., Canberra). We have solved exactly an infinite family of models in statistical mechanics. These are called the decorated hard hexagon models. The solution entails the study of several generalizations of the Rogers-Ramanujan identities. The original hard hexagon model solved by Baxter in 1980 [see R.J. Baxter, Exactly Solved Models in Statistical Mechanics, Academic Press, New York, 1982, Chapter 14]) led to series-product identities originally found by L.J. Rogers and others. All of these results related certain q-series to sums of quotients of the classical elliptic theta functions. In our more general setting we find that multi-dimensional theta series arise. We describe not only the solution but also emphasize related questions (some open) that arise in additive number theory and combinatorics.

R.W. CARTER: The left and right cells in affine Weyl groups of type A_n

Kazhdan and Lusztig have given a decomposition of any Coxeter group into equivalence classes called left cells, right cells and two-sided cells. This cell decomposition is important in a number of problems in representation theory.
The lecture will discuss recent work of J.Y. Shi, who has determined the cell decomposition explicitly when the Coxeter group is an affine Weyl group of type A_n.

R. DIPPER: On representations of general linear groups and Hecke algebras

Let $G = GL_n(q)$ and p be a prime not dividing q-1. Let (F,O,K) be a split p-modular system for G. The representation theory of G is closely connected with the representations of the Hecke algebra $R[W]_q$, where $R \in \{F,O,K\}$, and W denotes the Weyl group of G, which is isomorphic to S_n. In general $K[W]_q \cong KW = K[W]_1$, but $F[W]_q \ncong FW$. There is a special case, where $F[W]_q \cong FW$ too, namely if p divides q-1. In this case we may use the representation theory of symmetric groups to show that the p-decomposition matrix of G is lower unitriangular. If $p \nmid q-1$, partial results on representations of $O[W]_q$ and $F[W]_q$ indicate, that this holds in general.

V. DRENSKY: Representations of the symmetric group and polynomial identities of simple algebras

Let $K\langle X\rangle$ be the free associative algebra over a field of characteristic 0 and let P_n be the set of all multilinear polynomials in $x_1,\dots,x_n$. For any algebra R we denote by T(R) the T-ideal of all polynomial identities of R.
There is a natural action of S_n on P_n and $P_n \cap T(R)$ is a submodule. The sequence $\chi_n(R) = \chi_n(P_n/(P_n \cap T(R)))$, n = 1,2,..., is called the cocharacter sequence of R.
We compute $\chi_n(M_2(K))$ for the algebra $M_2(K)$ of 2×2 matrices. Some other quantitative results are obtained. Analogous results are established for Lie and Jordan algebras.

A. DRESS: The arithmetic structure of Burnside-rings, revisited

Der Burnside-Ring $\Omega(G)$ einer endlichen Gruppe G kann als Teilring seines zu einem direkten Produkt endlich vieler Kopien von $\mathbb{Z}$ isomorphen ganzen Abschlusses in $\mathbb{Q} \otimes \Omega(G)$ durch verschiedene, einfache, gruppentheoretisch formulierte Kongruenzen beschrieben werden. Aus diesen folgen fast alle bis heute bekannten arithmetischen Eigenschaften von $\Omega(G)$.

D. FOATA: Statistics on the symmetric group and q-series

Extensions of the classical identities on q-series (such as the q-binomial formula) are derived by means of the Schur function calculus. For each r and s $\geqslant 0$ let $(u;q_1,q_2)_{r+1,s+1} = \Pi(1-uq_1^{i-1}q_2^{j-1})$ (with i running from 1 to r+1 and j from 1 to s+1). Then the formula

$$\sum_n C_n(z,t_1,t_2,q_1,q_2) \frac{u^n}{(t_1;q_1)_{n+1}(t_2;q_2)_{n+1}}$$

$$= \sum_{r,s} t_1^r t_2^s \frac{(-zu;q_1,q_2)_{r+1,s+1}}{(u;q_1,q_2)_{r+1,s+1}}$$

is proved with C_n being a polynomial with positive integral coefficients of sum $2^n n!$. Moreover, this derivation yields a natural combinatorial interpretation for those polynomials. The results discussed here are taken from a joint work with Jacques Désarménien.

