

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 2/1985

Mathematische Optimierung

6.1. bis 12.1.1985

Die mathematische Optimierung hat als Teilgebiet der angewandten Mathematik in den letzten Jahren eine immer weiter zunehmende Bedeutung gewonnen und befindet sich in einer Entwicklung, deren Verlauf noch nicht abzusehen ist. So stieß auch die diesjährige, nun schon fast zur Tradition gewordene Tagung über Mathematische Optimierung in Oberwolfach auf lebhaftes Interesse unter den Kollegen im In- und Ausland. Unter der Leitung der Herren B. Korte (Bonn) und K. Ritter (München) trafen sich Teilnehmer aus 12 Ländern, um Probleme der mathematischen Optimierung zu diskutieren und eigene Forschungsergebnisse vorzustellen.

Die 48 Einzelvorträge beschäftigten sich mit Fragen, die das ganze Spektrum der Optimierungstheorie ansprachen. Dabei kamen Problemstellungen der diskreten oder kombinatorischen Optimierung in gleichem Maße zum Ausdruck wie Problemstellungen der stetigen Optimierung. Das zentrale Thema war, die algorithmische Lösbarkeit von Optimierungsproblemen zu untersuchen, bzw. neue Lösungsmethoden zu entwickeln. Neben konkreten Problemen, z.B. aus der Signaltheorie, der Physik und der Verkehrssteuerung, standen auch methodische Fragen ganzer Problemklassen im Vordergrund. Die Vorträge beinhalteten etwa neue Ergebnisse über Schnittebenenverfahren, Netzwerkalgorithmen und Updateverfahren bei nichtlinearen Programmen ebenso wie neue Verfahren zur quadratischen und semi-infiniten Programmierung (für einen umfassenderen Überblick sei auf die beigegeführten Vortragsauszüge verwiesen). Zusätzlich wurde an einem Abend ein informelles Seminar abgehalten, das dem neuen Algorithmus zur Lösung linearer Programme gewidmet war und sich eines besonders regen Zuspruchs erfreute.

Die Veranstalter und Teilnehmer sind dem Mathematischen Forschungsinstitut Oberwolfach sehr dankbar, ohne dessen Gastfreundschaft der Erfolg dieser Tagung nicht denkbar gewesen wäre.

Zum Schluß sei auch sonst Unbenennbares erwähnt: der Mittwochnachmittagsspaziergang mußte ausfallen. Der ungewöhnlich eisige Frost hatte sogar das Heizöl der Pension Rauber "einfrieren" lassen.

### Vortragsauszüge

#### A. BACHEM: Extension lattices of matroids and oriented matroids

The point extensions of a matroid  $M$  are in one-one correspondence to the set of all linear subclasses of  $M$  which, ordered by inclusion, form the so-called extension lattice of  $M$ . Two matroids are said to be extension equivalent if their extension lattices are isomorphic. In this joint work with W. Kern we show that for two extension equivalent (oriented) matroids  $M$  and  $M'$  of equal rank,  $M^*$  is an adjoint of  $M$  if  $M^*$  is an adjoint of  $M'$  and use this result to prove:  $M$  is representable over the field  $F$  iff  $M'$  is. We also report on the "sticky conjecture" of Turzik and Poljak which states that a geometric lattice is modular iff any two of its extensions can be glued together. It is known for rank 3 geometries and we show that it suffices to consider rank 4 geometries which do not have an "intersection property".

#### F. BARAHONA: Compositions in the acyclic subdigraph polytope

Given a digraph  $D$  that is a 2-sum of two digraphs  $D_1$  and  $D_2$ , we characterise a system of linear inequalities that define the acyclic subdigraph polytope of  $D$ , provided that such systems are known for  $D_1$  and  $D_2$ . This fact is used to characterise this polytope for symmetric digraphs not contractible to  $K_{3,3}$ .  
(jointly with A. R. Mahjoub)

M. J. BEST: A quadratic programming algorithm

An algorithm is presented which determines a global optimum of an  $n$ -dimensional convex quadratic programming problem and a strong local minimum of a nonconvex problem. At each iteration, the method maintains a set of conjugate directions which span the space orthogonal to the gradients of the active constraints. These conjugate directions are used to construct the Newton direction at each step. A Householder transformation is used to modify the conjugate directions when a previously inactive constraint becomes active. The updating at each iteration requires  $O(n^2)$  arithmetic operations.

