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Nukleare Fréchet-Räume

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The meeting was organized by E. Dubinsky (Potsdam, N.Y.) and D. Vogt (Wuppertal). 23 mathematicians from Austria, Brazil, Bulgaria, Czechoslowakia, Finland, Poland, Spain, United States and West Germany took part in the conference.

The 18 lectures that were presented dealt mainly with operators in (F)-spaces, structure theory of nuclear (F)-spaces and applications to analysis, for example algebras of entire functions, convolution operators, differential operators. The fruitful discussions during the meeting gave evidence of the pleasant atmosphere at the "Mathematische Forschungsinstitut Oberwolfach".

Abstracts

Banaszczyk, W.

Pontryagin duality in nuclear spaces

Let G be an abelian topological group. By G^{Λ} we denote the dual group, consisting of all continuous characters of G, with the topology of precompact convergence. G is called reflexive, if the canonical imbedding $G \to G^{\Lambda\Lambda}$ is a topological isomorphism. If $A \subset G$, then we define $A^{a} = \{\chi \in G^{\Lambda} : \chi(A) = \{0\}\}$. A reflexive group G is called strongly reflexive, if every closed subgroup and Hausdorff quotient of G and G^{Λ} is reflexive, and if for each closed subgroup H of G the canonical mappings $(G/H)^{\Lambda} \to H^{a}$, $G^{\Lambda}/H^{a} \to H^{\Lambda}$, $(G^{\Lambda}/H^{a})^{\Lambda} \to H$ and $G/H \to (H^{a})^{\Lambda}$ are topological isomorphisms. The main result is that every nuclear Fréchet space is a strongly reflexive additive topological group. This generalizes the result known for countable products of real lines, and even locally compact groups.

Djakov, P.

Complemented Block subspaces of Köthe spaces

Let E be a Köthe space and $\{e_i, i \in I\}$, $I = \{1,2,...\}$, be its natural basis. Suppose $I = \bigcup_{n=1}^{\infty} I_n$ is a decomposition of the set of indeces I into disjoint subsets and let E_n be the closed linear hull of vectors $\{e_i, i \in I_n\}$. Then we have $E = \bigoplus_n E_n$ in the sense, that every element $x \in E$ has a unique representation $x = \sum_n x_n$, where $x_n \in E_n$. In the following we call subspaces E_n blocks of E. We say that the subspace F is a block subspace with respect to the decomposition $E = \bigoplus_n E_n$ if we have $F = \bigoplus_n F \cap E_n$. The following result is obtained by the author and Prof. Ed Dubinsky:

Theorem: Suppose E is a Köthe space, $E = \bigoplus_n E_n$ is a decomposition of E into blocks and F is a complemented block subspace with respect to the given decomposition. If $\sup_n \dim E_n < \infty$ the subspace F is isomorphic to some coordinate subspace of E (i.e. subspace, generated by some part of the natural basis of E).

Remark: In nuclear case instead of $\sup_n \dim E_n < \infty$ it is enough to suppose $\sup_n \dim (F \cap E_n) < \infty$.

Floret, K.

Norms and Bases

An example of a strict inductive sequence (E_n) of nuclear Frêchet spaces with continuous norms was presented such that

(1) each E_n is not countably normed and does not have the bounded approximation property.

(2) The inductive limit E does not admit any continuous norm.

(3) E does not have a basis.

The main idea of the construction (in order to satisfy (2)), namely violating certain concordance properties of the norms on the step-spaces, implies automatically property (1). That E has no basis is a consequence of the structural property of strict inductive limits with an unconditional basis being a direct sum of complemented subspaces of the step-spaces - a result due to S. Dineen. Dualizing this result (e.g. to: certain strict Frêchet spaces with an unconditional basis are actually product-spaces) simplifies the proof that V.B. Moscatelli's example of a nuclear Frêchet space does not have a basis.

K. Floret: Continuous norms of locally convex strict inductive limit-spaces; preprint.

K. Floret - V.B. Moscatelli: On bases in strict inductive and projective limits of locally convex spaces; to appear in: Pacific J. Math.

