MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1984

Linear Operators and Applications

11.3. bis 17.3.1984

Tagungsleitung: I. C. Gohberg (Tel Aviv), B. Gramsch (Mainz)
H. H. Schaefer (Tübingen).

An der Tagung haben 43 Mathematiker aus 10 Ländern teilgenommen. Leider konnte Herr Schaefer an der Tagung selbst nicht teilnehmen.

Es wurden 33 Vorträge zur Operatorentheorie gehalten, u. a. zu den folgenden Themenkreisen: Wiener-Hopf-Faktorisierung, Toeplitzoperatoren, Integralgleichungen, Fredholmoperatoren, Operatorenideale und Eigenwertverteilungen, Fréchetalgebren von Pseudodifferentialoperatoren, Invariante Unterräume, Transmissionsprobleme.

Die anregende und harmonische Atmosphäre hat auch in vielen Diskussionen zum Gelingen der Tagung beigetragen.

Vortragsauszüge

ABDELAZIZ, N. H.:

Generation of n-parameter semigroups of linear operators of class A:

In [1], a characterization of commutativity of co semi-groups was obtained in terms of their generators. In the present note it is shown that this result holds also for semi-groups of class A .

The notion of an n-parameter semi-group of class A is introduced as a natural extension of the one parameter case; and the above mentioned result is utilized to establish a Hille-Yosida type of theorem for the present case. This would present an extension to the n-parameter case of the result of Hille-Phillips on the generation of one parameter semi-groups of class A ; (see [2] , P.P. 373, theorem 12.5.1).

[1] N. H. Abdelaziz, Commutativity and the Generation of ... etc. Houston Jour. of math, vol. 9, No. 2, 1983; P.P. 151-156.

[2] E. Hille and R. S. Phillips:Functional Analysis and semi-groups. AMS coll. publ. vol. 31. rev. ed. Providence (1951).

AHARON ATZMON:

Invariant subspaces

It is still an open problem whether or not every operator on an infinite dimensional complex Hilbert space has a non trivial invariant subspace. In this talk we described some Hilbert space operator which are suspected as being operators without invariant subspaces. These operators arise as the restriction of some operators without invariant subspaces on a nuclear Frêchet space which were described in our paper: "An operator without invariant subspaces on a nuclear Frêchet space". Annals of Math 117 (1983) 669-694.

JOSEPH A. BALL

Invariant subspace representations and Löwner interpolation

The talk will report on recent joint work with Bill Helton. We study a Pick-Löwner type of interpolation problem for matrix functions on the unit disk. Solutions are prescribed to satisfy interpolation constraints at a finite number of points in the closure of the unit disk, to have norm at most one on the unit circle, and to have at most ℓ poles in the open unit disk, where ℓ is to be determined. We determine the smallest ℓ for which solutions exist and for this ℓ obtain a linear fractional map parametrization for the set of all solutions. Our Grassmannian approach involves Krein space geometry and generalized Beurling-Lax representations (using the Liegroup $U(m,n)$ in place of $U(n)$); this approach was used previously for the simpler case where all interpolating nodes are in the open unit disk.

H. BART:

A method for solving Wiener-Hopf integral equations with symbol analytic in a strip

The talk is concerned with systems of convolution equations on the full or half line having a symbol that is invertible and analytic in a strip around the real line, the point infinity excluded. The method presented for solving such systems of equations is based on a realization of the symbol involving an unbounded state space operator whose main feature is that it is a direct sum of two infinitesimal generators of strongly continuous semigroups, one of which is defined on the positive half line and the other on the negative half line. The analysis is based on a perturbation theorem for operators of this kind. Necessary and sufficient conditions in order that the symbol admits a canonical Wiener-Hopf factorization (with respect to the real line) are discussed. Also, explicit formulas for the factors of such a factorization and for the resolvent kernels of the convolution equations are given in terms of the realization.

N.B.: The work reported on in the talk was done jointly with I. Gohberg (Tel Aviv) and M. A. Kaashoek (Amsterdam)

KEVIN CLANCEY:

Operators with rank one self-commutator

Let T be a Hilbert space operator whose self-commutator has the form $T^*T - TT^* = \varphi \otimes \varphi$. C. R. Putnam (1973) has shown that for every complex z the equation $(T-z)^*x = \varphi$ has a solution. The unique solution of this last equation in $[\text{Kernel } (T-z)^*]^\perp$ is denoted by $T_z^{*-1}\varphi$. The H-valued function $z \to T_z^{*-1}\varphi$ is weakly continuous and bounded $(\|T_z^{*-1}\varphi\| \leq 1)$ on $\mathbb{C}$.

