MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1984

Gruppentheorie

20.5. bis 26.5.1984

Die Tagung fand unter der Leitung der Herren K. W. Gruenberg (London) und O. H. Kegel (Freiburg) statt. Es wurden Fragen aus den verschiedensten Teilbereichen der Gruppentheorie diskutiert. Schwerpunkte bildeten hierbei vor allem Ergebnisse zur Struktur linearer Gruppen über Schiefkörpern sowie Beispiele und Resultate über endliche und unendliche p-Gruppen. Insgesamt wurden 34 Vorträge gehalten, wobei neben den oben genannten Schwerpunkten Ergebnisse aus der Blocktheorie, Kohomologietheorie und Darstellungstheorie ebenso zur Sprache kamen wie Überlegungen zur Struktur von Permutationsgruppen und zur Subnormalteilerstruktur in Gruppen.

Vortragsauszüge

N. Blackburn

On a manuscript of P. Hall

The manuscript, entitled 'Semi-regular p-groups' was discussed in the light of present day knowledge of the power-structure of p-groups. The chief point which had not emerged in the meantime was the sufficiency of considering 'admissible' sections G_1/G_2, in the sense that G_1, G_2 are $\mathfrak{P}$-invariant for some $\mathfrak{P} \in S_p(\mathrm{Aut}\ G)$.

A. Brandis

Über die Grade irreduzibler Darstellungen endlicher Gruppen

Eine Klasse Λ von endlichen Gruppen, die gegenüber Isomorphie abgeschlossen ist, heiße p-Klasse, wenn p eine Primzahl ist und:

(i) $E \in \Lambda \quad (|E| = 1)$

(ii) $X/Y_1 \in \Lambda,\ X/Y_2 \in \Lambda \Rightarrow X/Y_1 \cap Y_2 \in \Lambda$

(iii) $X \in \Lambda \Rightarrow X/O_{p'}(X) \in \Lambda$

(iv) $X/Y \in \Lambda,\ Y \trianglelefteq O_p(X) \Rightarrow X \in \Lambda$

Ist Λ eine p-Klasse, so heiße $\tilde{\Lambda}$ der p'-Abschluß von Λ, wenn

$$X \in \tilde{\Lambda} \Leftrightarrow X/O_{p'}(X) \in \Lambda.$$

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Beispiele: a) $\Lambda = \Pi$ die Klasse der p-Gruppen, $\tilde{\Lambda}$ die Klasse der p-nilpotenten Gruppen. b) $\Lambda = \tilde{\Lambda}$ die Klasse der p-auflösbaren Gruppen.
Es wird folgender Satz bewiesen:
Ist $\tilde{\Lambda}$ der p'-Abschluß einer p-Klasse Λ, G eine endliche Gruppe, $G \notin \Lambda'$.
Dann gibt es einen irreduziblen Charakter χ von G, so daß $G/\mathrm{Ker}\,\chi \notin \tilde{\Lambda}$ und $p \nmid \chi(1)$.
Anwendung: Sei G eine endliche Gruppe, p eine Primzahl, $p \mid \chi(1)$ für alle irreduziblen Charaktere χ, für die $G/\mathrm{Ker}\,\chi$ nicht auflösbar ist, so ist G p-auflösbar. (Einzelheiten siehe im J. of Algebra)

R. Brandl

On the commutator map

Let $\mathfrak{E}$ be the class of all hyper- (abelian or finite) groups G such that for all $x,y \in G$ there exist positive integers $r = r(x,y)$ and $d = d(x,y)$ (always assumed to be chosen minimal) with $[x,_r y] = [x,_{r+d} y]$ and let $\mathfrak{E}_d$ (resp. ϑ_r) be the class of all $\mathfrak{E}$-groups G satisfying $d(x,y) \mid d$ (resp. $r(x,y) \leq r$) for all $x,y \in G$.
Theorem 1. a) Let G be hyper- (abelian or finite). Then $G \in \mathfrak{E}$ if and only if G is locally (finite-by-nilpotent),
b) $\mathfrak{E}_1 = \mathfrak{E}_2$ and every group in $\mathfrak{E}_2$ is locally supersoluble.
Theorem 2. Let G be a finite group in V_2. Then $G/F(G)$ is supersoluble, metabelian and has abelian Sylow p-subgroups for p odd. Also, $\ell_p(G) \leq 1$ for $p \neq 2$ and $\ell_2(G^2) \leq 1$.
For $x \in G$ let $\Gamma(x) = (G,I)$ be the directed graph with incidence defined by $(a,b) \in I$ iff $[a,x] = b$. Let $\zeta(x)$ be the number of connected components of $\Gamma(x)$ and let $\zeta(G) = \sup\{\zeta(x) \mid x \in G\}$.
Theorem 3. Assume that G is finite.
a) If $\zeta(G) \leq 6$ then G is soluble,
b) If $\zeta(G) \leq 4$ then G is nilpotent-by-abelian.