Gramsch, B.

Perturbation theory in Frêchetalgebras of pseudodifferential operators

The results of Beals (1977), Cordes (1979), Connes (198o) and Dunau (1977) show that interesting Frêchetalgebras of pseudodifferential operators have an open group of invertible elements; therefore the inversion is continuous and the analytic functional calculus in several variables applies. Definition: A Frêchetalgebra ψ in a Banachalgebra $\mathcal{B}$ is called a ψ-algebra if for the groups ψ^{-1} and B^{-1} of invertible elements the condition $\psi \cap \mathcal{B}^{-1} = \psi^{-1}$ is satisfied. It "follows" that the connected components of $\mathcal{P} = \{p \in \psi : p^2=p\}$ are locally - ψ - rational homogeneous Frêchet manifolds. This sharpens in the case of Banachalgebras a result of Raeburn (1977). Similar results (much more complicated to prove) are valid for the set of operators in ψ with closed range ($\mathcal{B}$ a C^*- algebra). This makes it possible to apply in the Banach situation the Oka principle for homogeneous manifolds. The local non-linear problem in the proof of an Oka principle (Lemma of H. Cartan) can be treated for special Frêchet algebras resp. Frêchet Liegroups defined by means of generalized dynamical systems. Dynamical systems have been applied to the theory of pseudo differential operators by Cordes (1979) and Connes (198o). The implicit function theorem for Banach spaces is sharpened with parameterized families of automorphisms $\{ \alpha_t \; t \in \Omega\}$ through C^∞ - or A-smooth elements. There are some connections with the Nash-Moser method. Counterexamples show the limitations of the method with α_t- automorphisms which on the other hand is not restricted to Frêchet spaces.

Haslinger, F.

Spaces of holomorphic functions

Let $P = (p_m)_{m=o}^\infty$ be a family of functions $p_m : \mathbb{C}^k \to \mathbb{R}$, such that $p_m \geq p_{m+1}$ for $m \in \mathbb{N}$ and $\exp(p_1)$ is locally integrable. $\mathcal{F}_p$ denotes the space of all entire functions f such that $\|f\|_m := \sup_{z\in\mathbb{C}} |f(z)| \exp(-p_m(z)) < \infty$, $\forall m \in \mathbb{N}$. It is shown that $\mathcal{F}_p$ is a nuclear Frêchet space, if the following two conditions are satisfied : (i) $\forall m \; \exists 1$ such that $\int_{\mathbb{C}^k} \exp(-p_m(z) + p_1(z))\, d\lambda(z) < \infty$

(λ being the Lebesgue measure on $\mathbb{C}^k$), (ii) $\forall m \; \exists 1$ and a constant $C = C(m,1) > 0$ such that $\sup_{\xi\in B_z} \{\exp(p_1(\xi))\} \leq C \exp(p_m(z)) \; \forall z \in \mathbb{C}^k$, where

$B_z = \{\xi : |\xi - z| \leq 1\}$. Special cases are due to Mityagin and Wloka. Now let $p : \mathbb{C}^k \to \mathbb{R}^+$ be a plurisubharmonic, convex function and $(r_m)_m$ a sequence of real positive numbers with $r_m \searrow r_\infty > 0$. Define the weight functions by $p_m(z) = r_m p(z)$ and suppose that $\int_{\mathbb{C}^k} \exp(-r_\infty p(z))\, d\lambda(z) < \infty$. If the above

conditions (i) and (ii) are satisfied, the topology in $\mathcal{F}_p$ can also be determined by the sequence of Hilbert-norms

$$|f|_m^2 = \int_{\mathbb{C}^k} |f(z)|^2 \exp(-2r_m p(z))\, d\lambda(z) .$$

Let $E_m = \{f \text{ entire}: |f|_m < \infty\}$, $m = 0,1,\ldots, m = \infty$. Then it can be shown that E_∞ is dense in E_o, this is proven by using results of B.A. Taylor on polynomial approximation in weighted spaces of entire functions. Finally the problem of existence of a basis in $\mathcal{F}_p$ is discussed in connection with a method due to Mityagin and Henkin.

