

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 9/1984

Mathematische Stochastik

19.2. bis 25.2.1984

Die Tagung fand unter Leitung von Herrn W. Hazod (Dortmund) und Herrn G. Neuhaus (Hamburg) statt. Von 47 Teilnehmern aus 9 Ländern wurden insgesamt 38 Vorträge gehalten, die ein umfassendes Bild über den Forschungsstand in der Mathematischen Stochastik wiedergaben.

Die Hauptgewichte der Tagung lagen auf den Gebieten der empirischen Prozesse, der adaptiven- und Rangverfahren, der nicht-kommutativen Wahrscheinlichkeitstheorie, der speziellen Grenzverteilungen sowie der asymptotischen Statistik.

In den Vorträgen, den anschließenden Diskussionen sowie persönlichen Gesprächen konnten wissenschaftliche Kontakte angeknüpft und vertieft werden.

Vortragsauszüge

W. ALBERS:

A scale rank test using Helmert's transformation

In the problem of testing equality of scale of two distributions a rank test should be preferred over the F-test if it is not sure that the distributions involved are normal. However, if in addition

the distributions may also differ in location, it becomes necessary to first adjust the observations, and the rank test will then at best be asymptotically distributionfree, even if normality holds after all. In this paper it is demonstrated how using Helmert's transformation for the adjustment of the observations leads to a rank test which is exact under normality and asymptotically distributionfree otherwise.

E. BERGER:

The Komlós-Major-Tusnády approximation theorem in \mathbb{R}^d

A large deviation theorem for conditional moment generating functions is presented, from which the following theorem follows via a modified Komlós-Major-Tusnády type construction:

Theorem: Let (X_n) be a sequence of i.i.d. \mathbb{R}^d -valued random variables with $EX_1 = 0$ and $\text{Cov}(X_1) = I$ (= identity matrix), and let (Y_n) be a sequence of independent normally distributed random variables with mean 0 and covariance matrix I . Write $S_n = X_1 + \dots + X_n$ and $T_n = Y_1 + \dots + Y_n$. Suppose that $E(\exp(\langle t, X_1 \rangle)) < \infty$ for all t with $\|t\| < \epsilon$ and some $\epsilon > 0$. Then it is possible to redefine the sequences (S_n) and (T_n) on a new probability space such that for all $n \in \mathbb{N}$, $x > 0$ and suitable constants $C, K, \lambda \in \mathbb{R}^+$

$$P(\max_{k \leq n} \|S_k - T_k\| \geq C \log n + x) \leq Ke^{-\lambda x}.$$

The method also works for non-i.i.d. random variables (without smoothness conditions).

I. BERKES:

Exchangeability and limit theorems for subsequences of random variables

The following heuristic principle was formulated by Chatterji:

SUBSEQUENCE PRINCIPLE: Let T be a limit theorem valid for all i.i.d. sequences belonging to an integrability class L defined by the finiteness of a norm $\|\cdot\|_L$. Let $\{X_n\}$ be an arbitrary (dependent) sequence of r.v.'s satisfying $\sup_n \|X_n\|_L < +\infty$. Then there exists a subsequence X_{n_k} satisfying T in a mixed form.

Although special cases of this principle were known as long as 40 years ago, no general result existed in the field until 1977 when D. Aldous proved that the principle is valid for all a.s. and distributional type limit theorems T . This proof exploits exchangeability properties of subsequences of general r.v. sequences. In our paper we give a detailed study of the structure of subsequences of dependent r.v. sequences, obtaining best possible exchangeability properties of such systems. We then use these results to extend Aldous' theorem for a larger class of limit theorems and to give partial converses. We give necessary and sufficient conditions for a sequence $\{X_n\}$ to have a subsequence satisfying the central limit theorem and the analogous limit theorem with a limiting stable distribution in a mixed form. We then introduce the class of "I-type" limit theorems containing a.s. as well as distributional limit theorems and several further limit theorems of different type and prove the subsequence principle for this class.

H. CRÉMERS:

Weak convergence in Suslin path spaces

Under the assumption that a sequence of stochastic processes has paths in a Suslin function space we can prove the following: If convergence in the path space implies stochastic convergence, then tightness and convergence of the finite dimensional distributions of the stochastic processes are sufficient for weak convergence. The result in many cases implies a unification of the weak convergence proof: Such cases are C, D, Lip_α, L_p and \mathcal{D} , the space of distribution functions of finite measure. Further it is shown, that in L_p -spaces the tightness condition can be replaced by an apparently weaker one, involving uniform integrability instead of compactness. Thus, in $L_1(\nu)$ with finite ν , under the condition of weak convergence of the f.d.d., tightness in the weak topology implies tightness in the strong topology.

S. CSÖRGÖ:

A unified asymptotic theory for empirical reliability and concentration processes

An outline of a recently submitted research monograph, completed

jointly with Miklós Csörgő, Lajos Horváth and David M. Mason, will be given. The processes covered on the reliability side are the empirical mean residual life and the empirical total time on test processes with various (scaled, "from the first failure", two-sided) versions, while those on the economic concentration or diversity side are the empirical Lorenz and the Goldie concentration processes and their modifications such as the empirical Shannon and redundancy processes. For all these processes we prove almost sure uniform consistency, weak convergence and strong approximation results under appropriate, often necessary moment conditions. The unification is achieved by the observation that certain integral processes based on the uniform empirical and quantile processes are the main ingredients of all the processes in question. These integral processes are treated by a combination of weak and strong approximation and martingale techniques in conjunction with the Chibisov-O'Reilly theorem.

