MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 16/1984

Diophantische Approximationen

8.4. bis 13.4.1984

Die diesjährige Tagung stand unter der Leitung von P. Bundschuh (Köln) und R. Tijdeman (Leiden).
42 Teilnehmer aus 11 Ländern berichteten in 39 Vorträgen über die neueren Entwicklungen im Bereich der diophantischen Approximation. Schwerpunkte bildeten dabei die Themen: Gleichverteilung, Irrationalität, Transzendenz und algebraische Unabhängigkeit sowie diophantische Gleichungen.
In der traditionellen "problem session", die A. Schinzel leitete, wurden einige offene Probleme zur Diskussion gestellt.

Vortragsauszüge

J.BECK:

New lower and upper bounds in the theory of irregularities of point distribution

We extend some well-known results of K.F.Roth and W.M.Schmidt concerning the discrepancy of boxes, rotated boxes and balls for arbitrary convex bodies (domains). Our method is based on Fourier analysis. We mention also some applications in discrete geometry, in "lattice point problems" and in multi-dimensional probability theory.
Furthermore, we have some new upper bounds using combinatorial and probabilistic ideas.

P.-G.BECKER-LANDECK:

Maße für algebraische Unabhängigkeit im Zusammenhang mit der Mahlerschen Transzendenzmethode

Mahler zeigte in einer Reihe von drei Arbeiten (1929/30) unter anderem die Transzendenz und algebraische Unabhängigkeit von Zahlen, die sich als Funktionswerte holomorpher Funktionen an algebraischen Stellen ergeben. Diese Funktionen erfüllen Funktionalgleichungen:

$$f_i(\underline{z}) = a_i f_i(T\underline{z}) + b_i(\underline{z}) \ , \ i = 1,\dots,p \ ,$$

mit rationalen Funktionen $b_i(\underline{z})$ und einer Transformation $T\underline{z} = \underline{w}$, $w_j := \prod_{i=1}^{n} z_i^{t_{ij}}$, T ist eine n×n-Matrix aus nicht-negativen ganzen Zahlen. Sind nun die Funktionen $f_i(\underline{z})$ algebraisch unabhängig, und ist $\underline{\alpha} \in \bar{\mathbb{Q}}^n$, so kann man unter einigen zusätzlichen technischen Voraussetzungen zeigen:

Für alle $P \in \mathbb{Z}[X_1,\dots,X_p]$, $P \neq 0$, $\partial_{X_i} P \leq d$, $H(P) \leq H$, gilt mit einer von d und H unabhängigen Konstanten $k > 0$ und einer in ihrer d-Abhängigkeit ineffektiven Konstanten $c = c(d) > 0$

$$\log |P(f_1(\underline{\alpha}),\dots,f_p(\underline{\alpha}))| > -c - e^{k^{d^p}} \log H .$$

D.BERTRAND:

Irregular singularities and Shidlovsky's lemma

I shall report on the following multiplicity estimate, obtained in collaboration with F.Beukers. Let $f_0,\dots,f_m$ be an analytic vector solution of an homogeneous differential system (D) with rational functions as coefficients. Let N,h be two positive integers, and let P be an element of $\mathbb{C}[z,X_0,\dots,X_m]$, of degree $\leq N$ in z and homogeneous of degree h in the X_i's. Assume that the function $R(z) = P(z,f_0(z),\dots,f_m(z))$ is not identically zero, and denote by s the order of the minimal differential operator with rational functions as coefficients which vanishes on R.
Then the multiplicity of R at 0 is bounded from above by

$$sN + c_1 sh + c_2 s^2 ,$$

where c_1 and c_2 denote positive numbers depending only on (D).
The proof relies on a generalization to irregular singularities of a classical relation connecting the exponents of fuchsian differential equations.

D.W.BOYD:

Some conjectures suggested by computations on the Schinzel-Zassenhaus conjecture

Let α be an algebraic integer of degree d and let $\lceil\alpha\rceil$ denote the maximum of the absolute values of its conjugates. Schinzel and Zassenhaus have conjectured that, if α is not a root of unity, then $\lceil\alpha\rceil \geq 1 + \frac{c}{d}$ for some absolute constant c. The best results obtained in this direction by Smyth and Dobrowolski are based on lower bounds for $M(\alpha)$, the Mahler measure of α . It is known that the extremal α for Mahler's measure are reciprocal (have α^{-1} as a conjugate). We describe computations of the extremal α for $\lceil\alpha\rceil$ for $d \leq 11$ which suggest that, for this problem, the extremal α is non-reciprocal and has about 2/3 of its conjugates outside the unit circle. Combined with Smyth's result, this would prove the Schinzel-Zassenhaus conjecture.

W.D.BROWNAWELL:

The growth of linear combinations of functions

We extend a result of Nevanlinna on the grow of a sum of $\mathbb{C}$-linearly independent functions. This sharpens an earlier result of my own. We also remark that a similar method applies to Bessel functions (but with no composites).