THEOREM. The operator T is irreducible if and only if

$$\vee \{ T_z^{*-1}\varphi : z \in \mathbb{C} \} = H .$$

COROLLARY 1. If T is irreducible and the spectrum of T is nowhere dense, then φ is a rationally cyclic vector for T .

COROLLARY 2. If T is irreducible, there exists ψ in H such that $\{ p(T)\varphi + q(T)\psi : p, q - \text{polynominals} \}$ is dense in H .
Thus the operator T is 2-cyclic.

LEWIS A. COBURN

Toeplitz Operators and Quantum Mechanics (Joint work with C. A. Berger)

An interesting connection between some particular Bergman-type spaces of analytic functions and quantum mechanics was uncovered and explored by I. E. Segal and V. Bargmann in the early 1960's. On these spaces, with domain $\mathbb{C}^n$, the Fock boson creation operators are represented as multiplications by linear functions of the independent complex variables, z_j , $j = 1, 2, ----, n$. This connection was called to our attention by William Arveson several years ago when he gave a talk on the role of unbounded Toeplitz operators in quantum mechanics.

Since 1960, the study of bounded Toeplitz operators on a variety of domains has been systematized. In view of this, we have revisited the Segal-Bargmann spaces in order to clarify two questions:

(1) What is the structure of the algebras of bounded Toeplitz operators on these spaces,

(2) What is the relation between bounded Toeplitz operators and the Weyl operators of boson quantum mechanics.

R. G. DOUGLAS:

Hilbert Modules for Function Algebras

Although in operator theory one is used to considering the representation of algebras on Hilbert Space, in the domain of algebra it is more natural to consider modules over the algebra. In our talk we formulate theorems of von Neumann, Sz.-Nagy, Foias, Arveson and others in this language. A class of Hilbert Modules over function algebras called Šilov Modules is introduced and normal dilations results are shown to be equivalent to the existence of Šilov Representations. A result of Foias and the speaker on projectivity properties of Šilov Modules over the disk algebra is shown to be equivalent to the lifting theorem of Sz.-Nagy and Foias. Many questions, conjectures and hopes for future results were given in this context.

HARRY DYM

The Lossless Inverse Scattering Problem

The objective of the lossless inverse scattering problem is to find all linear fractional representations

$$S = (AS_L + B)(CS_L + D)^{-1}$$

for a given $n \times n$ matrix valued function S which is analytic and contractive in the open unit disc $\mathbb{D}$, where S_L (the "load") is of the same form as S and

$$\Theta = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is a 2n x 2n matrix valued function in $\mathbb{D}$ which is inner with respect to the signature matrix

$$J_0 = \begin{bmatrix} I_n & 0 \\ 0 & -I_n \end{bmatrix} :$$

$$\Theta(\omega)\, J_0\, \Theta(\omega)^* \leq J_0$$

for $\omega \in \mathbb{D}$, with equality a.e. on the boundary.
The talk reports on the solutions obtained in collaboration with Daniel Alpay. These results extend previous analysis carried out for rational Θ and scalar S, with P. Dewilde.

A. DYNIN

Multivariable Wiener-Hopf and Toeplitz Operators

It is well known that the Wiener-Hopf operators on stratified cones and the Toeplitz operators on stratified Siegel and Cartan domains are not pseudo-local. Therefore the standard technique of pseudo-differential operators is not applicable to such singular integral operators. Nevertheless a kind of non-commutative microlocalization is developed to answer the questions of inversion and regularization of such operators. It is based on the spectral analysis of the C^*-algebras generated by Wiener-Hopf and Toeplitz operators.

JÖRG ESCHMEIER

A duality result for several commuting operators

In a paper of 1959 E. Bischop showed that there is a close relationship between general spectral decompositions and duality for a single linear operator on a reflexive Banach space.
The aim of the talk is to prove that a finite commuting system of continuous linear operators on an arbitrary complex Banach space has a certain spectral decomposition property if and only if the adjoint system satisfies the same property.
For a single operator the property coincides with the notion of decomposability given by C. Foias, in several dimensions it is equivalent with the

existence of a Fréchet soft sheaf model in the sense of M. Putinar.

The talk generalizes results of E. Bishop, S. Frunză, M. Putinar and others. It covers part of a joint preprint written together with M. Putinar.

GILLES GODEFROY

Unicity of preduals for several non self-adjoint algebras of operators.

It is well-known that if A is a W^*-algebra, then one has existence and unicity of the predual A_* . It is natural to ask whether it is necessary to use the Hilbert space structure for obtaining such results. In a joint paper with P.D.SAPHAR [1], ist is shown that for any reflexive Banach space E, the space $E^* \tilde{\otimes}_\pi E$ is the unique predual of L (E), in the isometric sense; the main tool is a smoothness condition on the space L (E). Using the same technics, the following results are proved:

1) Under a "nice" smoothness condition on E, the space E^* has the metric approximation property if E has.
2) If X and Y are reflexive Banach spaces, the space K (X,Y) is reflexive or is not 1-complemented in L (X,Y).