R. FOSSUM: Formal group law invariants

Let k be field of characteristic $p > o$. Let P be a polynomial ring $P = k[x_o,...,x_n]$ with the usual grading. Let $e \in \mathbb{N}$ be such that $p^{e-1} < n+1 \leq p^e$. Define $D:P_1 \to P_1$ by $DX_i = X_{i-1}$ for $i > o$ and $DX_o = O$. Then P_1 becomes a $k[[T]]/(T^q)$-module by $T \mapsto D$. Extend D to ∞) a derivation on P or 2) an automorphism via 1+D (since D is nilpotent). In each case the homogeneous polynomials of degree r, denoted P_r, become $k[[T]]/(T^q)$ modules. I discussed the decomposition of the P_r into indecomposable $k[[T]]/(T^q)$-modules for both the derivation and automorphism action, as well as other formal group law actions.

C. GREENE: Standard tableaux and balanced tableaux

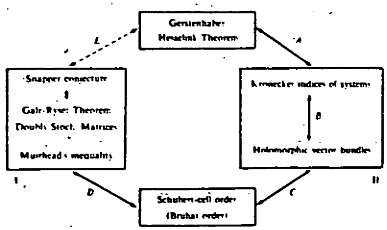
We define and discuss a new class of combinatorial objects called balanced tableaux. For a given shape λ, these are equinumerous with standard tableaux of shape λ, and we show this by purely combinatorial arguments. In the special case when λ has stair-case shape, balanced tableaux can be viewed as encodings of maximal chains (reduced decompositions) in the weak Bruhat order of S_n. Our theorem can be used in this special case to give a combinatorial proof of a theorem of R. Stanley, originally proved by algebraic methods. The arguments make heavy use of Schützen-berger's promotion and evacuation operators, as well as a new variant of the Robinson-Schensted correspondance. Several new contributions to these two areas are also mentioned.

M. HAZEWINKEL: Representations of the symmetric group, the specialization order, systems and Grassmann manifolds

A certain partial order on the set of all partitions of a given natural number n describes many containment, specialization or degeneration relations in the seemingly, rather disparate parts of mathematics dealing with permutation representations of S_n, the existence of (0,1)-matrices with prescribed row and column sums, symmetric mean inequalities, orbits of nilpotent matrices under similarity, Kronecker indices of control systems, doubly stochastic matrices and vectorbundles over the Riemann sphere. In this paper we discuss relations between all these subjects which show why the same ordering must appear all the time. Central to the discussion is the Schubert-cell decomposition of a Grassmann manifold and the associated (closure) ordering which is a quotient of the Bruhat ordering on the Weyl group S_n.
The ordering in question is defined as follows. Let p,q be n-vectors, $\Sigma p_i = \Sigma q_i$, $p_i, q_i \geq 0$ then

$$p \succ q \iff \bar{p}_1 \leq \bar{q}_1, \bar{p}_1+\bar{p}_2 \leq \bar{q}_1+\bar{q}_2, \ldots, \bar{p}_1+\ldots+\bar{p}_k \leq \bar{q}_1+\ldots+\bar{q}_k, \ldots$$

where $(\bar{p}_1,\ldots,\bar{p}_n)$ is $(p_1,\ldots,p_n)$ rearranged so that $\bar{p}_1 \geq \bar{p}_2 \geq \ldots \geq \bar{p}_n$. The pattern of relations takes the following form



Relation B associates the socalled Hermann-Martin vectorbundle to a system (A,B), A an n×n matrix, B an n×m matrix. Relation C is based on the classifying map of this bundle, and relation A takes the shape of a completely dual proof of the two theorems involved, the one dealing with the closure of nilpotent orbits (under similarity) and the other dealing with the closures of feed-back group orbits of systems.

Perhaps the most beautifull relation in this diagram is E. This one is discussed by H.-P. Kraft at this meeting and involves the socalled Springer representation (Springer, Springer-Hotta) and work of Procesi, deConcini, Kraft, Borho, MacPherson, S.I. Gelfand. The diagram also extends to involve such things as Verma modules.

For the relations indicated in box I the reader is referred to a paper of Harper & Rota (Progress in Probability Vol. 1) and for matters much related to it and further occurences of the order introduced above and its role in chemistry cf. E. Ruch's contribution to this meeting.

