MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1984

Theorie, Numerik und Anwendungen nichtlinearer Eigenwertprobleme

6.5. bis 12.5.1984

Die Tagung stand unter der Leitung von H.B. Keller (Pasadena), K. Schmitt (Salt Lake City) und H.-O. Peitgen (Bremen). Der Zielsetzung der Tagung entsprechend, wurden Ergebnisse aus der

- Theorie gewöhnlicher, partieller und zeitverzögerter Differentialgleichungen,
- Anwendungen aus der Hydrodynamik, Chemie, Biologie und Physik, und die
- Numerik nichtlinearer Eigenwert- und Verzweigungsprobleme

diskutiert.

J. ALEXANDER:

A Tale of Two Brusselators

The Brusselator is the system for $u = \binom{x}{y}$

$$\dot{u} = f(u) = \begin{pmatrix} A-(B+1)x+x^2y \\ Bx-x^2y \end{pmatrix}$$

where A, B > o are kinetic parameters. For fixed A, there is a "third" stationary point $x=1, y=B/A$. This point is stable for $B < A^2+1$. There is a Hopf bifurcation at $B=A^2+1$ and for $B > A^2+1$ there is a globally stable periodic limit cycle.

We consider two such Brusselator "cells" coupled by diffusion. That is we consider the 4 variable system of u_1, u_2 with

$$\dot{u}_i = f(u_i) + 2D(u_{i+1}-u_i) \qquad i=1,2 \bmod 2$$

where

$$D = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}$$

is a diffusion matrix.

We attempt to investigate the bifurcation diagram as A, B, D_1, D_2 are varied. The interesting parameters to vary are B, D_1. There are several types of orbits:

i) a new branch of asymmetric stationary points.

ii) synchronous orbits with the two cells oscillating identically in phase.

iii) antisynchronous orbits with both orbits oscillating identically, but $\frac{1}{2}$ out of phase.

iv) asynchronous periodic orbits.

v) possibly chaotic orbits.

The connections of the various branches and their bifurcations are investigated both analytically and numerically.

J. BEBERNES:

Miscellanea on $u_t - \Delta u = \lambda e^u$

The ignition model for a rigid explosive in a bounded container is: (I) $u_t - \Delta u = \lambda e^u$, $u(x,0) = 0$, $x \in \Omega$, and $u(x,t) = 0$, $x \in \partial\Omega$, $t > 0$. The Gelfand or associated steady state problem is:
(SS) $-\Delta u = \lambda e^u$, $u(x) = 0$ on $\partial\Omega$. Three related problems are discussed.
A) The small heat loss problem. This arises by considering the boundary layer affects of (I).
B) The development of the hot spot for (SS). In particular, we show that there exists $\bar{\lambda} < \lambda_{FK}$, the Frank-Kamenetski critical value, such that for $\lambda \in [\bar{\lambda}, \lambda_{FK}]$ all solutions of (SS) are bell-sheped.
C) L_1 blow-up. We compare thermal blow-up in the L_1 and L_∞-sense.

E. BOHL:

Numerical identification of an organizing center: The Gierer-Meinhard-model of Morphogenesis.

It is possible that Gierer -Meinhard-objects after their first differentiation fall back into an unstructured state before they develop into the state of the next level of differentiation. To understand this the steady states of the G-M-model are discussed. The bifurcation diagram with respect to growth and activator diffusion shows a singularity with unfolding pictures resembling those of the elliptic umbilic. The action of further chemical cell assemblages has been linked to various singularities.

St. CANTRELL:

Secondary bifurcation in the steady-state solutions to the Volterra-Lotka competition model with diffusion.