C. J. B. Brookes

Locally nilpotent groups

Let G be a torsion-free locally nilpotent group and B be a subgroup. The property 'all abelian subgroups of G of infinite rank intersect B non-trivially' will be considered for various choices of B. If B itself is trivial then this is the well-documented property that all abelian subgroups are of finite rank.
The motivation for this study is some recent work on primitive group rings. For example let G be countable and nilpotent and k be a field.

Then kG is primitive if and only if k is countable, G is torsion-free and all abelian subgroups A such that $A \cap Z = 1$ (where Z is the centre of G) are of finite rank. Perhaps more interesting is that this is related to whether the partial quotient ring $kG(kZ)^{-1}$ is Noetherian or not. For example if G is the free nilpotent group of class 2 on a countably infinite generating set then $kG(kZ)^{-1}$ is Noetherian, despite first appearances that it looks like a polynomial ring in infinitely many variables, a far from-Noetherian ring.

M. Broué

Local Theory for Blocks

The talk is a survey of methods and results issued from the local-theoretic point-of-view of Block Theory of Finite Groups, which was born in Alperin-Broué, Ann. of Math. 110 (143 - 157).

Let G be a finite group and let p be a prime. We denote by P, Q p-subgroups of G. A <u>subpair</u> of G is a pair (P,e) where e is a p-block of $C_G(P)$. There is a natural notion of containment between subpairs, which is the closure of a normal containment relation, and has the following property: Given P,Q such that $P \subset Q$, and given f block of $C_G(Q)$, there exists one and only one block e of $C_G(P)$ such that $(P,e) \subset (Q,f)$. Let b be a block of G; a <u>b-subpair</u> is a subpair (P,e) such that $(1,b) \subset (P,e)$. Now the so-called first and third Brauer's main theorems may be reformulated as follows:

1) (Sylow theorems) The maximal b-subpairs are conjugate under G. Moreover, a subpair (P,e) is maximal if and only if
   (i) it is maximal as a subpair of $PC_G(P)$,
   (ii) $N_G(P,e)/PC_G(P)$ is a p'-group.

2) Let $b_0(G)$ be the principal block of G. Then the map $P \mapsto (P, b_0(C_G(P)))$ is a bijection from the set of p-subgroups of G onto the set of $b_0(G)$-subpairs, which preserves inclusion and conjugation. Thus the theory of b-subpairs generalizes the ordinary "p-local theory" of finite groups, got as the particular case $b = b_0(G)$.

A number of results have been got along this line, as generalization to block theory of results from ordinary group theory. Some of them are presented: A Frobenius theorem (Broué-Puig, Inventiones Math. 56, 117 - 128) which generalizes the celebrated theorem of Frobenius (1907) on p-nilpotent groups, the structure of block algebras of p-solvable groups, Alperin's theorem on fusion, ...

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An application of this point of view is given to a new proof and a new light on Nakayama's conjectures on blocks of the symmetric group, giving Brauer's theorem as well as the complete local structure of blocks (this is a work of Marechal and Puig).

R. M. Bryant

Algebraic groups of automorphisms

Let k be a field of characteristic 0 and let K be an algebraically closed extension field of k. Let H be an algebraic subgroup of GL(n,K) defined over k, where $n \geq 2$. Then there is an n-generator nilpotent Lie algebra L over k such that, for every field k, with $k \subseteq k_1 \subseteq K$, the group of automorphisms induced by $\mathrm{Aut}(L \otimes_k k_1)$ on the derived factor algebra of $L \otimes_k k_1$ is (using a suitable basis) equal to $H \cap GL(n,k_1)$. Furthermore, L may be taken to have derived length at most 4. In the case where $k = \mathbb{Q}$ and $K = \mathbb{C}$ it follows that there is an n-generator torsion-free nilpotent group G, where G/G' is torsion-free, such that the group of automorphisms induced by Aut(G) on G/G' is (with respect to some basis) a subgroup of finite index in $H \cap GL(n,\mathbb{Z})$. Furthermore, G may be taken to have derived length at most 4.