M. DENKER:

An Invariance principle for two-sample linear rank statistics

Let $T_{n,m}(h) = \int_{-\infty}^{\infty} h(\frac{N}{N+1} \hat{H}_N(t)) d\hat{F}_N(t) - \int_{-\infty}^{\infty} h(H_N(t)) dF(t)$ be a 2-sample linear rank statistic with score function $h \in H$, where H denotes the class of all right continuous functions on $(0,1)$ of bounded variation on compact sets with $\|h\| = \int (|h_1(t)| + |h_2(t)|) (t(1-t))^{-1/2} dt < \infty$ ($h = h_1 - h_2$ according to the Jordan decomposition). For a certain subspace $\hat{H} \subset H$ (including absolutely cont. h) one can define a gaussian process $Z(h)$ and show the following invariance principles:

Theorem 1: If $h \in H$ is of class C_2 with $|h''(t)| \leq K(t(1-t))^{-1+n}$ then

$$S_N(h) \rightarrow Z(h) \text{ weakly in } D([0,1]^2) \text{ where } S_N(h,r,s) = \begin{cases} 0 & r < 1/N \\ N^{-1/2} [rN] T_{[rN]}, [sN] (h) & \end{cases}$$

Theorem 2: If $h \in \hat{H}$ then $S_N(h) \rightarrow Z(h)$ weakly in $(D(E), d)$, where $D(E)$ denotes the set of all r.c. functions on $E = \{(r,s) : r \neq 0 \text{ or } s \neq 0\} \cup \{0,0\}$ with no discontinuities of the 2nd kind and where d metrizes the topology of uniform convergence on compact sets.

Extensions and applications are briefly mentioned. The results were obtained jointly with M.L. Puri.

Y. DERRIENNIC:

On "subadditive" processes arising in first passage percolation

In the usual setting of first passage percolation on \mathbb{Z}^2 let $a_{n,m}$ be the point-to-point travel time between $(0,n)$ and $(0,m)$ and let $a'_{n,m}$ be the cylinder restricted point-to-point travel time. Consider

$$\begin{aligned} \phi_n(u) &= E(e^{-u a_{0,n}}) \\ \phi'_n(u) &= E(e^{-u a'_{0,n}}). \end{aligned}$$

Then we have the relations

$$\begin{aligned} \phi'_{n+m}(u) &\leq \phi'_n(u) \phi'_m(u) & u < 0 \\ \phi'_{n+m}(u) &\geq \phi'_n(u) \phi'_m(u) & u > 0 \\ \phi_{n+m}(u) &\geq \phi_n(u) \phi_m(u) & u > 0. \end{aligned}$$

(the last relation is a consequence of the fact that $a_{n,m}, a_{m,k}$ ($n < m < k$) although not independent are "associated"). To obtain limit theorems for $a'_{0,n}$ or $a_{0,n}$ it is essential to know the behaviour of the limit function of $\frac{1}{n} \log \phi'_n$ (resp. $\frac{1}{n} \log \phi_n$) in the vicinity of 0. If the limit has a derivative in 0 then it is not hard to obtain that $P(a_{0,n} < na)$ or $P(a'_{0,n} < na)$ go to 0 exponentially fast for

$\alpha < \mu = \lim_{n \rightarrow \infty} \frac{E(a_{0,n})}{n}$ and the exact rate of convergence is given by the Cramer transform of the limit function as it is for sums of i.i.d. r.v. But there are examples of subadditive processes with independent increments for which the limiting function has no derivative in 0. Therefore delicate estimations are required to obtain the preceding estimations for $a_{0,n}$ or $a'_{0,n}$. The preceding line of reasoning leads to the following proposition:

Let $(X_n)_n$ be a weakly stationary sequence of "mutually associated" identically distributed positive r.v.. If $\sum_{j=2}^{\infty} \text{cov}(X_1, X_j) < \infty$ then for every $\epsilon > 0$ $P(\sum_{i=1}^n X_i < n(E(X_1) - \epsilon)) \xrightarrow[n \rightarrow \infty]{} 0$ at exponential rate.

(The condition on covariances was introduced by Newman and Wright (Annals of Prob. Vol. 9 (1981) 671-675) to obtain a central limit theorem in this context).



E. DETTWEILER:

Transformation of martingales on Banach spaces

Let E, F denote two separable, real Banach spaces. An operator $T \in L(E, F)$ is called smooth, if there exists a constant $C > 0$ such that for every E -valued, square-integrable martingale (M_n) the following inequality holds:
$$\sup_n E \|TM_n\|^2 \leq C^2 \cdot \sum_{k \geq 0} E \|M_{k+1} - M_k\|^2$$

($E :=$ expectation operator). The space $S_2(E, F)$ of all smooth operators is a Banach space under a natural norm and always larger than the space $\Pi_2(E, F)$ of all 2-absolutely summing operators. With the aid of these classes of operators it is possible to obtain a "good" stochastic integration theory on arbitrary Banach spaces.

T. DRISCH:

Quantum integration and quantum Fourier transform

The problem of Quantum Integration is the following: Given a von Neumann algebra W and a "probability" P on the set A of projectors of W (e.g. a positive normed function $P: A \rightarrow \mathbb{R}$, σ -additive on sequences of pairwise orthogonal projectors), exist then an extension of P to a linear functional $E_P: W \rightarrow \mathbb{C}$? The answer is "yes" if the 2×2 -matrices do not constitute a direct summand of W and if the cardinality is not so great that it can not be incorporated in any model of Zermelo-Fraenkel's set theory (included the axiom of choice; it is unknown if we can postulate (as existence axiom) such "very big" cardinals).

The definition of Fourier Transform of a quantum probability is the following: Let T be a projective representation of the covariance group. G of the physical system; let W be the von Neumann algebra generated by $\{T(g)\}$; let P be a probability on the set of projectors of W . Then $\hat{P}: g \rightarrow E_P(T(g))$ is the Fourier Transform of P . Finally indications of Bochner and Lévy (continuity-) like theorems are given.

P. GÄNSSLER:

On parameter estimation in Pareto-type distributions

A preliminary report is given to a problem posed by Professor J. Teugels at the Oberwolfach-Meeting in November 1982. It is con-

cerned with asymptotic properties of a certain estimator for the tail behaviour of Pareto-type distributions. In contrast to a similar estimator introduced by Hill (1975) and further studied by Mason (1982) our estimator will be based on the empirical distribution function and so some of the main tools for proving its strong consistency, asymptotic unbiasedness and asymptotic normality are results from the theory of (weighted) empirical processes. The solution in its present form is mainly due to my co-worker Dr. Erich Haeusler (Univ. of Munich).

