MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 17/1984

Rekursionstheorie

15.4. bis 21.4.1984

Die Tagung fand unter der Leitung der Herren Prof. Dr. H.-D. Ebbinghaus (Freiburg), Prof. Dr. Gert Müller (Heidelberg) and Professor Gerald E. Sacks (Harvard Universität) statt. Im Mittelpunkt des Interesses standen Fragen aus allen Bereichen der Berechenbarkeitstheorie. An der Tagung haben 43 Wissenschaftler aus 11 Ländern teilgenommen. In 28 Vorträgen wurden neue Ergebnisse aus fast allen Teilbereichen der Rekursionstheorie vorgestellt, eingeschlossen Beiträge zur deskriptiven Mengenlehre und zu rekursionstheoretischen Aspekten der Beweistheorie. Schwerpunkte bildeten die Theorie der Unlösbarkeitsgrade, die Komplexitätstheorie und die verallgemeinerte Rekursionstheorie. Abgerundet wurde das Programm durch einen Abendvortrag von Prof. Sacks über zentrale offene Fragen in der Rekursionstheorie und eine von Prof. Jockusch geleitete 'Problem session'.
Die Veröffentlichung eines Tagungsbandes in der Reihe "Springer Lecture Notes in Mathematics" ist vorgesehen.

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Vortragsauszüge

K.AMBOS-SPIES: Polynomial-time degrees.

We present some new results on the algebraic structure of the polynomial time (many-one and Turing) degrees of recursive sets. Besides fairly general results about distributive sublattices of intervals of polynomial-time degrees, we distinguish the structure of the p-Turing degrees from that of the p-many-one degrees and exhibit nonisomorphic intervals in the latter structure. Moreover, we present some results on embeddings of nondistributive lattices and on infima and suprema of pairs of degrees. Finally we show that the elementary theory of the p-many-one degrees is not $\aleph_0$-categorical.

ANDREAS BLASS: The Kleene ordering of ultrafilters.

We consider non-principal ultrafilters over the set ω of natural numbers as type-two objects and ask when one of them is Kleene reducible to (i.e., recursive in a real and) another. This reducibility, $U \leq_K V$, clearly holds when U is the image of V under some function $f: \omega \to \omega$, (i.e. $U \leq V$ in the Rudin-Keisler ordering) and also when U is the limit with respect to V of a sequence of ultrafilters that are $\leq_K V$. I know of no instance of $\leq_K$ between ultrafilters that cannot be obtained by repeated application of these two facts and the transitivity of $\leq_K$. The conjecture that all instances of $\leq_K$ between ultrafilters are trivial in this sense takes a particularly simple form when V is selective, for then the only ultrafilters that are trivially $\leq_K V$ are those that are, up to isomorphism, obtainable from V by iterated summation, and they form a chain in the Rudin-Keisler ordering. The only case of the conjecture that has been proved is that in which both U and V are selective: Between selective ultrafilters, Kleene reducibility is equivalent to isomorphism. The proof depends on the following extension of a partition theorem of Mathias. If U and V are non-isomorphic selective ultrafilters, and if X is a Σ^1_1 family of infinite subsets of ω, then there exist $A \in U$ and $B \in V$ such that X contains all or none of the infinite subsets of ω of the form $\{a_0 < b_0 < a_1 < b_1 < \dots\}$ with all $a_i \in A$ and all $b_i \in B$.

W.BUCHHOLZ: An independence result for $(\Pi^1_1\text{-CA})+\text{BI}$.

It is shown that a certain combinatorial statement on finite trees (with labels $\leq\omega$) is independent of the subsystem $(\Pi^1_1\text{-CA})+\text{BI}$ of classical analysis. This result is an extension of an earlier independence result for Peano arithmetic by Kirby and Paris [Bull. London Math. Soc. 14 (1982), 285-293].

J.DILLER: An embedding of ID_α in $ID_\alpha(W)$.

We embed the theory of α-times iterated inductive definitions ID_α by Feferman's 1970 Π^1_1-translation in a second order system T_α not closed under arithmetic comprehension and containing bar-induction principles instead. T_α can be directly interpreted in the theory $ID_\alpha(W)$ of the hyperjump hierarchy $W_y = W^{W_{<y}}$ for $y<\alpha$, by relativization to RecW. Similar to Feferman 1982, this proves that ID_α is an extension by definitions of $ID_\alpha(W)$. This is work in co-operation with H.D.Wunderlich from Münster.