We discuss joint work (with our colleague Chris Cosner) on the system of equations

$$(1)\qquad \begin{aligned} -\Delta u &= u[a-u-cv] \\ -\Delta v &= v[d-eu-v] \end{aligned} \qquad \text{in } \Omega\,,\ \Omega\subset\mathbb{R}^n \text{ a smooth bounded domain}$$

$$u \equiv 0\,,\quad v\equiv 0 \qquad \text{on}\quad \partial\Omega$$

This system is usually interpreted as the steady-state system corresponding to the Volterra-Lotka competition model with diffusion. Nonnegative solutions (usually thought of as the only solutions with physical relevance) are sought with the aim of determining their stability. Some progress has been made on this problem by preceding investigators. Our work has been motivated by two of the principal preceding results. Namely, if c and e are arbitrary fixed positive constants and $a > \lambda_1$, where λ_1 is the first eigenvalue of the problem

$$(2)\qquad \begin{aligned} -\Delta w &= \lambda w \qquad &&\text{in } \Omega\,, \\ w &\equiv 0 &&\text{in } \partial\Omega \end{aligned}$$

coexistence states (both u and v positive in Ω) may be observed to emanate from extinction states (one of u and v identically zero) by varying the parameter d. Furthermore, other investigations have noted that unique coexistent states are stable. Our research advances understanding of the problem in several ways: first, we view the problem as being multiparameter in a and d and obtain estimates on the location of the secondary bifurcation curves, obtaining in case c and e and both less than 1, the sharpest necessary conditions to date for existence of coexistence states; second, we perform a bifurcation analysis which views (1) as a system, determining a formula for the initial parameter direction of secondary bifurcation; and third, we observe that more than one coexistence state is sometimes possible when $a \neq d$ if $e<1<c$ or $c<1<e$ (the result being known when $a=d$ and $c=e=1$).

P. CHOSSAT:

Bifurcation of rotating waves in the Couette-Taylor problem.

The Couette-Taylor problem concerns the description of states of fluid flow between two concentric rotating cylinders. Recent accurate experiments have shown a variety of patterns of wavy flows bifurcating either from the basic Couette flow or, as a secondary bifurcation, from the Taylor vortices, depending on the value of different physical parameters (angular velocities of the cylinders, aspect ratio ...). Bifurcation theory in the presence of symmetry provides a suitable frame for explaining these patterns, and has permitted to compute the possible solutions and to determine the conditions of their stability.

J. DESCLOUX:

On the rotating rod.

The stationary motion of a rotating rod clamped at one extremity, free at the other one is given by the equations

$$\begin{aligned} -v''(s) &= \lambda \sin\Theta(s) & & v'(0) = v(1) = 0 \\ & & 0 < s < 1 \,, & & , \\ -\Theta''(s) &= \lambda v(s)\cos\Theta(s) & & \Theta(0) = \Theta'(1) = 0 \end{aligned}$$

where λ is a parameter representing the angular velocity.

We study both from theoretical and numerical points of view the bifurcation diagram of this problem. In particular we show the global existence of one solution branch and obtain nodal properties for certain solutions.

We furthermore consider in connection with the Golubitsky-Schaeffer theory the perturbed problem obtained by replacing the boundary conditions $v'(0) = 0$, $\Theta(0) = 0$ by $v'(0) = \lambda\beta$, $\Theta(0) = \alpha$.

St. R. DUNBAR:

Traveling Waves in Diffusive Predator-Prey Equations: Periodic Orbits, and Point-to Periodic Heteroclinic Orbits.

We establish the existence of several related kinds of traveling wave solutions for reaction-diffusion systems based on a predator prey interaction model. In particular, we prove the existence of small amplitude periodic traveling wave solutions by means of the Hopf Bifurcation Theorem. We also prove that for high values of the wave speed, constant or periodic traveling wave solutions are the only possible traveling wave solutions satisfying natural amplitude restrictions, thereby excluding the existence of chaotic solutions. We also prove the existence of traveling wave front solutions connecting spatially homogeneous solutions. Finally, we prove the existence of traveling wave front solutions connecting a spatially homogeneous solution and a periodic solution.

B. FIEDLER:

Global Hopf bifurcation for Volterra integral equations.

An abstract bifurcation theorem for systems

$$x(t) = \int_0^\infty a_\lambda(s) f_\lambda(x(t-s))ds , \quad \lambda \in \mathbb{R} , \quad x \in \mathbb{R}^u$$

is presented. For the case of a unique stationary solution $x_o \equiv 0$, for example, the theorem asserts that any net change of stability of x_o between $\lambda = -\infty$ and $\lambda = +\infty$ leads to global bifurcation of periodic orbits from x_o . However, at present one has to exclude characteristic roots 0 - in particular steady state bifurcations. The theorem is applied to an epidemic model.

