MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 8/1984

Funktionentheorie

12. 2. bis 18. 2. 1984

Die Tagung fand unter der Leitung der Herren G. Frank (Dortmund), K. Strebel (Zürich) und St. Ruscheweyh (Würzburg) statt. Von den 47 Teilnehmern hielten 28 Vorträge; die Vortragsdauer variierte zwischen 30 und 45 Minuten. Wie aus der Teilnehmerliste ersichtlich ist, hatte die Tagung einen ausgesprochen internationalen Charakter.

Im Vordergrund der diesjährigen Tagung über Funktionentheorie stand die geometrische Richtung, d. h. konforme und quasikonforme Abbildungen. Neben Antworten auf alte und neue Probleme aus diesem Bereich wurde eine Reihe neuer Fragestellungen und Perspektiven vorgetragen. Rege Diskussionen rundeten den Verlauf dieser interessanten Tagung ab.

Teilnehmer

C. Andreian-Cazacu, Bucaresti
R. W. Barnard, Lubbock
J. Becker, Berlin
H. Begehr, Berlin
B. Bojarski, Warszawa
D. A. Brannan, Milton Keynes
P. Buser, Lausanne
P. Duren, Ann Arbor
H. Epheser, Hannover
R. Fehlmann, Helsinki
G. Frank, Dortmund
D. Gaier, Gießen
H. Grunsky, Würzburg
K. Habetha, Aachen
D. Hamilton, College Park
W. Hayman, London
A. Huber, Zürich
F. Huckemann, Berlin
J. A. Hummel, College Park
J. A. Jenkins, St. Louis
V. Kasten, Hannover
H. Kloke, Dortmund
W. Koepf, Berlin
G. Krzyż, Lublin
S. Kung, Beijing
M. Lehtinen, Helsinki
Y. J. Leung, Newark
T. H. MacGregor, Albany
O. Martio, Jyväskylä
K. Menke, Dortmund
D. Peschl, Bonn
A. Pfluger, Zürich
E. Reich, Minneapolis
H. M. Reimann, Bern
M. v. Renteln, Karlsruhe
S. Rickman, Helsinki
St. Ruscheweyh, Würzburg
F. J. Schnitzer, Leoben
G. Schober, Bloomington
K. Strebel, Zürich
T. Suffridge, Lexington
H. Tietz, Hannover
St. Timmann, Hannover
W. Tutschke, Halle
M. Vuorinen, Helsinki
J. Winkler, Berlin
J. Wirths, Braunschweig

Vortragsauszüge

C. Andreian-Cazacu: Eigenschaften der quasikonformen Abbildungen

Sei $f : G \to G^*$ eine Q-quasikonforme Abbildung zwischen zwei Gebieten G und G^* der Ebene, $w = f(z)$, z_o ein regulärer Punkt von f und $w_o = f(z_o)$. Wir betrachten die Transformation des Geradenbündels in z_o unter der zu f in z_o assoziierten affinen Abbildung und zeigen, daß das Verhältnis der Tangenzfunktionen der Winkel, die zwei Richtungen - insbesondere zwei konjugierte Durchmesser der charakteristischen Ellipse E_{z_o} - mit der großen Achse dieser Ellipse bilden, gleich dem Verhältnis der Tangenzfunktionen der Bildwinkel ist. Diese Eigenschaft wird benutzt, um das Hyperbolische System von linearen partiellen Differentialgleichungen erster Ordnung zu bestimmen, das eine quasikonforme Abbildung mit f. ü. in G vorgegebenen Richtungen der großen Achsen der charakteristischen Ellipsen E_{z_o} und E_{w_o} im verallgemeinerten Sinn befriedigt:

$$-\frac{\sin 2\theta}{\sin 2\theta^*} u_x + \frac{\cos 2\theta - \cos 2\theta^*}{\sin 2\theta^*} u_y = v_y ,$$

$$-\frac{\cos 2\theta + \cos 2\theta^*}{\sin 2\theta^*} u_x - \frac{\sin 2\theta}{\sin 2\theta^*} u_y = v_x .$$

Dabei sind $\theta(z)$ und $\theta(z)^*$ die Argumente der großen Achse von E_{z_o} bzw. E_{w_o} und E_{w_o} ist die charakteristische Ellipse von f^{-1} in w_o.

R. W. Barnard: The Omitted Area Problem

Let $U = \{z : |z| < 1\}$ and $S = \{f : f(z) = z + \ldots, f$ analytic and $1-1$ in $U\}$. The omitted area problem was first stated in 1949 by A. W. Goodman and has appeared in a number of problem sets since then including D. Brannan's more general question in 1976. The question is to find the maximum area omitted from U by a function f in S . Letting A_o equal this maximum, the bounds $.62\pi < A_o < .78\pi$ were obtained by Goodman and E. Reich in 1955. The author obtains a characterization of an extremal domain for this domain for this problem. Using this characterization an upper estimate to A_o of $.760\pi$ is found by numerical methods which appears to be a good approximation to A_o .