W. GAWRONSKI:

On the bell-shape of stable densities

The central result of this talk deals with a proof that all stable densities are bell-shaped (i.e. its k -th derivative has exactly k zeros on its support and they are simple) thereby generalizing a well-known property of the normal distribution and the associated Hermite polynomials.

H. HEYER:

Convolution semigroups on Sturm-Liouville structures

The topic of this talk refers to a hypergroup generalization of the well-known transience criterion for continuous convolution semigroups $(\mu_t)_{t>0}$ on an Abelian weakly compact group G in terms of weak integrability of $\frac{1}{t}$ for the negative definite function ψ corresponding to $(\mu_t)_{t>0}$. The work presented and the analytic preparations (Schreiberg correspondence, resolvent correspondence, support correspondence) stems from joint efforts of the speaker with W.R. Bloom from the Murdoch University, Perth, Western Australia. Instead of giving a detailed derivation of the mentioned result the talk was denoted for motivation and examples of hypergroups as special Sturm-Liouville structures (in the sense of S. Bochner, C. Genge and others). We just recall the main motivation due to M. Kennedy (1961). Let X be the birth and death process on \mathbb{Z}_+ defined by the transition matrix $(p_\lambda(i, j))_{i, j \in \mathbb{Z}_+}$ with parameter $\lambda \geq 0$

$$p_\lambda(i, i \pm 1) := \frac{1}{2} \left(1 \pm \frac{\lambda}{i + \lambda} \right), \quad p_\lambda(i, i) = 0$$

Then there exists a convolution $*_{\alpha}$ (with $\alpha = \lambda - \frac{1}{2}$) in $M^1(\mathbb{Z}_+)$ such that $X = X_{\alpha}$:= random walk defined by the transition kernel

$$(X, A) \rightarrow N(X, A) = \epsilon_X *_{\alpha} \epsilon_1(A)$$

on $(\mathbb{Z}_+, \mathcal{P}(\mathbb{Z}_+))$. The definition $*_{\alpha}$ is carried out via Gegenbauer polynomials $(Q_n^{\alpha})_{n \in \mathbb{Z}_+}$ of order α , in three steps:

$$(a) \quad Q_n^{\alpha}(x) Q_m^{\alpha}(x) = \sum_{r \in \mathbb{Z}_+} g^{\alpha}(m, n, r) Q_r^{\alpha}(x) \quad \forall x \in [-1, 1]$$

$$(b) \quad \epsilon_n *_{\alpha} \epsilon_m := \sum_{r \in \mathbb{Z}_+} g^{\alpha}(m, n, r) \epsilon_r$$

$$(c) \quad \mu *_{\alpha} \nu := \sum_m \sum_n \psi(m, \nu(n)) \epsilon_m *_{\alpha} \epsilon_n$$

[Here $g^{\alpha}(m, n, r) \geq 0$ are the coefficients defining the Gegenbauer polynomials]

After this motivation the axiomatic set up of Sturm-Liouville system has been given, then a list of examples followed; the rest of the talk was devoted to the transience criterion for convolution semi-groups of hypergroups.

M. HUŠKOVA:

Adaptive procedures based on ranks for testing independence

Let $(X_1, Y_1), \dots, (X_N, Y_N)$ be independent identically distributed bivariate random variables with continuous distribution function $H(x, y)$ and marginal distribution function $F(x)$ and $G(y)$. Consider the testing problem $H_0: H = F \cdot G$ versus $K: H \geq F \cdot G, H \neq F \cdot G$ (positive dependence). Optimal rank statistic (with respect to Bahadur efficiency) can be written in the following form: $\sum_{i=1}^N \log \bar{h}(R_i/N, S_i/N)$ where R_i and S_i is the rank of X_i and Y_i , respectively, among X 's, and Y 's, respectively, \bar{h} is the Lebesgue density of rv's $F(X_1), G(Y_1)$. An effective estimator \hat{h}_n (kernel estimator) of \bar{h} was proposed, some asymptotic properties were stated. The assertion on the asymptotic distribution of $\sum_{i=1}^N \hat{h}_n(R_i/N, S_i/N)$, under hypothesis and alternative were presented. An interesting feature of the asymptotic behaviour is that (under some regularity conditions) the variance of the proposed test statistic is of order N and n^{-2} under an alternative and hypothesis, respectively.

The presented results were obtained jointly with K. Behnen and G. Neuhaus.

A. JANSSEN:

Scale invariant exponential families and one-sided test problems

The main purpose of the talk is to investigate scale invariant exponential families. It is proved that $E = ((P_\theta)_{\theta \in \Theta})$ is scale invariant if and only if P_θ^f is stable where f is the sufficient statistic appearing in the densities of the exponential family.

It is pointed out that these experiment are limit points if a scale transformation $\delta_n, \delta_n \rightarrow 0$, is applied to the n -th product experiment. The results can directly be applied to asymptotic test problems for a simple hypothesis which belongs to the boundary of the parameter space.

J.-P. KREISS

On the existence and construction of adaptive estimates in ARMA-models

We consider the problem of constructing adaptive estimates for the parameter $\vartheta = (a_1, \dots, a_p, b_1, \dots, b_q)^T$ for the following ARMA(p,q)-model

$$X_t = a_1 X_{t-1} + \dots + a_p X_{t-p} + e_t + b_1 e_{t-1} + \dots + b_q e_{t-q}, \quad t \in \mathbb{Z},$$

where the white noise process $\{e_t; t \in \mathbb{Z}\}$ consists of i.i.d. random variables.

In a first part we proof the LAN-condition for our model. Therefore we need an explicit representation of the likelihood ratio. Then we derive a necessary condition for the LAM-property of estimates in our model and construct such estimates when the error-distribution is known. Therefore we use an \sqrt{n} -consistent initial estimator.

Finally we prove by construction the existence of *adaptive* estimates for the parameter ϑ , which posses the LAM-property and which are independent of the distribution of the white noise and demonstrate the practical relevance of such estimators in a small simulation study.