This is joint work with J. R. J. Groves. It extends work of F. Grunewald and D. Segal, and answers a question raised by them.

A. Camina

Block-Designs with Block transitive Automorphism groups

Let D be a 2 - (v,k,λ) design. That is a incidence structure with Ω as the set of points, B the set of blocks $|\Omega| = v$, $|B| = b$ and natural numbers r,k and λ, $k > 2$, $\lambda \geq 1$ such that each block is incident with k points, each point is incident with r blocks and two distinct points are incident with λ blocks.

We say that the pair (D,G) where D is a 2 - (v,k,λ) design and $G \leq \mathrm{Aut}(D)$ satisfies (A) if G is block transitive. For an arbitrary permutation group X define rank (X) as (π,π) where π is the permutation character.

Theorem 1 Let (D,G) satisfy (A) and let B be a block. G_B acts as a permutation group on the points incident with B. Let this action have rank m and t orbits. Then G has point-rank $\leq m - t + 1$.

Cor Let (D,G) satisfy (A). Then G has point-rank $\leq k^2 - k + 1$.

Theorem 2 Let (D,G) satisfy (A) and assume (1) G acts faithfully on points and (2) G has point-rank $k^2 - k + 1$. Then G has odd order, λ is odd and $(k^2 - k) \mid (v - 1)$. Further one of the following holds:

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(i) G acts as a regular group on both blocks and points, D is a projective design and $v = k^2 - k + 1$,

(ii) G is a Frobenius group and $\lambda = 1$, and

(iii) $G \leq A\Gamma L(1,p^a)$ for some prime power p^a.

Cor For fixed k and λ there exists a pair (D,G) satisfying (A) and G having point-rank $k^2 - k + 1$ implies that if $v > k^2(k - 1)^2$ then v is a prime power.

I. M. Chiswell

Right-Angled Coxeter Groups

A right-angled Coxeter group is a group G with a presentation $\langle x_1,\dots,x_n \mid x_i^2 = 1\ (1 \leq i \leq n),\ (x_i x_j)^2 = 1,\ (i,j) \in I\rangle$ for some subset I of $\{1,2,\dots,n\} \times \{1,2,\dots,n\}$. Either G is a direct product of cyclic groups of order 2, or else G has a decomposition $G = A *_B C$, where A,B and C are right-angled Coxeter groups with less than n generators. The class of accessible groups is defined recursively by the conditions (1) the trivial group is accessible, (2) $U *_W V$ is accessible if U,V and W are, (3) the HNN-extension $\langle t,U \mid tVt^{-1} = W\rangle$ is accessible if U,V and W are. Using the above decomposition one can give a simple proof that G is virtually accessible, a result of B. Levy. By similar arguments one can classify the finite subgroups of G.

M. J. Collins

Some subgroups of M_{24}

M_{24} has nine conjugacy classes of maximal subgroups.

Theorem There exist maximal subgroups L,M of M_{24}, isomorphic to $L_2(23)$ and $L_2(7)$ respectively, such that $L \cap M \cong S_4$.

A *regular* S_4 in M_{24} is a subgroup isomorphic to S_4, in which all non-identity elements are fixed-point-free. Class algebra constant calculations determine the number of regular S_4', inside M_{24}, and a group theoretical analysis forces the desired intersection.

Related general questions were discussed.

M. Curzio

Groups with a permutation property

Let G be a group and n an integer ≥ 2. We say that G has the n-permutation property $(G \in \mathfrak{P}_n)$ if, for any elements $x_1,x_2,\dots,x_n$ in G, there exists some permutation σ of $\{1,2,\dots,n\}$, $\sigma \neq$ id., such that

$$x_1x_2\cdots x_n = x_{\sigma(1)}x_{\sigma(2)}\cdots x_{\sigma(n)}.$$

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A finite group of order n belows to $\mathcal{P}_n$ and an abelian group has trivially the 2-permutation property; furthermore the groups $G \in \mathcal{P}_3$ are exactly the groups with derived subgroup of order at most 2.
A finitely generated group has the n-permutation property (for some n) if, and only if, it is abelian by finite, but there are groups $G \in \mathcal{P}_n$ which are not abelian by finite.
The main result is that a group belows to $\mathcal{P}_n$ (for some n) if, and only if, it is finite by abelian by finite, and consequently, if a torsion-free group $G \in \mathcal{P}_n$ (for some n), then it is abelian by finite.