J. Becker: Univalence criteria and Jordan domains
(joint work with Ch. Pommerenke)

We consider the following three univalence criteria for functions analytic in $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, in $H = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$, and in $\Delta = \{z \in \mathbb{C} : |z| > 1\}$ respectively:

a) $(1-|z|^2)\,|zf''(z)/f'(z)| \leq 1$, $f'(0) \neq 0$ $(z \in \mathbb{D})$

b) $2 \operatorname{Re} z \; |f''(z)/f'(z)| \leq 1$ $(z \in H)$

c) $(|z|^2-1)\,|zf''(z)/f'(z)| \leq 1$ $(z \in \Delta)$.

In case a) the image domain $f(\mathbb{D})$ is always a Jordan domain. In case b) and c) however there exist exceptional functions whose image domains $f(H)$ and $f(\Delta \cup \{\infty\})$ respectively are not Jordan domains in the closed plane. These functions can be all determined. Furthermore it can be proved that the constant 1 on the right side is best possible in each case, in case a) even if the factor z is omitted.

B. Bojarski: Remarks on Markov and Bernstein inequalities

Various forms of local inverse Hölder inequalities, inverse Sobolev inequalities and averaged Harnack type inequalities - crucial for the study of geometric and analytical properties of quasiregular mappings and weak solutions of elliptic equations with measurable coefficients are discussed as analogues of Markov, Bernstein and Nikolsky-type inequalities for polynomials and entire functions of finite type. It is also shown that these inequalities for polynomials and their various generalisations can be obtained from an elementary reproducing formula

$$f(x) = \int_{\mathrm{supp}\,\varphi} f(y)K(x,y;\varphi)\,dy$$

where $K = \sum_{|t|\le k} \frac{(\alpha-y)^t \varphi^{(t)}(y)(-1)^t}{t!} \binom{k+1}{t+1}$

$t = (t_1,\dots,t_n)$, $|t| = \Sigma t_i$, φ an arbitrary C_0^∞ function, $\int \varphi\,dy = 1$ and $\varphi^{(t)}(y)$ is the usual notation for derivatives, valid for arbitrary polynomials $f(x)$ of degree k in real (or complex) variables $x = (x_1,\dots,x_n)$. For the derivatives we have

$$f^{(\alpha)}(x) = \int_{\mathrm{supp}\,\varphi} [f(y)-p(y)]K_x^{(\alpha)}\,dy \quad \text{for an arbitrary}$$

polynomial $p(y)$ of order $<|\alpha|$. Minimizing the norms of the right hand side with respect to p and φ , say, with fixed $\mathrm{supp}\,\varphi = \omega$ and various pointwise or integral norms, local estimates for the derivatives $f^{(\alpha)}(x)$ in terms of the norms $(\int_\omega |f|^p)^{1/p}$, $0 < p \le +\infty$ can be obtained. These estimates are invariant under linear conformal transformations and, in particular cases, reduce to generalisations of Markov inequalities, (naturally, not with optimal constants). They can be also used to obtain various local forms of Landau-Kolmogorov estimates for intermediate derivatives.

D. A. Brannan: Extreme points of a class of polynomials

When looking at a class of functions for its extreme points the normalisation is all-important. There exists some information about extreme points of univalent polynomials and of polynomials of positive real part in the disk. I will discuss extreme points of polynomials in the disk that are bounded by M.

This represents joint work with Professor J. G. Clunie.

P. Duren: Truncation problems for univalent functions

A support point of the class S of univalent functions is a function which maximizes $\mathrm{Re}\{L\}$ over S for some continuous linear functional L not constant on S. It is known that each support point maps the disk onto the complement of an analytic arc. If the arc is truncated by removal of a subarc at the tip, the corresponding renormalized function is again a support point which uniquely maximizes the associated functional. For the pointevaluation functionals $L(f) = e^{-i\sigma}f(\zeta)$ and $L(f) = e^{-i\sigma}f'(\zeta)$ it will be shown that all support points are obtained by truncation from those which maximize $\mathrm{Re}\{L\}$ nonuniquely. It is conjectured that in general this property characterizes the terminal support points.

R. Fehlmann: Unique extremality of affine mappings with small dilatation in $\mathbb{R}^n$

This talk is on a joint-work with Stephen Agard (Minneapolis, University of Minnesota).

In 1975 Ahlfors introduced a new dilatation for quasiconformal mappings in space based on the Riemannian metric $ds^2 = \mathrm{tr}(Y^{-1}dYY^{-1}dY)$ in the space of positive definite matrices. It is called Earle-Ahlfors dilatation K_E. Then in 1981 Agard showed that the affine mapping is extremal with respect to this dilatation for the "classical rectangular box problem of Grötzsch" in $\mathbb{R}^3$, provided that K_E is small,

i. e. $K_E \leq \sqrt{e}$. We generalize this result to $\mathbb{R}^n$ and prove that in contrast to the nonunique extremality with respect to the outer and inner dilatations the affine mapping is uniquely extremal with respect to this new dilatation.