P.BUNDSCHUH:

Eine Klasse von Irrationalzahlen

Sind g, h natürliche Zahlen ≥ 2 und ist in h-adischer Darstellung gesetzt $a_h(g) := 0,(g^0)(g^1)(g^2)\ldots$, wobei (g^n) die Zahl g^n in h-adischer Entwicklung aufgeschrieben bedeutet, so wird die Irrationalität von $a_h(g)$ bewiesen. Dies verallgemeinert ein Resultat von K. Mahler (1981), allerdings mit ganz anderen Beweismethoden. Bei multiplikativ abhängigen g,h kann auch ein Irrationalitätsmaß angegeben werden, aus dem insbesondere folgt, daß $a_h(g)$ keine Liouville-Zahl ist.

W.W.L.CHEN:

Irregularities of distribution (and applications to approximate evaluation of integrals)

Let $U = [0,1]$, and let $K \geq 2$ be an integer. Denote by $J(K)$ the class of all functions of the type

$$C + \int_{B(\underline{x})} g(y)\, d\mu ,$$

where (i) μ is the Lebesque measure on U^K;

(ii) g is Lebesque integrable on U^K, non-zero in a set of positive measure;

(iii) $B(\underline{x}) = [0,x_1) \times \ldots \times [0,x_K)$, where $\underline{x} = (x_1,\ldots,x_K)$;

and (iv) C is any constant.

We wish to approximate functions in $J(K)$ by functions of the type

$$\phi(\underline{x}) = \sum_{i=1}^{M} m_i \chi_{B_i}(\underline{x}),$$

where (i) each B_i is a rectangular block in U^K with sides parallel to the sides of U^K;

(ii) χ_{B_i} is the characteristic function of B_i; and

(iii) m_i are real.

Then given any $f \in J(K)$, the study of the difference $\phi(\underline{x}) - f(\underline{x})$ can be shown to be a problem on irregularities of point distribution in U^K.

T.W. CUSICK:

Finding fundamental units in totally real fields

Let F be a totally real cubic field; for any α in F, define $T(\alpha)$ = trace (α^2). We define a pair of units ϵ_1, ϵ_2 as follows: Let ϵ_1 be a unit which gives the least value of $T(\epsilon)$ for any unit $\epsilon \neq \pm 1$ in F; let ϵ_2 be a unit which gives the least value of $T(\epsilon)$ for any unit $\epsilon \neq \pm \epsilon_1^n$.

Theorem. The units ϵ_1, ϵ_2 are a pair of fundamental units of F. This theorem was first stated by the speaker in Math. Proc. Camb. Phil. Soc. 92 (1982), 385-389, but with some possible exceptional cases which were conjectured never to occur. The exceptional cases were shown to be impossible by H.J. Godwin (same journal, to appear) via a clever and short argument. The speaker now gives a new and much simpler proof ot the theorem which uses geometry of numbers. The new method of proof extends at least to totally real quartic fields, but there are exceptional cases.

E.DUBOIS:

An application of P.V. numbers to Jacobi-Perron algorithm

We prove that in any real number field there exist a basis for which the Jacobi-Perron algorithm is periodic. For this we use a property of P.V. numbers.

J.H.EVERTSE:

Upper bounds for the numbers of solutions of diophantine equations

In our lecture we shall discuss upper bounds for the number of solutions of linear equations in two S-units and applications to other equations, such as the Thue-Mahler equation and special types of norm-form equations. The upper bounds for the numbers of solutions of the equations we consider have the advantage that they depend only on a few parameters. For instance, let K be an algebraic number field of degree d, let S be a finite set of valuations on K of cardinality s containing the non-archimedean valuations and let λ,μ be non-zero elements of K. Then the equation

$$\lambda x + \mu y = 1 \quad \text{in S-units } x,y$$

has at most $3\cdot 7^{d+2s}$ solutions.

E.HLAWKA:

Uniform distribution

Uniform distribution, especially with respect to harmonic weights and other weights with some applications.

M.LAURENT:

Diophantine exponential equations

I can prove, in the special case of linear forms, a general conjecture due to S. Lang, extending Mordell's conjecture. In terms of diophantine polynomial equations, we get a description of the solutions belonging to a given multiplicative subgroup. As example, we recover the classical Schlickewei-Schmidt's results about "Norm form equations".

L.LOVASZ:

Diophantine approximation algorithms

Suppose that we are given an oracle (subroutine) to compute, for any given $\epsilon>0$, a rational approximation of a real number α with error $<\epsilon$. Also suppose that we know that α is an algebraic number and its minimal polynomial has input size $\leq k$.

Theorem: The minimal polynomial of α can be computed in polynomial time.

Corollary: Given a polynomial $f\in\mathbb{Q}[x]$, the factorization of f into irreducible polynomials can be found in polynomial time. (Lenstra,H.W.jr.; Lenstra,A.K.; Lowász,L.)