R. Geoghegan

Infinite-dimensional finitely presented groups

Let T be the group of piecewise linear homeomorphisms of the circle, S^1, which have power-of-two slopes and dyadic rational singularities. R. J. Thompson showed that T is finitely presented, infinite and simple. Let F be the subgroup of T fixing $1 \in S^1$. F is finitely presented, infinite, torsion free and an iterated HNN extension over itself. F and T have infinite cohomological dimension. Recently, Ken Brown and I have built a free $\mathbb{Z}F$ resolution having two generators in every dimension, implying that F and T are of type FP_∞. We also obtained a general result of independent interest about the Mayer-Vietoris sequence of an HNN extension, which implies $H^*(F,\mathbb{Z}F) = 0 = H^*(T,\mathbb{Z}T)$. There are other recent results and conjectures about these and related infinite-dimensional finitely presented groups.

K. W. Gruenberg

Minimal free resolutions over finite simple groups

Let G be a finite group, A a $\mathbb{Z}G$-lattice and

$$\begin{tikzcd}[column sep=small] (\mathcal{P}C): \cdots \arrow[r] & \mathcal{P}_i \arrow[rr] \arrow[dr] & & \mathcal{P}_{i-1} \arrow[r] & \cdots \arrow[r] & \mathcal{P}_o \arrow[r] & A(=C_o) \arrow[r] & 0 \\ & & C_i \arrow[ur] & & & & & \end{tikzcd}$$

be a projective resolution of A. This is called minimal if C_i has no projective summand, all $i > 0$. A free resolution (E,D) of A is minimal if $d_G(E_i) = d_G(D_i)$, all $i \geq 0$.
The lecture led up to the following theorem: If G is a non-abelian simple group and A is a non-projective direct summand of C_r for some $r \geq 0$ and some $(\mathcal{P},C)$ of $\mathbb{Z} \oplus \mathbb{Z}G^{(m)}$, $m \geq 0$, then every minimal free resolution of A is automatically minimal projective.
Some indication of the proof was given.

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C. K. Gupta

Dimension subgroups of free centre-by-metabelian groups

Let F be a free group, arbitrary rank, $\mathbb{Z}F$ its integral group ring, $\underline{F}$ the augmentation ideal of F (generated by all 1 - f, f $\in$ F) and $\underline{r}$ the augmentation ideal of [F",F] in $\mathbb{Z}F$. We show that $F \cap (1 + \underline{\underline{r}} + \underline{\underline{F}}^{c}) = [F",F] \cdot \gamma_{c}(F)$ for any $c \geq 2$, where γ_{c} is the c-th term of the lower central series of F, which gives a positive solution to the dimension subgroup problem for free centre-by-metabelian groups. The proof uses the fact that $[F",F] \leq 1 + \underline{F}\,\underline{\mathfrak{a}}\,\underline{F}$, where $\underline{\mathfrak{a}}$ is the augmentation ideal of F' in $\mathbb{Z}F$ (Hurley, 1972), and as a preliminary result we also determine $F \cap (1 + \underline{F}\,\underline{\mathfrak{a}}\,\underline{F} + \underline{F}^{c})$ for all $c \geq 2$. (The corresponding subgroups $F \cap (1 + \underline{F}\,\underline{\mathfrak{a}} + \underline{F}^{c})$ have been determined by N. D. Gupta (1983)).

This is a joint work with F. Levin.

N. Gupta

Recursively presented infinite p-groups on two generators

For every prime p, a group with the title properties is constructed and some of its features discussed. The groups are similar to, but not the same as, the ones constructed (jointly with Said Sidki) using tree automorphisms.

B. Hartley

Finite skew linear groups

Some recent progress in this area will be discussed.

Theorem: If D is a division ring of char O and G a finite subgroup of GL(n,D) then G has a metabelian subgroup of index bounded in terms of n (Hartley & Shahabi).

Theorem (Banieqbal) Let H be a finite group, K a number field, V an irreducible KH-module and suppose $V^{G} = U$ is irreducible. Let $D = \mathrm{End}_{KG}U$. Then $\dim_{Z(D)}D$ is bounded in terms of $|H|$ only.

We also discuss the classification of finite subgroups of GL(2,D), recently essentially completed by Banieqbal.