Holmström, L.

Locally round FDDs

The work described is mostly done jointly with Ed Dubinsky:
Let E be a Fréchet space with a continuous norm and suppose (E_n) is an FDD of E. We say that (E_n) is locally round if there is a matrix $(a(n,k))$, $0 < a(n,k) \le a(n,k+1), n,k \in \mathbb{N}$, and a Hilbert norm $\| \|_{n_0}$ on each E_n such that the norms

$$\|x\|_k = \sum_{n=1}^{\infty} a(n,k) \|x_n\|_{n_0}, \quad x = \sum_{n=1}^{\infty} x_n \in E, \; k \in \mathbb{N},$$

define the topology of E. It is easy to see that if $F_n \subset E_n$ is a subspace, $n \in \mathbb{N}$, then $\overline{sp} \bigcup_n F_n$ (a "block subspace") is complemented and has a basis. Conversely, in the case E is a Fréchet Schwartz space and (E_n) is strong and absolute, the complementedness of all block subspaces(or just block basic sequences) implies that (E_n) is locally round. If (E_n) is obtained by grouping together coordinate basis vectors (in their natural order) in a weakly stable nuclear power series space, then the complementedness of all block subspaces (block basic sequences) and the existence of a basis in every block subspace both imply that (E_n) is locally round.

John, K.

Tensor products of several spaces and nuclearity

Let $X_1,\dots,X_r$ be locally convex spaces. We consider the question (G_r):
Suppose that the ε - and π- topologies coincide on the tensor product $X_1 \otimes\dots\otimes X_r$. Does it follows that at least one of the spaces $X_1,\dots,X_r$ must be nuclear?
We show that the answer to (G_r) is no for every $r \ge 2$. Non-nuclear (FS) spaces $X_1,\dots,X_r$ with bases are constructed such that $X_1 \otimes_\varepsilon \dots \otimes_\varepsilon X_r = X_1 \otimes_\pi \dots \otimes_\pi X_r$. Moreover, every continuous r-linear form on $X_1 \times \dots \times X_r$ is strongly nuclear. But if $r \ge 3$ and $X_1 = \dots = X_r = X$ then the answer to (G_r) is yes. Also if $X_1, \dots, X_r$ are Banach spaces and if the ε - and π - topologies coincide on $X_1 \otimes \dots \otimes X_r$ then at least $r - 2$ from $X_1, \dots, X_r$ have finite dimension.
The proofs are based on some results in ideals of multilinear operators, which were introduced by Pietsch in Leipzig 1983.

Langenbruch, M.

Sequence space representations for solution sheaves of partial differential equations

Let P(D) denote a hypoelliptic system of the form $P(D) := D_t Id_m - P_1(D_x)$, where $(x,t) = (x_1,x_2,t) \in \mathbb{R}^{N_1} \times \mathbb{R}^{N_2} \times \mathbb{R}$, P_1 is a $m \times m$ - system of PDO with constant coefficients and Id_m is the $m \times m$ - unit matrix. Let $Q_p(x,t) := \det P(x,t)$ and suppose that for some $\tilde{m} < m$

$$|Q_p(y+z,0)| \le C_1(|Q_p(y,0)| + |z_1|^{\tilde{m}} + |z_2|^m) \text{ for } y \in \mathbb{R}^{N-1},\ z \in \mathbb{C}^{N-1}$$

$$|Q_p(\operatorname{Re} z_1,0)|^{1/m} + |z_2| \le C(|\operatorname{Im}(z,\tau)| + 1) \quad \text{for } Q_p(z,\tau) = 0$$

Let $h(t) := \lambda\{\, y \mid |Q_p(y,0)|^{1/m} \le t\}$, λ = Lebesgue-measure and suppose that for any $C > 0$ $C\,h(t) \le h(C_1 t)$ for some $C_1 > 0$ and large t.