The proof of the theorem depends on the polynomial time solvability of the following problem. Given a lattice $L\subseteq\mathbb{Q}^n$, we can find in polynomial time a vector $b\in L$, $b\neq 0$ such that $\|b\|\leq 2^n\min\{\|x\| \mid x\in L,\ x\neq 0\}$. This algorithm can also be applied to solve the following problem: Given $\epsilon>0$, $\alpha_1,\dots,\alpha_n\in\mathbb{Q}$, find integers $b_1,\dots,b_n$, a such that $0<a<2^{n^2}\epsilon^{-n}$ and $|a\alpha_i-b_i|<\epsilon$ $(i=1,\dots,n)$.

J.H.LOXTON:

Linear forms in dilogarithms of algebraic numbers

The dilogarithm function is defined, for suitable z, by

$$\ell i_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2} = -\int_0^z \frac{\log(1-t)}{t}dt.$$

There is an embarrassing wealth of relations between the values of the dilogarithm function at certain algebraic points; three nice examples are the relation

$$3\ell i_2(\alpha) + 3\ell i_2(\alpha^2) - \ell i_2(\alpha^3) = \frac{7\pi^2}{18} - 3\log^2\alpha \quad (\alpha = \tfrac{1}{2}\sec\tfrac{\pi}{9})$$

and the "conjugate" relations involving $-\frac{1}{2}\sec(2\pi/9)$ and $-2\cos(4\pi/9)$. Such relations may have both a geometrical and a combinatorial significance, but a proper understanding of them seems hard to find.

R.G.MASON:

Norm form equations

In recent years a bound on the heights of solutions in S-units of the equation $u + v = 1$ over function fields has proved of great value in deriving algorithms for the resolution of various general families of equations. Recently a bound has been derived for the heights of solutions in S-units of $u_1+\ldots+u_n = 1$ for arbitrary n. This result has important consequences, and in particular the following has been obtained. Theorem: Let K be a finite extension of k(z) (k algebraically closed, characteristic zero, explicitly presented), and M a free non-degenerate k[z]-module in K. Then for each c in k(z), all the solutions x in M of $\mathrm{Norm}_{K|k(z)}(x) = c$ may be determined effectively.

D.W.MASSER:

Some remarks on diophantine approximation

(I) Let α be a real algebraic number of degree three and coefficient height H. We give a short elementary proof of the estimate $q_n \leq (13H)^{3(n+1)/2}$ for the denominator q_n of the n-th continued fraction convergent to α. This simplifies recent work of Wolfskill.

(II) We give a version of the proof of Siegel's Theorem on rational approximations that can be made effective in special cases. As an example we show that if $d \geq 22$ and $m \geq m_0(d)$ then the real number α satisfying $\alpha^d - m\alpha^{d-1} + 1 = 0$ has the property that $|\alpha - p/q| \geq cq^{-\lambda}$ for all integers $p,q \geq 1$. Here $\lambda = \frac{55}{14}(4+\sqrt{2})$, and $c = c(m,d) > 0$ is effectively computable. This improves recent examples obtained by E.Bombieri using similar methods.

M.MENDES-FRANCE:

Automata theory and formal power series

We present part of some work done jointly with A.van der Poorten

(I) Automata: Let $p \geq 2$ be an integer. A p-automaton is a finite automaton with p arrows leaving from each one of its states. The p-automaton sends the sequence of integers on an infinite sequence $\epsilon = (\epsilon_n) \epsilon (\mathbb{Z}/p\mathbb{Z})^{\mathbb{N}}$. Such a sequence is said to be automatic. Lemma: If ϵ is automatic, then the subsequences ϵ_{p^n} is eventually periodic.

(II) Formal power series: Let K be the field of formal Laurent series over the finite field $\mathbb{Z}/p\mathbb{Z}$ (p prime). An element $f \epsilon K$

$$f = \sum_{k=k_0}^{\infty} f_k X^k, \quad f_k \in \mathbb{Z}/p\mathbb{Z},$$

is said to be algebraic if there exists polynomials $a_0, a_1, a_2, \ldots, a_v \in K$ not all 0 such that

$$a_0 f^v + a_1 f^{v-1} + \ldots + a_v = 0$$

<u>Theorem:</u> (Christol,Kamae,MF,Rauzy) The sequence $\epsilon = (\epsilon_n)$ is automatic iff $\sum_0^{\infty} \epsilon_n X^n$ is algebraic in K.

If $(1+X)^{\lambda}$ is defined in a natural way, then it is proved

<u>Theorem:</u> $(1+X)^{\lambda}$ is algebraic in K iff λ is rational.

M.MIGNOTTE:

<u>Automatic resolution of some diophantine equations</u>

We consider equations of the type $x^2-k = a^n$ where a,k are fixed integers, $a>1$, and x and n are unknowns. Using Pell-Fermat equation $x^2-ay^2 = k$ we are lead to search the values of some linear recursive equations which are powers of a. Using suitable congruences in practice we find quickly all the solutions. Example: The equation $X^2+X+1 = 13^n$ has only the "trivial" solutions n=0 and n=1.

W.NARKIEWICZ:

<u>Problems in uniform distribution of sequences of integers</u>

Let M(f) be the set of $N \in \mathbb{Z}^+$ for which $\{f(m)\}_{m=1}^{\infty}$ is UD (mod N) and let $M^*(f)$ denote the corresponding set for WUD (mod N). Problems were posed concerning M(f) and $M^*(f)$ for various classes of functions f.

