MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Algebraic K-Theory

27.5. bis 2.6.1984

Die Tagung wurde von den Herren R.K. Dennis (Ithaca) und U. Rehmann (Bielefeld) geleitet. Die großen Fortschritte, die seit der letzten Konferenz in Oberwolfach vor vier Jahren in der algebraischen K-Theorie erzielt worden sind, wurden in den Vorträgen und Gesprächen sehr deutlich, insbesondere die tiefen Verbindungen zur algebraischen Geometrie, zur Zahlentheorie und zu der erst kürzlich entwickelten zyklischen Homologie.

Die große Beteiligung - auch diesmal wieder überwiegend aus dem Ausland - zeigte, wie gefragt und wie notwendig ein häufiger intensiver Gedankenaustausch in diesem sich immer noch rasch entwickelnden Zweig der Mathematik ist.

Vortragsauszüge

C.A. WEIBEL: K-theory of varieties with isolated singularities

Theorem: Let X be a quasiprojective variety [with isolated singularities]. Then

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(1) $A^*(X) \otimes \mathbb{Q} \cong K_o(X) \otimes \mathbb{Q} \cong \coprod H^p(X,\underline{\underline{\mathcal{K}}}_p) \otimes \mathbb{Q}$

(2) $H^p(X,\underline{\underline{\mathcal{K}}}_q) = 0$ for $p \geq 2+q$ and can be nonzero for $p = 1+q$ $(q \geq -1)$

(3) There is a spectral sequence

$$E_2^{pq} = H^p(X,\underline{\underline{\mathcal{K}}}_{-q}) \Rightarrow K_{-p-q}(X) \qquad (p \geq 0,\ q \in \mathbb{Z})$$

and the associated filtration on $K_o(X)$ coincides with the geometric filtration $K_o(X) \supset \tilde{K}_o(X) \supset SK_o(X) = Fil^2(X) \supset \ldots$ $\ldots \supset Fil^d(X) \supset 0$ where $(p \geq 2)$ $Fil^p(X)$ is generated by $[\mathcal{O}_Y]$, $\operatorname{codim}(Y) \geq p$ and $Y \cap \operatorname{sing}(X) = \emptyset$.

We conjecture that this theorem holds if [with isolated singularities] is removed, except that in (2) we must have

$$H^p(X,\underline{\underline{\mathcal{K}}}_q) = 0 \quad \text{if} \quad p \geq 2+q+\dim(\operatorname{sing} X) \quad (q \geq -1-\dim(\operatorname{sing} X)) .$$

In addition, we conjecture that $K_q(X) = 0$ if $q \leq -1-\dim(\operatorname{sing}(X))$.

After stating the theorem and the conjectures, we pointed out in our talk that the theorem immediately implied several results of Collino, Weibel, Pedrini-Weibel, and Levine. For example, if $\dim(X) = 2$, then we immediately obtain $H^2(X,\underline{\underline{\mathcal{K}}}_2) = SK_1(X)$. If $X = \operatorname{Spec}$ of a semilocal ring, $\dim(X) = 2$, then $SK_1(X) = SK_o(X)$ implies that $\underline{\underline{\mathcal{K}}}_2$ is flasque on X. This was the key step in showing that $CH^2(X,Y) \cong SK_o(X) \cong H^2(X,\underline{\underline{\mathcal{K}}}_2)$ when X is a surface with isolated singularities.

W. RASKIND: $\underline{\underline{K}}_2$-cohomology of algebraic varieties

Let X be a smooth, proper, geometrically connected algebraic variety over a field k. We use the work of Merkur'ev-Suslin, Suslin and the Weil conjectures as proved by Deligne to determine much of the structure of the $\underline{\underline{\mathcal{K}}}_2$-cohomology groups $H^i(X,\underline{\underline{\mathcal{K}}}_2)$ for $i = 0,1,2$. Among other things, we show:

<u>Theorem 1</u>: If k is a p-adic field then $H^0(X,\underline{\underline{K}}_2)$ is the direct sum of a finite group of order prime to p and a group which is uniquely divisible prime to p.

<u>Remark</u>: This theorem is a generalisation of and depends on results of Moore, Carroll and Tate on the structure of K_2 of a p-adic field.

<u>Theorem 2</u>: If k is a number field then for any positive integer n, the group $(H^0(X,\underline{\underline{K}}_2)/K_2k)/n$ is finite.

One expects that under the hypotheses of Theorem 2, the group $H^0(X,\underline{\underline{K}}_2)/K_2k$ is finitely generated. Because of the analogy between $H^0(X,\underline{\underline{K}}_2)/K_2k$ and the group of k-points of an abelian variety, we like to call Theorem 2 "The Weak Mordell-Weil Theorem" for $H^0(X,\underline{\underline{K}}_2)/K_2k$.