Let $\mathcal{N}(\mathcal{O}) := \{ f \in C^\infty(\mathcal{O})^m \mid P(D) f = 0 \}$ for $\mathcal{O}$ open in $\mathbb{R}^N$. With these assumptions the following is proved:

Theorem 1: $\mathcal{N}(\mathcal{O}_1 \times \mathbb{R}^{N_2+1})$ is (topologically) isomorphic to $(\Lambda_\infty(h^{-1}(n)))^{\mathbb{N}}$ for any $\mathcal{O}_1$ open in $\mathbb{R}^{N_1}$.

Theorem 2: Let $N_2 = 0$ in the above assumptions.

a) Then $\mathcal{N}(\mathcal{O}_1 \times \mathbb{R}_*)$ is isomorphic to

$$(\Lambda_\infty(h^{-1}(n)) \otimes \Lambda_1(h^{-1}(n)))^{\mathbb{N}} \quad \text{for any } \mathcal{O}_1 \text{ open in } \mathbb{R}^{N-1}.$$

b) Let $P_1(JX)\, I = -I\, P_1(x)$ for some matrices J and I of a special type. Then $\mathcal{N}(\mathcal{O}_1 \times \mathbb{R}_\pm)$ is isomorphic to $\mathcal{N}(\mathcal{O}_1 \times \mathbb{R}_*)$ for any $\mathcal{O}_1$ open in $\mathbb{R}^{N-1}$.

Theorem 3: Same assumptions as in Thm. 2. Then $\mathcal{N}(\mathcal{O}_1 \times (a,b))$ is isomorphic to $(\Lambda_1(h^{-1}(n)))^{\mathbb{N}}$ for any $\mathcal{O}_1$ open in $\mathbb{R}^{N-1}$.

The proofs rely on the introduction of a multiplicative structure on $\mathcal{N}(\mathcal{O})$, which is used to define certain continuous projections. They carry over to most of the classical localizable analytically uniform spaces and non hypoelliptic operators satisfying suitable conditions.

Meise, R.

On quotients of nuclear algebras of entire functions module closed ideals

Let $p: \mathbb{C} \to [0,\infty[$ be a continuous subharmonic function with the following properties:

(1) $\exists\, D > 0\ \forall z, \zeta \in \mathbb{C}$ with $|z-\zeta| \le 1$: $p(\zeta) \le D(1+p(z))$

(2) $\log(1+|z|^2) = O(p(z))$ resp. (2_0) $\lim_{|z|\to\infty} \frac{\log(1+|z|^2)}{p(z)} = 0$

(3) $p(z) = p(|z|)$ and $p(2z) = O(p(z))$.

(4) $\exists\, E > 0 : 2p(z) = O(p(Ez))$

Let $A(\mathbb{C})$ denote the space of all entire functions on $\mathbb{C}$ and put

$A_p := \{f \in A(\mathbb{C}) \mid \exists\, n \in \mathbb{N}\ \sup_{z\in\mathbb{C}} |f(z)|\, e^{-n\, p(z)} < \infty\}$ if (2) is satisfied

$A_p^o := \{f \in A(\mathbb{C}) \mid \forall\, n \in \mathbb{N} :\ \sup_{z\in\mathbb{C}} |f(z)|\, e^{-\frac{1}{n}p(z)} < \infty\}$, if (2_0) is satisfied.

Then the following theorem holds, which extends previous work of Schwartz, Ehrenpreis and Taylor. Part (b) of the theorem has been obtained in joint work with B.A. Taylor.

Theorem: (a) For every closed ideal J in A_p of infinite codimension, A_p/J is the dual of a power series space of infinite type and J is complemented in A_p.
(b) For every closed ideal J in A_p^o of infinite codimension, A_p^o/J is a power series space of finite type and J is not complemented in A_p^o.

Mujica, J.