D. Gaier: Räume konformer Selbstabbildungen - Ergebnisse und offene Probleme

Es sei G ein einfach zusammenhängendes, beschränktes Gebiet und $\Sigma(G)$ der Raum aller konformen Selbstabbildungen φ von G auf G. In $\Sigma(G)$ kann man neben der Gruppenstruktur eine Metrik einführen: $d(\varphi_1, \varphi_2) = \sup \{|\varphi_1(z) - \varphi_2(z)| : z \in G\}$. Fragen: Ist $\Sigma(G)$ kompakt, vollständig, lokal kompakt? Wann ist $\Sigma(G)$ zu $\Sigma(\mathbb{D})$ homöomorph? Wann ist $\Sigma(G)$ eine topologische Gruppe? Wie sehen die Zusammenhangskomponenten von $\Sigma(G)$ aus, wann ist $\Sigma(G)$ zusammenhängend oder total unzusammenhängend? Insbesondere: Wann ist id in $\Sigma(G)$ isoliert? Zu all diesen Fragen werden Antworten gegeben, die z. T. unvollständig sind. Bei Kammgebieten 1. Art ist id isoliert, bei solchen 2. Art und bei Schlangengebieten nicht. Hilfsmittel sind u. a. der Ahlforssche Verzerrungssatz und Sätze über das harmonische Maß.

D. H. Hamilton: The existence of extreme points of S which are not support points

A new variation based on quasiconformal mapping is used to investigate generalised linear problems on S arising from embedding S in a Hilbert space. Regularity results, analogous to the Schiffer Theory, are developed. It is shown that there are extreme points f of S such that the omitted area $\Gamma = \mathbb{C} - f(|z| < 1)$ has a subarc where the mapping function is nowhere N times differentiable. Consequently it is shown that there is no "nice" parametrisation of the class of support points of S .

W. K. Hayman: Das Wachstum von quasisymmetrischen Funktionen
(Zusammenarbeit mit A. Hinkkanen)

Es sei $f(x)$ ein wachsender Homöomorphismus der reellen Achse $\mathbb{R}$, sei $K > 1$ und

$$\frac{1}{k} \leq \frac{f(x+t)-f(x)}{f(x)-f(x-t)} \leq K \ , \quad x \in \mathbb{R}, \ t > 0 \ .$$

Solche Funktionen heißen K-quasisymmetrisch (K. q. s.). Nach einer Arbeit von Beurling und Ahlfors (Acta Math. 96 (1956)) sind die quasisymmetrischen Funktionen genau die, die eine Erweiterung auf eine quasikonforme Abbildung der ganzen Ebene ermöglichen. Es zeigt sich, daß f mindestens wie eine Potenz $|x|^{\alpha_2}$ und höchstens wie eine Potenz $|x|^{\alpha_1}$ wächst, wo $\alpha_1(k), \alpha_2(k)$ von Beurling und Ahlfors eingeführt sind und $0 < \alpha_2 < 1 < \alpha_1$ gilt. Ferner existieren die Grenzwerte

$$\lambda_j = \lim_{x\to\infty} \frac{|f(x)|}{|x|^{\alpha_j}} \ , \quad j = 1,2, \quad 0 \leq \lambda_1 < \infty \quad , \ 0 < \lambda_2 \leq \infty \quad ,$$

außer wenn $(\log K)/\log \{\frac{1}{2}(K+1)\}$ rational ist. Entsprechende Resultate gelten auch für die oberen und unteren Grenzen $M_o(x,K)$ und $m_o(x,K)$ von $f(x)$, wenn f durch $f(-1) = -1$, $f(1) = 1$ normiert ist.

A. Huber: Probleme der konformen Verheftung

Gegeben sei ein Homöomorphismus $\Phi : e^{i\vartheta} \to e^{i\varphi(\vartheta)}$ der Einheitskreisperipherie auf sich. Durch Identifikation entsprechender Punkte werden $E = \{z \mid |z| < 1\}$ und $A = \{w \mid |w| > 1\} \cup \{\infty\}$ zu einer zweidimensionalen Mannigfaltigkeit verheftet.

Fragen: (1) Ist Φ "konform zulässig", d. h. existieren eine Jordankurve Γ und zugehörige konforme Abbildungen $f : E \to \text{int}\ \Gamma$, $g : A \to \text{ext}\ \Gamma$ so, daß $f(e^{i\vartheta}) = g(e^{i\varphi(\vartheta)})$ für alle ϑ ?

(2) (Nachdem (1) mit JA beantwortet wurde)

Was kann über Γ ausgesagt werden?

Seien λ_1 und λ_2 rektifizierbare Jordankurven gleicher Länge, $p : E \to \text{int}\ \lambda_1$, $q : A \to \text{ext}\ \lambda_2$ konforme Abbildungen. Eine isometrische Verheftung längs λ_1 und λ_2 induziert eine Verheftung Φ längs des Einheitskreises. Für diese gilt fast überall $|p'(e^{i\vartheta})|d\vartheta = |q'(e^{i\varphi})|d\varphi$.

SATZ (A. Huber, Comment. Math. Helv. 51 (1976)). Es gibt isometrische Verheftungen längs rektifizierbarer Jordankurven, welche nicht konform zulässige Homöomorphismen Φ erzeugen.