R.E. BIXBY: On minors of 3-connected matroids

We consider the problem to find minors of a given matroid which contain a specified element and are isomorphic to a given minor.  
(jointly with C.R. Coullard)

R.E. BURKARD: Traffic matrix decomposition for multiple access systems

In communication satellites and other time-division multiple access systems the problem arises to decompose a given  $(n \times n)$ -matrix in a weighted sum of permutation matrices such that the sum of the weights becomes minimal. For finding an optimal solution for this problem, Birkhoff's theorem can be used. If the decomposition is restricted to  $n$  matrices the problem turns out to be NP-hard (F. Reindl). If a fixed set of permutation matrices is used (e.g.  $2n$  such matrices), an optimal solution for this modified problem can be found by solving an assignment problem. This approach can be generalized by admitting arbitrary Boolean matrices in the decomposition. The optimal weights turn out to be the values of dual variables in a special max flow min cost network flow problem.

V. CHVATAL: Perfect graphs

A relaxation and special cases of the strong perfect graph conjecture are discussed.

A.R. CONN: Semi-infinite programming

We describe an exact penalty function for semi-infinite programming. The function is a generalisation of the exact  $l_1$  penalty function for nonlinear programming and may be used as a merit function for semi-infinite programming methods. The theoretical results are based essentially upon results given in the case of the  $l_1$  penalty function, complicated by the presence of an infinite number of constraints. The proofs are constructive and they give rise directly to an implemented algorithm.

Nevertheless, some undesirable properties of the generalised exact penalty function lead us to consider an approach based upon a non-exact penalty function for which the usually associated asymptotic ill-conditioned problems can be avoided by a suitable rearrangement of the required algebraic systems that need to be solved.

W. COOK: Sensitivity results in integer linear programming

We consider integer linear programming problems with a fixed coefficient matrix and varying objective function and right-hand-side vector. Among our results, we show that, for any optimal solution to a linear program  $\max\{wx : Ax \leq b\}$ , the distance to the nearest optimal solution to the corresponding integer program is at most the dimension of the problem of the problem multiplied by the largest subdeterminant of the integral matrix  $A$ . Using this, we strengthen several integer programming "Proximity" results of Blair and Jeroslow; Graver; and Wolsey. We also show that the Chvátal rank of a polyhedron  $\{x : Ax \leq b\}$  can be bounded above by a function of the matrix  $A$ , independent of the vector  $b$ .

(jointly with A.M.H. Gerards, A. Schrijver and E. Tardos)

D. CRYSTAL: Tag systems: A combinatorial abstraction of integral dependence

We propose a structure which generalizes matroids and abstracts the notion of integral dependence of rational vectors. Characterizations are given for these structures when they are in fact matroids and then when they model integral dependence exactly.

W.H. CUNNINGHAM: Submodular function minimization

Let  $g$  be an integer-valued submodular function, defined on subsets of an  $n$ -element set  $E$  and satisfying  $|g(A)| \leq M$  for all  $A \subseteq E$ . Grötschel, Lovász, and Schrijver discovered an algorithm for finding a minimizer of  $g$  which runs in time polynomial in  $n$  and  $\log M$ . Their algorithm is based on the ellipsoid method. Bixby, Cunningham, and Topkis found a combinatorial algorithm based on augmenting paths. We prove that a modified version of that algorithm runs in time polynomial in  $n$  and  $M$ .

J.E. DENNIS, Jr.: Extracting useful secant conditions from inaccurate ones

Many successful methods for nonlinear optimization are based on quasi-Newton or secant updating of approximations to appropriate derivative matrices. This updating involves at each step projecting the matrix to be updated into the affine set of matrices that satisfy an appropriate secant equation  $As = y$ . The proper  $y$  is always subject to cancellation as the optimization procedure converges, but sometimes the proper  $y$  is computationally inconvenient. This talk will suggest some procedures for extracting a useful  $\hat{y}$  from an approximation to a proper  $y$ .

(jointly with Ph. Vu and H. Walker)

U. DERIGS: A primal matching algorithm

We describe a matching algorithm which has shown to be highly efficient i.e. fast for large-scale matching problems over dense graphs. This method can either be viewed as a two-phase shortest augmenting path approach or as a clever implementation of the primal negative cycle approach: Starting with an optimal matching in a sparse subgraph obtained by applying the shortest augmenting path method the optimal matching in the complete graph is constructed by successively introducing the missing edges and reoptimizing via the shortest augmenting path method if necessary.