Ref.: M. Hazewinkel, C.F. Martin, Eins. Math. 24 (1983), 53-87

J.F. HUMPHREYS: Projective representations of wreath products $G \wr S_n$ and $G \wr A_n$

A report was given on joint work with Peter Hoffman. Constructions for groups and representations were given which are important for our investigations of the complex projective representations of $G \wr S_n$ and $G \wr A_n$ (G a finite group). Let $\mathcal{G}$ be the set of finite groups Γ with a central involution z and a homorphism $s:\Gamma \to \mathbb{Z}/2$ with $s(z)=0$. Given two such objects Γ_1, Γ_2, we define another element of G, $\Gamma_1 \hat{\times} \Gamma_2$ by taking the cartesian product of Γ_1 and Γ_2 with twisted multiplication $(g_1,g_2)(h_1,h_2) = (z_1^{s_1(g_2)s_2(n_2)} g_1h_1, g_2h_2)$ and factoring out by the central subroup $\{(1,1),(z_1,z_2)\}$.

Given an element Γ of G, any $\mathbb{Z}/2$-graded representation of Γ, is a pair $\{V_o, V_1\}$ of finite dimensional vector spaces such that $V_o \oplus V_1$ is a representation of Γ with $gV_i = V_i$ if $s \in \ker s$ and $gV_i = V_{i+1}$ if $g \notin \ker s$ $(i \in \mathbb{Z}/2)$. The Grothendieck group generated by "negative" $\mathbb{Z}/2$-graded representation of Γ (ones on which z acts as -1) is denoted $GR^-\Gamma$ and that generated by "negative" representations of Γ is denoted $R^-\Gamma$. Let $W\Gamma$ be the $\mathbb{Z}/2$-graded abelian group with $W\Gamma^{(o)} = GR^-\Gamma$ and $W\Gamma^{(1)} = R^-\Gamma$. This can be made into a module over the ring $L := \mathbb{Z}[p]/(p^3=2p)$. Our main theorem is that we can define a product $W_1^\Gamma \otimes_L W_2^\Gamma \to W(\Gamma_1 \hat{\times} \Gamma_2)$ which is an isomorphism of L-modules.

P. HOFFMAN: <u>Induction algebras for S_n and A_n graded over $\mathbb{Z}/2 \times \mathbb{N}$</u>

If $G\wr S_n$:= pullback of $\left\{ \begin{matrix} (\text{doublecover of } S_n) \searrow \\ G\wr S_n \nearrow \end{matrix} S_n \right\}$, then $\widehat{G\wr S_n}$ is an object in G defined in the previous abstract of J. Humphreys. A monomorphism $\Phi: \widehat{G\wr S_i} \hat{\times} \widehat{G\wr S_j} \to \widehat{G\wr S_{i+j}}$ can be defined by analogy with Young subgroups. The theorem of Humphreys' abstract then gives us a product and a coproduct on $H := \bigoplus_{n=0}^{\infty} W(\widehat{G\wr S_n})$:

$$W(\widehat{G\wr S_i}) \otimes_L W(\widehat{G\wr S_j}) \overset{\cong}{\rightleftarrows} W(\widehat{G\wr S_i} \times \widehat{G\wr S_j}) \underset{\Phi^*}{\overset{\Phi^*}{\rightleftarrows}} W(\widehat{G\wr S_{i+j}}).$$

Here Φ_* (resp. Φ^*) is inducing (resp. restricting). Our structure theorem (with John Humphreys) is that H is a Hopf algebra over L with elements $h_i^{(A)} \in W^{(i+1)}(\widehat{G \wr S_i})$, coming from Clifford modules, such that, as an algebra,

$$H \hat{=} \mathrm{Alg}_L\{h_i^{(A)} : i \geq 1,\ A \in \text{Conj. Class } G\}/\langle(\text{CONN}), (\text{SQ})\rangle, \text{ where}$$

$$(\text{CONN}):\ xy = r^{\varepsilon\delta+ij}yx \text{ for all } x \in W^{(\varepsilon)}(\widehat{G \wr S_i}), y \in W^{(\delta)}(\widehat{G \wr S_j})$$

$$(\text{SQ})\ :\ (h_i^{(A)})^2 = (-1)^{i+1}p[h_{2i}^{(A)} + p\sum_{j=1}^{i-1}(-1)^j\, h_j^{(A)}\, h_{2i-j}^{(A)}].$$

Here $r := p^2-1$ in L. We find:

(i) for $n \geq 4$, monomial bases for groups of projective representations of $G \wr A_n$ (i.e. for $W^{(0)}$) and of $G \wr S_n$ (i.e. for $W^{(1)}$);

(ii) the action of p gives inducing and restricting between these;

(iii) the coalgebra map is $h_i^{(A)} \mapsto h_i^{(A)} \otimes 1 + 1 \otimes h_i^{(A)} + p\sum_{j=1}^{i-1} h_j^{(A)} \otimes h_{i-j}^{(A)}$ "branching rules".