Let us note as a conclusion that it is not clear whether or not K (X,Y) can be non trivially 1-complemented in L (X,Y). Indeed, the following problems seem to be open:

a) Does there exist X, Y reflexive Banach spaces such that K (X,Y) is reflexive and K (X,Y) $\neq$ L(X,Y) ?
b) Does there exist an infinite-dimensional Banach space X such that K (X) is reflecive ?

[1] : G. GODEFROY and P.D.SAPHAR: Smooth norms and the metric approximation property, to appear.

I. GOHBERG

Factorization, Integral operators and systems.

It was a review of recent results of H. Bart, M.A. Kaashoek, F. van Schagen and myself. It was described in detail the method of factorization and the method of reduction to systems for Wiener-Hopf Equations. Recent generalization for the irrational case and for semiseparable equations also presented. The talk was an introduction for three other talks (of H.Bart, M.A. Kaashoek and F. van Schagen) presented to this conference.

B. GRAMSCH

Multiplicative decompositions in Fréchetalgebras of operators

In the theory of pseudodifferential operators important classes of Fréchet-algebras have the property that the group of invertible elements is open (work of Beals 1977, Dunau 1977, Cordes 1979, Connes 1980). For some Fréchetalgebras and Fréchet-Liegroups the essential local difficulty to build up a theory of analytic operator functions can be overcome by proving the following generalization of "the" Lemma of H. Cartan:Thm. : Let G be a countable dense projective limit of Banach Liegroups, K', K'', $K = K' \cap K''$, $K' \cup K''$ rectangles in the complex plane. For each holomorphic function $f \in \mathcal{H}(U,G)$, $K \subset U$ there exist $f' \in \mathcal{H}(U',G)$, $U' \supset K'$, and $f'' \in \mathcal{H}(U'',G)$, $U'' \supset K''$, such that

$$f(z) = f'(z)\,(f''(z))^{-1} \quad \forall z \in K.$$

The proof uses methods of Arens 1966 (and Davie 1971); "the" use of the implicit function thoerem is avoided. The multiplicative decomposition of Fourier series with values in ψ-algebras poses a series of problems connected with "the" inverse function theorem for Fréchet spaces. An explicit formula (sharpening results of Masani, Gohberg, Krein and Leiterer) in the case of Banach algebras can be used to give positive results for Fréchet-C^∞-subalgebras of C^*-algebras and on the other hand counter-examples which show the limitations of multiplicative decompositions in Fréchet algebras. A suitable Fréchetalgebra ψ (following work of Cordes 1979 and Connes 1980) can be defined: Let Ω be a C^∞-manifold, $\mathcal{B}$ a Banachalgebra with 1 and

$$\alpha : \Omega \longrightarrow \mathrm{Aut}\,(\mathcal{B}),$$

$$\Psi :=\{a \in \mathcal{B} : \Omega \ni t \longrightarrow \alpha_t(a) \in C^\infty(\Omega, \mathcal{B})\}.$$

For these Fréchetalgebras the so called Oka-principle is discussed.

J. HEJTMANEK

Asymptotic Behavior of Strongly Continuous Semigroups, the Spectral Mapping Theorem for the Exponential Function and Applications to Linear Transport Theory.

Linear transport processes (like neutrons in a reactor, photons in a star or photons from an X-ray tube in Computerized Tomography) are solution of the linear Boltzmann equation. The asymptotic behavior of the semigroups generated by the Boltzmann operator is a one-parameter semigroup of positive operators in an L^1-Banach lattice. The spectral mapping theorem for the exponential function would give means to decide about the desired asymptotic behavior. The validity of the spectral mapping theorem is discussed and examples, similar to the Boltzmann equation, are presented.

M.A. KAASKOEK

Time varying systems with boundary conditions and integral operators

Time varying linear systems with boundary conditions and integral operators with semi-separable kernels are studied by exploiting their mutual connections. The concepts of transfer operator and realization are analysed. For systems with boundary conditions the transformations of similarity and reduction are considered and the invariants under these transformations are determined. Special attention is given to inversion and factorization of systems and of integral operators. The work reported on is joint work with I. Gohberg.

W.KABALLO

Decomposition of Fredholm Resolvents

This talk is on joint work with B. Gramsch. A main result is the following.