In particular:

1. Determine the behaviour of $M^*(\sigma_k)$ for large k.
2. Which stes $X \subset \mathbb{Z}^+$ appear as M(P) for $P \in \mathbb{Z}[x]$?
3. Determine $M(\{u_n\})$, $M^*(\{u_n\})$ for linear recurrences.

H.NIEDERREITER:

Distribution mod 1 of monotone sequences

Let (X_n) be a monotone sequence of reals. If (X_n) is uniformly distributed with respect to a Borel probability measure on the one-dimensional torus that is not a point measure, then it is shown that necessarily $\lim_{n\to\infty} |X_n|/\log n = \infty$. More generally, an analogous result is established for any summation method of weighted means. We show also that the later result is in a sense best possible. Furthermore, we obtain a characterization of those weighted means that admit uniformly distributed sequences.

C.F.OSGOOD:

Diophantine approximation and Nevanlinna theory

Let $w_1,\ldots,w_M$ denote formal power series solutions of linear homogeneous differential equations having coefficients in $\mathbb{C}[z]$. Suppose further that the $w_1,\ldots,w_M$ are linearly independent over $\mathbb{C}[z]$. Let ord denote the order of vanishing at $z = 0$. Let $\underline{A}$ denote an M-tuple of elements of $\mathbb{C}[z]$. Let $\deg A_j$ denote the degree of A_j ($\deg 0 := 0$).

<u>Theorem I:</u> For each nonzero $\underline{A}$

$$(+)\quad \mathrm{ord}\left(\sum_{j=1}^{M} A_j w_j\right) \le \sum_{j=1}^{M} \deg A_j + O\left(\left(\log\left(\sum_{j=1}^{M} \deg A_j\right)\right)^{\tau}\right) + O(1)$$

for some effectively computable τ, $0<\tau<1$.

Suppose now each occurance of $\mathbb{C}[z]$ is replaced by $\mathbb{Q}[z]$.

<u>Theorem II:</u> The implied constants in (+) are effectively computable.

<u>Remark:</u> The methods extend to Nevanlinna Theory. It is now possible to prove the n small function theorem without restrictions.

A.PETHŐ:

<u>Polynomial values in second order recurrences</u>

Die folgenden Resultate wurden gemeinsam mit I.Nemes bewiesen. Es seien G_0, G_1, A, B und d ganze Zahlen mit $|G_0|+|G_1|>0$ und $|B|=1$. G_n sei für $n\geq 2$ durch die Rekursion $G_n = AG_{n-1}+BG_{n-2}$ definiert. Es sei $P(x) \in \mathbb{Z}[x]$ mit Grad $P = k$. Dann gilt:

Wenn die Folge G_n nicht ausgeartet ist, dann hat die diophantische Gleichung

$$(1)\qquad dx^q + P(x) = G_n$$

nur endlich viele, effektiv berechenbare Lösungen $n, |x|>1, q>k+2$. Für $q\leq k+2$ kann (1) unendlich viele Lösungen haben. Es sei $dx^q + P(x) = R(x) = a_m x^m + \dots + a_0$. Wenn (1) unendlich viele Lösungen n,x hat, dann gilt $R(x) = \epsilon\sqrt{s}\,T_m\left(\frac{2a_m}{\eta\sqrt{E}}x + \frac{2a_{m-1}}{\eta\sqrt{E}}\right)$, mit Konstanten s und E abhängig von G_n und $R(x)$. $T_m(x)$ bezeichnet das m-te Chebishev-Polynom; ϵ und η sind 1 oder -1. In diesem Fall können wir sogar die Lösungen x von (1) gut charakterisieren.

P.PHILIPPON:

Critères d'indépendance algébrique

Nous présentons un nouveau critère d'indépendance algébrique.

Critère: Soient $\underline{\theta} = (\theta_1,\dots,\theta_n) \in \mathbb{C}^n$ et $C = C(n)$ un nombre réel suffisamment grand. Il n'existe pas de suite:

$$[I_N = (P_{1,N},\dots,P_{m(N),N})]_{N \geq N_0}$$

d'idéaux de $\mathbb{Z}[X_1,\dots,X_n]$ dont l'ensemble des zéros dans la boule $B(\underline{\theta},\exp(-3CN^{n+1}))$ de $\mathbb{C}^n$ soit de cardinal fini et que pour $N \geq N_0$ et $i = 1,\dots,m(N)$ on ait $t(P_{i,N}) \leq N$ et

$$0 < \max_{1 \leq i \leq m(N)} \{|P_{i,N}(\underline{\theta})|\} < \exp(-CN^{n+1}).$$

On peut ainsi montrer le résultat suivant:

Théorème: Soient $\alpha \in \bar{\mathbb{Q}} \setminus \{0,1\}$ et $\beta \in \bar{\mathbb{Q}} \setminus \mathbb{Q}$ de degré d on a

$$\mathrm{degtr}_{\mathbb{Q}}\, \mathbb{Q}(\alpha^{\beta},\dots,\alpha^{\beta^{d-1}}) \geq [d/2].$$

On propose également une version quantitative du critère précédent.