Das isometrische Verheften von Flächenstücken ist ein wichtiges Werkzeug der Alexandrowschen Flächentheorie. Aus Resultaten von A. D. Alexandrow, Zalgalla, Reschetnjak (1963) kann man schließen: Die isometrische Verheftung längs Kurven von beschränkter Drehung ist konform zulässig, und die entstehende Kurve Γ ist ebenfalls von beschränkter Drehung. Auf funktionentheoretischem Wege unter Zuhilfenahme von bekannten Resultaten über quasikonforme Abbildungen beweist man leicht:

SATZ (A. Huber, Comment. Math. Helv. 50 (1975)). Die isometrische Verheftung längs Jordankurven von beschränkter Drehung und ohne Nullwinkel induziert auf der reellen Achse eine quasisymmetrische Verheftungsfunktion. Folglich ist sie konform zulässig und Γ ist ein Quasikreis.

Im Weiteren wird darauf hingewiesen, wie der Beweis der Beschränktheit der Drehung von Γ potentialtheoretisch in Angriff zu nehmen ist. Es gibt Fälle, in denen sich das Problem der konformen Verheftung auf eine Orthogonalprojektion im Hilbertraum zurückführen läßt.

J. A. Hummel: Bounded univalent functions, the univalent Bloch constant, and numerical conformal mapping

The class of bounded univalent functions can be divided into subclasses of those which cover a fixed disc. Previous papers have characterized the mapping properties of those functions in the subclasses which maximized a_2, but did not indicate how the maximal function could be obtained. Upper bounds for the univalent Bloch constant can be found by giving specific mapping functions. The example given by R. Goodman (1945) gives the best known bound to date. We show that both of these questions can be attacked by a very simple method of numerical conformal mapping which uses only standard domain whose boundary is made up entirely of segments of radial arcs and of segments of concentric circles and which is symmetric with respect to the real axis.

J. A. Jenkins: On the limits of moduli for functions whose image has finite spherical area

The material for this talk is drawn form my joint work with Kôtaro Oikawa.

Many years ago Montel proved a result which may be formulated as follows. Let $f(z)$ be regular and bounded in a half-strip: $a < x < b$, $y > 0$ and let $f(\xi+iy)$ have a limit as $y \to \infty$ for a fixed ξ, $a < \xi < b$. The $f(x+iy)$ has the same limit as $y \to \infty$ for any x in (a,b), uniformly on any closed subinterval of (a,b). Shortly afterwards a somewhat more general result was given by Lindelöf. Hardy, Ingham and Pólya considered the same problem for $|f(x+iy)|$ rather than $f(x+iy)$ and found that the existence of this limit for one ξ was not enough but that if $\lim_{y\to\infty} |f(x+iy)|$ existed for $x = \alpha, \beta$ with $a < \alpha < \beta < b$ and $\beta - \alpha < \frac{1}{2}(b-a)$ analogous conclusions to Montel's result obtained. They also gave some extensions and further embellishments were made by Miss Cartwright and Hayman. If one replaces the condition of boundedness by that that the Riemann image of the mapping by f should have finite spherical area we readily see that the existence of the limit for the modulus of f on one line implies the same situation as in Montel's result, indeed the more general formulation of Lindelöf is almost immediate. Further one can weaken the assumption on the set for which the limit must exist, there is a natural interpretation in terms of cluster sets which has interesting consequences and all results are readily extended to quasiconformal functions.

W. Koepf: Extreme points and support points of families of nonvanishing analytic functions

We consider subsets of $N_o := \{f : \mathbb{D} \to \mathbb{C} \mid f$ anal., $f(o) = 1$, $f \neq o\}$, $\mathbb{D} := \{z \mid |z| < 1\}$. Subordination $\prec$ gives an ordering in N_o, if we identify functions f and their rotations $f(xz)$, $x \in X := \{x \mid |x| = 1\}$. Let be MaxF the set of maximal elements of F with respect to $\prec$.

We call f a BCK-function: $\Longleftrightarrow$ $f \in BCK$, if for every $g \prec f$ there is a representation $g(z) = \int_X f(xz)\, d\mu(x)$ with a probability measure μ over X. Brannan, Clunie and Kirwan generalized a well-known theorem of Herglotz, showing that

$$\left(\frac{1+xz}{1-z}\right)^{\alpha} \in BCK \quad \text{for} \quad \alpha \geq 1 \; , \; |x| \leq 1 \; , \; x \neq -1 \; .$$

Theorem. $F \subset N_o$ compakt, rotation-invariant with $\mathrm{MaxF} \subset BCK$. Then

(a) the extreme points of the closed convex hull $E\,\overline{co}\,F \subset \mathrm{MaxF}$,

(b) the support points $\mathrm{Spt}\,F \subset \mathrm{co}(\mathrm{MaxF})$.

We apply this theorem to many classes of convex, starlike, close-to-convex and symmetric functions. It turns out that in these cases $\mathrm{Spt}\,F \subset E\,\overline{co}\,F$.