W.FELSCHER: A lemma on the extension of strategies.

Let S_o, S_1 be two disjoint sets the elements of which are called (even and odd) statements. Assume that every v in S_o determines a natural number $\|v\|$. Assume that every v in S_i, i=0,1, determines a finite subset A(v) of S_{1-i} and that every w in A(v) determines a finite subset A(v,w) of S_i; the elements of A(v) are called attacks on v and the elements of A(v,w) are called answers to the attack w. Consider a sequence $s_o, s_1, \ldots$ of statements, $s_{2i}\in S_o$, $s_{2i+1}\in S_1$, each of which, except s_o, is specified as being either an attack upon a previous one or an answer to a previously specified attack. Such a sequence is a liberal game provided the following holds: (1) If s_i is specified as answer to an attack s_j and if $i>k>j$, $k-j$ even, and s_k is also specified as an attack, then there exists h such that $i>h>k$ and s_h is specified as an answer to s_k. (2) There must be no two specified answers referring to the same attack. (3) A statement v in S_o may be specified as being attacked at most $\|v\|$ times. - The sequence is an illiberal game if, in addition to (1) - (3), the following holds: (4) s_{2i+1} must refer to s_{2i}. - A (liberal or illiberal) game is won if it is finite, ends at an even position, and cannot be continued without ceasing to be a liberal or illiberal game respectively.

Lemma: Every winning strategy for illiberal games can be extended to a winning strategy for liberal games.

S.D.FRIEDMAN: Forcing in recursion theory.

In this talk we indicate how to show that the α-degrees form an undecidable partial ordering for all admissible α. This result was established for any Σ_3 admissible α by Dorer, using the minimal degree construction of Shore. Thus his approach was based on the possibility of realizing finite distributive lattices as initial segments. Our approach is based on a coding method due to Slaman-Woodin which is a finite initial segment construction, rather than one which requires perfect trees. Conjecture Thy(α-Degrees) is recursively isomorphic to 2^{nd} Order Thy $\langle L_\alpha, \in \rangle$ (if V=L). We can verify this conjecture when α is an L-cardinal of uncountable cofinality.

R.O.GANDY: Partial recursive functional of finite type.

Let $\mathcal{P}$ be the collection of all hereditarily consistent (= hereditarily monotone) partial functions of finite type, with $\omega \cup \{\perp\}$ as the only ground type. Platek [66] and Kleene [78] have given schemes intended to characterize the notion of deterministic computability for $\mathcal{P}$; non-deterministic schemes such as "strong or" are excluded. Kleene has also outlined a semantics for the evaluation of terms built up by application of these schemes. Computation of the term proceeds by "call by name"; if the result is defined then all numerical expressions occuring in the computation get a definite value. Kleene characterized the functions defined by such computations as "unimonotone", but there are difficulties at types greater than two.

For the case where $\mathcal{P}$ is replaced by the collection $\mathcal{C}$ of hereditarily continuous monotone partial functions we suggest an alternative approach; it uses the notion of an interrogation to determine the values of numerical expressions (in place of Kleene's oracles). Details have been worked out by N. Garcia (1983) for a different set of schemes. It is hoped that our approach will provide a useful analysis of "call by name" procedures at higher types, and a clear characterization of the subclass $\mathfrak{C}'$ of $\mathfrak{C}$ which is closed under the Kleene-Platek schemes. $\mathfrak{C}'$ is the class of all functions which can be computed by finite deterministic, sequential, means.

N.Garcia, New foundations for recursion theory, D.Phil.Thesis, Oxford 1983.

S.C.Kleene, Recursive functionals and quantifiers of finite type revisited I, in Generalized Recursion Theory II (Ed. J.E.Fenstad, R.O.Gandy & G.Sacks) North Holland 1978.

_____, Recursive functionals and quantifiers of finite type revisited II, in The Kleene Symposium (Ed. J.Barwise, H.J.Keisler & K.Kunen) North Holland 1980.

R.Platek, Foundations of recursion theory, Ph.D.Thesis, Stanford 1966.

J.Y.GIRARD and D.NORMAN: Embeddability of ptykes.

Ptykes are hereditary functors over ordinals preserving direct limits and pull-backs. The paper investigates various aspects of the relation "there is an embedding from A to B": $I(A,B) \neq \emptyset$.

1^o) Weak morphisms: when $I(A,B) \neq \emptyset$, then $I(A,B)$ has a smallest element, but this does not yield a simple construction of this morphism. The theory of weak morphisms gives a simple inductive way of constructing this morphism (if it exists) or to recognize that $I(A,B) = \emptyset$. The use of this theory is to reduce non denumerable embeddability problems to denumerable ones.