A.J.VAN DER POORTEN:

Constructing curves with prescribed singularities

In order to obtain effective improvements of Liouville's theorem by Thue's method it appears critical to be able to make very precise constructions of polynomials $P(X,Y) = \sum_j \sum_i a_{ij} X^i Y^j \in \mathbb{Z}[X,Y]$ which, together with many of their derivatives, vanish at given points. To apply the theorem of Bombieri and Vaaler, Bombieri and I during 1983 put considerable effort into analysing certain determinants that arise in the construction problem; we obtained some quite surprising results which suggest that "exact" constructions are not quite beyond our capabilities.

A.SCHINZEL:

An unusual metric in the group of real numbers

In the additive group of real numbers an invariant metric $\rho(x,y)$ is constructed with the following properties:

$\mathbb{R}$ under ρ is a complete non-discrete and non-separable space.

$\rho(x,0)$ is measurable and satisfies the Baire condition,

$\rho(x,0) = 1$ unless x is rational or a transcendental Liouville number.

H.P.SCHLICKEWEI:

Kleine Nullstellen quadratischer Gleichungen

Es sei $f(x_1,\ldots,x_n) = \sum_{i,j=1}^{n} f_{ij}x_ix_j$ eine quadratische Form mit $f_{ij}=f_{ji}$. Weiter sei $F = (\sum_{i,j=1} f_{ij}^2)^{1/2}$. Λ sei ein Gitter im n-dimensionalen euklidischen Raum mit Determinante Δ. Die Form f nehme auf Λ ganzzahlige Werte an. Dann wird bewiesen

Satz: Es sei $0 < d \leq n$. f verschwinde auf einem d-dimensionalen Teilgitter Γ von Λ. Dann gibt es ein d-dimensionales Teilgitter Γ_d von Λ, auf dem f verschwindet, mit

$$\det \Gamma \ll \Delta F^{\frac{n-d}{2}} .$$

Dabei hängen die Konstanten in $\ll$ nur von n ab.

Der Fall d=1 wurde 1956 von Cassels für das Gitter $\mathbb{Z}^n$ und 1958 von Birch und Davenport in der allgemeineren Form d=1 und mit beliebigem Gitter bewiesen.

W.M.SCHMIDT:

Kleine ganze, reelle und p-adische Nullstellen quadratischer Formen

Es gibt unendlich viele quadratische Formen mit ganzen Koeffizienten, der Gestalt $L_1^2+\dots+L_r^2-M_1^2-\dots-M_s^2$ mit gegebenem $r,s > 0$, deren ganzzahlige Nullstellen alle $|\underline{x}| \gg F^{r/2s}$ erfüllen, wo F der maximale Betrag der Koeffizienten ist. Damit wird ein Ergebnis von Watson verschärft, und es folgt, daß eine kürzlich durch Schlickewei erzielte obere Schranke bestmöglich ist. Genauer gilt $q(\underline{x}) \gg F^{r/2s}$ für jede reelle Nullstelle, wobei $q(\underline{x}) := \mathrm{Max}\,(|x_i|/|x_j|)$ über i,j mit $x_j \neq 0$. Auch ein p-adisches Analogon zu $q(\underline{x})$ wird untersucht.

J.SCHOISSENGEIER:

Diskrepanz von $(n\alpha)$

Es sei $\alpha \notin \mathbb{Q}$ und $D_N^*(\alpha) = \sup_{0\le x<1} |\frac{1}{N}\sum_{n=1}^{N} c_{[0,x)}(\{n\alpha\}) - x|$ die Diskrepanz der Folge $(n\alpha)_{n\ge 1}$. Man kann mit Hilfe der Kettenbruchentwicklung $\alpha = [a_0;a_1;\dots]$ und den Näherungsbrüchen $\frac{p_n}{q_n}$ eine explizite Formel für $D_N^*(\alpha)$ angeben. Approximativ lautet sie so: Ist $m = m_N$ so gewählt, daß $q_m \le N < q_{m+1}$, und ist

$$E_N^*(\alpha) = \sum_{2|j\le m} (a_{j+1}\{Nq_j\alpha\}(1-\{Nq_j\alpha\})+\{Nq_j\alpha\}(\{q_{j+1}N\alpha\}-\{q_{j-1}N\alpha\})),$$

$$O_N^*(\alpha) = \sum_{2\nmid j\le m} (a_{j+1}\{Nq_j\alpha\}(1-\{Nq_j\alpha\})+\{Nq_j\alpha\}(\{q_{j+1}N\alpha\}-\{q_{j-1}N\alpha\})),$$

so ist $N\,D_N^*(\alpha) = \max(E_N^*(\alpha),O_N^*(\alpha)) + O(1)$, wobei die O-Konstante von α nicht abhängt.