V. SNAITH: <u>Calculation of $K_3(\mathbb{F}_q[t]/t^i)$, $i = 2,3$</u>
(joint with J. Aisbett and E. Lluis-Puebla)

If A is a ring with an ideal I let $(1+I)^*$ denote units in A which are one mod I.

<u>Theorem</u>. Let p be an <u>odd</u> prime and $m \geq 1$. There are isomorphisms

(i) $$K_3(\mathbb{F}_{p^m}[t]/(t^3)) \cong K_3(\mathbb{F}_{p^m}) \oplus \left(\frac{1+t\,\mathbb{F}_{p^m}[t]}{(t^6)}\right)^* / \langle 1+at^3 ; a \in \mathbb{F}_{p^m} \rangle .$$

(ii) $$K_3(\mathbb{F}_{p^m}[t]/(t^2)) \cong K_3(\mathbb{F}_{p^m}) \oplus \left(\frac{1+t\,\mathbb{F}_{p^m}[t]}{(t^4)}\right)^* / \langle 1+at^2 ; a > \mathbb{F}_{p^m} \rangle .$$

Let F denote $\mathbb{F}_{p^m}$ and $F(n)$ denote $\mathbb{F}_{p^m}[t]/(t^n)$. Since $K_2(F(n)) = 0$, $K_3(F(n)) \cong H_3(SLF(n);Z)$ and the above calculation is accomplished by studying the homology spectral sequence of the extensions

$$(*) \quad M_t^o(n-1) \xrightarrow{i} SL_tF(n) \xrightarrow{\pi} SL_tF(n-1) \;, \quad (2 \le t \le \infty) \;.$$

Here $M_t^o(n-1)$ is the additive group of trace-zero matrices with entries in F with $SL_tF(n-1)$ acting through SL_tF via conjugation. The talk illustrated how to play off the similarities of the homological algebra related to (*) for different values of n.

A.O. KUKU: K_n, SK_n of integral group rings.

Let π be a finite group. For $n \ge 0$ let $SK_n(\mathbb{Z}\pi)$ be the kernel of the canonical map $K_n(\mathbb{Z}\pi) \to K_n(\mathbb{Q}\pi)$. It is well known that for $i = 0,1$, $K_i(\mathbb{Z}\pi)$ is finitely generated and that $SK_i(\mathbb{Z}\pi)$ is a finite group and answers to such finiteness questions for $K_n(\mathbb{Z}\pi)$, $SK_n(\mathbb{Z}\pi)$, $n \ge 2$, have been open for some time. We prove in this paper that for all $n \ge 2$, $K_n(\mathbb{Z}\pi)$ is finitely generated and $SK_n(\mathbb{Z}\pi)$ is finite. We then deduce that for a rational prime p, $SK_n(\hat{\mathbb{Z}}_p\pi)$ is finite for all $n \ge 1$.
If R is the ring of integers in a number field F, p a rational prime, $\mathfrak{p}$ the prime of R lying over p, $k = R/\mathfrak{p}$, π any finite group, then we show that for all $n \ge 1$,

(i) $K_{2n}(k\pi)$ is a finite p-group

(ii) The Cartan homomorphism $\varphi_{2n-1}: K_{2n-1}(k\pi) \to G_{2n-1}(k\pi)$ is surjective and $\operatorname{Ker} \varphi_{2n-1}$ is the Sylow p-subgroup of $K_{2n-1}(k\pi)$ where $K_{2n-1}(k\pi)$ is a finite group.

A. COLLINO: Torsion in the Chow group of codim 2: the case of varieties with isolated sigularities

Let X be a variety with one isolated singular point, the Bloch-Ogus theory goes through in this case as in the smooth case and as a consequence one has that the n-torsion subgroup of $H^2(X,\underline{\underline{K}}_2) = CH^2(X)$ is finite. If X is proper Bloch has remarked that $\varprojlim H^1(X,\mathcal{X}^2(\mu_n^{\otimes 2})) \tilde{\to} T_1\ CH^2(X)$, up to torsion kernel. Let $\tilde{X}$ be a desingularization of X, then the kernel of $T_1\ CH^2X \to T_1\ CH^2\tilde{X}$ is bounded by "topology". If X is a surface ${}_nCH^2(X) \simeq {}_nCH^2(\tilde{X})$ which was a result of Levine. Following an indication of Bloch we provide an example of a 3-fold X as above, for which the Tate group of the group of cycles of cod 2/"alg. equiv." is not 0 .

H . GILLET: Some new cases of Gersten's conjecture

Recall that Gersten's conjecture says that if R is a regular local ring and $\underline{M}^{(i)}(R)$ is the category of f.g. R-modules with codimension of support $\geq i$, then $K_*\underline{M}^{(i)}(R) \to K_*\underline{M}^{(i-1)}(R)$ is zero for $i \geq 1$. If Gersten's conjecture is true the Bloch's formula holds:

$$CH^p(X) \simeq H^p(X,K_p(\mathcal{O}_X)) .$$

Quillen proved Gersten's conjecture for regular local rings essentially of finite type over a field. The general case is still open. We now have some new results.