K. GEORG:

Error propagation for matrix modifications.

In many numerical methods (e.g. linear programming steps, Broyden update) one solves at each step a linear system with a matrix A, and in the next step the matrix $\tilde{A}$ is obtained by some simple modification of the form $\tilde{A} = A + (y-As)w^T$, $w^Ts = 1$. The idea is to use the work done on A also for $\tilde{A}$. As an example, consider a decomposition $AQ = L$, where Q is orthogonal and L lower triangular. Then $\tilde{A}\tilde{Q} = \tilde{L}$ is obtained in $O(N^2)$ arithmetic operations, see Gill-Golub-Murray-Saunders 1974. There are several ways to do this, e.g. (1) A is not used to obtain $\tilde{Q},\tilde{L}$ from Q,L (2) Q is not used to obtain $\tilde{L}$. In case (2), essentially 2/3 of the numerical effort of (1) is saved since Q is not updated. This latter method has been proposed by Gill-Murray 1973 for the simplex algorithm and by M. Todd 1980 for PL-algorithms. Error analysis: $(A+\Delta A)Q=L$, $(\tilde{A}+\Delta\tilde{A})\tilde{Q}=\tilde{L}$. The error in the next step comes from two sources: (a) new round off errors (b) propagation of previous errors. The latter can easily be calculated: $\Delta\tilde{A} = \Delta A(1-sw^T)$ in case (1), $\Delta\tilde{A} = \Delta A - (y-As)w^T\Delta A^T A^{-T}+O(||\Delta A||^2)$ in case (2). This shows: method (1) is "selfcorrecting" and method (2) is generally unstable. In fact, I can recommend it only in cases where one knows a priori that $||y-As||\mathrm{cond}(A) = O(1)$. Numerical tests confirm this error analysis.

M. GOLUBITSKY:

Hopf Bifurcation with Symmetry.

Using group theoretic techniques we obtain a generalization of the Hopf bifurcation theorem to differential equations with symmetry, analogous to a static bifurcation result of Cicogna. This is joint work with Ian Stewart.

As an application we study the bifurcation of fluid states in the Taylor problem describing fluid flow between two concentric counterrotating cylinders.

P. HESS:

Positive solutions of nonlinear periodic-parabolic eigenvalue problems

The linear periodic-parabolic eigenvalue problem

$$\begin{aligned} \frac{\partial u}{\partial t} + A(x,t,D)u &= \lambda m(x,t)u && \text{in} \quad \Omega \times \mathbb{R} \\ u &= 0 && \text{on} \quad \partial\Omega \times \mathbb{R} \\ u(\cdot,t) &= u(\cdot,t+T) && \text{on} \quad \overline{\Omega}, \forall t, \end{aligned}$$

with T-periodic, uniformly parabolic differential operator $L = \frac{\partial}{\partial t} + A(x,t,D)$ and given T-periodic (not necessarily positive) weight function m, is first investigated. It is shown that necessary and sufficient for the existence of a positive eigenvalue λ having a positive eigenfunction u is that $\int_0^T (\max_{x\in\overline{\Omega}} m(x,t))dt > 0$. Then applications to the nonlinear eigenvalue problem (a necessary and sufficient condition for bifurcation of positive solutions from the line of trivial solutions) and the construction of positive sub- and supersolutions are given.

H. HOFER:

Solutions of Hamiltonian systems having prescribed minimal period (report on a joint work with I. Ekeland, Paris)

Studying autonomous Hamiltonian systems $-J\dot{u} = H'(u)$ at least two problems are interesting concerning the existence of periodic solutions: Do there exist periodic solutions with a prescribed period T or with a prescribed energy E. Since the seminal work of Rabinowitz in 78 employing variational technique's a whole industry has studied these questions. However the most important and exciting questions are still unanswered today. A question raised in Rabinowitz's paper was if the T-periodic solutions found have actually minimal period T. The question was then raised in nearly all papers concerned with this subject but never satisfactorily answered. Up to now there are only a paper by Ambrosetti and Mancini and Clarke and Ekeland concerned with the minimal period problem. There is now an answer to that question for convex Hamiltonian systems: Always when the mountainpass theorem can be applied to the dual functional

$$u \to \int_0^T [\frac{1}{2} (Ju(t) , \int_0^t u(\tau)d\tau) + G(-Ju(t))]dt$$

(G the Fenichel transform of H) , then there exist periodic solutions having minimal period T .