K. LOHSE

On bootstrap-procedures

Let $T_n(\tilde{X}_n)$ be an estimator for θ , where \tilde{X}_n is a vector of n i.i.d. r.v. with values in \mathbb{R}^k . Wanted is the distribution of $\sqrt{n}(T_n(\tilde{X}_n) - \theta)$. If X_n is a sample, then the Bootstrap-Procedure leads to a distri-

bution of $\sqrt{n} (T_n(X_n) - T_n(\tilde{X}_n^*))$, where \tilde{X}_n^* is a vector of n i.i.d. r.v. with empirical distribution, and takes this for an estimator of $L(\sqrt{n}(T_n(X_n) - \theta))$.

Let $T_n(X_n)$ have the representation: $T_n(X_n) = T(\eta_n(X_n))$ where $\eta_n: \mathbb{R}^{kn} \rightarrow V$ not necessary measurable, $T: V \rightarrow \mathbb{R}^P$, $\theta = T(\eta)$ and where V is a normed vectorspace. Then under certain regularity conditions it can be shown that the Bootstrap estimator is consistent if T is differentiable (compact, Frechét) only at the point η , and if some boundedness or tightness conditions of $\sqrt{n}(\eta_n(\tilde{X}_n^*) - \eta)$ are fulfilled. As examples for the working of this theory were given M,L,R-estimates.

A. LUCZAK:

Martingale convergence in the state space of a C*-algebra

Let A be a separable C*-algebra of operators with unit 1, ultra-weakly dense in a von Neumann algebra M and let σ be the state space of A .

Let $\{M_n\}$ be an increasing sequence of von Neumann subalgebras of M such that $M = (\bigcup_{n=1}^{\infty} M_n)''$ and let $E_n: M \rightarrow M_n$ be a conditional expectation of M onto M_n . The sequence $\{E_n\}$ is called a martingale on M if, whenever $m \geq n$, $E_m E_n = E_n E_m = E_n$.

We show that, for a given martingale $\{E_n\}$ on M , there exists a sequence $\{\tilde{E}_n\}$ of transformations defined "almost everywhere" on σ , canonically induced by $\{E_n\}$. Moreover, for "almost all" ψ in σ

$$\lim_{n \rightarrow \infty} \tilde{E}_n \psi = \psi$$

with the limit in the weak*-topology of σ .

U. MÜLLER-FUNK:

On the centering of L-statistics and Bernstein-type operators

Some methods commonly employed in deriving the normal approximation to L-statistics $L_n = L(\hat{F}_n)$ require exact centering at their expectations (\hat{F}_n denoting the empirical d.f.). From a statistical point of view, however, L_n should be centered at the limiting functional $L(F)$ (F denoting the true d.f.). If L_n is based on the weights $g(\frac{1}{n+1})$ and on the score fct. h one realizes that

$$|E_F(L_n) - L(F)| \leq \int_0^1 |B_{n-1}(g)(t) - g(t)| |h \circ F^{-1}(t)| dt$$

where B_n stands for the (slightly modified) n^{th} Bernstein operator. Accordingly, we ask for conditions ensuring that the approximation error on the LHS is of the order $n^{-1/2}$. We state a theorem that allows for a broad class of functions g, h (possibly having jump discontinuities and/or being unbounded). A generalization to probabilistic operators based on steep exponential families is briefly indicated. The result extends works started by Hoeffding (1971, 1973).

G. PFLUG:

Log-Likelihood process and nuisance parameters

Let a family of densities $f(\vartheta, \psi)$ be given, where ϑ is the parameter of interest and ψ is a nuisance parameter. We call the model $\{f(\vartheta, \psi); \vartheta \in \Theta_0, \psi \in \Psi_0\}$ adaptive with respect to the first parameter, if there is an estimator sequence for ϑ , which is asymptotically efficient for every model $\{f(\vartheta, \psi_0); \vartheta \in \Theta_0\}$, $\psi_0 \in \Psi_0$, if (ϑ_0, ψ_0) is the underlying true parameter. The problem is to find necessary and sufficient conditions for a model to be adaptive.

If the experiment is smooth enough, then a necessary condition for allowing adaptivity is the orthogonality of the respective tangent cones (see Pfanzagl (1982)). For other experiments, necessary conditions can be formulated in terms of the limiting Gaussian or Poisson-experiment. For an infinite dimensional nuisance parameter these conditions are far from being sufficient.

A global condition of orthogonality is however sufficient to guarantee adaptivity. This is shown by proving that the log-likelihood process with estimated nuisance parameter converge to the same limiting process as if the nuisance parameter is exactly known. If this global orthogonality is fulfilled then consistent estimation of the nuisance parameter is sufficient, if the orthogonality is only of local nature then quick consistent estimates (in the sense of Strasser) are required.

W. PHILIPP:

Invariance principles for partial sum processes and empirical processes indexed by sets

Let $\{x_j, j \in \mathbb{N}^q\}$ be independent identically formed random elements with values in a Banach space B , indexed by q -tuples of positive integers. We obtain the almost sure approximation as well as the approximation in probability of the partial sum process $\{\sum_{j \in nA} x_j, A \in \mathcal{A}\}$ by a partial sum $\{\sum_{j \in nA} y_j, A \in \mathcal{A}\}$ as $n \rightarrow \infty$ uniformly over all sets A in a certain class \mathcal{A} of subsets of the q -dimensional unit cube. Here $\{y_j, j \in \mathbb{N}^q\}$ are independent identically distributed Gaussian random variables. These results are then applied to obtain the approximation of empirical processes over sets and indexed by sets by Gaussian partial sum processes indexed by sets.

D. PLACHKY:

A characterization of product-measurability of Radon-Nikodym derivatives by separability

Let \mathcal{A} be a σ -algebra of subsets of a set Ω , \mathcal{P} the set of all probability measures on \mathcal{A} , and \mathcal{A}^* the smallest σ -algebra on \mathcal{P} such that all mappings $P \rightarrow P(A), A \in \mathcal{A}$, are measurable. The following result is proved: There exists an $\mathcal{A} \otimes \mathcal{A}^* \otimes \mathcal{A}^*$ -measurable version of $(\omega, P, Q) \rightarrow \frac{dP}{dQ}(\omega)$, where P_Q denotes the Q -continuous part of the Lebesgue decomposition for P with respect to Q , if and only if the following condition holds:

(*) There exists a separable (i.e. countably generated) sub- σ -algebra \mathcal{A}_0 of \mathcal{A} such that $\mathcal{A}_0 = \mathcal{A}[P]$ for all $P \in \mathcal{P}$ holds.