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Ist

$\omega_m(\alpha) = \max(\sum_{2|j\leq m} a_{j+1}, \sum_{2\nmid j\leq m} a_{j+1})$ und $(a_j)_{j\geq 0}$ nicht C-1 beschränkt, so ist $\frac{1}{4} \leq \overline{\lim_{N\to\infty}} \frac{N\, D_N^*(\alpha)}{\omega_{m_N}(\alpha)} < \infty$.

Ist $(a_j)_{j\geq 0}$ C-1 beschränkt, so ist $0 < \overline{\lim_{N\to\infty}} \frac{N\, D_N^*(\alpha)}{\log N} =: \nu^*(\alpha) < \infty$.

Wenn alle a_j gerade sind, so ist $\nu^*(\alpha) = \frac{1}{4} \overline{\lim_{m\to\infty}} \frac{1}{\log q_m} \omega_m(\alpha)$.

Insbesondere: Ist $\alpha = [\overline{a_0;a_1;\ldots;a_{l-1}}]$, l die Periode, $2|a_j$ für $j \geq 0$, so ist

$$\nu^*(\alpha) = \frac{1}{4} \frac{1}{\log(q_{l-1}\alpha+q_{l-2})} \max(\sum_{2|j\leq l-1} a_j, \sum_{2\nmid j\leq l-1} a_j), \text{ falls } l \text{ gerade,}$$

und

$$\nu^*(\alpha) = \frac{1}{8} \frac{1}{\log(q_{l-1}\alpha+q_{l-2})} \sum_{j\leq l-1} a_j, \text{ falls } l \text{ ungerade.}$$

T.N.SHOREY:

Applications of linear forms in logarithms

Let $\{u_m\}$ be a non-degenerate binary recursive sequence. For an integer a with $|a| > 1$ debote by P(a) the greatest prime factor of a and by Q(a) the greatest square free factor of a.

Generalizing earlier results of Mahler, Schinzel and Stewart, Shorey proved:

Let $m > n$ with $u_n \neq 0$. Then

$$P\left(\frac{u_m}{(u_m,u_n)}\right) \geq C_1 \left(\frac{m}{\log m}\right)^{\frac{1}{d+1}}, \quad m \geq C_2,$$

where $C_1, C_2 > 0$ are effectively computable numbers depending only on $\{u_m\}$ and $d = [\mathbb{Q}(\alpha):\mathbb{Q}]$, where α is a root of the companion polynomial to $\{u_m\}$. Further Shorey proved:

$$\log Q(u_m) \geq C_1 \left(\frac{(\log m)^2}{\operatorname{loglog} m}\right), \quad m \geq C_2.$$

Let $f(X,Y) \in \mathbb{Z}[X,Y]$ be an irreducible binary form of degree $m \geq 3$ and let $g(X,Y) \in \mathbb{Z}[X,Y]$ be a binary form of degree $n < m$. Then the equation $f(x,y) = g(x,y)$ has only finitely many solutions in integers x and y. Further the result is effective. This is an immediate consequence of Baker's effective version of Thue's theorem.

G.L.STEWART:

<u>Irregularities of distribution in shifts and dilations of the sqares</u>

We shall discuss the following result: Let $N \in \mathbb{Z}^+$ and let $\epsilon_1,\ldots,\epsilon_N \in \mathbb{C}$ with $|\epsilon_i| = 1$. There exist positive integers a and q such that

$$\left| \sum_{\substack{j \geq 1 \\ a+j^2q \leq N}} \epsilon_{a+j^2q} \right| > \frac{1}{50} (\log N)^{1/2}$$

for N sufficently large.

R.F.TICHY:

Zur gewichteten Diskrepanz von Folgen

Es wird für Folgen $x_n \in [0,1) = E$ und positive Gewichte $P = (p_n)$ die Diskrepanz

$$D_N(P,x_n) = \sup_I \left| \frac{1}{P(N)} \sum_{n=1}^{N} p_n \chi_I(x_n) - |I| \right|$$

betrachtet; I läuft durch alle Teilintervalle von E, χ_I ist charakteristische Funktion und $|I|$ die Länge von I; $P(N) = \sum_{n=1}^{N} p_n$. Für diesen Diskrepanzbegriff werden allgemeine Abschätzungen nach oben und unten (analog zur Roth'schen) angegeben, ferner werden für spezielle Folgen Diskrepanzabschätzungen angegeben, z.B. metrische Sätze analog zum klassischen Satz von Koksma sowie Verallgemeinerungen auf Matrizenfolgen.

R.TIJDEMAN:

Rational approximations of p-adic numbers

The applications of p-adic approximations to finding all solutions of some diophantine equations and to error-free computation by so-called Hensel-codes using an electronic computer lead to the following questions: Does there exist an efficient algorithm to compute the best rational approximations to any p-adic number. How well can a p-adic number be approximated by rational numbers. A report will be given on some joint work with B.M.M. de Weger (Leiden) which can be considered as a continuation of investigations of Mahler. Analogues of classical results on best approximations to real numbers will be presented. A variant of Mahler's algorithm, which is based on Euclid's algorithm, turns out to yield all best rational approximations.