2^o) Amalgamations: when (A_i, T_{ij}) is an inductive system, indexed by the opposite of a tree, then we show the existence of a point at infinity (the amalgamation) for the system. The process is sufficiently effective and functorial for our applications.

3^o) Functorial boundedness: when f is a "sufficiently" definable function from $|\sigma|$ to $|\tau|$, we show that f can be bounded (pointwise w.r.t. embeddability) by a recursive ptyx $\Phi \in |\sigma \to \tau|$. Two versions are given, according to the acception of "sufficiently": 1 f is obtained from a Σ^1_1 graph 2 f is set recursive; in this case weak morphisms are used to reduce the general case to the denumerable one.

4^o) T_A-technology: T_A is the tree of descending sequences of A. We show the existence of a recursive Φ such that for all A, $I(T_A,B)\neq\emptyset \to I(A,\Phi(B))\neq\emptyset$. This enables us to generalize Kechris & Woodin's theorem on Σ_1 sets of ptykes to Σ^1_R sets.

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E.R.GRIFFOR: Π^1_2-logic and definable uniformization (w/ J.Y.Girard).

We give proofs of the following results using the tools of Π^1_2-logic:

(A) (Martin-Solovay) If all sharps exist, then every non-empty $\Sigma^1_3(x)$ set has a $\Delta^1_4(x)$ element.

and

(B) (Martin) If all sharps exist and 0^+ does not exist, let $x \in {}^\omega\omega$ then every $\Sigma^1_3(x)$ set of reals can be written as an ω_1-union of Borel sets.

Both proofs generalize to $\Sigma^1_n(x)$ for $n > 3$ modulo the existence, respectively nonexistence of certain universal ptykes.

L.HARRINGTON: Infinite injury.

We present a format for construing infinite injury priority arguments as a series of constructions: one recursive in 0; one recursive in 0'; and one recursive in 0".(For certain extensions of the infinite injury method, there will also be a construction recursive in 0"', and sometimes beyond). So, we have the constructions on three levels: level 0: Δ^0_1; level 1: Δ^0_2; level 3: Δ^0_3.

Level 0 will be a finite injury construction where, via a limiting process, the requirements and their priority listing are built at level 1. Similarly the construction on level 1 (which includes the requirements for level 0) is built by a finite injury priority argument, and the requirements for this level 1 construction are built on level 2.

All known examples of priority arguments can apparently be developed using this format, and it is suggested that there are certain mathematical and pedagogical advantages for doing so.

P.G.HINMAN: Random computations and oracles.

We consider the complexity classes NP, R, BPP, and related classes obtained by relativization and dualization. Proofs of inclusions $\mathfrak{A} \subseteq \mathfrak{B}$ among such classes are usually relativizable and yield also $\forall A . \mathfrak{A}^A \subseteq \mathfrak{B}^A$, which we write $\mathcal{A} \rightarrow \mathcal{B}$. It is, of course, much easier to prove negative results $\mathcal{A} \not\rightarrow \mathcal{B}$ than to show $\mathcal{A} \nsubseteq \mathcal{B}$. We survey known positive and negative $\rightarrow$-relationships including several new ones. For example, if $\Delta := NP \cap$ co-NP, we have $R \not\rightarrow \Delta$ and $NP \not\rightarrow \Delta^{BPP}, BPP^{\Delta}$. Proofs are based on characterizations of classes using polynomial relations and the quantifiers $\exists$, $\forall$, and $\exists^{+}$, where $\exists^{+} |u| \leq n \,.\, A(x,u)$ means that for some ε, $0 < \varepsilon < 1$, independent of x,n, A(x,u) holds for at least $\varepsilon \cdot 2^{n+1}$ many of the strings $u \in \{0,1\}^{*}$ of length at most n.

S.HOMER: Minimal degrees for polynomial reducibilities.

A new polynomial time reducibility is defined. This reducibility, "honest" polynomial-time Turing reducibility, is a strengthening of reducibilities which are usually considered. It is shown that no recursive set is minimal with respect to this reducibility. On the other hand, if no set recursive in 0'' is minimal then $P \neq NP$. The methods used here are recursiontheoretic and are extensions of those used to construct minimal Turing degrees.

G.JÄGER: Γ_0 revisited.