J. G. Krzyż: Conjugate holomorphic eigenfunctions and quasiconformal reflection

Let D and $D^* \ni \infty$ be simply connected domains whose common boundary is a Jordan curve Γ. If $\Gamma \in C^3$ then the eigenvalues of the Neumann kernel associated with Γ are called classical Fredholm eigenvalues of Γ. The present author introduced the notion of conjugate holomorphic eigenfunctions /CHE/ associated with Γ which enables us to define Fredholm eigenvalues without any regularity assumptions on Γ [Ann. Acad. Sci. Fenn., to appear]. The functions f, F holomorphic, non-constant and bounded in D, D^* continuous in $\bar{D}$ and $\overline{D^*}$, resp., whose boundary values satisfy $f(\xi) = L \circ F(\xi)$ on Γ, where $L(w) = (1-\lambda)^{-1}(w+\lambda\bar{w})$, are called CHE of Γ and the corresponding real constant λ has most of the well-known properties of classical Fredholm eigenvalues of Γ. If Γ admits a K-quasiconformal reflection then the Dirichlet integrals $\iint_D |f'|^2$, $\iint_{D^*} |F'|^2$ are both finite and $|\lambda| \geq (K+1)/(K-1)$ for any CHE.

S. Kung: A Remark on Möbius Transformations

Let $f(z)$ be an analytic function in $|z| < 1$. The explicit formula $f\left(\frac{\zeta + z}{1+\bar{z}\zeta}\right) = \sum_{n=0}^{\infty} g_n(z)\zeta^n$ is given when $|\zeta| < 1$. The explicit formula of the relationship between the derivatives $f^{(n)}$ and the covariant derivatives $\nabla^n f$ is given too. Besides these results we generalized the Landau theorem and prove that the Bieberbach conjecture, the estimations of $|g_n(z)|$, the estimations of $|f^{(n)}(z)|$ and the estimations of $|\nabla^n f(z)|$ of univalent functions are mutually equivalent to each other. There are lots of applications of these results. For example, we can give a differential geometrical meaning of $|a_3| \leq 3$, where a_3 is the third coefficient of $f(z) \in S$. The several complex variables case and the several real variables case are discussed, if the domain is a complex or real ball and the function is analytic in the ball. Using Möbius transformations and expanding the function, we find that the coefficients of second order in the expansion can be expressed as the covariant derivatives of the second order of the function. The covariant derivatives are formed with respect to the corresponding invariant differential metric.

M. Lehtinen : The maximal dilatation of the Beurling-Ahlfors extension

The best-known explicit construction of a quasiconformal extension f of a real k-quasisymmetric function h is the one given by A. Beurling and L. Ahlfors in 1956. Beurling and Ahlfors showed that f can be so chosen that it is k^2-quasiconformal, and T. Reed showed in 1966 that k^2 can be replaced by $8k$. A more detailed study of the implications of k-quasisymmetry allows one to lower the exponent 2 to values less than $3/2$ and replace the coefficient 8 by 2. On the other hand, the Beurling-Ahlfors extension always has a maximal dilatation at least k.

Y. J. Leung: Geometry of the coefficient region of univalent functions

Let S be the class of functions $f(z) = z+a_2z^2+\dots+a_nz^n+\dots$ analytic and univalent in $|z| < 1$. Let V_n be the set of points $\vec{a} = (a_2,\dots,a_n)$ corresponding to functions in S. We call $\vec{a}$ in V_n a support point if it supports a hyperplane with normal direction $\vec{\lambda} = (\lambda_2,\dots,\lambda_n)$. In other words, $\mathrm{Re} \sum_{k=2}^{n} \lambda_k a_k \geq \mathrm{Re} \sum_{k=2}^{n} \lambda_k b_k$ for all $\vec{b}$ in V_n. The set of support points in V_n is denoted by K_n.

A point $\vec{a}$ in K_n supporting a hyperplane with direction $\vec{\lambda}$ can be embedded into an analytic Löwner curve $\vec{a}(t) = (a_2(t),\ldots,a_n(t))$ in K_n satisfying the Löwner equation

$$\frac{d}{dt}\begin{vmatrix} a_1(t) \\ a_2(t) \\ \cdot \\ \cdot \\ a_n(t) \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & & 0 \\ 2K & 1 & 0 & & \cdot \\ 2K^2 & 4K & 2 & & \cdot \\ \cdot & & & & 0 \\ 2K^{n-1} & \cdot & \cdot & \cdot & n-1 \end{vmatrix} \begin{vmatrix} a_1(t) \\ a_2(t) \\ \cdot \\ \cdot \\ a_n(t) \end{vmatrix},$$

with $\vec{a}(o) = \vec{a} = (a_2,\ldots,a_n)$, $a_1(t) \equiv 1$. $K(t)$ has modulus one and is an analytic function of t in $[0,\infty)$. Define $\vec{\lambda}(t)$ by the adjoint Löwner equation

$$\frac{d}{dt}\begin{vmatrix} \lambda_1(t) \\ \lambda_2(z) \\ \cdot \\ \cdot \\ \lambda_n(t) \end{vmatrix} = -\begin{vmatrix} 0 & 2K & 2K^2 & \cdots & 2K^{n-1} \\ 0 & 1 & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & 0 & & & n-1 \end{vmatrix} \begin{vmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \cdot \\ \cdot \\ \lambda_n(t) \end{vmatrix},$$

with $\vec{\lambda} = \vec{\lambda}(o) = (\lambda_2(o),\ldots,\lambda_n(o))$ and $\lambda_1(o) = 0$.