J.VAALER:

Applications of a new form of Siegel's lemma

I will describe a recent improvement in Siegel's lemma obtained jointly with E. Bombieri and also indicate a few applications. In particular the problem of constructing a polynomial of degree $< N$ vanishing at an algebraic number α with multiplicity $\geq M$ and having coefficients in O_K for a number field K over $\mathbb{Q}$ will be described. In this problem a basis of such polynomials can be found having small average height (and hence one among this basis must have a small height). I will mention some numerical results on measures of irrationality which result from a two variable polynomial construction.

M.WALDSCHMIDT:

Algebraic independence of transcendental numbers

Let d_0, d_1, d_2 be non negative integers, with $d = d_0+d_1+d_2 > 0$. Define $G_0 = \mathbb{G}_a^{d_0}$, $G_1 = \mathbb{G}_m^{d_1}$, and $G = G_0 \times G_1 \times G_2$, where G_2 is a commutative algebraic group over $\bar{\mathbb{Q}}$ of dimension d_2. We assume that for i=1 and i=2, there is no isomorphic algebraic subgroup of G_i and G_2 of dimension ≥ 2. Let $\varphi: \mathbb{C}^n \longrightarrow G(\mathbb{C})$ be a n-parameter subgroup of G whose image is Zariski dense in $G(\mathbb{C})$. Let Y be a finitely generated subgroup of $\mathbb{C}^n$, and K a subfield of $\mathbb{C}$, of transcendence degree t over $\mathbb{Q}$, such that $\varphi(Y) \subset G(K)$.

Theorem 1: If $(d-n)\,\mu(Y,\mathbb{C}^n) > d_1+2d_2$, then $t \geq 1$.

Theorem 2: If $(d-2n)\,\mu(Y,\mathbb{C}^n) \geq 2(d_1+2d_2)$, then $t \geq 2$.

Theorem 3: Write $Y = \mathbb{Z}\,y_1+\ldots+\mathbb{Z}\,y_l$. Assume that for all $\varepsilon > 0$, there exists $M_0 > 0$ such that, for any $M \geq M_0$, any algebraic subgroup H of G defined (in a projective space) by equations of degree $\leq M$, any $y = h_1y_1+\ldots+h_ly_l$ with $|h_j| \leq M$ and $\exp_G y \notin H(\mathbb{C})$, and finally any $u \in T_G(\mathbb{C})$ with $\exp_G u \in H(\mathbb{C})$, $|u-y| > \exp(-M^{\varepsilon})$.

Then: $t+1 \geq \dfrac{d\mu(Y,\mathbb{C}^n)}{n\mu(Y,\mathbb{C}^n)+d_1+2d_2}$.

R.WALLISSER:

On irrationality of values of functions satisfying certain q-difference equations.

i) The method of Siegel-Shidlowski is applied to give measures of irrationality for the values at rational places of functions satisfying a q-difference equation of the form

$$f(z) = (a+bz)\,f(qz) + c, \quad q \in \mathbb{C},\ q \neq 1.$$

ii) With a method of W. Maier, results on irrationality and linear independence are given for the special q-hypergeometric series,

$$f(z) = \sum_{k=0}^{\infty} \frac{q^{\alpha\binom{k}{2}}}{Q(q^0)\ldots Q(q^k)}\, z^k, \quad Q \in \mathbb{Q}[z].$$

P.WARKENTIN:

On norm form inequalities

Let $M(\underline{x}) = \mu_1 x_1 + \ldots + \mu_n x_n$ be a linear form in $\underline{x} = (x_1, \ldots, x_n)$ with $\mathbb{Q}$-linearly independent coefficients $\mu_1, \ldots, \mu_n \in K$, where K is a numberfield of degree d. We call $M(\underline{x})$ degenerate iff the $\mathbb{Z}$-module $\mathfrak{M} = M(\mathbb{Z}^n)$ contains elements $\lambda\omega_1, \ldots, \lambda\omega_e$ such that $\omega_1, \ldots, \omega_e$ is the basis of a numberfield L, $L \neq \mathbb{Q}$ and $L \neq$ imaginary quadratic. There exists a $\gamma > 0$ such that the inequality

(*) $|\mathrm{Norm}_{K/\mathbb{Q}}(M(\underline{x}))| < \max(|x_1|, \ldots, |x_n|)^{\gamma}$

has at most finitely many solutions iff $M(\underline{x})$ is non-degenerate.

Theorem: Let $M(\underline{x})$ be non-degenerate, $\kappa := [\mathbb{Q}(\mu_2/\mu_1, \ldots, \mu_n/\mu_1):\mathbb{Q}]$, $\epsilon > 0$. (*) has at most finitely many solutions if

$n = 3$: $\gamma = \frac{d}{4} - \epsilon$

$\gamma = \frac{d}{3} - \epsilon$ for $\kappa \neq 4,6$

$n = 4$: $\gamma = \frac{d}{5} - \epsilon$

$\gamma = \frac{d}{4} - \epsilon$ for $\kappa \neq 5,7,10$

The first case is due to W.M.Schmidt. The theorem is best possible.