Theorem A (joint with M. Levine) Let R be a local ring smooth over a discrete valuation ring Λ, $\underline{MF}^{(i)}(R)$ the category of f.g. R-modules flat over Λ. Then

$$K_*\underline{MF}^{(i)}(R) \to K_*\underline{MF}^{(i-1)}(R)$$

is zero.

Corollaries i) Gersten's conjecture for R is true iff it is true for the discrete valuation ring which is the local ring at the generic point of the closed fibre.

ii) $K_o\underline{M}^{(i)}(R)$ is generated by classes of modules $[R/(f_1,\dots,f_i)]$ where $(f_1,\dots,f_i)$ is a regular sequence.

iii) $CH^p(X) \simeq \mathbb{H}^p(X, u_*K_p(\mathcal{O}_{X_F}) \xrightarrow{\partial} i_*K_{p-1}(\mathcal{O}_{X_k}))$ where

$u: X_F \longrightarrow X$, $i: X_k \longrightarrow X$ are the inclusions of the generic and closed fibres, for X smooth over a dvr. Λ .

<u>Theorem B</u> Let R be a dvr. with $\frac{1}{n} \in R$. Then $K_*(k,\mathbb{Z}/n) \to K_*(R,\mathbb{Z}/n)$ is zero $(k = \text{r.f. } R)$ i.e. Gersten's conjecture is true for the K-theory with coefficients of R .

<u>Corollary</u> i) Gersten's conjecture is true for the K-theory with $\mathbb{Z}/n$ coefficients for all regular local rings R smooth over a dvr. Λ with $\frac{1}{n} \in \Lambda$,

ii) $H^p(X, K_p(\mathcal{O}_X, \mathbb{Z}/n)) \simeq CH^p(X) \otimes \mathbb{Z}/n$ for X smooth over a dvr.

D.L. WEBB: <u>Quillen G-theory of abelian group rings</u>

Consider the Quillen K-theory $G_*(R\pi)$ of finitely-generated $R\pi$-modules, where R is a ring with 1 and π is a finite Abelian group. Given a cyclic quotient ρ of π , generated say by t , let $R(\rho) = \frac{R\rho}{(\Phi_{|\rho|}(t))}$, Φ_n the n^{th} cyclotomic polynomial, and $R\langle\rho\rangle = R(\rho)[\frac{1}{|\rho|}]$.

Then the following analogue of Lenstra's formula for $G_o(R\pi)$ is valid for all n:

(1) $SG_n(R\pi) \cong \bigoplus_\rho SG_n(R\langle\rho\rangle)$, ρ a cyclic quotient of π

For the case $R = \mathbb{Z}$, one obtains, via a vanishing theorem of Soulé,

(2) $SG_n(\mathbb{Z}\pi) = 0$.

Combining (2) and an explicit devissage description of the relative term in the localization sequence yields Lenstra's formula

(3) $G_n(\mathbb{Z}\pi) \cong \bigoplus_{\rho} G_n(\mathbb{Z}<\rho>)$.

The idea is to obtain a Heller-Reiner type presentation for $SG_n(R\pi)$ and use a map analogous to Lenstra's to map to a suitable localization sequence.

J. HURRELBRINK: A question on squares in K_2.

This is a report on a joint paper with R. Perlis and D. Estes. F always denotes an algebraic number field; recall:

$$1 \to K_2(F)^2 \to K_2(F) \to Br(F) \quad \text{is exact.}$$

Question 1: Given $\alpha \in F^*$ totally positive, when does there exist a positive $q \in \mathbb{Q}$ such that $\{\alpha,-q\} \in K_2(F)$?

Theorem 1: $\alpha \in F^*$ is totally positive

$\iff \exists$ positive $q \in \mathbb{Q} : \{\alpha,-q\} \in K_2(F)^2$

$\iff \exists$ infinitely many rational primes $q : \{\alpha,-q\} \in K_2(F)^2$.

Corollary: $\alpha \in F^*$ sum of squares in F

$\Rightarrow \alpha = x^2+y^2+\ldots+y^2$ for some $x,y \in F$.

Question 2: Which classes in the Witt ring $W(F)$ can be represented by trace forms over F ?

Theorem 2 (P. Conner): F totally imaginary $\Rightarrow$ every class in $W(F)$ can be represented by a trace form over F .

Remark: Theorem 1 implies Theorem 2.