Remark: (*) does not imply separability of \mathcal{A} (ex.: $\Omega = \mathbb{R}, \mathcal{A}_0 = \mathcal{B}, \mathcal{A} = \mathcal{B}_{\mathbb{N}}$, $\mathcal{B}_{\mathbb{N}}$ σ -algebra of universal measurable subsets of \mathbb{R}) and (*) does not imply $\mathcal{A} = \bigcap_{P \in \mathcal{P}} \mathcal{A}_0(P|\mathcal{A}_0)$, where $\mathcal{A}_0(P|\mathcal{A}_0)$ denotes the completion of \mathcal{A}_0 with respect to $P|\mathcal{A}_0$.

D. POLLARD:

Central limit theorems for functionals of the empirical measure

It was shown that maximal inequalities developed through recent work

in empirical processes have applications to other areas of asymptotics. In particular, consistency and central limit theorems for estimators defined through minimizations of $P_n f(\cdot, t)$ -where P_n = empirical measure and $\{f(\cdot, t) : t \in T\}$ is a family of real-valued functionals- depend upon bounds for $\mathbb{P}\{\sup_t |P_n f(\cdot, t) - Pf(\cdot, t)| > \epsilon\}$ and $\mathbb{P}\{\sup_t |P_n R(\cdot, t) - PR(\cdot, t)| > \epsilon\}$, where $R(\cdot, t)$ is a formal remainder term in a Taylor expansions of $f(\cdot, t)$. The talk made propaganda for empirical process methods.

R. PYKE:

Set-indexed partial-sum process and empirical process

Consider data of the form (x, m_x) in which x represents location and m_x represents a measurement taken at location x . Such data determines a signed measure $\mu(B) := \sum_{x \in B} m_x$ for $B \in \mathcal{B}$. If the locations are random and the masses constant (=1), μ becomes the empirical measure, F_n say, while if the locations are fixed μ is called the partial-sum measure. In particular, consider locations $\underline{j} \in \mathbb{N}^d$ with masses $\{X_{\underline{j}} : \underline{j} \in \mathbb{N}^d\}$ and set $S(B) = \sum_{\underline{j} \in B} X_{\underline{j}}$. A brief historical overview of the asymptotic theories for both was given emphasizing the SLLN, LIL and CLT with special emphasis upon the progress of the past 10 years for set-indexed cases. Recent results for partial-sum processes were presented including i) SLLN. $A \subset \mathbb{B}^d \cap [0, 1]^d$ satisfies $|A^\epsilon \setminus A^{-\epsilon}| \rightarrow 0$ uniformly in $A \in \mathcal{A}$, and $EX_{\underline{j}} = \mu$, $\{X_{\underline{j}}\}$ iid, implies $S(nA)/n^d \rightarrow \mu|A|$ uniformly in $A \in \mathcal{A}$. (Examples of \mathcal{A} and statements of the assumptions used in remaining results were given.) ii) CLT (Normal case) Let $Z_n(A) = n^{-d/2} \sum_{\underline{j} \in nA \cap C_{\underline{j}}} X_{\underline{j}}$, where $C_{\underline{j}} = (\underline{j}-\underline{1}, \underline{j}]$, be the smoothed partial-sum process. Under "total boundedness with inclusion" and $E|X_{\underline{j}}|^s < \infty$ with $s > 2(1-r)^{-1}$ where $r \in (0, 1)$ is the coefficient of metric entropy, $Z_n \xrightarrow{L} Z$ the appropriate Gaussian process. Extensions to non-iid, Poisson and empiricals were discussed. iii) LIL (Functional) A Skorokhod-type embedding for partial-sums results in this LIL under similar conditions to ii). iv) CLT (Stable case). Set $Y_n(A) = \sum_{\underline{j} \in nA} X_{\underline{j}}^1 / n^{d/\alpha}$ when $\{X_{\underline{j}}\}$ iid in d. of a. of a Stable- α distribution, and where $\{U_{\underline{j}}\}$ are independent $\text{Unif}(C_{\underline{j}})$ r.v.'s. Then to show $Y_n \xrightarrow{L} Y$ the existence of regular Lévy processes Y is studied and the meaning of \xrightarrow{L} defined. Here the



space $\mathcal{D}(A)$ is discussed. For convex sets C^d , a stable process on $\mathcal{D}(A)$ exists if $\alpha < \frac{d+1}{d-1}$. v) Lévy's Brownian Motion as a set function was briefly described. vi) Future directions were suggested to include statistical applications of set-indexed theories including interactive work station approaches to data analysis. Most of the results reviewed represent joint work with R.F. Bass, Mina Ossiander and K. Alexander; the result with the latter being the establishment of the CLT (Normal case) assuming only $EX_j^2 < \infty$.

J.P. RAOULT:

Some generalization of James Stein estimation

Let y be a n -dimensional gaussian observation, with mean θ in a known k -dimensional subspace of \mathbb{R}^n , θ , and regular variance $\sigma^2 \sum$ (\sum known, and σ^2 known or unknown); let $\varphi_0(y)$ denote the maximum likelihood (i.e. generalized least square) estimation of θ ; we define a generalized James-Stein (g. J.-S.) estimator as an estimator of θ of the form $\mu + [1 - \xi(y - \varphi_0(y), \varphi_0(y) - \mu)](\varphi_0(y) - \mu)$; $\mu (\in \theta)$ is called the pole, and ξ' is called the shrinkage function.

We first present a critical review of classical and Bayesian justifications of g. J.-S. estimators, with respect to the quadratic loss function $c(\theta, \hat{\theta}) = \frac{1}{\sigma^2} (\theta - \hat{\theta})' \sum^{-1} (\theta - \hat{\theta})$.