J. IZE:

Bifurcation and Symmetry

The notion of equivariant extensions of maps is used in order to prove existence of global branches of bifurcating solutions for equations which are equivariant under the action of some compact Lie group.

In case of one parameter problems one is reduced to the classical degree criterium or fixed point subspaces of isotopy subgroups.

For more parameters the problem is more complicated but can be completely studied for S' (as in Hopf bifurcation) and S^3 actions. In this case one uses obstruction theory or projective spaces to obtain integers which will determine the existence of bifurcating branches.

A. D. JEPSON:

A Numerical Approach for the Analysis of Multi-Parameter Nonlinear Systems.

Nonlinear problems of the form $\underline{f}(\underline{x},\lambda,\underline{\alpha}) = \underline{0}$ are considered, where $\underline{x} \in \mathbb{R}^n$ is the state variable, $\lambda \in \mathbb{R}$ is a bifurcation parameter, $\underline{\alpha} \in \mathbb{R}^p$ is a vector of auxiliary parameters, and $\underline{f}$ is a sufficiently smooth function. A numerical approach is developed to calculate the regions in auxiliary parameter space for which the problem has qualitatively similar bifurcation diagrams. The approach is based on recent work in singularity theory, which is used to construct equations and inequalities characterizing various types of singular points for the above equation. Numerical results, given for a model of a stirred tank chemical reactor, illustrate the power of the approach.

B. KAWOHL:

Rearrangements.

Rearrangements are mappings which transform a given function u into a function u* with prescribed symmetries. The talk contains a survey of various kinds of rearrangements (Steiner-, Schwarz-, circular symmetrization) as well as some standard properties.

Main result is a discussion of the equality sign in the inequality

$$(*) \qquad \int_\Omega |\nabla u^*|^p \, dx \leq \int_\Omega |\nabla u|^p \, dx .$$

This question had been dismissed as hopeless in the book of Polya and Szegö (onp. 186). For $p > 1$ and for a dense subset of $W_0^{1,p}$ one can show that the equality sign in (*) implies u=u*. A counterexample shows that this result cannot be extended to all of $W_0^{1,p}$.

As an application one can obtain new information on the multiplicity and shape of nonrotationally invariant solutions to semilinear elliptic equations on annuli. This extends recent results of C.V. Coffman.

H. B. KELLER:

Complex Bifurcation from Simple Quadratic Folds.

In a general Banach space setting the nonlinear problem $G(u,\lambda) = 0$ for $G: B \times \mathbb{R} \to B$ has a real solution path $\Gamma: \{u(s),\lambda(s)\}$. A point $[u(s_o),\lambda(s_o)] \equiv [u_o,\lambda_o]$ is said to be a:

FOLD POINT iff: i) $\eta(G_u^o) \neq 0$,

ii) $G_\lambda^o \notin R(G_u^o)$;

SIMPLE FOLD POINT iff: in addition

iii) $\dim \eta(G_u^o) = \operatorname{codim} R(G_u^o) = 1$;

QUADRATIC SIMPLE FOLD POINT iff: in addition

iv) $\psi G_{uu}^o \phi\phi \neq 0$.

Here: $\eta(G_u^o) = \operatorname{span}\{\phi\}$, $\eta((G_u^o)^*) = \operatorname{span}\{\psi\}$ and $G_u^o \equiv G_u(u_o,\lambda_o)$, etc.