Theorem. Let X be a Banach space, $\Omega \subseteq C^N$ a domain of holomorphy and $T : \Omega \longrightarrow L(X)$ be holomorphic with values in $\Phi^1(X) = \{ A \in L(X) : \dim N(A) < \infty$, R(A) complemented $\}$ such that $T(z_0)$ is injective for some $z_0 \in \Omega$. Then T has a meromorphic left inverse L on Ω with a global decomposition $L(z) = A(z) + S(z)$, where $A : \Omega \longrightarrow L(x)$ is holomorphic and $S : \Omega \longrightarrow J_q$ is meromorphic, $q > N-1$.

Here J_q is the ideal of ψ_q-nuclear operators, where $\psi_q(t) = (\log \frac{1}{t})^{-q}$, $o < q < \infty$, t near o. This theorem sharpens earlier decomposition theorems of B. Gramsch (1973;$\mathcal{S}(x)$ instead of J_q) and H. Bart, W. Kaballo, Ph. Thijsse (1980;J_q with $q > N$). After a sketch of the proof an example is presented showing that this is false, even locally, for J_{N-1}.

K.G.KALB

C^∞ -vectors and operator algebras

(Report on joint work with B. Gramsch.)
Let $\alpha : \Omega \longrightarrow \mathcal{L}(E)^{-1}$ be a function from a C^∞-manifold Ω into the unitary group of a Hilbert space E. Consider the space $\langle E, C^\infty(\Omega), \alpha \rangle =$ $\{f \in E: [t \rightarrow \alpha(t)f] \in C^\infty(\Omega,E)\}$ of C^∞-vectors and the algebra $\langle \mathcal{L}(E), C^\infty(\Omega), \check{\alpha} \rangle$ (where $\check{\alpha}(t) = \alpha(t)T\alpha(t)^{-1}$) of C^∞-operators with respect to α, concept introduced by B. Gramsch in connection with results of A. Connes (1980) and H.O. Cordes (1979).

Recall: ①. $T(\langle E, C^\infty(\Omega), \alpha\rangle) \subseteq \langle E, C^\infty(\Omega), \alpha\rangle$. - Our first theorem is: ②. If $T \in \langle \mathcal{L}(E), C^\infty(\Omega), \check{\alpha}\rangle$ is left semi Fredholm, then (under an additional density condition) $T^{-1}(\langle E, C^\infty(\Omega), \alpha\rangle) \subseteq \langle E, C^\infty(\Omega), \alpha\rangle$. Various examples for α are discussed, particularly actions of $\mathbb{R}^d$ on E which satisfy a weakened group law (Weyl relations) and one parameter groups generated by self adjoint operators with pure point spectrum. Combining a remark on C^∞-vectors with a theorem of H.O. Cordes on C^∞-operators with respect to Weyl-action, ② spezializes to the following: If T $(\in \mathcal{L}(L^2(\mathbb{R}^n)))$ is a left semi Fredholm pseudodifferential operator with symbol $a \in CB^\infty(\mathbb{R}^{2n})$, then $T^{-1}(\mathcal{S}(\mathbb{R}^n)) \subseteq \mathcal{S}(\mathbb{R}^n)$. Examples of C^∞-operators are given which are translation operators and hence neither pseudolocal nor hypoelliptic. This motivates (again for $E = L^2(\mathbb{R}^n)$) the construction (e.g. by superposition of vector-fields) of locally supported families $\alpha = \{\alpha_{x,\delta} : x \in \mathbb{R}^n, \delta > 0\}$, such that for $T \in \langle \mathcal{L}(E), C^\infty(\Omega), \check{\alpha}\rangle$ the assertions of ①. resp. ②. spezialize to pseudo-locality respectively hypoellipticity of T. Part of the results is proven in more general situations then indicated above.

Hans G. KAPER

Sturm-Liouville eigenvalue problems with indefinite weights

We shall discuss eigenvalue problems of the form $Au = \lambda Tu$ in a Hilbert space, where A is a selfadjoint positive operator generated by a second-order Sturm-Liouville differential expression and T a selfadjoint indefinite multiplicative operator which is one-to-one. In particular, we shall discuss the partial-range completeness properties of the eigenfunctions. We shall illustrate the theory with several examples from physics and engineering.

Heinz KÖNIG

The Conjugation Operator in the abstract Hardy Algebra Theory

Let (H, φ) be a Hardy algebra situation on (X, Σ, m) in the sense of Barbey-König, LNM Vol. 593 (1977) (cited as LN). One first extends a basic result of the theory (LN IV.3.10 with IV.2.5) from the bounded to arbitrary real-valued measurable functions. This is then applied in order to characterize the conjugable functions. The main result is: A function $P \in ReL(m)$ is conjugable (in a certain modified sense, that is, conjugable in the sense of LN and subject to a certain mild growth restriction) iff it can be written in the form $P = F - G$, where F and G are pointwise limits of increasing sequences of nonnegative functions in ReH which are bounded in L^1- norm with respect to the m-continuous representing measures for φ.