A class of g. J.-S. estimators uniformly better than φ_0 is obtained for $k \geq 3$ and a shrinkage function of the form

$$\eta(y - \varphi_0(y)) \rho((\varphi_0(y) - \mu)' \sum^{-1} (\varphi_0(y) - \mu)),$$

with function $u \rho(u)$ non decreasing and lower than $2(k-2)B$, where the constant B depends on function η .

We present numerical studies (due to Ben Mansour and Martin (1983)) of Bayes risk functions for g. J.-S. estimators with σ^2 known and shrinkage function of the form $\frac{c}{s^2} + [\frac{d}{s^2}]^a$, with $a > 1$, and $s^2 = (\varphi_0(y) - \mu)' \sum^{-1} (\varphi_0(y) - \mu)$; these estimators greatly improve those presented by Tze Fen Li (1982), with shrinkage functions of the form $\sum_{i=1}^n c_i (s^2)^{-1}$.

P. RÉVÉSZ:

Some new invariance principles for local time

Let X_1, X_2, \dots be a sequence of i.i.d. r.v.'s with $P(X_1=0) = 0$, $EX_1 = 0$, $EX_1^2 = 1$ and let $S_0 = 0$, $S_n = X_1 + X_2 + \dots + X_n$ ($n=1, 2, \dots$), $S(t) = S_n$ if $t=n$ piecewise linear otherwise. Define the local time of S_n by $\xi(x, n) = \#\{t: t \leq n, S_t = x\}$. Several theorems say that $\xi(x, n)$ can be well approximated by the local time $\eta(x, n)$ of a Wiener process $W(t)$. Beside some moment conditions most of these theorems assume that X_1 is either lattice or continuously distributed. We are interested to prove such a theorem what does not assume anything about the distribution of X_1 except some moment conditions. This goal is not achieved yet, our sufficient condition is: $P\{|S_n| < x\} \leq Cx^\alpha$ ($x > 0$, $n=1, 2, \dots$) for some $C > 0$, $\alpha > 0$. Having this condition and some moment restrictions we can prove the required closeness of $\xi(x, n)$ and $\eta(x, n)$. Note that the distribution $P(X_1 = \sqrt{2}) = \frac{1}{2+\sqrt{2}}$, $P(X_1 = -1) = \frac{\sqrt{2}}{1+\sqrt{2}}$ does, while the distribution $P(X_1 = \sqrt{3}) = P(X_1 = -\sqrt{3}) = P(X_1 = -1) = P(X_1 = +1) = 1/4$ does not satisfy our condition.

H. RIEDER:

Robust estimation and testing of functionals

We modify the nonparametric local asymptotic minimax bound of Koshevnik, Levit (1976) a) to allow for various kinds of infinitesimal neighborhoods (ϵ -contamination, total variation, Hellinger, Kolmogorov, $L^2(\nu)$), b) to suit the estimation of functionals T given by expansions $T(Q) = \int \psi dQ$, where ψ has the properties of an influence curve. We suggest to estimate that functional which can be estimated at lowest risk and satisfies certain robustness side conditions.

If a convex class of distribution functions on the real line is given, which has an element of smallest Fisher information, and if maximum risk of estimating T is to be minimized with respect to all points (each defining a location family) at which estimation of T makes sense, we arrive at an abstract version of Huber's (1964) saddle point result that counters all previous criticisms raised by Beran (1981, 1982), Millar (1981, 1982) since

- i) the corresponding estimator is optimum among all estimators,
- ii) infinitesimal contamination of all kinds is allowed (in

particular, no restriction to symmetric contamination is required)

iii) it is clear what to estimate outside the parametric model.

If the robustness condition is bounds on the oscillation of T, we obtain an extension of Hampel's (1968, 1974, 1978) theory whose advantages may again be described by i), ii), iii).

For testing multiparameter functionals an analogous optimum selection subject to robustness side conditions still seems to be beyond scope. In the one-dimensional case, however, the corresponding optimality results are available. Moreover, if the one-dimensional testing problem is of one-sided form, and if we use the maximal infinitesimal total variation balls possible, the Koshevnik-Levit least favorable direction catches the Huber-Strassen least favourable pairs, up to negligible remainders.

U. RÖSLER:

Asymptotic behaviour of stopping time of diffusions

Consider a diffusion on \mathbb{R}^+ with 0 an reflecting boundary. If X_t is transient, then under some conditions on a, b , $dX_t = a(X_t)dW_t + b(X_t)dt$, see Keller, Kersting, Rösler, the possible asymptotic behaviour of X_t is either $X_t - \mu_t$ converges a.e., $\frac{X_t - \mu_t}{\gamma_t} \rightarrow N(0,1)$ or $\gamma_t \ln \frac{X_t}{\mu} \rightarrow N(0,1)$. μ_t is the deterministic solution of $d\mu_t = b(\mu_t)dt$, γ_t some expl. known function. A similar result is true for the stopping times τ_4 , correctly normalized to $\tau_4^* = (\tau_4 - E_0 \tau_4) / \sqrt{\text{Var}_0 \tau_4}$.

Theorem: Let X_t be any diffusion on \mathbb{R}^+ or BAD on $\mathbb{R}, 0$ reflecting. Then $\tau_4^* \rightarrow \text{a.e.} \Leftrightarrow \lim \text{Var}_0 \tau_4 < \infty$.
Otherwise $\tau_4^* \rightarrow N(0,1) \Leftrightarrow E_0(\tau) / \sqrt{\text{Var}_0 \tau} \rightarrow \infty$
 $\tau_4^* + 1 \rightarrow \exp(\text{Par.1}) \Leftrightarrow E_0(\tau) / \sqrt{\text{Var}_0 \tau} \rightarrow 1$

There remains a small gap, the diffusions of the form $dX_t = \text{const} \sqrt{X_t} dW + dt$, which are not covered by the above theorem. These are exactly the processes with $X_t \stackrel{D}{=} r X_{tr^2}$, $r \in \mathbb{R}$.

H. ROST:

A limit dynamics for a system of many interacting diffusions

We report here on a recent result of K. Oelschläger.