THEOREM. Let $G(u,\lambda)$ be analytic in u at (u_o,λ_o). Let the real solution path $\Gamma_{\mathbb{R}}: \{u(s),\lambda(s)\}$ contain a quadratic simple fold at (u_o,λ_o). Then the COMPLEXIFIED problem:

$$G(u+iv,\lambda) = 0 \; ; \quad G: (B+iB) \times \mathbb{R} \to (B+iB)$$

has a bifurcation at $(u,v,\lambda) = (u_o,0,\lambda)$. A complex branch $\Gamma_{\mathbb{C}}$ bifurcates "orthogonally" from $\Gamma_{\mathbb{R}}$ and turns in the "opposite" direction.
[i.e. $\Gamma_{\mathbb{C}}$ exists for $\lambda > \lambda_o (< \lambda_o)$ if the fold on $\Gamma_{\mathbb{R}}$ opens left (right)].

H. KIELHÖFER:

Multiple Eigenvalue Bifurcation for Fredholm Operators.

For parameterdependent equations

$$G(\lambda,u) = 0 \ , \quad G: \mathbb{R} \times D \to E \ ,$$

where $D \subset E$ are real Banachspaces, we assume the trivial solutions

$$G(\lambda,0) = 0 \ , \quad \text{for} \quad \lambda \in \mathbb{R} \ .$$

The bifurcation of nontrivial solutions at $(\lambda_o,0)$ where $G_u(\lambda_o,0) = A(\lambda_o)$ is singular is linked to the eigenvalue perturbation of $A(\lambda) = G_u(\lambda,0)$ near the critical eigenvalue zero. It turns out that all results of "linearized bifurcation theory" (imposing only conditions on the Fredholm operators $A(\lambda)$) can be embedded into a single theorem: A sufficient condition for local bifurcation is fulfilled if an odd number of critical eigenvalues leave or enter the negative real axis through zero ("odd crossing number"). If in addition G is proper and if $G_u(\lambda,u)$ has only finitely many eigenvalues on the negative real axis then the bifurcating branch exists globally. This result is achieved by introducing the degree for nonlinear Fredholm operators in a simple and natural way.

T. KÜPPER:

Nodal properties of nonlinear Poisson equations.

By variational methods it has been shown that there are infinitely many radialsymmetric solutions for the nonlinear Poisson equation (or generalizations of it)

$$\Delta u + |u|^{\sigma} u + \lambda u = 0$$

$$u \in L^2(\mathbb{R}^n)$$

if $0 < \sigma < 4/(n-2)$ and that there are no solutions if $\sigma > 4/(n-2)$. It has long been suspected that these solutions are ordered by the number of zeros in the radial variable, but a proof was only known for the special case $n = 3$.

In a joint work with C. Jones it is shown by a shooting argument (which is also helpful for numerical calculations) that for each $M \in \mathbb{N}$ there exists a solution with exactly M zeros. In addition this approach provides a geometric explanation for the failure of existence at the critical exponent $\sigma = 4/(n-2)$. This result also gives a classification for the infinitely many "branches" bifurcating at the lowest point of the continuous spectrum of the linearized equation by nodal properties in a similar way as is known for problems on bounded domains.

J. MAWHIN:

<u>The forced pendulum equation with periodic boundary conditions: closed range and open problems</u>.

Some recent work of the author, Willem and Fournier is surveyed about the structure of the set $\mathcal{R}$ of $e \in L^1(0,T)$ such that the periodic problem for the forced pendulum equation

$$(1) \qquad \begin{aligned} & u''(t) + cu'(t) + A \sin u(t) = e(t) = \bar{e} + \tilde{e}(t) \\ & u(0) - u(T) = u'(0) - u'(T) = 0 \end{aligned}$$

with $\bar{e} = (1/T) \int_0^T e(t)dt$, $A > 0$ and $c \in R$, has at least one solution.

By the use of upper and lower solutions techniques, it is shown that $\mathcal{R}$ is closed and that for each $\tilde{e}$ there exists $\gamma_1 = \gamma_1(\tilde{e}) < \gamma_2 = \gamma_2(\tilde{e})$ such that (1) has a solution if and only if $\gamma_1 \leq \bar{e} \leq \gamma_2$, and has at least two distinct solutions when $\gamma_1 < \bar{e} < \gamma_2$. If $\gamma_1 = \gamma_2$, (1) has infinitely many solutions but it is unknown if such a $\tilde{e}$ exists.