Michel L. LAPIDUS

Product formula for imaginary resolvents and modified Feynman integral. Dominated convergence theorem for Feynman integral.

An abstract product formula for imaginary resolvents is proved for a pair of self-adjoint operators A, B of a complex Hilbert space. Here, A is assumed to be nonnegative and the positive part of B is arbitrary while its negative part is small with respect to A in the sense of quadratic forms. When specialized, this theorem establishes the convergence of the "modified Feynman integral" - recently introduced by the author - in the most general case for which the Schrödinger equation can be solved without ambiguity. Dominated convergence theorems for Feynman integrals are also investigated.

L. LERER

The Bezout equation and resultant operators for analytic functions.

Let D^+ be a finitely connected domain in the complex plane $\mathbb{C}$ and let $a_1(z),\ldots, a_n(z)$ be analytic functions in D^+ which have continuous extensions to the boundary Γ of D^+. Assume $a_n(t) \neq 0$ for $t \in \Gamma$. Our main aim is to produce convolution-type operators whose kernel have dimension equal to the number of common zeros of $a_i(z)$ in D^+. Such operators we call resultant operators. Our strategy is the following:

We first consider the Bezout equation

$$(1) \quad \sum_{i=1}^{n} a_i(z) \quad x_i(z) = c(z),$$

where $c(z)$ is a given function, analytic in D^+ and continuous in $D^+ \cup \Gamma$ and x_i (z) are unknown functions on which we impose certain conditions. These conditions allow us to transform (1) into an equation of form $(B\varphi)(t) = g(t)$, where B is a convolution type operator. Finally, the transpose $R = B^{tr}$ is the resultant operator. To prove this we use two methods. The first is based on factorization of functions and matrix-functions involved. The second one concentrates on studying eq.(1) and reducing it to some interpolation problem. The results are obtained jointly with B. Kon.

E. MEISTER

On the transmission problem of the Helmholtz equation for quadrants

Find the scattered electro-magnetic field in $\mathbb{R}^3$-space divided into four right-angled dielectric wedges V_j filled with different materials and excited by a line-source situated in the first wedge V_1 and with electric field vector $E_{pr} = (0,0,\Phi_{pr}(x,y))$! The problem leads to a transmission problem for the four wave-functions $\Phi_j(x,y)$ satisfying the four Helmholtz equations $(\Delta + \lambda_j)\Phi_j = 0$ in $\mathring{V}_j$, respectively. Across the common boundaries the total field functions and their normal derivatives have to pass continuously.

By means of the two-dimensional Laplace-transform and after factorizing the characteristic polynomials $u^2 + v^2 + \lambda_j$ with respect to u and applying the additive Wiener-Hopf decomposition, then symmetrizing with respect to v and applying the inverse Laplace-transform with respect to v one ends up with two representations of the one-dimensional L-transform of the boundary values of Φ_{tot} on $y = 0$. Putting them equal leads - after some algebraic manipulations - to a one-dimensional singular integral equation for the L-transform of the normal derivative of Φ_{tot} on $y = o$. It is shown that this eq. may be solved - at least by the fixed point principle - for small values of $|\lambda_1 - \lambda_2|$ and $|\lambda_3 - \lambda_4|$ in $L^q(R)$, $q \geq 2$, when $\operatorname{Im} \lambda_j > 0$, $j = 1,2,3,4$.

A. PIETSCH

Eigenvalue distributions and tensor stability

A quasi-Banach operator ideal $\mathfrak{A}$ is said to be of Riesz type l_r if all operators $T \in \mathfrak{A}(E,E)$, where E is an arbitrary complex Banach space, are Riesz such that the eigenvalue sequence $(\lambda_n(T))$ belongs to l_r. Then there exists a constant $c_r \geq 1$ such that

$$\left(\sum_n |\lambda_n(T)|^r\right)^{1/r} \leq c_r \, \|T|\mathfrak{A}\|.$$

Here $\|\cdot|\mathfrak{A}\|$ denotes the quasi-norm of $\mathfrak{A}$.
A quasi-Banach operator ideal $\mathfrak{A}$ is called stable with respect to a tensor norm α if $S \in \mathfrak{A}(E,E_0)$ and $T \in \mathfrak{A}(F,F_0)$ imply $S \hat{\otimes}_\alpha T \in \mathfrak{A}(E \hat{\otimes}_\alpha F, E_0 \hat{\otimes}_\alpha F_0)$. Then there exists a constant $c_\alpha \geq 1$ such that

$$\|S \hat{\otimes}_\alpha T|\mathfrak{A}\| \leq c_\alpha \|S|\mathfrak{A}\| \, \|T|\mathfrak{A}\|.$$

If the constants c_r and c_α are chosen as small as possible, then we have

$$c_r \leq c_\alpha .$$

This inequality can be used to show that the Banach ideal of absolutely r-summing operators with $2 \leq r < \infty$ is of Riesz type l_r.
Proofs are to be found in a paper which will appear in a Volume dedicated to the 60^{th} birthday of L. Nachbin, North-Holland. See also my forthcoming book on "Eigenvalues and s-numbers", Cambridge Univ. Press, 1985+x.