Polynomial approximation in nuclear Frêchet spaces

Let K be a polynomially convex compact subset of a Frêchet-Schwartz space E. By combining results of Matyszczyk and Ligocka we show that if E has the bounded approximation property then each function which is holomorphic on a neighbourhood of K can be uniformly approximated by polynomials on a suitable neighbourhood of K. Next we show that the hypothesis of the bounded approximation property can be deleted whenever the space E is nuclear. As an application of this result we show that if $\mathcal{U}$ is a polynomially convex open subset of a nuclear Frêchet space, then the compact-ported topology τ_ω coincides with the compact-open topology τ_o on the space $\mathcal{H}(\mathcal{U})$ of all holomorphic functions on $\mathcal{U}$.

Nurlu, Z.

Compact operators and embeddings between nuclear Köthe spaces

In a paper which appeared in Studia Math. in 1975, Crone and Robinson show that if there exists a (linear, continuous) non-compact operator between two nuclear power series spaces of the same type then there is a common (up to isomorphsm) step space of the two spaces. We give more general examples of pairs of nuclear Köthe spaces enjoying this property.

Let $(E,F) \in \mathcal{A}$ denote the property that either each operator $E \to F$ is compact (denoted by $(E,F) \in K$) or E and F have a common step space. Then in each of the following cases for the nuclear Köthe spaces E and F, $(E,F) \in \mathcal{A}$ holds:

i) F is isomorphic to a closed subspace of the regular space E

ii) E is isomorphic to a quotient space of the regular space F

iii) $\mathrm{Ext}^1(F,E) = 0$ (Here we should remark that recently (1983) D.Vogt has obtained the result : $(E,F) \in K \Rightarrow \mathrm{Ext}^1(F,E) = 0$)

Pelczynski, A.

On Mazur's problem related to Cauchy multiplication of series

S. Mazur (Scottish Book Problem No 8) has asked whether every scalar series summable by the method of first means can be represented as a Cauchy product of two convergent series. This question reduces to the following.

Given a convergent sequence $\underset{\sim}{z} = (z_n)$. Do there exist convergent sequences $\underset{\sim}{x} = (x_n)$ and $\underset{\sim}{y} = (y_n)$ such that

$$(1) \qquad z_n = \frac{1}{n} \sum_{j=1}^{n} x_j \, y_{n+1-j} \qquad \text{for } n = 1,2,\ldots$$

Definition: We say that a $\underset{\sim}{z}$ is represented by sequences $\underset{\sim}{x}$ and $\underset{\sim}{y}$ whenever (1) holds. A sequence $\underset{\sim}{z} = (z_n)$ is weakly representable if there exists a matrix (a_{ij}) representing an element of the projective tensor product $l^\infty \hat{\otimes} l^\infty$ such that

$$(2) \qquad z_n = \frac{1}{n} \sum_{j=1}^{n} a_{j,n+1-j} \qquad \text{for } n = 1,2,\ldots$$

Theorem 1: If $\underset{\sim}{z} \in l^2$ then $\underset{\sim}{z}$ is weakly representable, moreover the matrix (a_{ij}) satisfying (2) can be chosen as a Hankel matrix representing some element of the tensor product $c_0 \hat{\otimes} c_0$.

Theorem 2: There exists a sequence $\underset{\sim}{w} \in \bigcap_{p>2} l^p$ which is not weakly representable.

Corollary: The sequence $\underset{\sim}{w}$ of Theorem 2 is not representable by any two bounded sequences.

Thus the answer on Mazur's problem is no.

The above results are obtained jointly by S. Kwapien and the author.

Petzsche, H.J.