When $c = 0$, variational techniques show that $\gamma_1 < 0 < \gamma_2$ and that the set of $\tilde{e}$ such that $\gamma_1 < 0 < \gamma_2$ is dense in the L^1-norm. Special results in this direction when $c \neq 0$ are also discussed.

R. D. NUSSBAUM:

Global continuation and complicated trajectories for periodic solutions of a differential-delay equation.

This talk represented a report on some aspects of joint work with John Mallet-Paret on the equation

$$(1)_{\varepsilon} \quad \varepsilon\dot{x}(t) = \varepsilon\frac{dx}{dt} = -x(t) + f(x(t-1)), \quad \varepsilon > 0,$$

or equivalently $(\lambda = \varepsilon^{-1})$

$$(2)_{\lambda} \quad \dot{x}(t) = -\lambda x(t) + \lambda f(x(t-1)), \qquad \lambda > 0.$$

Assume that there exist positive numbers A,B such that $f([-B,A]) \subset [-B,A]$, $f|[-B,A]$ is C^2, $xf(x) < 0$ for all $x \in [-B,A] - \{0\}$ and $f'(0) = -k < -1$. Define a periodic solution $x(t)$ of eq. $(2)_{\lambda}$ to be a "slowly oscillating periodic solution" if there exist numbers $q > 1$ and $\bar{q} > q + 1$ such that $x(0) = 0$, $x(t) > 0$ for $0 < t < q$, $x(t) < 0$ for $q < t < \bar{q}$ and $x(t+\bar{q}) = x(t)$ for all t. We prove that equation $(2)_{\lambda}$ has a global continuum of such periodic solutions bifurcating from 0 at some value $\lambda_0 > 0$. We are interested in the behaviour (particularly "boundary layer" behaviour) of such solutions of $(1)_{\varepsilon}$ as $\varepsilon \to 0^+$. Space allows me to mention only one such theorem. <u>Theorem</u>: If f is as above, there exists a constant C such that the minimal period p of any slowly oscillating periodic solution $x(t)$ of $(1)_{\varepsilon}$ satisfies $p < 2 + C\varepsilon$ (C independent of ε).

H. OTHMER:

Bifurcation Phenomena in Coupled Oscillatores.

We present results of joint work with D. Aronson and E. Doedel on the bifurcation in a two-parameter family of coupled oscillators. The governing equation are

$$\frac{dx_i}{dt} = x_i + \beta y_i - x_i(x_i^2+y_i^2) + \delta(x_{i+1}-x_i+y_{i+1}-y_i)$$

$$\frac{dy_i}{dt} = -\beta x_i + y_i - y_i(x_i^2+y_i^2) + \delta(x_{i+1}-x_i+y_{i+1}-y_i) \qquad i = 1,2 \ (\mathrm{mod}\ 2)$$

When $\delta = 0$ there is an invariant torus in R^4 that is covered by periodic solutions, and this torus persists under weak coupling. The method of averaging is used to show that the only periodic solutions that persist are those which correspond to a phase difference of 0 or π radians between the oscillators. The former is uninteresting in that it exists for a $\delta > 0$, is asymptotically stable in R^4, and is globally stable with respect to the invariant manifold defined by $\{(x_i,y_i)\,|\,x_1=x_2, y_1=y_2, (x_i,y_i) \neq (0,0)\}$. The latter disappears via a double heteroclinic bifurcation along $\beta = 2\delta$ for $\beta \in (0,1)$. Near $(\beta,\delta) = (1/2, 1/4)$ there is an infinite sequence of bifurcations from this periodic solution that accumulates at $(1/2, 1/4)$.

M. REEKEN:

Asymmetrically suspended rotating strings.

Rotating elastic strings which are suspended from points on the axis of rotation give rise to continua of solutions parametrized by the angular velocity and nodal properties. If suspended from points not in a plane through the axis of rotation nodal properties are destroyed. They are replaced by a topological invariant which has a geometrical interpretation. These continua emanate from limit configurations of infinite extension. The non-elliptic character of the underlying equations gives rise to a wealth of exotic solutions all forming analogous branches.