Let $X_1(t), \dots, X_N(t)$ be defined by the stochastic diff. equ.

$$dx_i = dw_i - \frac{1}{N} \sum_{j \neq i} \nabla V^N(x_j - x_i) dt, \quad i=1, \dots, N$$

(values in \mathbb{R}^d , V^N a given "potential" such that

$$V^N(x) = N^{\beta d} V(N^\beta x), \quad \text{where } \beta > 0 \text{ and}$$

$$V \geq 0, \int V dx < \infty, V(x) = V(-x), \text{ plus something.})$$

Then, in the limit $N \rightarrow \infty$, the process of empirical measures $\frac{1}{N} \sum \delta_{X_i}(t)$ converges weakly in prob. towards a solution of the nonlinear parabolic equation

$$\frac{\partial f}{\partial t} = \nabla \cdot \left(\left(\frac{1}{2} + cf \right) \nabla f \right)$$

where $c = \int V(x) dx$, provided $\beta < \frac{1}{d+2}$.

L. RÜSCHENDORF:

Wasserstein metric and strong approximation

We introduce the notion of markov construction as a practical modification of the Wasserstein distance. This notion allows inductive constructions of good approximations of two random variables with given marginals. We illustrate this method by strong approximation results for φ -mixing, weak and very weak Bernoulli rv's and also by approximations w.r.t. dependent sequences. Finally we solve the problem to find the optimal martingale approximation to a given sequence of random variables.

W. SENDLER:

A path property of the Brownian motion

We ask how the fluctuation of the Brownian paths near 0 can be measured. Besides the well-known results like log log-laws or Hölder-continuity the following idea is discussed: Let $g: (0,1] \rightarrow \mathbb{R}$, $g \in C$, g isotonic; how bad may g behave near 0 such that nevertheless $\lim_{t \rightarrow 0} \int_{(t,1]} W dg$ exists?

Th. 1: $\lim_{t \rightarrow 0} \int_{(t,1]} W dg \exists \Leftrightarrow \int_{(0,1]} g^2 d\lambda < \infty$

This result is a special case of the following (we use the usual notation of martingale theory as proposed in, e.g., Metevier's book on Semimartingales):

Th. 2: For an L^2 -Semimartingale M with $M_0 = 0$, an increasing cadlag process V , $V \leq 0$, M and V adapted to a filtration $F = \{F_t: t \geq 0\}$, the following statements are equivalent:

- (i) $E\left(\int_{(0,1]} V^2 d[M]\right) < \infty$.
- (ii) $\int_{(\tau_n, 1]} M dV$ converges a.s. and L^2 for every sequence of stopping times τ_n , with $0 < \tau_n \leq 1$ and $\tau_n \rightarrow 0$ a.s.

R. SERFLING:

On the Glivenko-Cantelli and Oscillation theory of empirical processes of non-classical structure, and some problems in maximal inequalities

A large class of statistics can be represented effectively as functionals of empirical d.f.'s of non-classical structure, in particular U-statistic structure for example. For applications such as the SLLN, LIL, CLT, Berry-Esseen thm, etc, various forms of Glivenko-Cantelli theorem are fundamental. For applications such as sequential fixed-width confidence intervals, the oscillation theory is relevant. These new empirical processes and the relevant Glivenko-Cantelli theory and oscillation theory will be discussed, along with some new problems in general maximal inequalities, which are stimulated by their application to the oscillation theory.

E. SIEBERT:

Operator-semistable laws on Euclidean spaces

Operator-stable and operator-semistable laws on $V = \mathbb{R}^d$ are defined as limiting distributions of properly normed sums of i.i.d. random vectors (M. Shrope respectively R. Jajte). In the one-dimensional case both concepts are due to P. Lévy (Semi-)stability also can be characterized algebraically by a decomposability condition. Non-infinitely divisible probability law μ on V is operator-semistable

iff $\mu^\beta = B\mu * \varepsilon_B$, where $0 < \beta < 1$, B a linear transformation of V and $b \in V$. It turns out that semistability is a more elementary concept than stability; nevertheless both classes have similar properties. This was illustrated by considering some basic properties of a semistable law: Lévy measure, Lebesgue density, moments, zero-one-laws and holomorphy. In this context some results of A. Luczak have been extended.

K. URBANIK:

Joint distributions in the non-commutative probability theory

Given a system $A = (A_1, A_2, \dots, A_k)$ of observables and a state T , we say that a probability measure P_T^A on \mathbb{R}^k is the joint probability distribution of the system A at the state T if for every $a = (a_1, a_2, \dots, a_k) \in \mathbb{R}^k$ the projection of P_T^A onto the real line defined by $x \rightarrow (a, x)$ ($x \in \mathbb{R}^k$) coincides with the probability distribution of the observable $\sum_{j=1}^k a_j A_j$ at the state T . Let $S(A)$ be the set of all states T for which the joint distribution P_T^A exists. We say that A fulfills the probabilistic commutation condition if there exists a system B consisting of commuting observables such that $P_T^A = P_T^B$ for all $T \in S(A)$. We prove that every system consisting of one-sided bounded observables with purely point spectrum fulfills the probabilistic commutation condition. This result can not be extended to all systems of observables. Namely, the pair of canonical observables does not fulfill the condition in question.

W.R. VAN ZWET:

Estimating a parameter and its score function

We consider the problem of estimating a real-valued parameter θ in the presence of an abstract nuisance parameter η , such as an unknown distributional shape. Attention is restricted to the case where the "score functions" for θ and η are orthogonal, so that fully asymptotically efficient estimation is not a priori impossible. For fixed sample size we provide a bound of Cramér-Rao type. The bound differs from the classical one for known η by a term involving the integrated mean square error of an estimator of a multiple of the score function

for θ for the case where θ is known. This implies that an estimator of θ can only perform well over a class of shapes η if it is possible to estimate the score function for θ accurately over this class.

This work is joint with C.A.J. Klaassen and will be published in the proceedings of the Neyman-Kiefer symposium.