P. Hauck

Distinguished conjugacy classes of subgroups

(Report on joint work with J. C. Beidleman and B. Brewster).

A Fitting functor f is a map which assigns to each finite (soluble) group

G a set f(G) of subgroups of G such that for any monomorphism $\alpha:G \to H$ with $\alpha(G) \trianglelefteq H$ the following holds: $\alpha(f(G)) = f(H) \cap \alpha(G)$. - A Fitting functor is called conjugate if f(G) consists of a single conjugacy class in each group G. Clearly, conjugate Fitting functors satisfy the Frattini argument, i.e. for all G, all $N \trianglelefteq G$, and all $X \in f(G)$: $G = N \cdot N_G(X \cap N)$. Finally, for an integer $m \geq 2$, a Fitting funtor f is said to satisfy condition (α_m) if for all G each conjugacy class of $f(G^m)$ contains a member T such that: $(X_1,\ldots,X_m) \in T$ implies $(X_1,1,\ldots,1,\underset{\uparrow j}{X_1^{-1}},1,\ldots,1) \in T$ for all j.

Theorem: For a Fitting funtor f the following are equivalent: (1) f satisfies the Frattini argument. (2) $f = \bigcup_{\lambda\in\Lambda} f_\lambda, f_\lambda$ conjugate. (3) f satisfi[illegible] (α_m). As one of the consequences of this result one can construct a Fitting functor f^* closely related to f (satisfying the Frattini argument) such that $V = (V \cap G) \times (V \cap H)$ for all finite groups G,H and all $V \in f^*(G \times H)$.

T. O. Hawkes

Groups with bounded subnormal defect

The following result was discussed:

Theorem. Let H be a finite soluble group with a chief series of length n. Then H can be embedded in a group of nilpotent length 2n in which each subnormal subgroup has defect at most 3.

H. Heineken

Subgroups of finite index in T-groups

Report on results acchieved jointly by John C. Lennox and H. Heineken. It was the aim to show that there is a bound on the defect of subnormal subgroups of subgroups indicated in the title.

Results: If H is a subgroup of a T-group G such that |G:H| is finite and H contains some term of the derived series of G, then H is a T-group. Suppose that n is a positive integer. Then there exists a function f(n) such that if H is a subgroup of index n in a T-group G, all subnormal subgroups of H are of defect f(n) at most.

The corresponding statement is false for groups G all of whose subnormal subgroups have defect at most 2.

D. F. Holt

Computation of Cohomology Groups

We describe a method for the mechanical computation of the first and second cohomology groups, $H^1(G,M)$ and $H^2(G,M)$, where G is a finite group acting on a finite module M over a prime field. These groups are calculated as the subgroups of stable elements of $H^i(P,M)$, for $P \in \mathrm{Syl}_p(G)$, where p is the characteristic of M. The principal tool used in the computation of $H^i(P,M)$ is the nilpotent quotient algorithm. $H^1(P,M)$ is a subgroup of the p-multiplicator of $P \ltimes \overline{M}$, where $\overline{M}$ is the dual of M. $H^2(P,M)$ is computed as a factor group of $\mathrm{Hom}_P(FM(P),M)$, where $FM(P)$ is the so-called Frattini module of P. $FM(P) = R/[R,R]R^P$, where $P = F/R$, and F is free on the same number of generators as P.

T. C. Hurley

On Commutators and Powers

The connection between power laws or relators and commutator laws or relators has often been studied in group theory mainly in connection with Burnside's problem. For example if n is a power of a prime p and x^n is a law in a finitely generated (f.g.) group G, then Burnside's problem is whether $[x_1,x_2,\dots,x_m]$ (m depending on n) is also a law in G. It is not known whether x^5 in a f.g. group implies such a commutator law but for p large enough x^p does not imply such a law. Many other problems and results connected with Burnside's problem can be formulated in such a way. On the other hand not much is known on the converse of this problem i.e. on whether commutator laws can imply power laws or relators. It is known for example that a nilpotency law plus a polynilpotency law does not imply any power relators. On the other hand $[y,x,x] \Rightarrow [y,x,z]^3$; $[[x,y],[x,z]] \Rightarrow [[x,y],[z,t]]^2$; and higher Engel laws imply 5-torsion. Note that these varieties have repeated entries in their laws. Regarding independent entries it is known that 2-torsion exists in centre-by-metabelian varieties.