K. Rybakowski:

Homotopy index and differential equations.

An introduction to Conley's homotopy index theory is given. Starting with Wazewski Principle, we define isolated invariant sets and isolating blocks and show how the latter are used to define the index. The homotopy index has the important homotopy invariance property which make it a useful tool in perturbation problems. This is illustrated by a simple application to finding equilibria of ODEs.
Finally an extension (due to this author) of Conley's theory to infinite dimensional systems and some applications to parabolic equations are mentioned.

J. SCHEURLE:

Successive Bifurcations and Unfolding of Singularities.

There are various types of sequences of successive bifurcations known which the state of a dynamical system might undergo when parameters are varied. Although these are global phenomena in general, to some extent it is possible to understand them from a more local point of view, namely by unfolding a singularity.

As an example a two-parameter unfolding of a smooth vector field in $\mathbb{R}^3$ is considered which has a singular point at the origin generated by a zero and a pair of purely imaginary eigenvalues. Explicitely computed non-degeneracy conditions are given which guarantee the occurrence of three successive bifurcations along generic paths through parameter space: trivial equilibrium $\rightarrow$ primary branch of non-trivial equilibria $\rightarrow$ secondary branch of periodic orbits $\rightarrow$ tertiary branch of invariant two-tori. In particular, it turns out that there are continuous paths along which all the bifurcating tori carry quasiperiodic flow. But a generic path will meet resonance zones (Arnol'd tongues) of periodic tori in between points of quasi periodic tori. Explicite exchange of stability results are given too.

M. STRUWE:

Variational problems without compactness.

For the nonlinear eigenvalue problem on $\Omega \subset\subset \mathbb{R}^n$, $n > 2$, with $\lambda \in \mathbb{R}$, $2^* = \frac{2n}{n-2}$

$$(1) \quad -\Delta u - \lambda u = u|u|^{2^*-2} \quad \text{in } \Omega \ , \ u_{|\partial\Omega} = 0$$

the classical Palais-Smale compactness condition does not hold globally. However, it may be shown that the compactness properties of (1) are determined by the spectrum of "energies" of solutions to the "limiting problem" associated with (1)

$$(2) \quad -\Delta u = u|u|^{2^*-2} \quad \text{in } \mathbb{R}^n \ , \ u(x) \rightarrow 0 \ (|x| \rightarrow \infty) \ .$$

As an application, existence results "in the large" for problem (1) may be formulated, extending recent results of Brezis and Nirenberg. The method extends e.g. to the Dirichlet and Plateau problems for surfaces of constant mean curvature.

H.-O. WALTHER:

Bifurcation of periodic solutions of the second kind in differential delay equations.

Consider eq.

$$(af) \qquad \dot{x}(t) = af(x(t-1))$$

with a function $f: \mathbb{R} \to \mathbb{R}$ having period $B - A > 0$, $f(A) = 0 = f(0) = f(B)$, f positive on $(A,0)$ and negative on $(0,B)$. This represents a state variable on a circle, with one rest point and with a delay in the response to deviations. The parameter $a > 0$ may be transformed into the delay.

In 1978, T. Furumochi proved existence of periodic solutions of the 2nd kind:

$$x(t) \equiv x(t+p) - (B-A), \quad \text{with} \quad p > 0,$$

for certain nonlinearities f and certain parameters a. This means that the state variable on the circle performs a periodic rotation. Numerical experiments suggested that these movements are stable and attractive.

We show how such solutions arise: For a suitable class of functions f there exists a parameter a_0 so that there is a heteroclinic solution of $(a_0 f)$ with $x(t) \to A$ as $t \to -\infty$, $x(t) \to B$ as $t \to +\infty$. For $a > a_0$ (and close to a_0) there is no connection from A to B. There exists a sequence $a_n \searrow a_0$ with periodic solutions of the second kind of eq. $(a_n f)$, and these converge to the heteroclinic orbit as $n \to +\infty$.

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