<u>Theorem 1.</u> Each $\vec{a}(t)$, for $t > 0$, is the <u>unique</u> point in K_n that supports the hyperplane with direction $(\lambda_2(t),\ldots,\lambda_n(t))$.

It is known that $\vec{a}(t)$ can be analytically continued for some t in $(-\infty,0)$. We have

<u>Theorem 2.</u> Each $\vec{a}(t)$ in K_n, when continued analytically backward for negative t, has to leave K_n eventually.

T. H. MacGregor: <u>A peaking and interpolation problem for univalent functions</u>

Suppose that $\alpha_1 < \alpha_2 < \ldots < \alpha_n < \alpha_1+2\pi$ and $\beta_1 < \beta_2 < \ldots < \beta_n < \beta_1+2\pi$ and let $\Delta = \{z: |z| < 1\}$. There is a polynomial p that is univalent in $\bar{\Delta}$ and satisfies

(1) $|p(z)| < 1$ when $|z| \le 1$ and $z \neq e^{i\alpha_k}$ $(k=1,2,\ldots,n)$;

and

(2) $p(e^{i\alpha_k}) = e^{i\beta_k}$ $(k=1,2,\ldots,n)$.

O. Martio: <u>On radial uniqueness of analytic functions and their generalizations</u>

A closed set E on the boundary of the unit disk B^2 is called a set of uniqueness if for all bounded analytic functions $\lim_{t\to 1} f(tz) = 0$ for all $z \in E$ implies $f \equiv 0$. Harmonic measure is used to give a short proof of a theorem of F. and M. Riesz: $m_1(E) > 0$ implies that E is a set of uniqueness. This result is generalized to bounded quasiregular mappings of the unit ball in R^n. Category type uniqueness results can also be extended to quasiregurlar mappings.

A. Pfluger: <u>The Fekete-Szegö inequality (1932) by variational method</u>

Using Loewner's differential equation Fekete and Szegö determined the exact bound of $|a_3-\lambda a_2^2|$ over S. This method doesn't give information about the mapping properties of functions which maximize the above functional.

Application of the variational method and of a device Garabedian and Schiffer used for the coefficient a_3 however lead to a full description of the domain onto which an extremal function maps the unit disk.

E. Reich: Some extremal problems for parabolic regions

As is well known, the extremal quasiconformal maps of regions with pointwise specified boundary values need not be unique. We study this situation in detail for the case of qc mappings F of the parabolic region $\Omega = \{(z,y): y > x^2\}$, where the boundary values are those of the affine stretch, $A_K = kx + iy$, $(K > 1, K$ fixed). It is known that A_K is an extremal mapping in the class F, but it is not uniquely extremal. To understand this phenomenon it is useful to consider the subclass of extremal mappings which shift the focus, $x = 0$, $y = \frac{1}{4}$, onto a specified point, $u = 0$, $v = b'$. For a certain positive number h, all values b' in the interval $\frac{1}{4} - h \le b' \le \frac{1}{4} K^2$ are achievable by means of extremal maps of the class F. If $\frac{1}{4} - h < b' < \frac{1}{4} K^2$, the extremal mappings are not unique. For $b' = \frac{1}{4} K^2$, there is only one extremal mapping.

H. M. Reimann: Quasiconformal mappings on the Heisenberg group (joint work with A. Korányi)

Multiplication in the Heisenberg group $H = \{g=(z,t): z \in \mathbb{C}, t \in \mathbb{R}\}$ is given by

$$(z_1,t_1)(z_2,t_2) = (z_1 + z_2, t_1 + t_2 + 2\mathrm{Im} z_1 \bar{z}_2)$$

and the left invariant metric is defined by

$$d(g,h) = |g^{-1}h|$$

$$|g| = |(z,t)| = (|z|^4 + t^2)^{1/4} .$$

A homeomorphism $\varphi : H \to H$ is a K-quasiconformal mapping on the Heisenberg group, if

$$\overline{\lim_{r\to 0}} \frac{\sup\limits_{d(g,h)=r} d(\varphi g, \varphi h)}{\inf\limits_{d(g,h)=r} d(\varphi g, \varphi h)} \le K .$$

This definition was introduced by Mostow (1973) in connection with his proof of the rigidity theorem on symmetric spaces of rank one.

Differential geometric methods permit the construction on non trivial quasiconformal mappings. They are obtained as flows generated by certain special vectorfields.

The geometric implications of the definition are investigated. Furthermore it is shown that the mappings satisfy a system of differential equations, which is related to the classical Beltrami equation. Methods from the theory of analytic functions on the unit ball allow the deduction, that the smooth 1-quasiconformal mappings are given by the action of SU(2,1).