W. DEUBER: Fibonacci k-nim

The following game is discussed: 2 players alternately take objects from a pile. The first player takes  $1 \leq a_1 < n$  objects. For subsequent moves  $1 \leq a_{i+1} \leq ka_i$ . The player who takes the last object is winner. The kernel positions are given by generalized Fibonacci numbers  $f_{i+1} = f_i + \min\{f_j \mid f_j \geq \lceil f_i/k \rceil\}$  in particular for  $k = 2$ ,  $f_{i+1} = f_i + f_{i-1}$ . For sufficiently large piles we have  $f_{i+1} = f_i + f_{i-c(k)}$ ; where  $c(k) = \Omega(k \log k)$ .

D. DE WERRA: Odd chain packings and coverings

An odd chain packing  $P$  (ocp) is a collection of edge-disjoint chains of odd length whose endpoints are all distinct. The cardinality  $|P|$  of  $P$  is the number of chains in  $P$ .

An augmenting chain theorem characterizing ocp's with maximum cardinality is given with a straightforward extension to the weighted case.

It is shown that if a graph has an ocp  $P$ , then there exists an ocp  $P'$  with  $|P'| = |P|$  where all chains are elementary.

Considering odd chain coverings  $C$  (each node is the endpoint of some chain of odd length in  $C$ ), one can give a variation of a theorem of Berge, Norman, Rabin.

(joint work with W. Pulleyblank)

K.-H. ELSTER: On a new concept of cone approximations in nondifferentiable optimization

In the lecture necessary optimality conditions for nonsmooth nonconvex optimization problems are given using the approach of Dubovitzkij/Miljutin, where cone approximations of sets and generalized differentiability of extended real functions are involved. The notion of an abstract local cone approximation  $K$  gives the possibility to introduce  $K$ -directional derivatives and  $K$ -subdifferentials, which are true generalizations of the corresponding notion known from convex analysis. By this framework optimality conditions of the John-type and the Kuhn-Tucker-type can be obtained, where several interesting results with respect to smooth resp. convex resp. Lipschitzian optimization problems are included.

U. FAIGLE: Sorting and recognizing ordered sets

The general sorting problem consists in determining an unknown order  $P$  under the condition  $P \cong P_0$  and the general recognition problem asks for an answer to the question " $P \cong P_0$ ?". Both problems reduce to the standard sorting problem when the pattern order  $P_0$  is a chain. In our computational model, we assume the existence of an "oracle" which gives the correct relation between any two elements of the underlying set. What is the minimal number of calls of the oracle needed to solve the above problems? We discuss the general complexity of these problems and show, for example, that the recognition problem with respect to ordered sets of bounded width attains the information-theoretic complexity bound. Also, open problems are presented.

(jointly with Gy. Turán)

A. FRANK: A combinatorial application of the simultaneous approximation algorithm

In combinatorial optimization it is often the case that a problem can be solved in polynomial time when the objective function  $w$  is integral (or rational) but the running time depends on the size of  $w$ .

Here we show how the existing methods can be used in certain cases to solve the given optimization problem in strongly polynomial time for arbitrary  $w$ .

In particular, strongly polynomial algorithms are presented for the submodular flow problem, for finding a maximum weight clique in a perfect graph, and for the membership problem in a matroid polyhedron. (The last problem was solved earlier by W. Cunningham).

The method relies on the simultaneous approximation algorithm of Lenstra, Lenstra and Lovász.  
(jointly with E. Tardos)

M.D. GRIGORIADIS: On fast heuristics for matching and postman problems

We first show that the worst-case time complexity of any exact algorithm for the minimum-weight perfect matching problem for complete Euclidean graphs with  $n$  vertices, is bounded below by  $\Omega(n \log n)$ . This bound also applies to any  $f(n)$ -approximate heuristic for this problem. Next, we consider the undirected Chinese postman problem which is solvable in  $O(n^3)$  time. For large instances arising in practice, it is often advantageous to consider low-order heuristics. We propose an MST-based heuristic and show that it produces solutions with relative error of at most  $2/n$  for complete graphs. For sparse graphs, a-posteriori bounds for this error are defined and they are further refined by a minimum-cost network flow computation. Experimental results with large graphs of various densities, with Euclidean as well as non-Euclidean edge weights indicate that the heuristic produces tours with attractive relative errors.



M. GRÖTSCHEL: Cuts, cycles and spin glasses

In this talk we describe recent work with F. Barahona and A. R. Mahjoub on the characterization of the polytope of bipartite subgraphs of a graph, the polytope of cuts of a graph and the polytope of cycles in a binary matroid. We give an excluded minor characterization of those binary matroids whose cycle polytope is characterized by the hypercube and the cocircuit constraints. Finally we discuss preliminary computational results on determining the ground states of spin glasses (a problem in theoretical physics (phase transitions in magnetism) which has a formulation as a combinatorial optimization problem). These results were obtained by a cutting plane algorithm for the max-cut problem which is based on the polyhedral results for the cut polytope mentioned above.

J.-B. HIRIART-URRUTY: Prolegomenon on nonconvex duality: a general formula on the conjugate of the difference of functions

The conjugacy operation which associates  $f^*$  to  $f$  is a key-tool in formulating variational principles in non-convex optimization and calculus of variations as well as deriving duality schemes for such problems. Calculating  $f^*$  in terms of  $f_i^*$  when  $f$  has been constructed from other  $f_i$  constitute the body of calculus rules on conjugate functions. Our aim here is to give the exact expression of  $(g-h)^*$ ,  $h$  convex, in its most general setting. We thereby shed a new light on Toland's duality results associating  $\inf_x \{g(x) - h(x)\}$  and  $\inf_{x^*} \{h^*(x^*) - g^*(x^*)\}$ .

E.L. JOHNSON: Mappings and liftings for group and semigroup problems

We study pairs of polyhedra related by a mapping having certain properties. The mapping gives a way of lifting valid inequalities from

one polyhedron to the other. The two main questions of interest are: (1) when do facets in the image lift back to facets in the original polyhedron; and (2) which facets of the original polyhedron come from such liftings. Examples are taken from covering and partitioning problems. We show some sufficient conditions for facets from the group relaxation to give facets for the integer programming problem.

P. KALL: Approximations to stochastic linear programs with recourse

The problems to be solved are of the type

$$(1) \quad \min \{c'x + Q(x) \mid Ax = b, x \geq 0\}$$

where

$$(2) \quad Q(x) = E Q(x, \xi(w))$$

$$(3) \quad Q(x, \xi(w)) = \inf_y \{q'y \mid Wy = h(w) - T(w)x, y \geq 0\}$$

$$\xi(w) = (h(w), T(w)),$$

and  $W$  is assumed to be a complete fixed recourse matrix. Since for discrete distributions of  $\xi(w)$  problem (1) is a linear program with dual decomposition structure, we approximate  $\xi(w)$  in a particular way such that we get lower and upper bounds for (2) and hence (1) by the Jensen inequality and by the Edmundson-Madansky inequality respectively. Computational tests have been made for various strategies of discretizing  $\xi(w)$ , the results of which are reported.

B. KORTE: Pivoting, connectivity and homotopy properties

Pivoting, i.e. exchanging exactly one element in a basis, is a fundamental step in the simplex algorithm for linear programming. This operation has a combinatorial analogue in matroids and greedoids. We call two bases of a matroid of a greedoid adjacent if they differ by exactly one element. The operation of obtaining from a given basis an adjacent one is called pivoting or basis deformation. The adjacency

relation of bases gives rise to a graph which we will call the bases graph. It is an easy consequence of the bases exchange axiom of matroids that the bases graph for any matroid is connected. This is no longer true for greedoids. We introduce an appropriate connectivity definition for greedoids.

Theorem: The basis graph of a two-connected greedoid is connected.

For shelling structures (= special classes of greedoids) this result is true in general. For  $k$ -connectivity we have to use homotopy theory in order to get similar results. We get results like this:

Theorem: The chain complex of dominating sets (or feasible sets) of a  $k$ -connected greedoid is  $(k-2)$ -connected.

Theorem: The convex cell complex of basis (bases polyhedron) of a  $k$ -connected interval greedoid is  $(k-2)$ -connected.

(jointly with A. Björner and L. Lovász).

W. KRABS: On the determination of the worst sampling error in a system for pulse-amplitude-modulated signals

The system for pulse-amplitude-modulated signals consists of a transmitter, a channel, a receiver and a sampler. It picks up discrete signals at a fixed time distance and modulates them into a continuous signal which is sampled behind the receiver in form of discrete signals again at the same time distance. If the sampling is done with a random error, the transfer function of the system can be chosen in such a way as to minimize the mean square error of the sampled signals in comparison with the input signals. The purpose of the lecture is to study the worst case where the sampling error is such that the corresponding least mean square error becomes maximal.

C. LEMARECHAL: On first order approximation of convex-valued multifunctions

Let  $G(x)$  be a multi-application, with  $G(x)$  convex for all  $x$ ; suppose we want to solve  $0 \in G(x)$  ( $G(x)$  could be the approximate subdifferential of a convex function to be minimized).

A Newton method needs to define a first-order approximation of  $G(x + d)$  near  $x$ , to obtain an iterative sequence  $x_{n+1} = x_n + d$ .

We give some results about such approximations, where the support function to  $G$  and the "contingent derivative" (introduced by Aubin) are central.

TH. M. LIEBLING: Steiner's problem on 2-trees

We examine special cases of Steiner's problem that yield efficient algorithms and polyhedral descriptions. One such example is Steiner's problem on two-trees. The approach shows a possible connection between problems amenable to efficient dynamic programming algorithms and such that have a simple polyhedral description.

L. LOVASZ: A non-linear relaxation of vertex packing

In this joint work with M. Grötschel and A. Schrijver, we consider a non-polyhedral convex set  $TH(G)$  for every graph  $G$ , which includes the vertex packing polytope and is contained in the fractional vertex packing polytope. The two important properties of  $TH(G)$  are: (1) The antiblocker of  $TH(G)$  is just  $TH(\bar{G})$ , where  $\bar{G}$  is the complementary graph of  $G$ . (2) One can optimize any linear objective function over  $TH(G)$  in polynomial time. This latter result implies that maximum weight stable sets can be found in perfect graphs in polynomial time.

T.L. MAGNANTI: When do nonlinear programming algorithms solve variational inequalities?

We consider the convergence of nonlinear programming algorithms adapted to solve systems of equations and variational inequality problems like those that arise in the analysis of transportation and economic equilibria. We assume that the underlying problem mapping is monotone, but not necessarily a gradient mapping. (If it is a gradient mapping, then the variational inequality problem is equivalent to a convex minimization problem.) We discuss conditions on the problem mapping under which the analogue of steepest descent converges for asymmetric systems of equations, and a contracting ellipsoid algorithm converges for variational inequalities. The results help to explain the role of symmetry in solving variational inequalities and systems of equations as equivalent convex optimization problems.

O. L. MANGASARIAN: Solving bounds in mathematical programming

Solution bounds are given for linear programs, monotone linear and nonlinear complementarity problems. Essentially it will be shown that feasibility implies boundedness of some solution components. In addition explicit numerical bounds on Lagrange multipliers of stationary points of nonconvex differentiable programs will be given in terms of a standard constraint qualification. Finally bounds on polyhedra will be given by means of solving a single linear program. It will also be shown that finding the largest element of the polyhedron using any  $p$ -norm for integer  $p \geq 1$  is an NP-complete problem.  
(jointly in part with L. McLinden and T.-H. Shian)

L. McLINDEN: Variational inequalities, monotone operators and stability

Variational inequalities associated with monotone operators (possibly nonlinear and multivalued) and convex sets (possibly unbounded) are studied in reflexive Banach spaces. A variety of results are given which

relate to a stability concept involving a natural parameter. These include characterizations useful as criteria for stable existence of solutions and also several characterizations of surjectivity. The monotone complementarity problem is covered as a special case, and the results are sharpened for linear monotone complementarity and for generalized linear programming. The proofs make some use of new tool theorems concerning the maximality and the range of the sum of two monotone operators.

J. J. MORE: Software for estimating sparse Hessian matrices:  
a graph coloring approach

The solution of a nonlinear optimization problem often requires an estimate of the Hessian matrix for a function  $f$ . In large scale problems the Hessian matrix is usually sparse, and then estimation by differences of gradients is attractive because the number of differences can be small compared to the dimension of the problem. For example, the number of differences is not more than the semi-bandwidth of the Hessian matrix. In this talk we show that the estimation problem has a natural graph coloring interpretation, and that this point of view leads to efficient algorithms and software. We then describe a set of subroutines whose purpose is to estimate the Hessian matrix with the least possible number of gradient differences. The emphasis of the talk is on the ideas and open problems which surround the software and not on the implementation details of the software.

D. PALLASCHKE: The steepest descent method for quasi-differentiable  
optimization problems

The problem to determine  $\epsilon$ -stationary points of quasi-differentiable optimization problems is considered. Based on a notion of  $\epsilon$ -differentiability a gradient-type solution method is introduced.

M.J.D. POWELL: Recursive quadratic programming and differentiable exact penalty functions for constrained optimization

We consider the use of Fletcher's differentiable exact penalty function as a line search function in a REQP algorithm for constrained optimization. An obvious difficulty is that descent conditions involve second derivatives of the functions of the main calculation. Therefore suitable difference approximations are introduced, which allow global convergence to be achieved using only first derivatives of the objective and constraint functions. Some numerical results are given, but the work so far is only for the case where all constraints are equalities. (jointly with Y. Yuan)

A. PREKOPA: Algorithmic solutions of stochastic network design problems

In this talk we discuss the planning problem for electrical power plants and water reservoirs with respect to stochastic consumption. Discretization of the problem yields an algorithmic solution for practical problems in Hungary. In particular, we generalize the Gale-Hoffman conditions for feasible flows to electrical networks, where the special constraints imposed by physical conditions are taken into account.

W.R. PULLEYBLANK: Minimum weight 2-connected spanning networks

Consider a set  $V$  of vertices with a nonnegative symmetric distance function defined which satisfies the triangle inequality. We wish to construct a 2-connected spanning subgraph for which the sum of the distances of the edges is minimized. (Note that, under these conditions, there exists a minimum cost 2-edge connected graph which is 2-vertex connected.)

We characterize the structure of the optimum solutions to the problem and show that if there exists a solution of value  $v$ , then there exists a

solution to the corresponding travelling salesman problem of value at most  $4/3$  v. We also show that the worst case bound  $(3/2)$  of Christofides' heuristic for the travelling salesman problem is the same when his algorithm is applied to this problem. (This extends earlier work of Fredrickson and Ja'Ja'.)  
(jointly with C.L. Monma and B.S. Munson)

C. RICHTER: Hybrid methods of nonlinear programming

The paper discusses hybrid methods of nonlinear programming. The coupling of methods, the construction of hybrid directions and principles of regularization are considered from the view-point of the guarantee of desirable numerical properties.

S. M. ROBINSON: A framework for looking at nonlinear programming

We present a conceptual framework for analyzing nonlinear programming problems. It is based on simple geometric insights, and it allows all major analytical properties of both linear and nonlinear programming problems to be recovered in a unified way, without resort to special devices. It also permits us to establish additional results not previously known. Some of these new results will be presented here.

K. SCHITTKOWSKI: Some remarks on optimization software

First some general remarks illustrate typical features of mathematical programming software coming from commercial institutions, libraries, or academic institutions. More detailed information will be given on linear and integer programming and, in particular, on the main areas in nonlinear programming. One of the conclusions of the author is that future nonlinear programming software will contain decision support systems, model builders, report writers, etc., comparable to the situation we have now in linear programming, to facilitate the search for a suitable algorithm, the construction of the underlying



mathematical model, its numerical solution, and the processing of the obtained results. As an example, some details of a special nonlinear programming system will be presented, which is developed at present at the University of Bayreuth.

R. SCHRADER: On simplicial elimination in combinatorial structures

In this joint work with U. Faigle and O. Goecke a general setting for simplicial elimination in combinatorial structures is introduced. It is shown that any two simplicially irreducible kernels are isomorphic. This implies known results about perfect elimination bipartite graphs, chordal graphs, strongly chordal graphs and crown-free ordered sets. We furthermore discuss how the concept of simplicial elimination relates to transposition greedoids.

A. SCHRIJVER: Vertex packing and cutting planes

The vertex packing problem: given an undirected graph  $G = (V, E)$  and a weight function  $v : V \rightarrow \mathbb{Z}_+$ , find a coclique of maximum weight, is NP-complete. We discuss several relaxations of this problem, each giving rise to an upper bound for the problem, which can be used in a branch-and-bound procedure for the vertex packing problem. Some of these upper bounds can be computed in polynomial time. The results are related to cutting plane theory, and as a side result we have that each matrix has finite Chvátal rank. We give some classes of matrices with Chvátal rank 1, again yielding relaxations of the vertex packing problem.

(jointly with W.J. Cook, A.M.H. Gerards, M. Grötschel, L. Lovász, P.D. Seymour, E. Tardos and K. Truemper)

D. SHANNO: Newton's method and variants for robust regression

Robust regression estimates regression parameters  $\beta_i$ ,  $i = 1, \dots, n$  by minimizing a loss function which gives less weight to data points corresponding to large residuals. This is done to lessen the influence

on the parameter estimates of possibly contaminated data. Inherent in the problem is the problem of estimating a scale factor which determines which residuals are large. The talk describes how Newton's method and iteratively reweighted least squares can be used to simultaneously estimate both the  $\beta$ 's and the scale factor. Computational results show that Newton's method converges generally very rapidly on both the Huber and biweight loss functions for two different scale factors, the median absolute value of the residuals and the Winsorized variance.

J. STOER: Three simple observations on sequential quadratic programming methods

Sequential quadratic programming methods for solving a nonlinear program

$$(P) \quad \min f(x) \\ x : g(x) = 0$$

generate a sequence  $\{x_k\}$  by means of an iteration  $x_{k+1} := x_k + s_k :=: \phi_k(x_k)$ , where  $s_k$  is an optimal solution of a quadratic program

$$(P_k) \quad \min \{ Df(x_k)s + 1/2 s^T B_k s \mid g(x_k) + Dg(x_k)s = 0 \},$$

depending on  $x_k$  and a symmetric matrix  $B_k = B_k^T$ . The Kuhn-Tucker (K.T.) points  $x_*$  of (P) are fixed points,  $\phi_k(x_*) = x_*$ . The convergence behaviour of the iteration near a K.T. point  $x_*$  is studied by analyzing the Jacobian  $D\phi_k(x_*)$  of  $\phi_k$ . In this way, one can show that for pos. def.  $B_k \equiv B$ , The K.T. points  $x_+$  satisfying the 2nd order sufficiency condition for a minimum of (P) are attractive fixed points, where the K.T. points  $x_*$  violating the 2nd order necessary condition for a minimum of (P) are repulsive fixed points of the iteration  $x_{k+1} = \phi(x_k)$ .

Moreover, using the explicit formulae for  $D\phi_k(x_*)$  one obtains rather simple proof of the Boggs-Tolle-Wang (1981) criterion for the Q-superlinear convergence of the  $\{x_k\}$  (i.e.  $\|x_{k+1} - x_*\| / \|x_k - x_*\| \rightarrow 0$ ), and of the Powell-criterion (1978) for the 2-step Q-superlinear convergence of the  $\{x_k\}$  (i.e.  $\|x_{k+1} - x_*\| / \|x_{k-1} - x_*\| \rightarrow 0$ ).

R. TAPIA: Secant updates for constrained optimization

In this talk we investigate the use of BFGS secant update in the SQP secant method for constrained optimization when the Hessian is not assumed to be positive definite. We derive a new secant update and show how the augmented Lagrangian can be used in an effective manner.

E. TARDOS: A strongly polynomial algorithm for combinatorial linear programs

Khachiyan and recently Karmarkar gave polynomial algorithms to solve the linear programming problem. These algorithms have a small theoretical drawback, namely, the number of arithmetic steps depends on the size of the input numbers. Here we present a polynomial linear programming algorithm whose number of arithmetic steps depends only on the size of the numbers in the constraint matrix, but is independent of the size of the numbers in the right hand side and objective.

In particular, it gives an algorithm for the minimum cost flow and multicommodity flow problems whose number of arithmetic steps is independent of the size of the input numbers.

The algorithm makes use of any known polynomial linear programming algorithm.

G. TINHOFER: Graph isomorphism, doubly stochastic matrices and linear programming

Graph Isomorphism is one of the rare problem classes which have not been proved either to belong to P or to be NP-complete. I consider a relaxation of the isomorphism relation between undirected graphs which is polynomially decidable. Two graphs  $G_A$  and  $G_B$  are called ds-isomorphic iff there is a doubly stochastic matrix  $X$  such that  $XA = BX$  holds where  $A$  and  $B$  are the adjacency matrices of  $G_A$  and

$G_B$ . It is shown that

- (a) the vertex number, the edge number and the sorted degree vector form an (in some sense) complete set of invariants with respect to this weaker isomorphism relation,
- (b) the polytop  $P_{A,B}$  consisting of all ds-isomorphisms between  $G_A$  and  $G_B$  has nonintegral vertices if the 'degree partition' of A (respectively B) is not the automorphism partition
- (c) if  $G_A$  is a tree, the  $P_{A,A}$  is the convex hull of the automorphisms of  $G_A$ . This statement can be considered as a generalization of Birkhoff's well known theorem on doubly stochastic matrices.