(iv) the irreducibles are obtained by applying Gram-Schmidt to bases as in i), suitably ordered. Schur (in 1911) proved essentially i), ii) and iv) for the case G = {1} by different methods.

G.D. JAMES: <u>Representations of general linear groups</u>

We discussed a result in the representation theory of $G_n = GL_n(q)$, similar to one from the theory for the symmetric group S_n. Let T_λ be the λ-tableau in which numbers increase from left to right in each row and from one row to the next, and suppose that $T_\lambda \pi_\lambda$ ($\pi_\lambda \in S_n$) is the λ-tableau having entries increasing from top to bottom in each column and from one column to the next. For the representation theory of S_n over a field K one may begin with the right ideal M_λ of K S_n generated by ρ_λ, the sum of the elements in the row stabilizer of T_λ. If κ_λ denotes the signed sum of the elements in the column stabilizer of $T_\lambda \pi_\lambda$, we have $M_\mu \kappa_\lambda = 0$ unless $\lambda \trianglerighteq \mu$, and $M_\lambda \kappa_\lambda$ is the one-dimensional space spanned by $\rho_\lambda \pi_\lambda \kappa_\lambda$.

Now suppose that K is a field of characteristic not equal to p, where q is a power of p, and assume that K contains a primitive p-th root of unity. Now let M_λ be the right ideal of KG_n generated by $\bar{P}_\lambda$, the sum of the matrices in a parabolic subgroup corresponding to λ. We define a certain idempotent E_λ in KU^-, where U^- is the group of lower unitriangular matrices, and for this element we get $M_\mu E_\lambda = 0$ unless $\lambda \trianglerighteq \mu$, and $M_\lambda E_\lambda$ is the one-dimensional space spanned by $\bar{P}_\lambda \pi_\lambda E_\lambda$. It turns out that the KG_n-module $\bar{P}_\lambda \pi_\lambda E_\lambda (KG_n)$ enjoys many properties analogous to those of the Specht module $\rho_\lambda \pi_\lambda \kappa_\lambda (KS_n)$ for the symmetric group.

T. JÓZEFIAK: <u>Symmetric functions and Koszul complexes</u>

R. Stanley has recently asked for a representation-theoretic interpretation of the following formula of D.E. Littlewood

$$\prod_i (1-x_i) \prod_{i<j} (1-x_i x_j) = \sum_I{}' (-1)^{(|I|+r(I))/2} s_I ,$$

where $|I|$ is the weight of a partition I, $r(I)$ is the rank of I, s_I is the Schur function corresponding to I and the sum ranges over all self-conjugate partitions.

We derive and interpret the formula using the Koszul complex of a symmetric algebra $S.(U+\wedge^2 U)$ where U is a vector space over a field K of characteristic zero.

Using a free resolution of the residue field K over an exterior algebra $\wedge^{\cdot}(U+S_2U)$ we obtain a new identity

$$\prod_i (1+x_i)^{-1} \prod_{i\leq j} (1+x_ix_j)^{-1} = \sum_{I\in B} (-1)^{(|I|+p(I))/2} s_I$$

where $p(I)$ is the number of odd parts of I and B is the set of all partitions I satisfying

$$i_K - i_{K+1} \equiv 0 \text{ or } 1 \pmod 4 \text{ for } i_K \text{ even,}$$
$$i_K - i_{K+1} \equiv 1 \text{ or } 2 \pmod 4 \text{ for } i_K \text{ odd.}$$

These results were obtained in collaboration with J. Weyman.

M. KLEMM: <u>On the square root bound for the minimum distance of codes</u>

Let C be an (n,k) code, invariant under an abelian group $(A,+)$, acting transitively on the basis of the ambient space over a field F with char $F \nmid n = |A|$.