There is a problem then of whether for a given prime p there exists a variety defined by commutators which has proper p-torsion; and preferably such a variety would be defined by commutators with independent entries. We show that such a variety exists and discuss some problems.

Let m be an integer ≥ 2 and $n = n(p,m) = mp + p + m$. F is a free group and $\gamma_i(F) = \gamma_i$ is the i-th term of the lower central series of F. Define $G = F/([\gamma_n,\gamma_m]\cdot[\gamma'_m,P\gamma_{m+1}])$. Then G has proper p-torsion. For $a \in \gamma_{m+1}$, $b,c \in \gamma_m$ define

$$w = \prod_{t=1}^{p-1} [a,b,(p - t - 1)a;\ a,c,(t - 1)a]^{\binom{p}{t}/p} \quad , \quad p > 2$$

$$w = [[a,b],[a,c]] \ , \quad p = 2.$$

Then $w^p \equiv 1 \bmod [\gamma_n,\gamma_m]\cdot[\gamma'_m,p\gamma_{m+1}]$ and when a is a basic commutator of weight (m + 1), and b,c are basic commutators of weight m, $b \neq c$, then $w \not\equiv 1 \bmod [\gamma_n,\gamma_m]\cdot[\gamma'_m,p\gamma_{m+1}]$.

F. Leinen

Existenziell abgeschlossene lokal endliche p-Gruppen

Eine LF_p-Gruppe G heißt existenziell abgeschlossene LF_p-Gruppe, wenn je[illegible] System endlich vieler Gleichungen und Ungleichungen mit Koeffizienten aus G, das in einer LF_p-Obergruppe von G lösbar ist, bereits eine Lösung in G besitzt. Nach B. Maier gibt es bis auf Isomorphie genau eine abzählbare, existenziell abgeschlossene LF_p-Gruppe E_p. Unter Verwendung geeigneter Permutationsprodukte läßt sich zeigen, daß G genau dann eine existenziell abgeschlossene LF_p-Gruppe ist, wenn jede endliche Teilmenge von G in einer zu E_p isomorphen Untergruppe von G enthalten ist. Es folgt, daß es zu jedem $K > \aleph_0$ genau 2^K paarweise nicht-isomorphe existenziell abgeschlossene LF_p-Gruppen der Mächtigkeit K gibt. Mittels einer neuen Einbettungstechnik für abzählbare Gruppen in Kranzprodukte kann der folgende Satz bewiesen werden.

Satz. Für jede existenziell abgeschlossene LF_p-Gruppe G gilt:

(a) G besitzt genau eine Hauptreihe Σ, der Ordnungstyp von Σ ist eine dichte Ordnung ohne Endpunkte, und die Normalteiler von G bilden eine Kette.

(b) Ist M/N ein Hauptfaktor von Σ, so ist N eine existenziell abgeschlossene LF_p-Gruppe.

(c) Tritt $1 \neq K \trianglelefteq G$ nicht in Σ auf, so sind K und G/K existenziell abgeschlossene LF_p-Gruppen.

(d) Jeder Subnormalteiler von G ist bereits ein Normalteiler von G.

Ferner kann die Struktur der Automorphismengruppe von E_p bestimmt werden.

A. Lichtman

Linear groups over fields of fractions of enveloping algebras

Let D be the field of fractions of the universal envelope U(L) of a finite dimensional Lie algebra or a group ring KH of a soluble residually torsion free nilpotent group.

We prove that for arbitrary natural n the matrix ring D_n has the following properties.

1) If S is a finitely generated subring of D_n then the Jacobson radical of S is nilpotent of index $\leq$ n.

2) If T is a finitely generated subring of D_n such that any element of T is a sum of nilpotent elements then $T^n = 0$.

3) Any noncentral normal subgroup of $GL_n(D)$ contains a noncyclic free subgroup.

4) Any periodic subgroup $G \subseteq GL_n(D)$ is locally finite.

5) If char K = 0 then any periodic subgroup $G \subseteq GL_n(D)$ contains an abelian normal subgroup of index $\leq p(n)$, where p(n) is any Jordan function.

A. Mann

The local hypercentre

Characterizations of finite-by-locally-nilpotent groups will be given, which parallel known characterizations of finite-by-nilpotent groups. Sample results:

1. The following are equivalent, for a group G:

a. G is finite-by-locally-nilpotent.

b. The local hypercentre of G has a finite index.

c. All subgroups of G are almost locally ascendant, with bounded "finite defect".

d. G is covered by finitely many locally nilpotent subgroups.