Several theories for iterated admissible sets of strength Γ_0 were presented which share the feature of having fairly strong set existence axioms on the price of being weak with respect to induction principles available.
One typical example: Let KPU be Kripke-Platek set theory above the natural numbers as urelements. KPU^{0} is obtained from KPU by restricting complete induction on $\mathbb{N}$ to Δ_0 formulas and omitting $\in$-induction completely. Then $KPU^{0} + \forall x \exists y (x \in y$ & y is admissible) is the "strongest" theory of strength Γ_0.

A.S.KECHRIS: Examples of Π^1_1 sets and norms.

We present some new examples of Π^1_1 non-Borel sets, such as (1) The set of continuous functions with everywhere convergent Fourier series and (2) (Solovay) The set of closed sets of uniqueness. We also discuss natural Π^1_1-norms on on these and other examples (such as on the set of everywhere differentiable functions, due to H.Woodin and the author). In addition we explain some new methods for proving non-Borelness results. These also give simplified proofs for some older examples such as the class of nowhere differentiable functions or the Besicoritch functions (The original proofs of these were due to Mauldin). Other classifications of sets in the projective hierarchy are given, including Woodin's result that the set of continuous functions satisfying the Mean Value Theorem is Π^1_2 but not Σ^1_2, and those satisfying Rolle's Theorem is Σ^1_1 but not Borel. Finally we discuss an example of a natural Borel inseparable pair of Π^1_1 sets due to H.Becker.

A.KUCERA: Degrees of complete extensions of Peano arithmetic.

In many constructions in recursion theory the concept of 1-genericity plays the key role. E.g. constructions showing that degree $\underset{\sim}{0}'$ has the cupping property, that any degree below $\underset{\sim}{0}'$ is complemented etc. Studying structural properties of initial segments of the form $\mathcal{D}(\leq \underset{\sim}{a})$, where $\underset{\sim}{a}$ is a degree of a complete extension of PA,the situation is quite different. In fact, there is a degree of a complete extension of PA which does not bound any 1-generic degree. Thus constructions using a complete extension of PA as an oracle must, in general, use other techniques. One of the convenient tools is the use of Π^0_1-classes (of sets). As an example, the following theorem holds.

Theorem. Any degree of a complete extension of PA has the cupping property. Some other results and open questions are mentioned too. One of them is of special interest.

Theorem. There is a degree $\underset{\sim}{a}$ of a complete extension of PA and a degree $\underset{\sim}{b}$ such that 1) $\underset{\sim}{b} \neq \underset{\sim}{0}$ & $\underset{\sim}{b} < \underset{\sim}{a}$ and 2) for every degree $\underset{\sim}{c}$ containing a complete extension of PA and satisfying $\underset{\sim}{c} \leq \underset{\sim}{a}$, $\underset{\sim}{b} < \underset{\sim}{c}$ holds.

Question. Given $\underset{\sim}{a}$ and $\underset{\sim}{b}$ with the just mentioned properties, can $\underset{\sim}{b}$ be (nontrivially) cupped to $\underset{\sim}{a}$?

A.H.LACHLAN: Codes for countable, ω-stable, ω-categorical structures.

On the basis of recent progress in the study of the class $\mathcal{C}$ of countable ω-stable, ω-categorical structures, a notion of code for structures $\in \mathcal{C}$

is proposed. To have a code a structure $M \in \mathfrak{C}$ must have a finite language. In this case a code for M consists of a finite L-structure $N \subseteq_{hom} M$ and a fifnite sequence $\langle \varphi_i : 1 \leq i \leq n \rangle$ of pairs of L-formulas which characterize the isomorphism type of M over N.

Conjecture 1. Every $M \in \mathfrak{C}$ has a finite language.

Conjecture 2. Every $M \in \mathfrak{C}$ has a code.

Conjecture 3. The class $\mathfrak{C}$ is strongly recursively enumerable, i.e. there exists an r.e. set C of codes such that every $M \in \mathfrak{C}$ has a code in C.

W.MAASS: Computational complexity.

Development of techniques for proving quadratic lower bounds for deterministic and nondeterministic Turing machines. Use of Ramsey's Theorem for lower bound arguments.

G:ODIFREDDI: The structure of m-degrees.

We give an recursion theoretical proof of an algebraic characterization of the m-degrees (obtained by Ershov) and deduce answers for all the kinds of global questions that are usually asked for degree-theoretical structures.

W.POHLERS: Inaccessible cardinals and recursive ordinals.