S.C. GELLER: On injectivity of K_i of a 1-dim integral domain into K_i of its field of fractions ($i \leq 3$)

Let A be a 1-dimensional integral domain and F its field of

fractions. When does $K_i(A)$ inject into $K_i(F)$? For $i = 2$, A seminormal, local, equicharacteristic $K_i(A) \hookrightarrow K_i(F) \iff A$ is regular. For $i = 1,3$, many non-regular A have $K_i(A) \hookrightarrow K_i(F)$. For example $K_i(k \oplus (t-\alpha_1)^{a_1} \ldots (t-\alpha_s)^{a_s} k[t]) \hookrightarrow K_1(k(t))$ where $\alpha_1, \ldots, \alpha_s \in k$ are distinct, $a_i \geq 1$ and $k = \overline{\mathbb{F}}_p$, $\mathbb{F}_q$ or $\overline{\mathbb{Q}}$. Also, $K_3(k \oplus (t-a_1) \ldots (t-a_r) k[t]) \hookrightarrow K_3(k(t))$ where $a_1, \ldots, a_r \in k$ are distinct and $k = \mathbb{F}_q$, $\overline{\mathbb{F}}_p$.
In fact, $K_{i+1}(k) \subseteq \ker(K_i(k \oplus (t-a_1) \ldots (t-a_r) k[t]) \to K_i(k(t)))$ for all $i \geq -1$ and all fields k . Other examples of both injectivity and non-injectivity were given.

B. LIEHL: <u>Bounded word length in matrix groups over arithmetic Dedekind rings</u>

The following result was presented: Let k be an algebraic number field, not totally imaginary; $A \subset k$ an arithmetic Dedekindring with infinitely many units, and let q_1, q_2 be ideals of A . Then by a theorem of Vaserstein (1972) every matrix $\alpha \in G(q_1, q_2) = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(A) \mid a-1, d-1 \in q_1 q_2 , b \in q_1 , c \in q_2 \}$ can be written as a product of elementary matrices $\varepsilon_j \in \{ \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} , \begin{pmatrix} 1 & 0 \\ y & 1 \end{pmatrix} \mid x \in q_1 , y \in q_2 \}$. Suppose there exists a natural number l , such that every prime p with $p \equiv -1 \pmod{l}$ splits completely in k , then it is proved, that the number of factors ε_j necessary to express α is a bounded function on $G(q_1, q_2)$. The bound can be chosen independently from q_1, q_2 . Further one gets similar results about elementary word length in SL_n $(n \geq 3)$ and Sp_{2n} $(n \geq 2)$.

G. TADDEI: Normality of elementary groups in Chevalley groups over a ring

Let $\mathcal{L}$ be a complex semi-simple Lie algebra, let Φ be the root system of $\mathcal{L}$ with respect to a (fixed) Cartan sub-algebra H and let Γ be a lattice of H which contains the root weight lattice P_r and is contained in the weight lattice P of Φ. Associated with Φ and Γ there is a group scheme $G_{\Phi,\Gamma} = G$ over $\mathbb{Z}$, called the Chevalley-Demazure group scheme.
For all roots α we have an injective natural map $\mathbb{G}_a \xrightarrow{e_\alpha} G$ and for a commutative ring R with unit we define $E(R)$ to be the subgroup of $G(R)$ generated by the images of the $e_\alpha(R)$'s.

Theorem: If Φ does not contain a component of type A_1 then for any commutative ring R, $E(R)$ is a normal subgroup of $G(R)$.

K. VOGTMANN: Outer Automorphisms of Free Groups (joint with M. Culler)

Let F_n be the free group of rank N. We study the group Γ_n of outer automorphisms of F_n by constructing a space X_n analogous to the Teichmuller space used for studying the mapping class group of a Riemann surface or the homogeneous space used for studying an arithmetic group. Points in X are equivalence classes of marked graphs (g,G), where G is a metric graph with $\pi_1 G = F_n$ (and G is connected and has no nodes of valence 1 or 2) and g is a homotopy equivalence from a standard bouquet of circles to G.

Theorem 1 X_n is contractible, and Γ_n acts on X_n discretely with finite stabilizers.

To prove the contractibility of X_n, we use the following

Theorem 2 There is a contractible simplicial deformation retract K_n of X_n such that

(i) K_n is Γ_n-invariant

(ii) $\dim K_n = 2n-3$

(iii) K_n/Γ_n is a finite CW complex .

Corollary (i) Γ_n is VFL

(ii) $VCD(\Gamma_n) = 2n-3$.

This corollary has also been recently proved by S. Gersten.