(Unfamiliar terms will be defined in the talk.)

2. G is an extension of a finite group by a group satisfying the normalizer condition, if and only if G contains only finitely many self-normalizing subgroups.

P. M. Neumann

Some Infinite Permutation Groups

A subset Σ of an infinite set Ω will be called a <u>moiety</u> of Ω if $|\Sigma| = |\Omega - \Sigma|$. Let Ω be a countably infinite set and G a primitive permutation group on Ω such that all G-orbits in the set of moieties of Ω are uncountable.

<u>Theorem:</u> Either G is highly transitive on Ω (that is, k-fold transitive for every positive integer k) or there is a prime-power q such that

(a) Ω can be so identified with infinite-dimensional affine space over GF(q) that $G \leq A\Gamma L(\infty, q)$ or

(b) Ω can be so identified with infinite-dimensional projective space over GF(q) that $G \leq P\Gamma L(\infty,q)$.

I shall talk about the provenance, the proof and some ramifications of this theorem.

M. L. Newell

Metabelian Groups of Prime-Power Exponent

The orders and nilpotency class of some finitely generated relatively free metabelian groups of prime-power exponent are described. A brief summary of earlier results is included. Recent results, some of which depend critically on a computer inplementation of the nilpotent quotie[illegible] algorithm, are given together with indications of a general computer-free approach.

W. Plesken

Soluble pro-p-groups of finite coclass

The coclass of a finite p-group P of order p^n and nilpotency class c is $ccl(P) = n - c$, and for a pro-p-group $\mathbf{P} = \varprojlim P_i$ one defines $ccl(\mathbf{P}) = \lim ccl(P_i)$ ($\leq \infty$). The following conjecture by C. R. Leedham-Green and M. F. Newman is proved:

For given $p,r \in \mathbb{N}$ there are only finitely many isomorphism classes of (infinite) soluble pro-p-groups $\mathbf{P}$ with coclass r.

The proof can be reduced to a special class of soluble pro-p-groups, namely to uniserial p-adic space groups for which the following is proved: The dimension of a uniserial p-adic space group of coclass r is given by $(p - 1)p^{\rho-1}$ with $\rho - 1 \leq r$.

(Joint work with C. R. Leedham-Green and S. McKay)

A. Rae

Extending derivations of soluble groups

Problem: Given a locally finite group G and a kG module V where $p = \text{ch } k \notin \pi(G)$ (the set of primes dividing the order of G). When can we say that every derivation from a subgroup H of G into V extends to G?

Motivation: In the Schur-Zassenhaus theorem for locally finite groups this is saying that every p'-subgroup is contained in a complement; equivalently that every Sylow p'-subgroup is a complement.

Results: For G soluble a necessary and sufficient condition is that $[x,V]^G$ (normal closure under G) is a min-C G-module (minimum condition for centralizers of subgroups G) for every $x \in G$. This has powerful implications for the structure of G. For example if G is a q-group for some prime q and V is irreducible then G is the finite extension of a divisible abelian group etc.

D. J. S. Robinson

<u>Groups with virtually trivial automorphisms</u>

An automorphism α of a group G is <u>virtually trivial</u> if $|G : C_G(\alpha)|$ is finite. The structure of groups all of whose automorphisms are virtually trivial is investigated. Necessary and sufficient conditions are found, and several examples are constructed.

D. Segal

<u>Local and global equivalence of binary forms</u>

Define k-<u>equivalence</u> of binary forms F,G by $F \underset{k}{\sim} G \Leftrightarrow G(x,y) =$ $= \lambda F(ax + by, cx + dy)$ for some $\lambda \in k^*$ and $\binom{ab}{cd} \in GL_2(k)$, where k is a field. I will discuss some results regarding the question:

$$\text{does } F \underset{\mathbb{Q}_p}{\sim} G \quad \underset{\text{primes}}{\forall p} \quad \Rightarrow \quad F \underset{\mathbb{Q}}{\sim} G \text{ ?}$$

This depends on Galois cohomology of finite subgroups of PGL_2. There are positive results for forms of degree coprime to 6 and negative results in other cases (including some forms of degree 4).