Let K_o denote the class of principal ordinals and let Φ_o be its ordering function. Define K_{s+1} := "closure of the regular fixed points of Φ_s in the order topology of the ordinals" and $K_\lambda = \bigcap_{\xi<\lambda} K_\xi$ for limit λ and let Φ_s be the ordering function of K_s. Denote by κ_s the least ordinal in K_s and put Λ := least ordinal χ such that $\kappa_\chi = \chi$. We define an ordinal notation system which internalizes the ordinals below Λ and gives notations to a large segment of the recursive ordinals.

J.SHINODA: Admissible ordinals relative to absolute type two objects.

G.E.Sacks showed that every countable admissible ordinal $>\omega$ can be represented as the first admissible ordinal $>\omega$ relative to some real X. We shall give an extension of this theorem. We say that a normal type 2 object F on ω is absolute with parameters $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ if there exist a Σ_1-formula $\Phi(x,\vec{y})$ and a Π_1-formula $\Psi(x,\vec{y})$ such that for every admissible set M with $\vec{\xi} \in M$, M is F-admissible iff

$$(\forall f \in M \cap \omega^\omega) \; M \models (\Phi(f,\vec{\xi}) \leftrightarrow \Psi(f,\vec{\xi}))$$

and $F \cap M = \{ f \in M \cap \omega^\omega : M \models \Phi(f,\vec{\xi})\}$. If F is absolute,then a theorem similar to that of Sacks holds for F-admissible ordinals with some restrictions. As a corollary we can show:

Suppose $\xi <$ the first recursively Mahlo ordinal. Then every countable ξ-recursively inaccessible ordinal can be represented as the first ξ-recursively inaccessible ordinal relative to some real X.

S.G.SIMPSON: Recursion-theoretic aspects of the dual Ramsey theorem.

We begin by reviewing the contents of "A dual form of Ramsey's theorem", by T.J.Carlson and S.G.Simpson (to appear in Advances in Mathematics). Ramsey's theorem is concerned with subsets of the natural numbers; the dual form is concerned with partitions of the natural numbers. We also consider the generalization to A-partitions, where A is a finite alphabet. If we force with infinite recursive A-partitions, we obtain an initial segment of the Turing degrees which is isomorphic to the (nondistributive) lattice of partitions of A. This initial segment will be biimmune-free provided Carlson-Simpson Lemma 2.4 is true recursively. Is every finite lattice isomorphic to a biimmune-free initial segment of the Turing degrees?

T.A.SLAMAN and J.R.STEEL: Degree invariant constructions on the real numbers.

Definition. (i) A property P holds a.e. in D, the Turing degrees, if $\exists a \; \forall b \geq a \;\; P(b)$.

(ii) $F: \omega^\omega \to \omega^\omega$ is degree invariant (resp. order preserving) if $x \equiv_T y \to F(x) \equiv_T F(y)$ (resp. $x \leq_T y \to F(x) \leq_T F(y)$).

(iii) $f: D \to D$ is representable if $\exists F: \omega^\omega \to \omega^\omega$ so that $f(\deg(x)) = \deg(F(x))$ a.e.

Martin has conjectured: (ZF+AD) Suppose $f: D \to D$ is representable then

(I) If $f(d) \not\geq d$ a.e. then f is constant a.e.

(II) Say $f \geq_M g$ if $f(d) \geq g(d)$ a.e. $\leq_M$ wellorders the representable functions f so that $f(d) \geq d$ a.e.

On the other hand, Sacks has conjectured that there is a degree invariant solution to Post's Problem (which contradicts (II)).

Theorem 1. (ZF+AD) If $f: D \to D$ and $f(d) < d$ a.e. then f is constant a.e.

Theorem 2. (ZF+AD) If $f: D \to D$, $a \leq b \to f(a) \leq f(b)$ a.e. and $f(d) \geq d$ a.e. then either f(d) is greater than any degree hyperarithmetic in d a.e. or there is a countable ordinal α so that $f(d) = d^\alpha$ a.e.

Remark. In the above AD may be replaced by "f is Borel".

Theorem 3. There is a degree invariant solution to Post's problem for those reals of degree recursively enumerable in 0'.

R.I.SOARE: Finite automata and parallel computation in recursion theory.