The edge-of-the-wedge theorem

A proof of the following version of the edge-of-the-wedge theorem was given:
Theorem: Let $\theta_j \in \mathbb{R}^N$, $|\theta_j| = 1$, $j = 1,2$ let $\Omega_{\theta_j} \subset \mathbb{R}^N \text{ xi } \{\lambda\theta_j \mid \lambda > 0\}$ relatively open with $\mathbb{R}^N \cap \partial\Omega_{\theta_1} = \mathbb{R}^N \cap \partial\Omega_{\theta_2} = U$. Assume f_j to be generalized functions (i.e. distributions, hyperfunctions) with $\frac{\partial}{\partial \bar{z}_{\theta_j}} = 0$ where $z_{\theta_j} = z\,\theta_j$, $z \in \mathbb{C}$. Then the following are equivalent:

(a) $b_{\theta_1} f_1 = b_{\theta_2} f_2$

(b) It exists a generalized function f on $\Omega_{\theta_1,\theta_2}$ ($\Omega_{\theta_1,\theta_2} \subset \mathbb{R}^N \text{ xi } \{\lambda\theta_1 + \mu\theta_2 \mid \lambda, \mu > 0$ relatively open) such that $\frac{\partial}{\partial \bar{z}_{\theta_j}} f = 0$, $j = 1,2$ and $b_{\theta_2} f_1 = f$, $b_{\theta_1} f = f_2$.

Hereby $b_\theta f$ denotes the boundery value of f in direction θ. The existence of the boundary value $b_\theta f$ provided f is holomorphic in z_θ was discussed. The proof given was based on a translation of the theorem's statement into the $\bar{\partial}$ - language i.e. replacing boundary values by suitable equations for $\frac{\partial}{\partial \bar{z}_{\theta_j}}$, and Hartogs theorem.

Finally a more general version of the theorem involving arbitrarily many boundary values was given.

Prada, J.

On idempotent operators on Frêchet spaces

The complemented subspaces of ExF, cartesian product of two Frêchet spaces E and F such that $(E,F) \in \mathcal{H}$ (all mappings on E into F are compact) are determined. For that, it is used the fact that the idempotent operators modulo-compact, that is the elements [D] of $\mathcal{L}(E,F)/\mathcal{H}(E,E)$ such that $[D]^2 = [D]$, where $\mathcal{L}(E,E)$ resp. $\mathcal{H}(E,E)$ designs the space formed by the continuous resp. compact linear operators on E into E, E being a Frêchet space, are the projections on E into E.

Ramanujan, M.S.

Strongly nuclear sets and maps

The topic discussed is the following Arzela-Ascoli type problem: Given Banach spaces E and F and a subset $H \subset L(E,F)$ when exactly is H a s-nuclear set of s-nuclear operators? Instead of the Kolmogorov and Gelfand numbers we use the entropy numbers (e_n) to characterize s-nuclear maps. Also we define the sequence $(v_n(H))$ of variation measures, viz.,

$$v_n(H) = \inf \{M > 0 : \exists \text{ a } 2^{n-1} \text{ - cover, } (B_1, \dots B_{2^{n-1}}) \text{ of the unit ball } B_E,$$

$$B_E \ni \forall T \in H, \forall x,y \in \text{(same) } B_i, \; \|Tx - Ty\| < M\} .$$

The following theorems are proved.

Theorem A: For $H \in L(E,F)$, the following are equivalent ('FAE'):
1) $H(B_E)$ is s-nuclear; 2) $(v_n(H')) \in s$; 3) $\exists$ a seq. $(F_n') \subset F'$, codim $F_n' \leq n$ and $\|H'|_{F_n'}\| \leq \lambda_n$, $(\lambda_n) \in s$.

Theorem B: $H \subset L(E,F)$ be a bounded set. Then 'FAE': 1) H is a s-nuclear set of uniformly s-nuclear maps. 2) $H(B_E)$ and $H'(B_{F'})$ are s-nuclear. 3) $H(B_E)$ is s-nuclear and H is of equi-s-variation. 4) H is of equi-s-variation and $(H(x), x \in B_E)$ is uniformly s-nuclear.
The results can be extended to $\Lambda_\infty(\alpha)$ replacing s if $\Lambda_\infty(\alpha)$ is multiplicatively stable (and this stability is shown to be 'almost' necessary).
This work is done jointly with K. Astala (Helsinki).

Taylor, B.A.