<u>Theorem 1</u> (J. Comb. Theory A, to appear).

Assume that C contains the repetition code and that $\dim(C \cap C^{\perp}) = k-1$. Denote by d^* the minimum distance of $C^* = C \setminus (C \cap C^{\perp})$. Then

$$d^{*2} - d^* + 1 \geq n$$

with equality if and only if the supports of vectors of weight d^* in C^* form a projective plane of order d^*-1.

The case $d^{*2}-d^*+1 = n$ can often be excluded with Hall's multiplier theorem on projective planes, a theorem which follows from part a) of the following statement.

<u>Theorem 2</u>

<u>a</u>) Let $F = GF(p)$. Then the minimum weight structure of C is invariant under the automorphism $a \to pa$ of the basis group $(A,+)$.

<u>b</u>) Let $F = \mathbb{Q}$. Then the minimum weight structure of C is invariant under the automorphisms $a \to ma$ $(a \in A)$, $(m,|A|) = 1$.

H. KRAFT: <u>Conjugacy classes and representations of Weyl groups</u>

This talk was a survey on results of Springer, Hotta-Springer, DeConcini-Procesi, Borho-Macpherson. We only considered the case of matrices $M_n(\mathbb{C})$ and the symmetric group S_n. <u>First construction</u>: Let C_p be the nilpotent conjugacy class in $M_n(\mathbb{C})$

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associated to the partition $p \vdash n$. Denote by R_{C_p} the coordinate ring of the schematic intersection $\bar{C}_p \cap D$, D the subspace of diagonal matrices $\subset M_n(C)$. By construction R_{C_p} is a graded, finite dimensional, S_n-algebra.

Theorem 1 (DeConcini-Procesi; Invent. 64, 1981):

a) $R_{C_p} \cong \mathrm{Ind}_{S_{\hat{p}}}^{S_n} 1$, $\hat{p}$ dual partition, $S_{\hat{p}} = S_{\hat{p}_1} \times \ldots \times S_{\hat{p}_K} \subset S_n$.

b) max. degree of R_{C_p} is $d_{\hat{p}} = \Sigma \binom{p_i}{2}$, and $(R_{C_p})_{max} \cong M_{\hat{p}}$ the irred. rep. associated to $\hat{p}$.

Springer's correspondence: Let F be the flag variety of complete flags in $\mathbb{C}^n$. One has a natural S_n-action on F and an isom. $H^*(F) \simeq R_{reg}$ the regular representation of S_n. For a nilpotent $x \in M_n(\mathbb{C})$ define $F_x := \{F \in F \,/\, xF = o\}$. Let $x \in C_p$.

Theorem 2 (Springer, Hotta-Springer; Invent. 36, 41, 44)

a) There is a repres. of S_n on $H^*(F_x)$ compatible with the restriction $H^*(F) \to H^*(F_x)$.

b) The max. cohomology is $H^{2d_x}(F_x) \cong M_p$, $d_x = d_p = \sum_i i p_{i+1}$. DeConcini and Procesi show in addition, that $R_{C_{\hat{p}}} \simeq H(F_x)$ in a natural way.

Borho-MacPherson's approach: Via intersection homology (C.R. 292, 1981) they reprove Springer's results and give more precise informations about the other cohomology groups:

Theorem 3: Let $x \in C_p$, $q \vdash n$: $\mathrm{mult}(M_q, H^{2i}(F_x)) = \dim \mathbb{H}_x^{2i-2d_q}(\bar{C}_q)$.