The purpose of this work is to use concepts of computer science to explain and simplify constructions in recursion theory, especially the $\emptyset'$, $\emptyset''$, and $\emptyset'''$-priority arguments. One first designs a strategy to meet a single requirement in isolation. One designs a finite automaton (chip) to implement this strategy. Secondly one analyzes all possible outcomes $\Lambda = \{a_1, \dots, a_n\}$ of this chip including possible infinite cycles. An appropriate ordering $<_\Lambda$ is put on Λ. The second step is to construct the full machine by placing a copy of the basic chip at each node in the tree $T = \Lambda^{<\omega}$ of possible outcomes. This corresponds to the computer architecture. Finally, one specifies the operating system which specifies exactly how and when each chip acts and interacts with the others.

J.STERN: Measurability of (lightface) Σ^1_2 sets.

The measure problem is the following: how can one build non-measurable subsets of $\mathbb{R}$ and how complicated are these sets?

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From the point of view of set theory the problem has been solved by the work of SOLOVAY and SHELAH. If our base theory is ZF+DC then the measurability of all sets of reals and the existence of an inaccessible cardinal are equiconsistent statements.

Although no inaccessible cardinal is involved, the measurability of $\mathbf{\Sigma}^1_2$ sets is itself a strong statement; it implies that all $\mathbf{\Sigma}^1_2$ sets have the property of Baire. This result has no converse: given a model of ZFC, one can build a generic extension in which

i) $\mathbf{\Sigma}^1_2$ sets have the property of Baire

ii) $\mathbf{\Sigma}^1_2$ sets are Ramsey

iii) some Σ^1_2 set is not measurable.

S.S.WAINER: Subrecursive hierarchies.

The aim is to find an 'exact' way of assigning ordinals to computations - as a measure of their complexity.

The "slow-growing" hierarchy G collapses countable tree-ordinals α onto number-theoretic functions $G(\alpha)$ such that nicely structured α's are the direct limits of their corresponding $G(\alpha)$'s.

G collapses Kleene-style computations over tree-ordinals onto identical computations over integers, and this provides a method of assigning ordinals in a natural way. For example the ordinals assigned to "fast-growing" functions at levels $|ID_n|$, $n < \omega$ are $|ID_{n+1}|$, $n < \omega$ respectively.

G.WECHSUNG: Sparse complete sets.

The following known results show to what extent it is unlikely that NP or coNP have $\leq^P_m$-complete sparse sets:

(1) $P = NP \leftrightarrow$ there exists a $\leq^P_m$-complete sparse set in coNP (Fortune).

(2) $P = NP \leftrightarrow$ there exists a $\leq^P_m$-complete sparse set in NP (Mahaney).

Nondeterministic and random versions of polynomial time reducibility (denoted by $\leq_\gamma$ and $\leq_R$, resp.) have been studied by L.M.Adleman and K.Manders in 1979. Since $\leq^P_m \subseteq \leq_R \subseteq \leq_\gamma$ it could be possible that there are $\leq_R$-complete ($\leq_\gamma$ -complete) sets which are not $\leq^P_m$-complete ($\leq_R$-complete).

We give evidence for sparse $\leq_R$-complete ($\leq_\gamma$-complete) sets not to exist in NP and coNP:

(3) NP $\cup$ coNP = R $\cap$ coR $\Leftrightarrow$ there exists a $\leq_R$-complete sparse set in NP.

(4) NP = coNP $\Leftrightarrow$ there exists a $\leq_\gamma$-complete sparse set in NP.

We get also true statements by replacing NP with coNP on the right hand sides of statements (3) and (4).

H.WOODIN: Aspects of complementation.

Suppose $x \subseteq \omega$. Let σ^x denote the hyperjump of x, i.e. σ^x is the complete $\Sigma^1_1(x)$ subset of ω. Let HYP denote the set of hyperarithmetic subsets of ω.

Theorem 1. Suppose $x \subseteq \omega$ and $x \notin$ HYP. Then there exists $y \subseteq \omega$ such that σ^x, σ^y are each recursive in the pair $\langle x,y \rangle$. In fact y can be chosen so that σ^y is recursive in $\langle \sigma, y \rangle$.

The main theorem behind theorem 1 is a complementation result about Turing degrees where the join is taken as the usual Turing join, however the meet is strengthened to the hyperarithmetic meet.

There is a version of theorem 1 for constructible degrees.

Theorem 2. Suppose $x \subseteq \omega$ and $x \notin$ L. Assume $x^\#$ exists. Then there exists $y \subseteq \omega$ such that $x^\#$, $y^\#$ are each recursive in $\langle x,y \rangle$. As before y can be chosen so that $y^\#$ is recursive in $\langle y, 0^\# \rangle$.

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