E.WIRSING:

c log n mod $\mathbb{Z}$

Nach Erdős (1946) ist jede additive Funktion $f : \mathbb{N} \longrightarrow \mathbb{R}$ mit der Eigenschaft $\Delta f(n) := f(n+1) - f(n) \longrightarrow 0$ notwendig von der Form $f(n) = c \log n$ (c = const.). Kátai (1984) vermutet, daß die multiplikativen Funktionen $f : \mathbb{N} \longrightarrow \mathbb{C}$ mit $\Delta f \longrightarrow 0$ genau diejenigen Funktionen f sind, die $f(n) \longrightarrow 0$ erfüllen, oder die Form $f(n) = n^{\sigma+i\tau}$ mit $0 \leq \sigma < 1$ haben. Unter schärferen Voraussetzungen ist dies bei Kátai ein Satz. Tatsächlich ist die Vermutung in vollem Umfang richtig. Der wesentliche Schritt zum Beweis ist der

Satz:

Ist $F : \mathbb{N} \longrightarrow \mathbb{R}/\mathbb{Z}$ additiv und gilt $\Delta F(n) \longrightarrow 0$, so ist $F(n) = c \log n/\mathbb{Z}$ mit einer Konstanten $c \in \mathbb{R}$. Dieser Satz selbst läßt sich - etwas überraschend - auf Erdös's Satz zurückführen.

G.WÜSTHOLZ:

Algebraische Punkte auf algebraischen Gruppen und diophantische Approximationen auf algebraischen Gruppen

In dem Vortrag wurden zunächst algebraische Gruppen eingeführt und zusammen mit der zugehörigen Exponentialabbildung anhand von Beispielen erläutert. Danach wurde das folgende Problem behandelt: Gegeben seien zwei kommutative algebraische Gruppen G,G', die zusammenhängend und über $\overline{\mathbb{Q}}$ definiert seien. Sei $\varphi: G' \longrightarrow G$ ein analytischer Homomorphismus, der ebenfalls über $\overline{\mathbb{Q}}$ definiert sei, d.h. die Tangentialabbildung $d\varphi : T(G') \longrightarrow T(G)$ sei ein $\overline{\mathbb{Q}}$ - Homomorphismus. Dann kann man sich fragen, wie die Menge $\varphi(G')(\overline{\mathbb{Q}})$ aussieht. Man wird dann auf den folgenden Satz geführt:

Satz 1:

Der Zariski-Abschluß $\overline{\varphi(G')(\bar{\mathbb{Q}})}$ von $\varphi(G')(\bar{\mathbb{Q}})$ ist in $\varphi(G')(\mathbb{C})$ enthalten.

Als Konsequenz dieses Satzes erhält man die Lösung eines Problems von Grothendieck. Dazu sei X eine quasiprojektive algebraische Varietät definiert über $\bar{\mathbb{Q}}$, $\xi \in \Gamma(X,\Omega_X^1)$, $d\xi = 0$ und $\gamma \in H_1(X,\mathbb{Z})$.

Satz 2:

$\int_\gamma \xi$ ist 0 oder transzendent.

V.G.SPRINDZUK:

Herr Sprindzuk, der nicht an der Tagung teilnehmen konnte, sandte folgenden Vortragsauszug ein:

Arithmetic properties of algebraic functions

Let $\mathbb{K}$ be a field of algebraic numbers, $[\mathbb{K}:\mathbb{Q}] = k$, S the full set of places of $\mathbb{K}$, $k_\nu = [\mathbb{K}_\nu:\mathbb{Q}_\nu]$. For $0 \neq x \in \mathbb{K}$ put $(x)_\nu = \max(1,|x|_\nu^{-k_\nu})$, $H_{\mathbb{K}}(x) = \prod_{\nu\in S} (x)_\nu$. Let $F(x,y)$ be an irreducible polynomial in $I_{\mathbb{K}}[x,y]$ of degree $n \geq 2$ in y and $f(x)$ a power series over $\mathbb{K}$ satisfying $F(x,f(x)) = 0$, $f(0) = 0$. For any $x_0 \in \mathbb{K}$ denote by $S(x_0)$ the subset of S formed by all ν with $\theta_\nu = (f(x_0))_\nu$ finite and set

$$\lambda_\nu = \frac{nk}{k_\nu} \frac{\log (x_0)_\nu}{\log H_{\mathbb{K}}(x_0)} - e_0 + \epsilon$$

where $e_0 = 1$ for infinite ν and $e_0 = 0$ for finite ν. Then the systems of inequalities

$$|P(\theta_\nu)|_\nu < \lceil P \rceil^{-\lambda_\nu}, \quad \nu \in S(x_0)$$

has only a finite number of solutions in $P(y) \in I_{K}[y]$, $\deg P \leq n-1$, if $\epsilon>0$ and $H_{\mathbb{K}}(x_0) > (1+\lceil F \rceil)^{c/\epsilon^2}$, $c = c(k, \deg F)$. The number of solutions is infinite if $\epsilon<0$.

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