S. Rickman: Sets with large local index of quasiregular mappings

It has been conjectured that for a nonconstant K-quasiregular mapping $f: G \to R^n$, $n \geq 3$, the points x with large local index $i(x,f)$ play the same role as the branch points play for $n = 2$. In particular it was believed that there exists a constant $c = c(nK)$ such that the set $E_c = \{x \in G : i(x,f) \geq c\}$ consists of isolated points only. This question is now settled in dimension three in the negative: There exists $K > 1$ such that for every c there exists a K-quasimeromorphic mapping $f: \bar{R}^3 \to \bar{R}^3$ for which E_c is a Cantor set. The proof is based on the author's construction of a nonconstant quasiregular mapping $f: R^3 \to R^3$ omitting more than one point in R^3.

G. Schober: Univalent Harmonic Mappings

Univalent, complex-valued, harmonic, orientation-preserving mappings will be discussed. In analogy to the familiar univalent classes S and Σ, one can pose problems of distortion, Fourier coefficients and structure. J. Clunie, T. Sheil-Small and R. R. Hall have initiated these ideas and made considerable progress in the study of families analogous to S and its subclasses. Together with W. Hengartner, we study such harmonic mappings of a disk onto a strip. In addition, we consider the family of harmonic mappings analogous to the class Σ and also the subfamily of mappings whose omitted sets are real line segments.

T. Suffridge: Bounded non-vanishing functions

Consider the family B of functions that are analytic in the unit disk of the complex plane with values in the punctured disk (i. e. they are bounded by one and are never zero). Then $-\log(f)$ can be defined as an analytic function with positive real part when f is in the family B. If f is chosen to be a function in the class that maximizes the real part of the nth coefficient in the power series then we may assume

$$-\log f(z) = \sum_{j=1}^{m} t_j (1+e^{i\alpha_j}z)/(1-e^{i\alpha_j}z)$$

where $m \leq n$ and each $t_j \geq 0$. A certain system of n equations (2n real equations) that is linear in the coefficients t_j must be satisfied.

W. Tutschke: Generalization of the Cauchy-Kovalevska theorem to the case of generalized analytic functions

In view of the classical Cauchy-Kovalevska theorem the initial value problem

$$\frac{du}{dt} = L\,u \;;\; u(\cdot,0) = u_o$$

is solvable if the initial function u_o is holomorphic and if further, L is a first order differential operator with holomorphic coefficients. Applying the theory of generalized analytic functions this theorem can be generalized: The initial function must only be a generalized analytic function in the sense of I. N. Vekua defined by an equation of type $\frac{\partial w}{\partial \bar{z}} = a(z)w + b(z)\bar{w}$ (Soviet Math. Dokl. Vol. 25, No 1, 201 - 205). The choice of the coefficients $a(z), b(z)$ depends on L (Bull. Acad, Sc. Georg. SSR, vol. 107, No 3, 481 - 484, 1982).

Other generalizations of the classical Cauchy-Kovalevska theorem concern generalized analytic functions in the space and so on. The construction of the solution of the initial value problem mentioned above is based on an abstract version of the Cauchy-Kovalevska theorem (cf. F. Treves, Basic linear partial diff. equations, Academic Press 1975).

M. Vuorinen: Conform invariants and quasiregular mappings

In this talk I wish to show that most of the known distortion and growth results concerning n-dimensional quasiconformal or quasiregular mappings together with several new results can be proved in a unified fashion by studying two conformal invariants λ_G and μ_G defined in a domain $G \subsetneq R^n$, $n \geq 2$. By definition, λ_G and μ_G are certain extremal quantities involving the moduli of some particular curve families in G. Some quantitative estimates for λ_G and μ_G are given in terms of the quasihyperbolic metric k_G.

Theorem. A quasiregular mapping $f: G \to fG \subsetneq R^n$ is uniformly continuous as a mapping $f: (G, k_G) \to (fG, k_{fG})$ iff $\exists\, C \geq 1$ such that $d(f(x), \partial fG)/d(f(y), \partial fG) \leq C$ whenever $x, y \in G$ and $|x - y| \leq d(x, \partial G)/2$.

Question. Is this result new if $G = D = \{z \in \mathbb{C} : |z| < 1\}$, $n = 2$ and f is analytic?

Berichterstatter: G. Frank (Dortmund)

Adressen der Tagungsteilnehmer

Frau
Prof. Dr. Cabiria Andreian-Cazacu
Institutul de Mathematica
Str. Academiei 14

Bucaresti

Romana

Prof. Dr. R. W. Barnard
Dept. of Mathematics
Texas Tech University
Box 4319

Lubbock, Texas 79409

U. S. A.

Prof. Dr. J. Becker
Technische Universität
Fachbereich Mathematik FB 3
Straße des 17. Juni 135

1000 Berlin

Prof. Dr. H. Begehr
Freie Universität
Institut f. Mathematik I
Hüttenweg 9

1000 Berlin

Prof. Dr. B. Bojarski
Dept. of Mathematics
Warsaw University
PKiN IX p.