J.D. PINCUS

A Local Index Theory for certain Banach Algebras

A report on joint work with Richard Carey

We introduce a local index theory for a class, called p-analytic of maximal ideals in a finitely generated commutative Banach algebra $\mathfrak{A}$ of $\mathcal{L}(H)$. The ideals we distinguish are associated with a p-dimensional (locally defined) analytic space V lying in the Taylor spectrum of the generators $\{T_1,\dots,T_n\}$.

We introduce, first locally, certain holomorphic chains defined in a neighborhood of such an ideal whose density gives a new index. This index is not constant on the Fredholm components of the generating operator n-tuple, but jumps on singular points of the variety V.

For p=1 let m be the maximal ideal generated by $\{T_j - z_j\}_{j=1}^{n}$. The density of the holomorphic chain $\Sigma\, g_\alpha [\, V\,]_\alpha$ is shown to be

$$i(m) = \lim_{n\to\infty} \dim m^{*\,n}H / m^{*\,n+1}H - \lim_{n\to\infty} \dim m^{n}H / m^{n+1}H = \Sigma\, g_\alpha\, m_{V_\alpha}(z)$$

where $m_{V_\alpha}(z)$ denotes the multiplicity of the local ring of the irreducible component V_α g V at z. The integers g_α are determined as

$$g_\alpha = \mathrm{length}_{\hat{\mathfrak{A}}^m_p} [\hat{H}^{m^*} \times \hat{\mathfrak{A}}^m_p] - \mathrm{length}_{\hat{\mathfrak{A}}^m_p} [\hat{H}^m \times \hat{\mathfrak{A}}^m_p],$$

a difference of lenghts coming from certain m-adic completions of the Hilbert and algebra localized to the prime p which corresponds to V_α. The principal current $\Sigma\, g_\alpha [V_\alpha]$ is unchanged by trace class perturbations of $\{T_j\}_{j=1}^{n}$ which preserve commutativity. Under the condition that $\sigma_{ess}(T_1,\dots,T_n)$ is a scarred one manifold and $[T_i, T_j^*]$ is compact, it is shown that the current extends to a globally defined current, the principal current J, which is invariant under compact perturbations. It is shown that $dJ(\frac{df}{f})$ = Index $f(T_1,\dots,T_n)$ for meromorphic f non-vanishing on $\sigma_{ess}(T_1,\dots,T_n)$. This determines $K_1(\sigma_{ess}(T_1,\dots,T_n))$. Higher dimensional and non-commuting situations are also treated.

F. van SCHAGEN

Integral Equations with a non-compact semi-separable Kernel

In this talk we discuss integral equations of the second kind on a half-line and a full line with non-compact kernels of semi-separable type. The analysis is based on connections with time varying systems with boundary conditions and dichotomy plays an essential role. Necessary and sufficient conditions for invertibility will be given together with explicit formulas for the resolvent kernel. The Fredholm characteristics will be described in terms of dichotomy. Special attention will be paid to kernels with an exponential representation. The discrete analogue is also included.
The talk reports on joint work with I. Gohberg (Tel-Aviv, Amsterdam) and M.A. Kaashoek (Amsterdam).

H. SCHRÖDER

On the regular group of a finite continuous W^*-algebra and continuous index theorems

For a W^*-algebra M let GM denote the group of regular elements. If M has no finite discrete part we prove the following theorem

Theorem 1 $$\pi_k(GM) = \begin{cases} 0 & k \text{ even} \\ K(M) & k \text{ odd} \end{cases}$$

(For M properly infinite this was proved by Brüning and Willgerodt - note that the algebraic K-group K(M) = 0 in this case.)
We then deduce an index theorem for Toeplitz operators on S^{2n-1} whose symbols take values in a II_1-factor M.

Theorem 2 For $\Phi \in C(S^{2n-1},M)$ T_Φ is Fredholm (in the sense of Breuer) iff $\Phi \in C(S^{2n-1},GM)$. In this case one has $\mathrm{ind}(T_\Phi) = (-1)^n\, t_n\,([\Phi])$ ($t_n : \pi_{2n-1}(GM) \longrightarrow K(M) \cong \mathbb{R}$ denoting the isomorphism of Theorem 1.)