J.D. LOUCK: A class of polynomials characterizing SU(3) tensor operators

A class of polynomials $G_q^t(\Delta;x)$ ($q \in \mathbb{Z}^+$, $t = 1,\ldots,q$), which depend on three parameters $\Delta = (\Delta_1,\Delta_2,\Delta_3)$ with $\Delta_i \in \mathbb{Z}^+$ and $\Delta_i \geqslant q$ and on three coordinates $x = (x_1,x_2,x_3)$ belonging to the Möbius plane $\mathbb{M}$ (barycentric coordinates), occur in the calculation of the canonical Clebsch-Gordon-Wigner coefficients of the unitary group SU(3). The symmetries and zeros of these polynomials are discussed in some detail. In particular, it is shown that, for each prescribed Δ, the polynomial $G_q^t(\Delta;x)$ has a zero at each (lattice) point of the weight space of a U(3) irreducible representation $(q-t,o,-t+1)$, where the correspondence between points $x \in \mathbb{M}$ and weights $w \in (q-t,o,-t+1)$ is $x_1 = \Delta_3-t+1-w_1$, $x_3 = \Delta_2-t+1-w_3$, $x_2 = -x_1-x_3$. Moreover, the proof that the multiplicity of a zero is at least the multiplicity of a weight (Kostka number) is given. The role of the generalized Gauss hypergeometric function and the associated generalized Saalschütz identity [J. Math. Analysis and Appl. 59 (1977), 423-431] in establishing this result is sketched.

G. MURPHY: On generalized Young tableaux

We consider the analogue of the Specht module over the field $\mathbb{Q}$ for an array of nodes which cannot be reduced to a proper or skew diagram by any rearrangement of rows or columns. The smallest such diagram has six nodes.

By looking at some small examples we list various methods for finding composition factors. The James-Peel method (J. Alg. 56.2, Feb. 1979) seems most likely to succeed. Garnir relations for related tableaux of skew shape, together with interchanges of elements in the same column generate part of the annihilats ideal, but additional relations involving row interchanges are needed. The standard tableaux only form part of the basis found.

G.E. MURPHY: On the calculation of the p-modular decomposition matrices of the symmetric groups

Let Γ be the Gram matrix for a Specht module S^{μ} for a partition μ of n and p an arbitrary prime. For each p-modular composition factor D^{λ} of S^{μ} of multiplicity $d_{\mu\lambda}$ there is a set of integers $\{k^{i}_{\mu\lambda} : i=1,2\ldots d_{\mu\lambda}\}$ such that the exponent of p in the prime decomposition of det Γ is

$$\nu_p(\det \Gamma) = \sum_{\substack{\lambda \rhd \mu \\ i \leqslant d_{\mu\lambda}}} k^{i}_{\mu\lambda} \dim(D^{\lambda}) \qquad (*)$$

Using the results of [James, Murphy, J. Algebra 59(1), 1979] we derive an expression similar to (*), which we conjecture to be identical with it; in the case that all decomposition numbers $d_{\gamma\lambda}$ are known for $\nu \rhd \lambda$ we can calculate the sum $k'_{\mu\lambda}+\ldots+k^{d_{\mu\lambda}}_{\mu\lambda}$. If it is known that $d_{\mu\lambda}$ is 0 or 1, we can determine which is the case. By duality, we show that

$$k^{i}_{\mu\lambda} \leq \nu_p \text{ (product of the hooks of } [\mu]).$$

For some special cases the coefficients $k^{i}_{\mu\lambda}$ can be calculated; in particular, when μ has 2 parts.

J.B. OLSSON: Symbols, hooks and degrees of unipotent characters

According to Lusztig the unipotent characters of the finite classical groups (symplectic or orthogonal) are indexed by so-called symbols. In order to be able to establish certain character correspondences for the modular r-blocks of characters of these groups (joint work with G. Michler) it is necessary to know the exact power of r dividing the degree of a unipotent character. The study of these character degrees indicates that it is reasonable to generalize the wellknown concept of e-hooks, e-cores and e-quotients for partitions to symbols. For symbols there is also a theory of cohooks, cocores and coquotients which has to be considered. These concepts and their relation to the characters was the topic of the talk.

P. PAULE: Two q-transformations

Three applications of the two q-identities (δ=0 or 1):

$$\sum_{h=-\infty}^{\infty} c_h \binom{a+c+\delta}{a+h}\binom{a+b+\delta}{b+h}\binom{b+c+\delta}{c+h} = \sum_{j=o}^{\infty} \frac{(q)_{a+\delta+c+\delta-j}\, q^{j^2+\delta j}}{(q)_{a-j}(q)_{b-j}(q)_{c-j}(q)_{2j+\delta}}$$

$$\sum_{h=-\infty}^{\infty} \binom{2j+\delta}{j+h} q^{-h^2+\delta k} \cdot c_k$$

were given: 1.) In the field of Rogers-Ramanujan type identities

2.) A generalization of Andrews' q-Dyson conjecture: $u = 3$

3.) Two conjectures of Kadele for the case $u = 3$

W. PLESKEN: <u>Sublattices of Specht modules over $\mathbb{Z}$</u>

The investigations of submodules of Specht modules over $\mathbb{Z}/p\mathbb{Z}$ by James, Murphy, and Peel can be extended to the corresponding question for Specht modules over $\mathbb{Z}$. For partitions of the form $(n-k,k)$ the lattice of submodules is shown to be distributive and can be partially described by using Schaper's extension of the description for the Gram determinant of a Specht module given by James and Murphy. For k smaller than 4 a full description is given and explicite $\mathbb{Z}$-bases for the submodules can be given. (Joint work with D. Stockhofe).

E. RUCH: <u>The direction distance</u>

The direction distance $d[x/y]$ is introduced to obtain a quantity that accounts for a "normspecific similarly" of directions in vectorspaces with any given norm

$$d:\begin{cases}\mathbb{E}\times\mathbb{E} \to \mathbb{D}\subset\mathbb{R}^{\geq 0}(\mathbb{R}^{\geq 0}\times\mathbb{R}^{\geq 0}) \\ (x,y)\mapsto d[x/y]\end{cases},\quad d[x/y]:\begin{cases}\mathbb{R}^{\geq 0}\times\mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0} \\ (\alpha,\beta) \mapsto \|\alpha x_o-\beta y_o\|\end{cases},\ z_o=\frac{z}{\|z\|}$$

with the order relation $d[x/y] \prec d[x'/y'] \Longleftrightarrow \|\alpha x_o-\beta y_o\| \leq \|\alpha x_o'-\beta y_o'\|$

$(\mathbb{D},\prec)$ becomes a set, that is not totally ordered in general. It can be shown that this ordering is total if and only if the norm satisfies the parallelogram equation. Therefore in cases of Prehilbertspaces there is a normspecific angle, while in all other cases the except represents the similarily of directions in a quantitative sense. In case of L^1 and l^1-norms and in the case of density operators the interpretation is "mixing distance of" and "information distance" and can be shown to be of physical relevance in the context with statistics.

B. SAGAN: <u>Shifted tableaux, Schur's Q-functions, and a conjecture of R. Stanley</u>

We provide a new version of the shifted Schensted algorithm (differing from Sagan, J.C.T.A, 1979) which enjoys many of the properties of the original Robinson-Schensted correspondence, generalizes to a Knuth-type algorithm for tableaux with repeated entries, and provides a proof of a conjecture of Stanley.

If P is a shifted partial Young tableau (distinct entries) and x is a positive integer, $x \notin P$, we define a new tableau $P' = I_x(P)$ by inserting x in P as follows. Elements are bumped from row to row in the usual way until one of two things happen. Either an element comes to rest at the right end of some row (a Schensted move) or a diagonal entry $P_{ii} \in D_P$ is displaced (a non-Schensted move). In the latter case the algorithm continues by inserting P_{ii} in column i+1 and continuing column insertion until an element comes to rest at the end of a column. Using this process we can prove

<u>Theorem 1</u> There is a bijection $\pi \leftrightarrow (P(\pi);Q(\pi))$ between S_n and pairs of shifted standard tableaux or SST (entries 1...n) of the same shape with Q having a subset of its off-diagonal elements circled (an element $i \in Q$ is circled if the i^{th} insertion is non-Schensted). □

Let T be a *costrip* (column strict reverse plane partition) and write $T \cong P$, P an SST, if T =  . Let $R_x(T)$ (respectively $C_x(T)$) denote the usual row (resp. column) insertion where x displaces the least element $\geqslant x$ (resp. $> x$), then to "lift" properties of the original Schensted map, use

<u>Lemma</u> If $P \cong T \Rightarrow I_x(P) \cong R_x \circ C_x(T) = C_x \circ R_x(T)$ □

<u>Corollary 1</u> $P(\pi_1) = P(\pi_2) \Longleftrightarrow \pi_1$ can be obtained from π_2 by (1) Knuth transpositions and/or (2) interchanging the first two elements. □

Let T^* be a shifted generalized tableau (SGT), i.e., weakly increasing along rows and columns and strictly along diagonals. The set of <u>special</u> <u>elements</u> in $S_{T^*} = \{T^*_{ij} \mid T^*_{ij-1} < T^*_{ij} < T^*_{i+1j}\}$, then a modification of I_x yields