Warszawa 00901

Poland

Prof. Dr. D. A. Brannan
Faculty of Mathematics
The Open University
Walton Hall

Milton Keynes, AK7 6AA

Großbritannien

Prof. Dr. P. Buser
Institut für Mathematik
ETH Lausanne

CH-1000 Lausanne

Prof. Dr. P. Duren
Dept. of Mathematics
University of Michigan

Ann Arbor, MI 48109

U. S. A.

Prof. Dr. H. Epheser
Lehrstuhl f. Angewandte Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Dr. R. Fehlmann
Dept. of Mathematics
University of Helsinki

00100 Helsinki

Finnland

Prof. Dr. G. Frank
Abteilung Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. D. Gaier
Mathematisches Institut
der Universität Gießen
Arndtstr. 2

6300 Gießen

Prof. Dr. H. Grunsky
Mathematisches Institut
der Universität Würzburg
Am Hubland

8700 Würzburg

Prof. Dr. J. A. Hummel
Dept. of Mathematics
University of Maryland

College Park, MD 20742

U. S. A.

Prof. Dr. K. Habetha
Lehrstuhl II für Mathematik
der RWTH Aachen
Templergraben 55

5100 Aachen

Prof. Dr. J. A. Jenkins
Dept. of Mathematics
Washington University

St. Louis, MO 63130

U. S. A.

Prof. Dr. D. Hamilton
Dept. of Mathematics
University of Maryland

College Park, MD 20742

U. S. A.

Dr. V. Kasten
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Prof. Dr. W. Hayman
Imperial College of Science and
Technology, Dept. of Mathematics
Queen's Gate

London SW7 2BZ

Großbritannien

H. Kloke
Abteilung Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. A. Huber
ETH Zürich
Mathematisches Institut
ETH-Zentrum

CH-8092 Zürich

Wolfram Köpf
Freie Universität Berlin
Fachb. Mathematik - FB 19
Institut f. Mathematik I
Arnimallee 3

1000 Berlin 33

Prof. Dr. F. Huckemann
Technische Universität
Fachbereich Mathematik
Straße des 17. Juni 135

1000 Berlin

Prof. Dr. G. Krzyż
Matematyki Instytut
Uniwesytet Marii-Curie-
Sklodowskiej, Nowotki 10

20-031 Lublin

Poland

Prof. Dr. Sheng Kung
Institute of Applied Mathematics
Academia Sinica
Friedship Hotel

Beijing

China

Prof. Dr. D. Peschl
Mathematisches Institut
der Universität Bonn
Wegelerstr. 10

5300 Bonn

Prof. Dr. M. Lehtinen
Dept. of Mathematics
University of Helsinki
Hallituskatu 15

00100 Helsinki 10

Finnland

Prof. Dr. A. Pfluger
ETH-Zürich
Mathematik ETH-Zentrum

CH-8092 Zürich

Prof. Dr. Y. J. Leung
Dept. of Mathematics
University of Delaware

Newark, DE 19711

U. S. A.

Prof. Dr. E. Reich
Dept. of Mathematics
University of Minnesota

Minneapolis, Minnesota 55455

U. S. A.

Prof. Dr. T. H. MacGregor
Dept. of Math. & Statistics
University of New York at Albany
1400 Washington Avenue

Albany, NY 1222

U. S. A.

Prof. Dr. M. Reimann
Mathematisches Institut
der Universität Bern

CH-3000 Bern

Prof. Dr. O. Martio
Dept. of Mathematics
University of Jyväskylä
Seminaarinkatu 15

SF-40100 Jyväskylä

Finnland

Prof. Dr. M. v. Renteln
Mathematisches Institut
der Universität Karlsruhe
Englerstr. 2

7500 Karlsruhe

Prof. Dr. K. Menke
Abteilung Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. S. Rickman
Dept. of Mathematics
University of Helsinki

00100 Helsinki

Finnland

Prof. Dr. St. Ruscheweyh
Mathematisches Institut
der Universität Würzburg
Am Hubland 3

8700 Würzburg

Dr. St. Timmann
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover

Prof. Dr. F. J. Schnitzer
Institut für Mathematik
und Angewandte Geometrie
Montanuniversität Leoben
Franz-Josef-Straße 18

A-8700 Leoben

Prof. Dr. W. Tutschke
Sektion Mathematik
Martin-Luther-Universität
Halle-Wittenberg
Universitätsplatz 10

DDR-4000 Halle

Prof. Dr. G. Schober
Dept. of Mathematics
Indiana University

Bloomington, IN 47405

U. S. A.

Prof. Dr. M. Vuorinen
Dept. of Mathematics
University of Helsinki
Hallituskatu 15

00100 Helsinki 10

Finnland

Prof. Dr. K. Strebel
Mathematisches Institut
Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. J. Winkler
Technische Universität
Fachbereich Mathematik
Straß des 17. Juni 135

1000 Berlin

Prof. Dr. T. Suffridge
Dept. of Mathematics
University of Kentucky

Lexington, Kentucky 40506-0027

U. S. A.

Prof. Dr. J. Wirths
Institut C für Mathematik
der Technischen Universität
Pockelsstr. 14

3300 Braunschweig

Prof. Dr. H. Tietz
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover