

T a g u n g s b e r i c h t 25/1984

Orders and their applications

3.6. bis 9.6.1984

The organizers of this "Tagung" were Irving Reiner (Urbana) and Klaus Roggenkamp (Stuttgart).

In the past 4 years since the last meeting, much progress has been made in various aspects of the topic, and most of it has been reported in the lectures of our meeting:

I. GALOIS MODULE STRUCTURE, NON-ABELIAN CLASSFIELD THEORY AND ANALYTIC NUMBER THEORY OF ORDERS

J.Brinkhuis (Rotterdam) Galois modules and embedding problems

C.Bushnell (London) Principal orders II

M.Desrochers (Cambridge) Torsion Galois modules

A.Fröhlich (London) Principal orders I

L.McCulloh (Urbana) Stickelberger relations, monoid rings and Galois module structure

A.Raggi (Urbana) Zeta-functions of 2-sided ideals in arithmetic orders

I.Reiner (Urbana) Zeta- and L-functions, a survey

M.Taylor (Cambridge) Galois modules and elliptic functions

II. K-THEORY OF ORDERS AND CONNECTION WITH ALGEBRAIC GEOMETRY

M.Auslander (Waltham) Almost split sequences and isolated singularities

J.Brzezinski (Göteborg) Algebraic geometry of quaternion orders

- J. Carlson (Athens) Nilpotent elements in the Green ring
W.v.d.Kallen (Utrecht) The Merkurjev-Suslin Theorem
A.Kuku (Ibadan) K-theory of group rings over maximal
orders in division algebras
R.Oliver (Aarhus) A survey of $K_0(\mathbb{Z}G)$
I.Reiten (Trondheim) Presentations of Grothendieck groups
P.Salberger (Göteborg) Class groups of orders in algebras over
function fields
D.Webb G_0 of dihedral and quaternion groups

III. APPLICATIONS TO GROUP THEORY AND GROUP REPRESENTATIONS

- G.Janusz (Urbana) Units in crossed product orders
W.Kimmerle (Stuttgart) Decomposition of relation cores of non-
soluble groups
R.Mollin (Calgary) The Schur group of a commutative ring
W.Plesken (Aachen) Finite unimodular groups of prime degree
and circulants
J.Ritter (Augsburg) On a Zassenhaus conjecture about units in
group rings
K.W.Roggenkamp (Stuttgart) Isomorphisms of p-adic group rings I
R.Sandling (Manchester) Computer calculations of units in modular
group algebras
L.L.Scott (Charlottesville) Isomorphisms of p-adic group rings II
P.Webb (Manchester) Permutation modules and group cohomology
H.Zassenhaus (Columbus) On A-group rings

IV. CLASSIFICATION OF INDECOMPOSABLE LATTICES

- G.Cliff (Edmonton) Crossed product orders
- E.Dieterich (Bielefeld) The Auslander-Reiten quiver of a non-
domestic tame group ring
- R.Guralnick(Los Angeles) Isomorphism of modules under ground
ring extensions
- W.Gustafson (Lubbock) Hereditary orders
- L.Levy (Madison) Modules over Dedekind-like rings
- W.Rump (Eichstätt) Representations of tiled orders and
module valuations
- A.Wiedemann (Stuttgart) Auslander-Reiten quivers of local
Gorenstein orders of finite type

V. APPLICATIONS TO CONCRETE PROBLEMS

- H.Lenstra (Amsterdam) Applications of ring theory to number-
theoretic algorithms

Summarizing, we are proud to say that many of the people interested in our subject were present at this meeting. However, since the last meeting 4 years ago - a too long time - there was not enough time to cover the recent interesting developments thoroughly.

There were always lively mathematical discussions, which led to new insights and sometimes to new results.

In the evenings we socialized together over glasses of wine and continued our discussions in a friendly atmosphere.

Vortragsauszüge

M.AUSLANDER: Almost split sequences and isolated singularities

Let R be a fixed complete regular local ring. An R -algebra Λ which is a finitely generated free R -module, is called an isolated singularity, if for all nonmaximal prime ideals $\mathfrak{p} \subset R$ we have that $\text{gl.dim } \Lambda_{\mathfrak{p}} = \text{gl.dim } R_{\mathfrak{p}} = \dim R_{\mathfrak{p}}$. Suppose Λ is an R -algebra and finitely generated free R -module and let $\mathcal{P}_R(\Lambda)$ be the category of Λ -modules which are free R -modules. Then we have the following theorem: Λ is an isolated singularity if and only if $\mathcal{P}_R(\Lambda)$ has almost split sequences.

J.BRZEZINSKI: Algebraic geometry of quaternion orders

Let R be a Dedekind ring with field of quotients K and A a central simple algebra over K of dimension n^2 . Each R -order Λ in A defines a $\text{Spec}R$ -scheme X_{Λ} . The set of R' -rational points, for a commutative R -algebra R' , is the set of R' -projective left Λ' -ideals I' of R' -rank n such that Λ'/I' is also R' -projective. We look at the schemes X_{Λ} in the particular case of quaternion algebras A . In this case, X_{Λ} is integral if and only if Λ is Gorenstein, normal if and only if Λ is Bass, and regular if and only if Λ is hereditary. Each Gorenstein order Λ defines in a natural way a Bass order $B(\Lambda)$ such that $X_{B(\Lambda)}$ is the normalization of X_{Λ} . For each Bass order Λ there is a sequence of orders $\Lambda = \Lambda_0 \subset \Lambda_1 \subset \dots \subset \Lambda_r$ such that $X_{\Lambda_{i+1}}$ is an elementary

transform of X_{λ_i} at singular points in one of its fibers (suitable blowings-up followed by a suitable contraction) and X_{λ_r} is a regular scheme.

J. BRINKHUIS: Galois modules and embedding problems

We combine the embedding problem with the problem of Galois module structure of rings of integers. We derive a necessary condition for the solvability of the "embedding problem with prescribed free Galois module structure". This is analogous to the classical condition of Hasse-Wolf for the embedding problem. Our approach leads to a number of explicit results, for example: no cyclic extension of the rationals of odd prime power order has a normal integral basis over any proper intermediate subfield. A basic tool is a map from a Hochschild-Serre sequence to a Fröhlich-Wall sequence. One intriguing feature of this diagram is that two of its vertical maps are not in general a homomorphism, but only have a weak multiplicative property.

J. CARLSON: Nilpotent elements in the Green ring

Let G be a p -group and let R be an integral domain in which p is not a unit. Let $\mathfrak{N}(RG)$ be the Green ring or representation ring of RG -lattices. The speaker and David Benson have found a new method for finding nilpotent elements in $\mathfrak{N}(RG)$ for many p -groups G . The method improves on that used by Zemanek in that it gives an infinite number of examples and it substitutes

a cohomology calculation for the more difficult tensor product calculation.

Let K be a field of characteristic $p > 0$. Benson has shown that if M is an absolutely indecomposable KG -module then, for any indecomposable N , K is a direct summand of $M \otimes N$ if and only if $N \cong M^*$ and p does not divide the dimension of M .

Let $\mathfrak{A}(KG, p)$ be the subgroup of $\mathfrak{A}(KG)$ generated by all $[M]$ such that p divides the dimension of every component of $K' \otimes M$ for any extension K' of K . Then $\mathfrak{A}(KG, p)$ is an ideal in $\mathfrak{A}(KG)$ and $\mathfrak{A}(KG)/\mathfrak{A}(KG, p)$ has no nilpotent elements. The speaker working with M. Auslander has discovered a proof of Benson's result that appears to extend these results to $\mathfrak{A}(RG)$ for R a complete D.V.R.

G. CLIFF: Crossed product orders

Let K/\mathfrak{t} be a finite Galois extension of local fields (with $[\mathfrak{t} : \mathbb{Q}_p]$ finite) with Galois group G , and rings of integers $\mathcal{O}; \mathfrak{o}$ (resp.). Let $A = (K/\mathfrak{t}, \rho)$ be a crossed product algebra where ρ is a factor set on G with values in \mathfrak{o}^* , and let $\Lambda = (\mathcal{O}/\mathfrak{o}, \rho)$ be the crossed product order in A .

Let $\Lambda_0 = \Lambda$, and $\Lambda_{i+1} = \mathcal{O}_1(J(\Lambda_i))$ be the left order of the Jacobson radical of Λ_i . Then

$$\Lambda_0 \subsetneq \Lambda_1 \subsetneq \Lambda_2 \subsetneq \dots \subsetneq \Lambda_s = \Lambda_{s+1} = \dots = \Lambda_\infty.$$

It is shown that $s = d - (e-1)$, where $\mathfrak{D}_{K/\mathfrak{t}} = \mathfrak{P}^d$, \mathfrak{P} the maximal ideal of \mathfrak{o} , $\mathfrak{D}_{K/\mathfrak{t}}$ the different, $e = e(K/\mathfrak{t})$ the ramification index. Also, the type numbers of the hereditary order Λ_∞ are $\underbrace{(f, f, f, \dots, f)}_{e/m \text{ times}}$ where $f = f(K/\mathfrak{t})$ is the inertial degree, and m is the Schur index of A .

M. DESROCHERS: Torsion Galois modules

Let K/\mathbb{Q} be a finite Galois extension of algebraic number fields with Galois group Γ , and let \mathcal{O} be the ring of integers of K , \mathfrak{o} , the ring of integers of \mathbb{Q} . The trace map gives an $\mathfrak{o}\Gamma$ -isomorphism between \mathfrak{C} , the codifferent of the extension K/\mathbb{Q} , and $\text{Hom}_{\mathfrak{o}}(\mathcal{O}, \mathfrak{o})$, the dual of \mathcal{O} , enabling one to use the torsion module $T = \mathfrak{C}/\mathcal{O}$ to "measure" the difference between \mathcal{O} and its dual. More precisely, if S is a fixed set of primes of \mathfrak{o} , let $G_{\mathfrak{O}}^S(\mathfrak{o}\Gamma)$ (respectively $K_{\mathfrak{O}}^S(\mathfrak{o}\Gamma)$) denote the Grothendieck group corresponding to the category of finitely generated \mathfrak{o} -torsion free $\mathfrak{o}\Gamma$ -modules (respectively also locally projective at the primes of \mathfrak{o} outside S), with the relations arising from short exact sequences splitting at the primes of \mathfrak{o} outside S . By computing the class of T in a suitably chosen Grothendieck group one obtains a purely algebraic proof of:

Theorem (Cassou-Noguès, Queyrut): If $S_{\mathbb{Z}}$ contains all the rational primes with a divisor in \mathfrak{o} wildly ramified in K , then $[\mathcal{O}] = [\text{Hom}_{\mathfrak{o}}(\mathcal{O}, \mathfrak{o})]$ in $K_{\mathfrak{O}}^{S_{\mathbb{Z}}}(\mathbb{Z}\Gamma)$; as well as another theorem computing the difference $[\text{Hom}_{\mathfrak{o}}(\mathcal{O}, \mathfrak{o})] - [\mathcal{O}]$ in $G_{\mathfrak{O}}^S(\mathfrak{o}\Gamma)$, where S contains the primes of \mathfrak{o} wildly ramified in K . This difference is seen to depend only on the ramification groups $\Gamma_{\mathfrak{p}, \mathfrak{o}}$ and $\Gamma_{\mathfrak{p}, \mathfrak{i}}$, and on the class of \mathfrak{p} in the ideal class group of \mathfrak{o} , where \mathfrak{p} runs through the primes of \mathfrak{o} ramified in K , \mathfrak{P} is a prime of \mathcal{O} above \mathfrak{p} , and $\Gamma_{\mathfrak{p}, \mathfrak{i}} = \{\gamma \in \Gamma \text{ such that } \gamma(x) - x \in \mathfrak{P}^{i+1} \text{ for all } x \in \mathcal{O}\}$. In particular, $[\mathcal{O}] = [\text{Hom}_{\mathfrak{o}}(\mathcal{O}, \mathfrak{o})]$ in $G_{\mathfrak{O}}^S(\mathfrak{o}\Gamma)$ if all the primes of \mathfrak{o} ramified in K are principal.

E. DIETERICH: The Auslander-Reiten quiver of a non-domestic tame group ring

Let C_3 be the cyclic group of order 3, R a complete discrete valuation ring in which the prime number 3 has valuation 4, and let $\Lambda = RC_3$ be the corresponding group ring. Let k be the residue class field of R and put $\mathfrak{S} = \{\text{monic irreducible polynomials in } k[X]\} \cup \{\infty\}$. Then the stable Auslander-Reiten quiver $\mathfrak{A}_S(\Lambda)$ of the lattice category \mathfrak{L}_Λ consists of a $\mathbb{P}_1(k)$ -family of \mathfrak{S} -tubular series:

$$\mathfrak{A}_S(\Lambda) = \bigcup_{\beta : \alpha \in \mathbb{P}_1(k)} \mathfrak{I}_{\beta : \alpha}, \quad \mathfrak{I}_{\beta : \alpha} = \bigcup_{\lambda \in \mathfrak{S}} \mathfrak{I}_{\beta : \alpha}(\lambda). \quad \text{Each } \mathfrak{S}\text{-tubular series}$$

$$\mathfrak{I}_{\beta : \alpha} \text{ is of tubular type } \begin{cases} \widetilde{D}_4 & \text{if } k \text{ is a splitting field for } C_3 \\ \widetilde{C}D_3 & \text{if } k \text{ is not a splitting field for } C_3 \end{cases}$$

Moreover, let β and α be relatively prime integers, not both equal to zero. Then the dimension-type of the primitive series of $\mathfrak{I}_{\beta : \alpha}$ is given by $|\beta|b + |\alpha|a \in \mathbb{Z}^4$, where $b = (2,1,0,1)$ and $a = (1,0,1,0)$:

A. FRÖHLICH - C. BUSHNELL: Principal orders I and II

The basic arithmetic properties of a principal order \mathfrak{A} (where $\mathfrak{J}_\mathfrak{A}$ (its Jacobson radical) is left (hence right) principal) were noted and the congruence Gauss sums for admissible representations of the normaliser, of \mathfrak{A} , $G = G(\mathfrak{A})$ were introduced. The significance of these in connection and comparison with Galois Gauss sums was discussed.

continued by C. Bushnell

The relation between the Gauss sum $\tau(\rho)$ attached to a representation ρ of the normaliser $\mathcal{O}(\mathfrak{N})$ of a principal order \mathfrak{N} in a p-adic simple algebra A , and the Godement-Jacquet local constant $\mathcal{E}(\pi, s)$ attached to a representation π of A^* was given:

$$\mathcal{E}(\pi, s) = (-1)^{n(d-1)} N_{\mathfrak{N}}(D_{\mathfrak{N}} f(\rho))^{(1/2 - s)/n} \frac{\tau(\rho)}{\sqrt{Nf(\rho)}}$$
$$n^2 = \dim_{Z(A)}(A), A \cong M_m(D), D \text{ a division algebra.}$$

R.GURALNICK: Isomorphism of modules under ground ring extensions

Let D be a Dedekind domain and R a module finite D -algebra. If M is a finitely generated R -module, there is an equivalence of categories between the classes of modules which are summands of M^e for some e and finitely generated projective R' -modules where R' is a D -order in a semi-simple algebra. In particular, this equivalence preserves the genus. Hence for D the ring of algebraic integers in a number field, one obtains a generalization of Jacobinski's cancellation theorem and a variation of his extension of base ring theorem (M and N are in the same genus if and only if $M \otimes_D D' \cong N \otimes_D D'$ for some larger ring of algebraic integers D'). We also discuss another proof of the extension theorem which depends essentially only on the fact that one is in the stable range of the ring of all algebraic integers.

W.GUSTAFSON: Hereditary orders

We show that an order is hereditary if and only if all of its artinian factor rings are of finite representation type. The

proof depends on a theorem about artin algebras that may be of independent interest. The work was done jointly with Edward L.Green.

G.JANUSZ: Units in crossed product orders

Let K be either a totally real number field, normal over \mathbb{Q} or a totally imaginary quadratic extension of such a field. Then K admits complex conjugation $x \rightarrow \bar{x}$. Let G be a finite group and J the involution on KG defined by $J(\sum \alpha_g g) = \sum \bar{\alpha}_g g^{-1}$. R_K = algebraic integers in K .

Theorem: Let Λ be any R_K -order in KG which contains $R_K G$. Then $U_J(\Lambda) = \{\lambda \in \Lambda : \lambda J(\lambda) = 1\}$ is a finite group containing G . If $G \subseteq H \subseteq (KG)^*$ with H finite, then $H \subseteq U_J(\Lambda)$ for some order Λ .

Consider the case $G =$ Frobenius group of order $p(p-1)$, p an odd prime. One can explicitly determine the orders in $\mathbb{Q}G$ containing ZG . They can be indexed as $\Gamma_1, \dots, \Gamma_{p-1}$ and for these orders $U_J(\Gamma_i)$ can be determined. In 4 cases this group is $\langle -1 \rangle \times \text{Sym}(p)$. If $p+1$ is divisible by 4, 6, 8 or 12, then some groups are $\langle -1 \rangle \times \text{PGL}(2, p)$ (p at most 8). The remaining groups are $\langle -1 \rangle \times G$.

W.v.d.KALLEN: The Merkurjev-Suslin theorem

Let n be a positive integer, F a field so that $1/n \in F$. The Merkurjev-Suslin theorem states that the Galois symbol

$$\alpha_F : K_2(F)/n K_2(F) \rightarrow H^2(F, \mu_n^{\otimes 2})$$

is an isomorphism.

See Math. USSR Izvestiya Volzi, 1983, 307 - 340. Discussed was Merkurjev's more elementary proof of this theorem, based on Hilbert 90 for K_2 and specialization arguments similar to those used in his original proof for the case $n = 2$.

W.KIMMERLE: Decomposition of relation cores of non-soluble groups

Theorem. Denote by S_n the finite symmetric group of degree n . Then relation cores of S_n decompose if, and only if, $n = p$ or $n = p + 1$, where p is an odd rational prime.

Remarks.

- a) The proof closes one gap left by Gruenberg and Roggenkamp on this topic and establishes that there exist finite insoluble groups with decomposable relation cores.
- b) The proof uses a criterion in terms of H^1 given by Gruenberg and Roggenkamp and yields the appropriate estimations for H^1 . In the case when $n = p$ (and only in this case) the proof requires the classification of the finite simple groups.
- c) The proof applies to many other finite groups. e.g. it follows that relation cores of every Zassenhaus group are decomposable.

A.KUKU: K-theory of group rings over maximal orders in division algebras

If B is a regular ring, π a finite group, $n \geq 0$, let $K_n(\pi, P(B))$ be the K_n -group of the category $[\pi, P(B)]$ of

π -representations in the category $P(B)$ of finitely generated projective B -modules. Swan proved that if R is a semi-local Dedekind domain with quotient field F , then the canonical map $K_0(\pi, P(R)) \rightarrow K_0(\pi, P(F))$ is an isomorphism. We prove that $K_0(\pi, P(A)) \xrightarrow{\delta} K_0(\pi, P(D))$ is not injective if A is a maximal order in a central division algebra D over a p -adic field F thus showing that Swan's result is not true in general if R is non-commutative. We then compute $\text{Ker } \delta$.

We also compute for all $n \geq 0$, $K_n(\pi, P(A))$, $K_n(\pi, P(A_p))$, $SK_n(\pi, P(A))$, $SK_n(\pi, P(A_p))$, $SK_n(\pi, P(\hat{A}_p))$ where $A_p(\hat{A}_p)$ is the localization (completion) of A at a prime p of R if R is the ring of integers in a number field.

H.LENSTRAS: Applications of ring theory to number theoretic algorithms

In this lecture it is shown how Galois theory for finite rings underlies most practical primality testing methods. Let A be a Galois extension of $\mathbb{Z}/n\mathbb{Z}$ with group G ; i.e., A is a $\mathbb{Z}/n\mathbb{Z}$ -algebra, commutative, that is f.g. free as a $\mathbb{Z}/n\mathbb{Z}$ -module. and $A \otimes A \rightarrow \prod_{\sigma \in G} A$, $a \otimes b \mapsto (a \circ (b))_{\sigma \in G}$ is an isomorphism. Assume G is abelian. Then for every prime r dividing n there is a unique $\varphi_r \in G$ (the Artin symbol) with $\forall x \in A$ $\varphi_r(x) \equiv x^r \pmod{rA}$. Extend this definition to all $r|n$ by $\varphi_{rr'} = \varphi_r \varphi_{r'}$. The decomposition group $D \subset G$ is defined to be the subgroup of G generated by all φ_r , $r|n$. Clearly $\langle \varphi_n \rangle \subset D$, with equality if n is prime. Many primality testing methods can be interpreted as attempting to show that

$\langle \varphi_n \rangle = D$. For example, if there is a ring homomorphism $A \xrightarrow{\langle \omega_n \rangle} \mathbb{Z}/n\mathbb{Z}$ (mapping 1 to 1) then we must have $D = \langle \varphi_n \rangle$. Applying this to $A = \mathbb{Z}[\zeta_s]/(n)$ (cyclotomic, with $\gcd(s, n) = 1$) with $G \cong (\mathbb{Z}/s\mathbb{Z})^*$ this leads, if n passes certain tests, to the information that $\forall r|n : \exists i : r \equiv n^i \pmod{s}$. If s is large and $\#\langle n \pmod{s} \rangle$ is small this can be used to check whether n is prime. The best methods used nowadays rely on the same ideas but are somewhat more involved. For $n \lesssim 10^{100}$ one can use

$$s = 2 \cdot 5040 \cdot \prod_{q \text{ prime}, q-1|5040} q = 2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 29 \cdot 31 \cdot \dots \cdot 1009 \cdot 2521 \approx 1.5 \cdot 10^{52};$$

then $\#\langle n \pmod{s} \rangle \leq 5040$, for $\gcd(n, s) = 1$. The resulting test runs for approximately 45 seconds.

L.LEVY: Modules over Dedekind-like rings

There are very few commutative Noetherian rings, all of whose (finitely generated) modules are known. Dedekind-like rings are a generalization of Dedekind domains. Their modules can be described, together with the direct-sum behavior and local-global behavior of these modules. Examples of Dedekind-like rings are

- (i) Group rings $\mathbb{Z}G_n$, G_n cyclic of square-free order n ;
- (ii) some rings of algebraic integers that are not integrally closed in their field of fractions; and
- (iii) many subrings of $\mathbb{Z} \oplus \dots \oplus \mathbb{Z}$.

Artinian homomorphic images of Dedekind-like rings (the module-structure theory obviously applies here, too) include the well-

discussed ring $K[x,y]/(x,y)^2$, Nazarova and Roiter's dyad of two discrete valuation rings, and all (commutative) artinian PIRs.

Main results: As with Dedekind domains, we associate an ideal class $cl M$ with each R-module M in such a way that $M \cong N \Leftrightarrow M$ is locally $\cong N$ and $cl M = cl N$. Moreover, $cl(M \otimes N) = (cl M)(cl N)$, and $cl M$ is projective of rank 1 over some ring between R and its integral closure.

L.McCULLOH: Stickelberger relations, monoid rings and Galois module structure

Let K be a number field with ring of integer \mathfrak{o} , and G a finite abelian group. To G , one can associate a certain commutative monoid E and a Stickelberger submodule S^* of the dual $ZE^* = \text{Hom}_{\mathbb{Z}}(ZE, \mathbb{Z})$ such that

(i) if $G \sim (l^n, \dots, l^n)$, l an odd prime, then

$$|Cl(ZG)^{-}| = [ZE^{*-} : S^{*-}] \text{ and}$$

(ii) if G is any abelian l -group (l odd), then

$$|{}^{\circ}D(ZG)| = |(ZE^{*+}/S^{*+})_{\text{tors}}| \text{ or more generally, if } G \text{ has odd order or has cyclic 2-primary component, then}$$

$$|(ZE^{*+}/S^{*+})_{\text{tors}}| = |A_{\mathfrak{o}}(G)| \quad (A_{\mathfrak{o}}(G) = \text{the cokernel of Artin induction}).$$

If $R(\mathfrak{o}G)$ (resp. $R_d(\mathfrak{o}G)$) is the subset of $Cl(\mathfrak{o}G)$ consisting of the Galois module classes of tame (resp. domestic) extension L/K with $\text{Gal}(L/K) \cong G$, then one can define an action of ZE^* on the group I' of $\mathfrak{o}G$ -ideals relatively prime to $|G|$ in terms of which one can characterize the elements of $R_d(\mathfrak{o}G)$.

Moreover, one can show that $R_d(oG) \supseteq$ the image of $(I')^{S^*}$ under the natural surjection $I' \rightarrow Cl(oG)$. In particular $Cl(ZG)^{S^*} = (1)$. One can also show $R_d(oG)$ is a group.

R.MOLLIN: The Schur group of a commutative ring

We define the Schur subgroup $S(R)$ (for a commutative ring R with identity), of the Brauer group $B(R)$, to consist of those classes having a representative A such that there exists a finite group G and an R -algebra epimorphism $f: RG \rightarrow A$. If R is a commutative ring of non-zero characteristic then $S(R) = (0)$. On the other hand, any finite abelian group is the Schur group of a commutative ring which is finitely generated as an algebra over the rational integers. We generalize several standard facts about the Schur group of a field to commutative rings with finitely many idempotents. We also investigate two subgroups of $S(R)$, one generated by cyclotomic algebras and the other by homomorphic images of separable group algebras.

(joint work with Frank De Meyer)

R.OLIVER: A survey of $K_0(ZG)$

The current state of knowledge about the $D(ZG) \subseteq Cl(ZG) \subseteq K_0(ZG)$ finite G was summarized. The groups

$D(ZG)$	G a 2-group
$D(ZG)^-$	G a p -group, p odd
$D(ZG)^+$	G a p -group, p odd and regular

are now fairly well understood: formulas have been derived for

their orders, and relatively simple algorithms for computing their structure are known. Also, Steve Ullom has results which describe $D(\mathbb{Z}G)^+$ in many cases where G is a cyclic p -group and p an irregular prime.

The difficult problem is thus to understand the kernel groups $D(\mathbb{Z}G)$ for G not of prime power order. Results on this problem worth mentioning include:

- (1) Matchett has computed $|D(\mathbb{Z}C_n)|$ when n is squarefree
- (2) Martin Taylor has described $D(\mathbb{Z}S_n) (=Cl(\mathbb{Z}S_n))$: at least modulo 2-torsion
- (3) Milgram has made computations in the $D(\mathbb{Z}G)$ for certain semi-direct products: $G \cong C_{pq} \rtimes Q(8)$ ($p \neq q$ odd primes); and succeeded in determining whether or not certain projective modules arising topologically are stable free.

W.PLESKEN: Finite unimodular groups of prime degree and circulants

The maximal finite irreducible subgroups of $GL(p, \mathbb{Z})$ for prime degrees $p \leq 23$ are classified up to conjugacy. Due to the fact that the p -th cyclotomic field has class number 1 for $p \leq 19$ they can be represented as integral automorphism groups of quadratic forms whose Gram matrix is a circulant. The additional cases in dimension 23 are related to the Leech lattice. For dimensions $p \leq 11$ and $p = 19$ all groups are essentially reflection groups. For dimensions 13, 17 and 19 it was necessary to compute the integral automorphism groups of some quadratic forms by machine.

A. RAGGI: Zeta-functions of 2-sided ideals in arithmetic orders

The work begins with an introduction to the theory of Z- and L-series. The basic plan is to compare these series with a Z-integral whose analytic properties are more accessible, and then use these properties to obtain some analogous ones of Z- and L-series. Next the theory of two-sided ideals is studied. First we translate the general theory just developed to our context; then we obtain explicit formulas for the zeta functions of orders in simple algebras, and we calculate the zeta functions for a quaternion algebra and for the integral group ring of a dihedral group of order $2p$. We also study, in the simple case, the behavior of the zeta functions at their largest pole. We conclude with a discussion of some possible generalizations of the prime ideal theorem to two-sided ideals of arithmetic orders in simple algebras.

I. Reiner: Zeta- and L-functions, a survey

Let Λ be an R-order in a f.d. semisimple K-algebra A , where $R = \text{alg.int.}\{K\}$, $\dim_{\mathbb{Q}} K$ finite. L. Solomon defined a zeta function $\zeta_{\Lambda}(s) = \sum_X (nX)^{-s}$, where X ranges over all left ideals of Λ of finite norm $nX = (\Lambda : X)$. Let $g(\Lambda) = \text{genus of } \Lambda = \text{set of locally free ideals } X$, and let $Cl \Lambda$ be the locally free class group of Λ , whose elements are stable isomorphism classes $[M]$, $M \in g(\Lambda)$. For each linear character $\psi : Cl \Lambda \rightarrow \mathbb{C}^*$, we introduce an L-function

$$L_{\Lambda}(s, \psi) = \sum_{X \in g(\Lambda)} \psi(X) (nX)^{-s}, \quad \text{Re } s > 1.$$

There are Euler products

$$\zeta_{\Lambda}(s) = \prod_P \zeta_{\Lambda_P}(s), \quad L_{\Lambda}(s, \psi_P) = \prod_P L_{\Lambda_P}(s, \psi_P)$$

where P ranges over all maximal ideals of R and $\psi_P : A_P \rightarrow \mathbb{C}^*$. In the local case, $L_{\Lambda_P}(s, \psi_P)$ can be expressed as a zeta integral $\int_{A_P^*} \hat{\psi}(x) \|x\|^s \psi(x) d^*x$ over the locally compact group A_P^* , with $\hat{\psi}$ a locally constant function of compact support, $\|x\| = P$ -adic absolute value, $d^*x =$ Haar measure. A key theorem shows the existence of a "common denominator" $L_{A_P}(s, \psi_P)$ for all zeta integrals (as $\hat{\psi}$ varies). When $\hat{\psi} = 1$ on $(\Lambda'_P)^*$, where $\Lambda'_P =$ maximal order, $L_{\Lambda}(s, \psi_P) = L_{\Lambda'_P}(s, \psi_P)$.

The above implies that $\zeta_{\Lambda_P}(s)/\zeta_{\Lambda'_P}(s) \in \mathbb{Z}[P^{-s}]$, with analogous results for L -functions. Combining these facts with the Euler products, we obtain analytic continuation of zeta and L -functions in the global case, as well as their behavior at $s = 1$. Consequences:

- 1) Given a left ideal M of Λ , the number of $X \subset \Lambda$ with $X \simeq M$ and $nX \leq T$ is asymptotic (as $T \rightarrow \infty$) to

$$\frac{1}{(r-1)!} \frac{(M^{-1} : \Lambda)}{(\Lambda' : M)} (\Lambda'^* : \text{Aut}_{\Lambda} M) b_{\Lambda}, \quad T(\log T)^{r-1}, \quad \text{where}$$

$r =$ number of simple components of A , $\Lambda' =$ maximal order, and $M^{-1} = \{x \in A : Mx \subset \Lambda\}$. Here $b_{\Lambda} > 0$ depends on Λ' but not on M or Λ .

- 2) Given a class $c \in \text{Cl } \Lambda$ and an integer $k \geq 1$, the number of $X \in g(\Lambda)$ with $X \in c$, composition length $l(\Lambda/X) = k$, $nX \leq T$ is asymptotic to

$$\frac{N(k, c)}{(k-1)! k_{\Lambda}} \frac{T}{\log T} (\log \log T)^{k-1}$$

as $T \rightarrow \infty$, where $k_{\Lambda} = |\text{Cl } \Lambda|$, and $N(k, c) =$ non-negative

integer. (Further, $N(k, c) = 0$ if and only if there are no $X \in c$ with $l(\Lambda/X) = k$)

- 3) Given $M \in g(\Lambda)$, let M_1, \dots, M_k represent the isomorphism classes in the stable class $[M]$. Then $\sum_i (\Lambda^* : \text{Aut}_\Lambda M_i)$ is an explicitly computable constant, independent of the choice of $M \in g(\Lambda)$.

I. REITEN: Presentations of Grothendieck groups

We introduce the concept of coherent pair $(\underline{A}, \underline{B})$ of additive categories over a commutative ring R . We use Quillens long exact sequence of K -groups to study the Grothendieck group $K_0(\text{mod } \underline{B})$, where $\text{mod } \underline{B}$ is the category of finitely presented contravariant functors from \underline{B} to $\text{Mod } R$. We show that $K_0(\text{mod } \underline{A}/\underline{B}) \rightarrow K_0(\text{mod } \underline{A})$ is a monomorphism if $\text{mod } \underline{B}$ is regular or if every object in $\text{mod } \underline{B}$ has finite length, or if $\underline{B} = \text{mod } \Lambda$ where Λ is a classical order of finite lattice type in a simple algebra. We further show that if G is a finite subgroup of $GL(n, \mathbb{C})$ acting naturally on $\mathbb{C}[[X_1, \dots, X_n]]$, and the action of G on $V \setminus \{0\}$ ($V =$ corresponding n -dim. vector space) is free, then $K_0(\text{mod } R) = \mathbb{Z} \oplus$ finite group, where $R = \mathbb{C}[[X_1, \dots, X_n]]^G$.

J. RITTER: On a Zassenhaus conjecture about units in group rings

For a unit u of finite order in the integral group ring $\mathbb{Z}G$ of a finite group G one of the Zassenhaus conjectures states the existence of a group element g such that $u = \alpha g \alpha^{-1}$ with some invertible $\alpha \in \mathbb{Q}G$. It is shown that this conjecture

is true if $G = \langle a \rangle \rtimes \langle x \rangle$ is split metacyclic with either $(\text{ord } a, \text{ord } x) = 1$ or $\text{ord } a = p^m$, $\text{ord } x = pq$. p and $q \mid (p-1)$ being prime numbers here. Moreover, if G is a nilpotent class 2 group or a metacyclic p -group, then actually $u = \alpha g \alpha^{-1}$ where $n \equiv g \pmod{W}$, where W is any Whitcomb ideal in $\mathbb{Z}G$.

K.ROGGENKAMP - L.SCOTT: Isomorphisms of p -adic group rings I, II

Let G be a finite p -group. Our main result is that there is only one conjugacy class of subgroups of order $|G|$ in the group of normalized units (augmentation 1) of the p -adic group ring $\hat{\mathbb{Z}}_p G$. As a consequence we obtain a positive answer to the isomorphism problem for group rings over \mathbb{Z} of finite nilpotent groups, as well as any extension of a finite abelian group by a finite p -group. In the nilpotent case a conjecture of Zassenhaus, that the isomorphism may be achieved by a group automorphism followed by conjugation with a unit in the group ring over \mathbb{Q} , is verified.

The main result holds also with $\hat{\mathbb{Z}}_p$ replaced by \mathbb{Z}_p or \mathbb{Z}_π where π is a finite set of primes containing p . The consequences above also hold with \mathbb{Z} replaced by \mathbb{Z}_π , if π contains each prime divisor of the group order.

For such a \mathbb{Z}_π with G finite nilpotent, we have $\text{Piccent } \mathbb{Z}_\pi G = \prod_{ij} R_{ij} P_i \supseteq \prod \text{Outcent } (P_i)^{n_i}$ where P_i is a Sylow p -subgroup and R_{ij} is the center of a component of $\mathbb{Z}_\pi P'_i$, where $G = P_i \times P'_i$. As a consequence we show that the analogue of our main result for $\mathbb{Z}_\pi G$ need not hold, and that

there are non-isomorphic groups E, E' , extensions of G by abelian groups A, A' (for some G), with isomorphic "small" group rings.

$$\mathbb{Z}_\pi E/I(A) \cdot I(E) \simeq \mathbb{Z}_\pi E'/I(A') \cdot I(E')$$

W.RUMP: Representations of tiled orders and module valuations

Die Darstellungstheorie der vollreduziblen Ordnungen (tiled orders) soll als spezielles Beispiel für die Theorie der Modulbewertungen, die hier erstmalig vorgestellt wird, erläutert werden. Hierzu führen wir zunächst den allgemeinen Begriff der Modulbewertung, vom speziellen Fall herkommend, schrittweise ein. Es wird sich dann zeigen, daß diejenigen Modulbewertungen, die von Darstellungen vollreduzierbarer Ordnungen herkommen, von besonders einfacher Bauart sind, nämlich gewisse Funktionen auf Vektorräumen mit Werten in einem distributiven Verband, auf dem die unendliche zyklische Gruppe lokalendlich operiert. Als weiteres Beispiel soll der Fall der Darstellungen kommutativer Ordnungen kurz umrissen werden.

P.SALBERGER: Class groups of orders in algebras over function fields

Theorem: Let k be a field. $R = k[t]$, $K = k(t)$ and Λ be a hereditary R -order in a central simple K -algebra of prime index l . Then $Cl(\Lambda)$ is finite if

- (a) k global, $l \neq \text{ch}(k)$ or
- (b) k f.g. over \mathbb{Q} and there is a maximal left Λ -ideal M

such that $1 \nmid \dim_k \Lambda/M$.

To prove the theorem we use Galois descent. We choose a Galois extension k' of k such that $K' := k'(t)$ splits A and consider a group homomorphism $\phi : \text{Cl}(\Lambda) \rightarrow H^3(k'/k, K_q(\Lambda'))$, where $\Lambda' := k' \otimes_k \Lambda$. The proof of the theorem then consists of two parts. First we prove that $\text{Im } \phi$ is finite by using class field theory in case (a) and a theorem of Mordell-Weil-Néron in case (b). Then we prove that ϕ is injective if (a) or (b) holds by using a result of Merkurjev-Suslin and some calculations on $H_{\text{et}}^3(k, \mu_e^{\otimes 2})$.

R.SANDLING: Computer calculation of units in modular group algebras

Let G be a finite p -group, I the augmentation ideal of $\mathbb{F}_p G$, $V = 1+I$ the Sylow p -subgroup of the group of units of $\mathbb{F}_p G$. Using Fortran programs to calculate in $\mathbb{F}_p G$ (for $|G| \leq 27$), I have obtained workable presentations for V . Group theoretic properties of V are then investigated by use of the software packages CAYLEY (from Sydney) and SOGOS (from Aachen). Such experimentation has suggested new theoretical results such as:

Theorem: If G is of nilpotency class 2 and has elementary abelian commutator subgroup, then

- (i) V has a normal complement to G and
- (ii) G is determined by $\mathbb{F}_p G$.

M.TAYLOR: Galois modules and elliptic functions

Let K be an imaginary quadratic number field in which 2 splits. We let \mathcal{O}_K denote the ring of integers of K and we fix $\pi \in 1+4\mathcal{O}_K$ such that $(\pi, \bar{\pi}) = 1$. For $\alpha \in \mathcal{O}_K$ we let $K(\alpha)$ denote the ray classfield of K with conductor $\alpha\mathcal{O}_K$. We then construct an elliptic function \mathfrak{f} and a $4\pi^2$ division point for the complex torus \mathbb{C}/\mathcal{O}_K with the property that $\mathfrak{f}(\alpha)$ generates the ring of integers of $K(4\pi^2)$ as a Galois module over the associated order for the extension $K(4\pi^2)/K(4\pi)$.

D.WEBB: G_0 of dihedral and quaternion groups

Let R be a ring with 1, G a finite group, $G_0(RG)$ the Grothendieck group ^{of} finitely generated RG -modules. Methods used by H.Lenstra in the calculation of $G_0(RG)$ for G abelian are employed to obtain computations of $G_0(RG)$ for various non-abelian groups; for example, for the dihedral group D_{2n} of order $2n$, $\tilde{G}_0(ZD_{2n}) \cong \bigoplus_{\substack{d|n \\ d>2}} \text{Pic}(Z[\zeta_d, \frac{1}{d}]_+)$,

where $+$ denotes "maximal real subring".

A similar formula is obtained for generalized quaternion groups of order $4m$. Finally, if G is a finite group in which every element has prime-power order and $M \subseteq \mathbb{Q}G$ is a maximal \mathbb{Z} -order containing ZG , then the transfer $G_0(M) \rightarrow G_0(ZG)$ is an isomorphism. Similar computations are possible for the functors G_n , for $n > 0$.

P.WEBB: Permutation modules and group cohomology

The following theorem provides a means of computing the p-part of cohomology from p-local subgroups.

Theorem: Let the finite group G act simplicially on the simplicial complex Δ such that for each simplex σ ∈ Δ the isotropy group G_σ fixes σ pointwise. Suppose that for each subgroup H ≤ G with H/O_p(H) cyclic the fixed point complex Δ^H has Euler characteristic χ(Δ^H) = 1. Then

(a) $Z(p) \equiv \sum_{\sigma \in \Delta/G} (-1)^{\dim \sigma} Z(p) \uparrow_{G_\sigma}^G \pmod{\text{projectives}}$ in the Green ring $A(Z(p), G)$.

(b) For any ZG-module M and integer n

$$H^n(G, \mu)_p = \sum_{\sigma \in \Delta/G} (-1)^{\dim \sigma} H^n(G_\sigma, \mu)_p \text{ in } K_0(\text{finite abelian groups}, 0)$$

The hypotheses of the theorem are satisfied when Δ is either the simplicial complex of elementary abelian p-subgroups of G, or of all p-subgroups of G, or Δ is a Tits building of a finite Chevalley group in defining characteristic p.

A.WIEDEMANN: Auslander-Reiten quivers of local Gorenstein orders of finite type

We derive necessary conditions for the configuration of a Gorenstein order Λ of finite type. In particular these conditions give rise to a complete list of possible finite Auslander-Reiten quivers of local Gorenstein orders. Concrete examples show that the above conditions are also sufficient.

If Λ is a local Gorenstein order in the algebra $A = \prod_{i=1}^s (D_i)_{n_i}$, then s, n_1, \dots, n_s are already determined by the combinatorial structure of the AR-quiver of Λ. Then for a Λ all n_i 's are 1 and the valuation of all arrows in the AR-quiver is (1,1) if and only if Λ has the "same" AR-quiver as a simple plane curve singularity over C.

H. ZASSENHAUS: On A-group rings

The Brauer conjecture ($ZG \approx ZH$; G, H finite groups; $\Rightarrow G \approx H$) seems to be intimately connected with the conjecture of the Brauer invariance of the group ring ZG of a finite group G (For any automorphism α of ZG there is $x \in UQG$ such that $\alpha G = xGx^{-1}$) as is demonstrated in the case that G is an A-Sylowtower group

($G = G_0 \supset G_1 \supset \dots \supset G_s = 1$, $(G_{i-1}, G_{i-1}) \subseteq G_i$, $|G_{i-1} : G_i| = p_i^{n_i} > 1$, $(1 \leq i \leq s)$, p_1, \dots, p_s distinct prime numbers). It is shown that such groups are both Brauer invariant and affirmative for the Brauer conjecture. The methods seem to be suitable for showing the same thing for A-groups (Taunt 1947) which, it is suggested, are simply defined as finite groups in which every Sylow subgroup is abelian (equiv. every nilpotent subgroup is abelian). The case $s = 1$ is dealt with by D.G. Higman's thesis - Induction over s . Applying a theorem of Schur-Zassenhaus one uses the induction argument to the proof of the following theorem: Let

$G = A \rtimes B$; $(A, A) = 1$, $|A| = p^n > 1$, $p \nmid |B|$, p prime,
 $\alpha \in \text{Aut}_{\mathbb{Z}_p}(Z_p G)$, $\alpha(b) = b(b \in B)$, $\alpha a \equiv a \pmod{W_p}$ ($a \in A$) where
 $W_p = \Delta_{\mathbb{Z}_p} B \Delta_{\mathbb{Z}_p} A + \Delta_{\mathbb{Z}_p}^2 A$ is the Whitcomb ideal; let it also be known that α merely permutes the class sums $C_i (1 \leq i \leq s)$ over the G -conjugacy classes. Then $\alpha(C_i) = C_i$. Use of lattice theory, tensoring.

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