Theorem 2 There is a bijection $M \leftrightarrow (T^*, U^*)$ between matrices $M = (M_{ij})$, $M_{ij} \in \mathbb{N}$, with a subset of positive M_{ij} circled and pairs of SGT of the same shape with subsets of $S_{T^*} \cup D_{T^*}$ and S_{U^*} circled. □

Corollary 2 Let $P_\lambda(x) = \sum_{T^* \in \lambda} 2^{|S_{T^*}|} m(T^*)$, where $m(T^*)$ is the monomial of T^* and let $Q_\lambda(x) = 2^{l(\lambda)} P_\lambda(x)$, then $\sum_{\lambda \vdash n} Q_\lambda(x) P_\lambda(y) = \prod_{i,j} \frac{1+x_i y_j}{1-x_i y_j}$ □

Theorem 3 (Stanley's Conjecture) If $e_\lambda(x)$ is the usual Schur function and $P_\lambda(x) = \sum_{\mu \vdash n} \kappa_{\lambda\mu} e_\mu(x)$ then $\kappa_{\lambda\mu} \in \mathbb{N}$. □

L. SOLOMON: A character formula for Coxeter groups

(joint work with Peter Orlik and Hiroaki Terao)

Let V be a vector space of dimension l over $\mathbb{R}$ and let $G \subset GL(V)$ be a finite irreducible reflection group. If $g \in G$ let $k(g)$ be the dimension of the fixed point set of g. Let $d_1, \ldots, d_l$ be the degrees of the basic polynomial invariants of G and let $m_i = d_i - 1$. Shepard and Todd proved that

$$\sum_{g \in G} t^{k(g)} = (t+m_1) \ldots (t+m_{l-1})$$

We prove

$$\sum_{g \in G} tr(g) t^{k(g)} = l(t-1)(t+m_1) \ldots (t+m_{l-1})$$

where tr means trace. The argument uses Terao's theory of tree arrangements of hyperplanes.

D. SVRTAN: Decompositions of the graded tensor representations of symmetric groups via symmetric functions

We consider here an infinite family $(\rho^n_{T,k} : T \subset \mathbb{Z}_+, k \in \mathbb{Z}_+)$ of representations of the symmetric groups S_n, $n \geqslant o$, which are defined as follows. Let $\mathcal{T}$ be the tensor algebra of a free, over the set $\mathbb{Z}_+$, complex vector space and for given subset T of $\mathbb{Z}_+$ let $\mathcal{T}_T$ be the sub algebra of $\mathcal{T}$ generated by T. Now, $\rho^n_{T,k}$ is defined to be the natural representation of S_n on a subspace of $\mathcal{T}_T$ spanned by all simple tensors of length n and weight k. The main result describes explicitly all decompositions of $\rho^n_{T,k}$: mult $\xi_\alpha(\rho^n_{T,k})$ is equal to Res$(q^{-k-1}$characteristic $(\xi^{\perp}_\alpha)(q^T))$. Particular choices of T give (old and new) decompositions of the natural representations of S_n on the, for example, 1) k-subsets of [n] (T = {0,1});2) weighted tensor powers W^n, dim W = m (T = [m]);3) polynomial rings (T = $\mathbb{Z}_+$) (including a result of He(1982) and a general formula for the inner plethysm $\chi^{(k)} \odot \chi^{(n-1,1)}$, generalizing some computations of Littlewood $(k \leqslant 4)$;4) truncated polynomial algebras (T = {0,1,...,m}) etc.

G. VIENNOT: A local definition of the Robinson-Schensted correspondance and an invariant of the plactic monoid

We give an interpretation of the value located at the (i,j)-cell of the P and Q-symbol of the Robinson-Schensted correspondance. This local definition is symmetric in rows and columns and follows from work of C. Greene and Frank.

We introduce the new concepts of "(i,j)-grid" and "extendable (i,j)-grid". These concepts can be also defined for skew-Young tableaux, and we show their invariance under the "jeu de taquin" of A. Lascoux and M.P. Schützenberger. From that, we can rederive the plactic monoid theory.

Also, generalization for arbitrary posets can be done, in relation with recent work of Fomin and Gansner.

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