C. PEDRINI: Bloch's Formula for singular varieties

Let X be a quasi-projective variety over a field k . If X is non-singular it is well-known that $CH^p(X) \simeq H^p(X,\underline{\underline{K}}_p)$ (Bloch's formula) where $CH^p(X)$ is the Chow group of codimension p cycles modulo rational equivalence and $\underline{\underline{K}}_p$ is the sheaf associated to the pre-sheaf $U \to K_p(U)$ on X . If X is singular we assume: $Y = \text{Sing } X$ is contained in an open affine set and we define $CH^p(X,Y)$ to be the cokernel of the "cycle map":

$$\coprod_{x \in X_*^{p-1}} k(x)^* \to \coprod_{x \in X_*^p} \mathbb{Z}$$

where $X_*^p = \{x \in X^p \;/\; \{\bar{x}\} \cap Y = \emptyset\}$. Define: $X_Y = \cap U$, U open affine such that $Y \subseteq U$ and $X-U$ is a divisor. Then there is a long exact sequence:

$$\ldots H^{p-1}(X,\underline{\underline{K}}_p) \to H^{p-1}(X_Y,\underline{\underline{K}}_p) \to CH^p(X,Y) \to H^p(X,\underline{\underline{K}}_p) \to H^p(X_Y,\underline{\underline{K}}_p) \to 0$$

In the case $p = 2$ and X has only finitely many singular points we prove the following results (jointly with C. Weibel):

Theorem: Let X be a quasi-projective irreducible variety over a field k having only finitely many singular points. Then the groups $H^1(X_Y,\underline{\underline{K}}_2)$ and $H^2(X_Y,\underline{\underline{K}}_2)$ have finite exponent; in particular $CH^2(X,\text{Sing } X) \otimes \mathbb{Q} \simeq H^2(X,\underline{\underline{K}}_2) \otimes \mathbb{Q}$.

Corollary: Let X as in the theorem and assume $\dim X \leq 3$. Then

$$CH^2(X, \text{Sing } X) \simeq H^2(X,\underline{\underline{K}}_2)$$

R. CHARNEY: K-theory and moduli spaces (joint work with R. Lee)

We consider applications of K-theory techniques to moduli spaces. We show that certain families of moduli spaces are related by sum operations or by collapsing maps which can be used to define, respectively, "+-construction" and "Q-constructions" for moduli spaces. This is applied to moduli spaces of Riemann surfaces and moduli spaces of abelian varieties to produce stable rational cohomology classes and to compute the effect of the Jacobian map on these classes. In particular, for

A_g^* = Satake compactification of moduli space of principally polarized abelian varieties of dim g

M_g^* = moduli space of stable curves of genus g

$J : M_g^* \to A_g^*$ the (extended) Jacobian map

$J^* : H^*(A_g^*;\mathbb{Q}) \to H^*(M_g^*;\mathbb{Q})$ the induced map

we prove

Theorem: For $g \geq i+1$, $H^i(A_g^*;\mathbb{Q})$ is the i-dimensional subspace in the polynomial algebra $\mathbb{Q}[x_{4j-2}] \otimes \mathbb{Q}[y_{4j+2}]$, $(j \geq 1)$.

E. Miller proves that J^* is an injection on $\mathbb{Q}[x_{4j-2}]$.

We prove

Theorem: $J^*(y_{4j+2}) = 0$.

M. KAROUBI: Cyclic homology

Let G be any discrete group. One can extract from the double complex defining the cyclic homology of the ring $\mathbb{Z}[G]$ another double complex (take the product of the g_i's to be equal to 1) which homology in dimension n is $H_n(G) \oplus H_{n-2}(G) \oplus \ldots \overset{\text{def}}{=} HC_n(G)$.

If now $G = GL_r(A)$, where A is a ring, we define group maps $H_i(G) \to HC_{i+2p}(A)$ as the composition of the maps

$$H_i(G) \to HC_{i+2p}(G) \to HC_{i+2p}(\mathbb{Z}G) \to HC_{i+2p}(M_r(A)) \xrightarrow{\text{Trace}} H_{i+2p}(A) .$$

From this we can deduce easily the "Chern character"

$$K_i(A) \xrightarrow{ch_p} HC_{i+2p}(A) .$$

In fact, these maps are compatible with the S operator of Connes and we have more precisely maps

$$K_i(A) \longrightarrow \varprojlim_p HC_{i+2p}(A) .$$

This can be generalized easily to the "relative" situation:

$$K_i(A,I) \longrightarrow \varprojlim_p HC_{i+2p}(A,I) .$$

In particular, if $p = 0$, one has a commutative diagram

$$\begin{array}{ccc} & \overset{ch_o}{\nearrow} & HC_i(A,I) \\ K_i(A,I) & & \uparrow \\ & \underset{D}{\searrow} & H_i(A,I) \end{array}$$

where D is the map defined by Dennis some years ago (1976).

Let us assume now that A is a $\mathbb{Q}$-algebra, then one can define "regulator maps"

$$K_i(\hat{A},I) \xrightarrow{R} HC_{i-1}(\hat{A},I)$$

which makes the following diagram commutative

$$\begin{array}{ccc} K_i(\hat{A},I) & \longrightarrow & HC_{i-1}(\hat{A},I) \\ & \underset{D}{\searrow} & \downarrow B \\ & & HC_i(\hat{A},I) \end{array}$$

where B is the Connes operator and $\hat{A} = \varprojlim A/I^n$. This is defined via differential geometry technics.

R. OLIVER: $\underline{K_2(\hat{\mathbb{Z}}_pG)\text{ and cyclic homology}}$

For any prime p and any abelian p-group G, there is a short exact sequence

$$0 \to Wh_2^{top}(\hat{\mathbb{Z}}_pG) \xrightarrow{\Gamma_2} HC_1(\hat{\mathbb{Z}}_pG) \xrightarrow{\omega_2} \tilde{H}_2(G) \to 0 . \qquad (1)$$

Here, $Wh_2^{top}(\hat{\mathbb{Z}}_pG) = \varprojlim_n (K_2(\mathbb{Z}/p^n[G])/\{\pm G,\pm G\})$, $\tilde{H}_2(G) \cong (G\otimes G)/\langle g\otimes h+h\otimes g\rangle$, $HC_1(\hat{\mathbb{Z}}_pG) \cong (G\otimes \hat{\mathbb{Z}}_pG)/\langle g\otimes g\rangle$, and with these identifications $\omega_2(g\otimes h) = g\otimes g^{-1}h$. A comparison with the exact sequence

$$HC_1(\hat{\mathbb{Z}}_pG) \to \tilde{H}_2(G) \to K_1(\hat{\mathbb{Z}}_pG)/(\pm G) \xrightarrow{\Gamma} HC_0(\hat{\mathbb{Z}}_pG) \to H_1(G)\times \mathbb{Z}/(p,2) \to 0$$

(for <u>any</u> p-group G) suggests the existence of some still longer sequence containing both.

The immediate problem now is to construct an exact sequence similar to (1) for non-abelian p-groups. As one immediate consequence, this would yield a simple combinatorial algorithm describing $SK_1(\mathbb{Z}G)$ for any finite G : at least up to exponent 2 . Another consequence of (1) (and eventual generalizations to the nonabelian case) is a lower bound for $|K_2(\mathbb{Z}G)|$:

<u>Theorem</u> If G is an abelian p-group, then

$$|SK_2(\mathbb{Z}G)| \geq |SK_2(\hat{\mathbb{Z}}_pG)| = \frac{|SK_1(\mathcal{O}G)|\cdot|HC_1(\hat{\mathbb{Z}}_pG)|\cdot|K_2^{top}(\hat{\mathbb{Z}}_p)|}{|K_2(\hat{\mathbb{Q}}_pG)_{(p)}|} =$$

$$|SK_1(\mathcal{O}G)|\cdot\frac{|G|^{|G|}}{\prod_{\substack{C\leq G\\ C\text{ cycl.}}}|C|^{\varphi(|C|)+1}}, \text{ where } \mathcal{O} = \mathbb{Z} \text{ if } p>2,\ \mathcal{O} = \mathbb{Z}\zeta_3$$

if $p = 2$. In particular, for any $n \geq 1$,

$$|SK_2(\mathbb{Z}C_{p^n})| \geq |SK_2(\hat{\mathbb{Z}}_p C_{p^n})| = p^N \text{ , where } N = \frac{p^n-1}{p-1} - \frac{n(n+1)}{2} \text{ .}$$

This last formula complements results of Chaładus, who found lower bounds for $|K_2(\mathbb{Z}G)/SK_2(\mathbb{Z}G)|$ for cyclic p-groups G .

R. STAFFELDT: Rational algebraic K-theory of truncated polynomial rings over rings of integers

Let $\mathcal{O}$ = ring of integers in a number field K

A = $\mathcal{O}$-algebra, f.g. projective as an $\mathcal{O}$-module, augmented $A \overset{\varepsilon}{\to} \mathcal{O}$ and with nilpotent augmentation ideal $\bar{A} = \text{Ker } \varepsilon$.

Let $R = \mathbb{Q} \otimes_{\mathbb{Z}} A$, $\bar{R} = \mathbb{Q} \otimes_{\mathbb{Z}} \bar{A}$.

Theorem: $\dim_{\mathbb{Q}}(K_*(A)/K_*(\mathcal{O})) \otimes \mathbb{Q} = \dim_{\mathbb{Q}} HC_{*-1}(\bar{R})$, where the cyclic homology of $\bar{R}$ is computed viewing $\bar{R}$ as a K-algebra.

We analyse the spectral sequence of the fibration

$BG(\bar{A}) \to BGL(A) \to BGL(\mathcal{O})$, where $G(\bar{A}) = \text{Ker } \varepsilon = \{I+M(\bar{A}) \mid M(\bar{A})$ = matrices with entries in $\bar{A}$, mostly zero.$\}$

Rational homotopy theory gives a model for the local system of coefficients $\{H_*(BG(\bar{A}))\}$ since $G(\bar{A})$ is nilpotent, Borel's vanishing theorem for cohomology of arithmetic groups with coefficients in non-trivial irreducible algebraic representations implies the spectral sequence collapses, and the Loday-Quillen theorem applied to the rational homotopy theory model $\Lambda_*(gl(h) \otimes_k \bar{R})$ for the chains on the fibre gives the computation of the primitives in the coinvariant homology of the fibre in terms of cyclic homology.

To calculate the example $A = \mathcal{O}[T]/T^{n+1}$

$$\dim K_*(A)/K_*(\mathcal{O}) \otimes \mathbb{Q} = \begin{cases} [K:\mathbb{Q}]\cdot n & * \text{ odd} \\ 0 & * \text{ even} \end{cases}$$

we applied F. Goodwillies

Theorem: Suppose R is a commutative $\mathbb{Q}$ - algebra with $\mathbb{Q}$-derivation D D induces an endomorphism L_D of $H^{DR}(R)$ and moreover $L_D = 0$.

Z. WOJTKOVIAK: Two lattices in the complex fundamental group of an algebraic variety

We show that iterated integrals of Chen behave like polynomial functions on the fundamental group made nilpotent. Using this property we define the algebraic de Rham fundamental group $\pi_1^{alg\ dR}(\ )$ for affine, smooth varieties. Then we describe some part of the image of the transcendental $\pi_1(\ )$ in the algebraic de Rham fundamental group for elliptic curves minus a point and for the complex plane minus 0 and 1 .

D. RAMAKRISHNAN: K_1 of Hilbert modular surfaces and values of L-functions

For any smooth projective variety X over $\mathbb{Q}$, there are higher regulators (with values in Deligne cohomology)

$$r_{a,b} : Gr^{(b)}K_{2b-a}(X) \otimes \mathbb{Q} \to H^a_{\mathcal{D}}(X_{\mathbb{R}}, \mathbb{R}(b)) ,$$

as defined by A.A. Beilinson (and H. Gillet, ...). Here $Gr^{(b)}$ denotes the graded piece of weight b with respect to Adams operations. These regulators generalize the classical Dirichlet regulator on the group of units in a number field. The higher regulator on K_2 of curves was first defined and studied by Bloch, after Borel's striking work relating higher odd K-groups of rings of integers of any number field F to the values at negative integers of $\zeta_F(s)$.

If $2b-a \geq 1$, then there is an exact sequence (cf. Beilinson)

$$0 \to F^b H_{DR}^{b-1}(X) \otimes \mathbb{R} \xrightarrow{\pi_{b-1}} H_B^{a-1}(X(\mathbb{C}), \mathbb{R}(b-1))^+ \to H_{\mathcal{D}}^a(X_{\mathbb{R}}, \mathbb{R}(b)) \to 0$$

where H_B (resp. H_{DR}) denotes Betti (resp. De Rham) cohomology and π_{b-1} denotes the map on complex cohomology induced by the morphism $\mathbb{C} = \mathbb{R}(b-1) \oplus \mathbb{R}(b) \to \mathbb{R}(b-1)$. There is a natural $\mathbb{Q}$-structure on $\Lambda^{max} H_{\mathcal{D}}^a(X_{\mathbb{R}}, \mathbb{R}(b))$ by taking the quotient of $\Lambda^{max}(H_B^{a-1}(X(\mathbb{C}), \mathbb{Q}(b-1))^+)$ by $\Lambda^{max}(F^b H_{DR}^{b-1}(X))$.

We now specialize to the case of $\dim X = 2$, and consider the regulator: $r_{3,2} : Gr^{(2)} K_1(X) \otimes \mathbb{Q} \to H_{\mathcal{D}}^3(X_{\mathbb{R}}, \mathbb{R}(2))$. Combining this with the cycle map on the Néron-Severi group $NS(X)$ of divisors modulo algebraic equivalence, we get the modified regulator

$$r : Gr^{(2)} K_1(X) \otimes \mathbb{Q} \oplus NS(X) \otimes \mathbb{Q} \to H^{1,1}(X(\mathbb{C}), \mathbb{R}(1))^+ .$$

<u>Conjecture</u> (Beilinson) (weak form)

(a) $r \otimes \mathbb{R}$ is surjective

(b) $Im(r)$ contains a $\mathbb{Q}$-lattice whose volume is (up to a non-zero rational number) the leading term at $s = 1$ of $L^{(2)}(X,s)$.

Here $L^{(2)}(X,S)$ denotes the H^2-piece of the Hasse-Weil zeta function of X . Bloch proved this conjecture when $X = Jac(X_o(37))$, where $X_o(37)$ is the standard modular curve of level 37. Then Beilinson proved this for $X = M \times M$ where M is <u>any</u> modular curve/$\mathbb{Q}$.

<u>Theorem</u>: Let X be any Hilbert modular surface/$\mathbb{Q}$. Then the conjecture of Beilinson is true for X .

We use the techniques and results of Harder, Langlands, Rapoport and Beilinson, and some techniques from representation theory.

R.W. THOMASON: Equivariant algebraic K-theory

Consider a linear algebraic group G acting on a noetherian scheme X. The exact category of G-coherent modules on X gives rise to equivariant algebraic K-theory spectra $G(G,X)$. There are analogues of the localization theorem, the calculation of the K-groups of G-linear projective space bundles, and a homotopy "axiom" for torsers under a G-linear vector bundle.

For schemes over a field, there is a G-equivariant Riemann-Roch theorem for the map of the algebraic K-theory spectrum to the topological K-theory spectrum. This yields a general Lefschetz-Riemann-Roch formula, extending previous results of Atiyah, Bott, Segal, Baum, Fulton, Quart, and Nielsen.

For schemes X of finite type over an algebraically closed field, the IG-adic completion of the ring of mod ℓ^n equivariant algebraic K-groups localized by inverting the Bott element is isomorphic to the ring of equivariant topological K-homology groups.

M. KNEBUSCH: Semialgebraic K-theory

Report on some work with Hans Delfs. Let M be a semialgebraic subset of the set $V(R)$ of rational points of an algebraic variety over a real closed field R. We define orthogonal K-groups $KO^i(M)$ which have properties similar to the orthogonal K-groups in topology, and in fact coincide with them for $R = \mathbb{R}$. At least for V affine the ring $KO^o(V(R))$ has a close connection to the Witt ring $W(V)$ of V (Brumfiel). If X is a variety over an algebraically closed field C of characteristic zero we have, after choice of a field R with $R(\sqrt{-1}) = C$, KO-groups $KO^i_R(X)$ since we can regard $X(C)$ as a semialgebraic set over R. The dependence of these groups on the choice of R is still mysterious.

S. CHAŁADUS: The l-rank of the $K_2 o_F$ for a number field F

1. If G_F denotes the quotient group H_2F/H_2^lF , then it is possible to calculate from the description of J. Brovkin the group G_F for several number fields. We find this group for $F = \mathbb{Q}(\zeta_{p^r})$, $\mathbb{Q}(\sqrt[p]{d})$, $\mathbb{Q}(\zeta_p, \sqrt[p]{d})$ and $\mathbb{Q}(\zeta_{p^r})^+$ - the maximal real subfield of the cyclotomic field $\mathbb{Q}(\zeta_{p^r})$.

2. Let $\Gamma = \{a \in F^* : \{\zeta_l, a\} \in K_2 o_F\}$ and $j(l) = rk_l(Cl(F)/Cl_l(F))$, where $Cl_l(F)$ is the subgroup of the ideal class group $Cl(F)$ of F, generated by divisors of the ideal (l) in o_F .

Theorem 1: If $\zeta_l \in F$, then $rk(\Gamma/(F^*)^l) = r_1+r_2+g(l)+j(l)$, where $g(l)$ is the number of different prime factors of the ideal (l) and $r_1 (r_2)$ is the number of real (complex) places of F .

Theorem 2: If $\zeta_l \in F$, then $rk_l(K_2 o_F) = r_1-1+g(l)+j(l)$.

The proof of theorem 1 runs as the proof of the theorem of J. Browkin (case $l = 2$). We need the Dirichlet-Hasse-Chevalley theorem on units and a very simple lemma. Theorem 2 follows from theorem 1.

M. KOLSTER: On the 2-primary part of the Birch-Tate conjecture

Let o be the ring of integers in a totally real number field E . The Birch-Tate conjecture predicts $|K_2(o)| = w_2(E) \cdot |\zeta_E(-1)|$, where ζ_E is the ζ-function of E and $w_2(E) = 2 \cdot \prod_p p^{n(p)}$, $n(p)$ being the largest integer n such that $E(\zeta_{p^n})/E$ is quadratic.

Theorem 1: If the 2-Sylow-subgroup of $K_2(o)$ is elementary abelian the 2-primary part of the Birch-Tate conjecture holds.

This theorem is deduced using results of K.S. Brown on the 2-fractional part of $\zeta_E(-1)$ from the following structure theorem:

Let $F = E(\sqrt{-1})$, S = set of dyadic and infinite primes of E , $C^S(E)$ (resp. $C^S(F)$) the 2-Sylow-subgroup of the class-group of the ring of S-integers in E (resp. F).

Theorem 2: The 2-Sylow-subgroup of $K_2(o)$ is elementary abelian if and only if the dyadic primes of E are undecomposed in F and the kernel of the norm map $C^S(F) \to C^S(E)$ is elementary abelian of order $rk(C^S(E))$.

Among others the proof uses a generalized relative genus theory.

B. MAGURN: Reviews in K-Theory

Through lengthy consultations with every conference participant, a comprehensive subject classification of algebraic K-theory has been developed for use in organizing an American Mathematical Society publication of collected "Reviews in K-Theory", and for consideration in amending the MR-ZBL subject classification of mathematics, to take into account the emergence of K-theory as a major, active field of research. Experts in each specialty of K-theory contributed much time and effort in these consultations. The "Reviews in K-Theory" will be completed and published early in 1985.

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