U. Stammbach

<u>On the integral homology of metabelian groups; an example</u>

Baumslag, Dyer, Miller III have given a description of sequences of abelian groups that appear as integral homology sequences of finitely presented groups. This description is in terms of recursion theory. The analogous problem for finitely presented metabelian groups seems to be open. Thus there is interest on examples of such groups with not "well-behaved" homology. Gilbert Baumslag and I constructed a finitely presented metabelian group $E = E(n)$, $n \geq 1$ whose integral homology is as follows:

$$H_m(E) = \begin{cases} \text{free abelian of rank } \binom{n+1}{m} \text{ for } 0 \leq m \leq n, \\ \mathbb{Z} \oplus \mathbb{Z}/(n!)^{n+1} - 1 \text{ for } m = n+1, \\ \overset{\infty}{\oplus} \mathbb{Z}/(n!)^m - 1 \quad \text{for} \quad m \geq n+2. \end{cases}$$

The case n = 1 is due to Baumslag-Dyer. -

The key step is the construction of a module A over the free abelian group Q on n + 1 generators with the following property: For $k \leq n$ the module $\Lambda^k A$ is generated by one element and non-free; for $k = n + 1$ the module $\Lambda^k A$ is free on one element, and for $k \geq n + 2$ the module $\Lambda^k A$ is free on infinitely many elements.

S. E. Stonehewer

Subnormal subgroups of factorised groups

Wielandt has shown that if a finite group G = HK, where H and K are subgroups, and if X is a subnormal subgroup of H and K, then X is subnormal in G. (G = ⟨H,K⟩ is not sufficient to guarantee this result.) When G is infinite, the result is still true for (i) nilpotent-by-abelian groups, (ii) nilpotent-by-polycyclic-by-finite groups, (iii) soluble minimax groups. Finding an infinite G for which the result fails seems to be harder. Some general constructions with H and K nilpotent are shown not to yield a counterexample. In particular, when G is finite, the subnormal defect of X in G often turns out to be bounded in terms of the defects o[illegible] X in H and K, thus preventing the construction of an infinite counterexample via direct products.

S. Thomas

Complete existentially closed locally finite p-groups

In a course of lectures in the 60s, Phillip Hall considered classes of groups C satisfying:

(+) if G is an infinite group such that $G < H \in C$, then there exists $H' \in C$ such that $G < H' < H$ and $|G| = |H'|$.

Hall showed that various classes of groups, such as simple groups and characteristically simple groups, have this property. He asked:

Question: Does the class of complete groups satisfy (+)?

Assuming Jensen's principle ◊, the answer is "No!". To prove this, it is enough to construct an uncountable complete group which has no countable complete subgroup. Such groups are given by the following two results.

Theorem 1

There are no countable complete locally finite p-groups.

Theorem 2 (◊)

There exist 2^{ω_1} nonisomorphic complete existentially closed locally fini[illegible] p-groups of cardinality ω_1.

Theorem 2 also provides counterexamples to a conjecture of Kegel that every existentially closed locally finite p-group is characteristically simple.

U. Webb

Almost all p-groups have automorphism group a p-group

Suppose that G is a finite p-group, and that A(G) is the image of Aut G under restriction to the Frattini quotient of G, so that $A(G) \leq GL(d,p)$

where d is the rank of $\overline{G}$. It is known that if $d > 1$ any subgroup of GL(d,p) can arise in this way, although it is hard to construct examples and A(G) generally seems to be large. I have shown that in fact A(G) is almost always one, so that Aut G is almost always a p-group. To be more precise, let $A_{n,d}$ be the set of groups G of Frattini class n where $\overline{G}$ has rank d. Then

$$\lim_{d\to\infty} \frac{|\{G \in A_{n,d} \mid A(G) = 1\}|}{|A_{n,d}|} \to 1 \quad .$$

B. A. F. Wehrfritz

Absolutely irreducible skew linear groups

Let D be a division ring with centre F, n a positive integer and G a subgroup of GL(n,D). The object of the talk is to justify calling G absolutely irreducible if F and G together generate the whole matrix ring $D^{n\times n}$. In doing this I shall discuss the following two theorems.

Suppose G is an absolutely irreducible skew linear group.

i) If $G \in P(\mathfrak{A} \cup L\mathfrak{F})$ then $G \in \mathfrak{A}\cdot L\mathfrak{F}$.

ii) If $H \triangleleft G$ is locally finite then $G/C_G(H)$ is locally finite.

Neither of these conclusions are valid for irreducible skew linear groups.

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