E. SCHROHE

Complex Powers of Elliptic Pseudifferential Operators

Suppose M is a compact n-dimensional manifold. Complex powers can be defined of large classes of elliptic pdo of order $m>0$ on sections of a vector bundle over M. The power A^s, $s \in \mathbb{C}$, of A is again a pdo. We are mainly interested in its kernel $k_s(x,y)$.

Seeley(1967), Šubin (1978): If $A \sim \sum_{j=0}^{\infty} Op(a_{m-j})$ and $a_{m-j}(x,\xi)$ is homogeneous of degree m-j in ξ, then $k_s(x,y)$, $x \neq y$, can be continued analytically to $\mathbb{C}$, $k_s(x,x)$ meromorphically with only simple poles in $\{0 \neq s_j = (j-n)/m : j \in \mathbb{N}_0\}$.

Gilkey (1981), Šubin (1978), Wodzicki (1982): Under similar assumptions the eta function of a selfadjoint operator is meromorphic in $\mathbb{C}$. There is no singularity in 0.

Considering perturbations, the holomorphy of $k_s(x,y)$, $x \neq y$, turns out to be a rather stable property whereas the meromorphy of $k_s(x,x)$ depends on the asymptotic expansion of the symbol $\sigma(A)$. Minor changes can make continuation to $\mathbb{C}$ impossible. Similar results hold for zeta and eta functions.

E. SEMENOV

Operator Blocks, Interpolation Theory, Geometry of Banach Spaces, Operators connected with Haar Series

Operator Blocks. Let E be a functional Banach lattice on [0,1] and Q_e be an operator of the multiplication by the characteristic function of a measurable set $e \subseteq [0,1]$. For each linear operator $T \in \mathcal{L}(E,E)$ let us denote

$$\sigma(T,E) = \inf_{me, mf > 0} \|Q_e \, T \, Q_f\|_E$$

and

$$D(E) = \{T : T \in \mathcal{L}(E,E), \sigma(T,E) > 0\}$$

We shall talk of solved and unsolved problem connected with the structure of D(E). The main problem is under what assumptions D(E) is empty.

(Herr SEMENOV mußte leider die Teilnahme an dieser Tagung absagen.)

F. SPECK

General Wiener-Hopf factorization and boundary problems in a half-space

We consider factorizations of a general Wiener-Hopf operator $W = T_P(A) = PA|_{PX}$ where X is a Banach space, A, $P \in \mathcal{L}(X)$, A invertible, $P^2 = P$. Introducing the notation of a cross factor with respect to P by $C \in \mathcal{L}(X)$ invertible such that $p_1 = C^{-1}PCP$, $p_3 = C^{-1}QCQ$ $(Q = I-P)$ are idempotent and $Pp_1 = p_1$, $Qp_3 = p_3$ hold, it can be proved, that the following assumptions are equivalent:

(i) $A = A_- C A_+$ with some invertible factors, $A_+PX = PX$, $A_-QX = QX$ and a cross factor C

(ii) $WVW = W$ for some $V \in \mathcal{L}(PX)$

(iii) $W = P\tilde{C}|_{PX}$ with some cross factor $\tilde{C}$.

This concept is used to solve several convolution equations with discontinuous symbols in a half-space as well as boundary value and transmission problems.

B. Sz.-NAGY

Contractions without cyclic vectors

It is proved that if T is a completely nonunitary contraction operator on Hilbert space such that T^{*n} does not converge strongly to 0 as $n \longrightarrow \infty$, there is an integer $N_T > 0$ so that none of the powers T^{*m} with $m > N_T$ has a cyclic vector. The result subsists even for contractions T with a nontrivial unitary part, provided this unitary part has its spectral measure absolutely continuous with respect to Lebesgue measure.

Reference: B.Sz.-NAGY - C. FOIAS: PAMS, vol 87.(1983), 671-674.

L. WEIS

On the essential spectra of L_1-operators

We give a formula for the essential spectral radius of operators in L_1 (Ω,μ) - spaces in terms of the quantity

$$\Delta\ (T) = \overline{\lim_{\mu(A)\to 0}} \ \|\chi_A T\|$$

and apply it to problems concerning

- the linear transport equation, in particular the asymptotic behaviour of its solutions
- the Doeblin condition as a criterion for uniform ergodicity in the theory of Markov chains
- the spectra of multipliers, in particular to the question: when does the spectra of a measure $\mu \in M(G)$ equal the closure of its Fourier transform

The proofs use the special structure properties of L_1 and this method also gives:
For a positive operator in $L_1(\Omega,\mu)$ the essential spectral radius always belongs to the essential spectrum.