Right inverses for the $\bar\partial$ operator and the property (DN) in spaces of entire functions

For p(z) a continuous plurisubharmonic function on $\mathbb{C}^n$, let A_p denote the algebra of all entire functions f on $\mathbb{C}^n$ such that $|f(z)| \leq A \exp(Bp(z))$ for some constants $A,B > 0$. Let C_p^∞ denote the corresponding space of infinitely differentiable functions, each of whose partial derivatives are bounded by $A \exp(Bp(z))$, and let $C_p^\infty(0,q)$ denote the space of differential forms of bidegree (0,q) with coefficients from C_p^∞. We discuss the splitting of the complex,

$$0 \to A_p \xrightarrow{\text{id}} C_p^\infty \xrightarrow{\bar\partial} C_p^\infty(0,1) \xrightarrow{\bar\partial} \dots\dots \xrightarrow{\bar\partial} C_p^\infty(0,n) \to 0.$$

Under mild technical assumptions on p, the complex always splits at the stage of (0,q) forms where $q \geq 1$. At the stage C_p^∞, however, the complex splits if and only if the strong dual $(A_p)'$ of the DF space A_p has the property (DN) of Vogt and Wagner. Some known cases when $(A_p)'$ has (DN) are presented. These include many, but not all, cases when $p(z) = p(|z|)$ and when p is convex. Applications are given to the splitting of

$$0 \to I \to A_p \to A_p/I \to 0$$

where I is a closed ideal in A_p.

Tidten, M.

On a Theorem of W.Wojtynski

The following theorem is well known since 1966:

Theorem (Wojtynski): Let E be a countably-Hilbert space and T: G → L(E) be a group action of the compact group G on E admitting a cyclic vector e_0 in E, i.e. the orbit $\{T(g)e_0 | g \in G\}$ is total in E. Then E has a basis.

In this talk there is shown that this theorem is incorrect, at least in the noncommutative case. There is described a nuclear (F)-space Z and an action of a suitable group G on Z satisfying the hypothesis of Wojtynski's theorem. But Z fails to have a weaker property than having a basis. More precisely Z has not the property (*) described by Mityagin in his paper with C. Bessaga in 1976. This property is situated much nearer to the bounded approximation property (BAP) than the existence of a basis. On the other hand Wojtynski's hypothesis are sufficient for (BAP) and in particular the space Z constructed in this talk has the (BAP). Concerning the construction of Z it should be mentioned that this is done by a modification of Mityagin's construction of a nuclear (F)-space without the property (*) mentioned above. The group action on Z makes use of the special arrangement of the finite-dimensional "blocks" in Z.

Wagner, M.

Subspaces and Quotients of regular Köthe spaces

The results of this lecture are obtained together with Prof. M.S. Ramanujan. Let λ(A) a Köthe space with continuous norm.

Theorem 1: A Köthe space $\lambda(B)$ is isomorphic to a block embedded subspace of $\lambda(A) \Leftrightarrow$

There exists a matrix C equivalent to B, a mapping $\pi : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ in the second component injective, and constants C_n with

$$\frac{c_{j,n}}{c_{j,n}} \geq C_n \cdot \frac{c_{\pi(n,j),k}}{c_{\pi(n,j),n}} \quad \text{for all } n,j,k.$$

If $\lambda(A)$ is regular, it can be assumed that $a_{j,k+1} = a_{j,k}\, f_k\left(\frac{a_{j,k}}{a_{j,k-1}}\right)$ with $f_k : \mathbb{R}^+ \to \mathbb{R}^+$ increasing.

Theorem 2: A Köthe space $\lambda(B)$ is isomorphic to subspace of a stable regular Köthe space $\lambda(A) \Leftrightarrow$

There exists a matrix C equivalent to B with

$$c_{j,k+1} \geq c_{j,k} \cdot f_k\left(\frac{c_{j,k}}{c_{j,k-1}}\right) \text{ and } \frac{c_{j,2}}{c_{j,1}} \geq \frac{a_{j,2}}{a_{j,1}}.$$

Related results in the case of quotient spaces are also discussed.

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