W. VON WALDENFELS:

The central limit theorem in quantum stochastics

The object of the talk was to present the most elementary non-trivial example of the central limit theorem. We use moment methods in the frame of algebraic quantum stochastics. The analogue of a probability measure on a probability space is a state on a *-algebra A , i.e. a linear functional $\rho: A \rightarrow \mathbb{C}$ such that $\rho(1) = 1$ and $\rho(f^*f) \geq 0$ for $f \in A$. The (infinitesimal) Weyl algebra $\mathcal{W}(p, q, \hbar)$ is the *-algebra generated by p, q , $p^* = p, q^* = q$ with the relation $pq - qp = \frac{\hbar}{i}$. Let Q be a 2×2 -matrix then a gaussian state on $\mathcal{W}(p, q, \hbar)$ is given by the familiar condition that all moments can be reduced to the second order moments and these are given by Q .

Now let $A = \mathbb{C}^{2 \times 2}$ the algebra of all 2×2 -matrices. Define the spin matrices by $S_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $S_2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $S_3 = \frac{1}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and the state ρ on A given by the density matrix

$$\begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \quad \rho_1, \rho_2 \geq 0, \quad \rho_1 + \rho_2 = 1$$

which we denote by ρ again. Consider $A^{\otimes N}$ and $\rho^{\otimes N}$ on $A^{\otimes N}$. Define

$$S_i^{(N)} = S_i \otimes 1 \otimes \dots \otimes 1 + 1 \otimes S_i \otimes 1 \otimes \dots \otimes 1 + \dots + 1 \otimes 1 \otimes \dots \otimes S_i.$$

Then

$$\frac{S_i^{(N)}}{\sqrt{N}} \rightarrow \rho(S_i), \quad \rho(S_1) = \text{Tr} \rho S_1 = 0, \quad \rho(S_2) = 0, \quad \rho(S_3) = \frac{1}{2}(\rho_2 - \rho_1) = \sigma.$$

The convergence is weakly, i.e. for all moments.

$\frac{S_i^{(N)} - N\rho(S_i)}{\sqrt{N}}$ converges in distribution (i.e. for the moments) to the following gaussian state γ_Q on $\mathcal{W}(p, q, \sigma) \otimes \mathcal{C}[t]$: With

$$Q = \begin{bmatrix} \frac{1}{4} & -i\frac{\sigma}{2} & 0 \\ i\frac{\sigma}{2} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} - \sigma^2 \end{bmatrix}$$

$$\gamma_Q = \gamma_{Q_1} \otimes \gamma_{Q_2}$$

$$\gamma_{Q_1} : \mathbb{C}(p, q, \sigma) \rightarrow \mathbb{C}, \quad Q_1 = \begin{pmatrix} \frac{1}{4} & -i\frac{\sigma}{2} \\ +i\frac{\sigma}{2} & \frac{1}{4} \end{pmatrix}$$

$$\gamma_{Q_2} : \mathbb{C}[t] \rightarrow \mathbb{C}, \quad \mathbb{C}[t] \text{ set of polynomials in } t$$

$$\gamma_{Q_2}(f) = \int \frac{1}{\sqrt{2\pi\alpha^2}} e^{-\frac{t^2}{2\alpha^2}} f(t) dt, \quad \alpha^2 = \frac{1}{4} - \sigma^2.$$

This result may be generalized to the general case of an algebra and may be proven not only by moment methods but by Fourier transform methods too.

H. WALK:

On a.s. convergence of Kiefer-Wolfowitz type processes

A pathwise consideration of the stochastic approximation procedure $X_{n+1} = X_n - a_n \nabla f(X_n) + a_n W_n$ in a Hilbert space H with $0 \leq a_n \rightarrow 0$, $\sum a_n = \infty$ and $f: H \rightarrow \mathbb{R}$ bounded from below shows that a.s. convergence properties of weighted means of the - systematic and random errors W_n together with certain conditions on ∇f are sufficient for a.s. convergence of $\nabla f(X_n)$ to 0 and of $f(X_n)$. There are further investigated convergence of a sequence in a Banach space which appears in filtering theory and is recursively defined by $X_{n+1} = X_n - n^{-1}(A_n X_n - b_n)$ with convergent arithmetic means of A_n resp. b_n , and convergence of sequential estimates of saddle-points in a convex constrained optimization problem concerning regression functions on \mathbb{R}^k . The theorems generalize and sharpen results of Ljung (1978), Györfi (1980) and Hiriart-Urruty (1977).

S. WEINRYB:

Homogenization problems for stochastic processes with penetrable boundaries

We have tried to extend the results on equations with a small parameter to processes with penetrable boundaries. The aim of this study is to model the behaviour of a particle in an inhomogeneous medium which can be constituted by some drops of liquid in a gaseous phase. First we consider the microscopic point of view through a law Q which is associated to circles in the plane, with a fixed radius and a fixed distance between each of them; then by a change of scale, we can study a sequence of laws $(Q^n)_n$ which are associated with a great density of smaller and smaller circles. It seems natural to consider the weak limit of this sequence as an approximation of the physical phenomenon.

We have proved two types of results that depend on the behaviours of the operator on the boundaries. First we have supposed that there was a reduction with n of the surface effect; then we have supposed that this effect was independent of n and in this case we need a centering condition. Under these assumptions we have proved the weak convergence of the sequence to a Markov process which is associated with a generator with drifts in the first case and with a second order generator in the second case.

J.E. YUCKICH:

Almost sure uniform rates of exact convergence for classes of functions

Let (X, \mathcal{A}, P) be a probability space. Let $X_i, i \geq 1$, be i.i.d. random variables with values in X and with distribution P . Let $P_n = n^{-1}(\delta_{X_1} + \dots + \delta_{X_n})$ be the empirical measures for P and let $G_n, n \geq 1$, be a sequence of classes of real-valued functions on X . Using a metric entropy condition I find sufficient conditions on the G_n such that

$$0 < U \leq \overline{\lim}_{n \rightarrow \infty} \sup_{g \in G_n} |R(n) \int g(dP - dP_n)| \leq L < \infty,$$

where U and L are finite, strictly positive constants, and where the rate of convergence $R(n)$ depends upon the metric entropy of the G_n . The result provides exact a.s. rates of uniform convergence for the

empirical characteristic functions over expanding intervals. It also establishes exact rates of convergence for density estimators of the general form $\hat{g}_n(x) := n^{-1} \sum_{i=1}^n g(x, X_i)$.

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