H. WIMMER

Polynomial matrices, modules and linear pencils

An algebraic framework is developed in which polynomial matrices can be reduced to matrix pencils. Let $L \in F^{nxn}[z]$ be a polynomial matrix over a field F, which has a proper rational inverse.
Theorem: (a) There exists a finitely generated $F[z]$-module $V_L \subseteq F^n[z]$ such that $V_L \cong F^n[z]/_{LF^n[z]}$. As a vector space V_L has a dimension given by $\dim V_L = \deg\det L(=:r)$. V_L and V_{L^T} are dual spaces with respect to a scalar product $[\cdot,\cdot]$.

b) Let the columns of the matrix $P \in F^{nxr}[z]$ form a basis of V_L, let the columns of Q^T be a basis of V_{L^T} dual to P and let A be the matrix of the shift S^+, $S^+a = z \cdot a$, $a \in V_L$. Then $L(z) = P(z)(zI-A)Q(z)$. L and $(zI-A)$ have the same elementary divisors.

G. WITTSTOCK

Completely compact operators on C^*-algebras

Let A,B be C^*-algebras, $X \subseteq A$ a linear subspace, $\varphi : X \to B$ a completely bounded linear map. φ is called completely compact if for $\varepsilon > 0$ there exists a finite dimensional linear subspace $U \subseteq B$, such that $\inf \|\varphi_n(x) - M_n(U)\|_{M_n(B)} < \varepsilon$ for all $x \in M_n(X) \subseteq M_n(A)$, $\|x\| \leqslant 1$. ($M_n(A)$ is the C^*-algebra of nxn-matrices with entries in A and $\varphi_n[x_{ij}] = [\varphi(x_{ij})]$ is the n-th multiplicity map). It is shown that nuclear C^*-algebras have the $(1+\varepsilon)$ extension and the $(1+\varepsilon)$- decomposition property for completely compact maps. A completely bounded map $\varphi : X \to C(\mathcal{H})$ (the algebra of compact operators on $\mathcal{H}$) is completely compact iff there is a representation $\pi : A \to B(\mathcal{K})$, $\mathcal{K}$ a Hilbert space, and compact operators $V,W : \mathcal{H} \to \mathcal{K}$, $\|V\| \cdot \|W\| \leqslant (1+\varepsilon)\sqrt{\|\varphi\|_{cb}}$ such that $\varphi(x) = V^*\pi(x)W$. A completely positive, compact map $\varphi : A \to C(\mathcal{H})$ is automatically completely compact.

M. WOLFF

A characterization of the generator of a positivity preserving semigroup

Let E be an arbitrary Banach lattice with positive cone E_+. Consider the generator (A, D (A)) of a C_0-semigroup of operators T_t on E of type $\omega(A) = \inf(\frac{1}{t} \ln \|T_t\|)$.

Theorem: The following conditions are equivalent:

a) All operators T_t are positive (i.e. $T_t(E_+) \subseteq E_+$)

b) (i) $D(A^2) \cap E_+$ is dense in E_+

(ii) There exists a constant $M \geqslant \omega(A)+2$ such that for all $h \in D(A^2)$ with nonvanishing positive part h^+ the relation $(Ah)^+ \ngeq M \cdot h^+$ holds; that means: there exists a positive linear functional φ on E satisfying $0 = \varphi(h^-) \neq \varphi(h^+)$ and $\varphi(Ah) \leqslant M\varphi(h)$.

Some applications to differential operators are given.

J. ZEMANEK

Asymptotic formulas for the semi-Fredholm radius

Let T be a bounded linear operator on a Banach space. The semi-Fredholm radius $s(T)$ is the supremum of all $\varepsilon \geq 0$ such that the operators $T - \lambda I$ are semi-Fredholm for $|\lambda| < \varepsilon$. We obtain asymptotic formulas expressing $s(T)$ in terms of various geometric characteristics of T. Some of these quantities extend the classical sequences of Gelfand, Kolmogorov, Bernstein, and Mitiagin numbers, and so the results show that there is a natural passage from the spectral radius through the essential radius to the semi-Fredholm radius. Finite rank perturbations of the reduced minimum modulus are compared to these characteristics. To formulate a sample result we denote by $m(T)$ and $q(T)$ the minimum and surjection modulus, respectively, and define $m_\infty(T) = \sup m(T+F)$ and $q_\infty(T) = \sup q(T+F)$, where F varies over the finite rank operators. Then $g(T) = \max\{m_\infty(T), q_\infty(T)\}$ is positive iff T is semi-Fredholm, and we have

$$\lim_n g(T^n)^{1/n} = s(T).$$

Related open questions may also be discussed.

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