L. E. TROTTER: Polynomial time computation of Hermite normal form using modulo determinant arithmetic

Traditional algorithms for computing the Hermite normal form of an integral matrix are known to exhibit dramatic growth in the size of numeric data over the course of the computation. We present algorithms which perform this computation using modulo arithmetic. The modulus for arithmetic computation can be taken as the determinant of the original matrix (assumed square and nonsingular, with no loss in generality). It follows that intermediate numeric entries are bounded in size (number of bits) by  $n(\log_2 n + \log_2 a)$ , where  $n$  is the dimension of the original matrix and  $a$  is the magnitude of its largest entry. The algorithms presented are thus polynomially time bounded. (jointly with P. Domich and R. Kannan)

V.K. VUJICIC: An interior globally convergent semiinfinite programming method

A discretization method for solving semiinfinite programming problems of the type

$$\begin{aligned} \min f(x), \quad X = \{x \in \mathbb{R}^n \mid c(x,t) \leq 0, \quad t \in C\} \\ x \in X \end{aligned}$$

where  $C \subseteq \mathbb{R}^r$  is compact,  $f$  continuous,  $c$  convex with respect to  $x$  and satisfies Lipschitz condition with respect to  $t$ , is proposed. The method generates feasible sequence of points whose cluster points are solutions to the problem. The main idea behind the method is to use analytic information about the constraint function  $c$  in order to refine the discretization. Under additional assumptions on the constraint function  $c$  and the index set  $C$  computational complexity and numerical stability of the method are analyzed. Some numerical results are also presented.

M. YUE: On the unified approaches in nonlinear programming

There are quite a lot of algorithms in nonlinear programming unified approaches to them are necessary. There are two such approaches: the point-to-set mapping theory from Zangwill (1969) and the parametric theory from Huang (1969). Our talk is about two aspects: (i) by introducing a  $l$ -increasing family of point-to-set maps and establishing a theory, to prove that Zangwill's and Huard's (1975) theories are included in it, and to show, by examples, that they are its real special cases; (ii) Huang's theory and others following it are dealing with unconstrained optimization problems. We establish a theory to deal with the constrained case and show, by examples, that many known algorithms are included in it.

Berichterstatter: U. Faigle (Bonn)

Tagungsteilnehmer

Professor Dr. A. Bachem  
Mathematisches Institut  
Universität zu Köln  
Weyertal 86-90  
5000 Köln 41

Professor A. R. Conn  
University of Waterloo  
Faculty of Mathematics  
Dept. of Computer Science  
Waterloo, Ontario  
KANADA N2L 3G1

Professor F. Barahona  
Dpto. de Matematicas  
Universidad de Chile  
Casilla 5272  
Santiago 3  
CHILE

Dr. W. Cook  
Institut für Operations Research  
Universität Bonn  
Nassestr. 2  
5300 Bonn 1

Professor M.J. Best  
University of Waterloo  
Dept. of Combinatorics & Optimization  
Waterloo, Ontario  
KANADA N2L 3G1

Dr. D. Crystal  
Institut für Operations Research  
Universität Bonn  
Nassestr. 2  
5300 Bonn 1

Professor R. E. Bimby  
Dept. of Mathematical Sciences  
Rice University  
P.O. Box 1892  
Houston, TX 77251-1892  
USA

Professor W.H. Cunningham  
Dept. of Mathematics and Statistics  
Carleton University  
Ottawa, Ontario  
KANADA K1S 5B6

Professor Dr. R. E. Burkard  
Institut für Mathematik  
Kopernikugasse 24  
A-8010 Graz  
Österreich

Professor J.E. Dennis  
Dept. of Mathematics  
Rice University  
P.O. Box 1892  
Houston, TX 77251  
USA

Professor V. Chvátal  
Mc Gill University  
Dept. of Mathematics  
P.O. Box 6070, Station A  
Montreal, Quebec  
KANADA H3C 3G1

Dr. U. Derigs  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor Dr. W. Deuber  
Universität Bielefeld  
Fakultät für Mathematik  
Universitätsstr. 1  
4800 Bielefeld 1

Professor Dr. M. Grötschel  
Lehrstuhl für Angewandte Mathematik II  
Universität Augsburg  
Memminger Str. 6  
8900 Augsburg

Professor Dr. K.-H. Elster  
Technische Hochschule Ilmenau  
Sektion Mathematik, Rechentechnik  
und Ökonomische Kybernetik  
Ehrenberg, Block G  
DDR 6300 Ilmenau

Dr. J.-B. Hiriart-Urruty  
Univ. Paul Sabatier (Toulouse III)  
U.E.R. Mathématiques, Informatiques  
118, Route de Narbonne  
F-31062 Toulouse Cedex  
Frankreich

Dr. U. Faigle  
Institut für Operations Research  
Universität Bonn  
Nassestr. 2  
5300 Bonn 1

Professor Dr. K.-H. Hoffmann  
Institut für Mathematik  
Universität Augsburg  
Memminger Str. 6  
8900 Augsburg

Dr. J. Fischer  
Technische Universität München  
Institut für Statistik und  
Unternehmensforschung  
Arcisstraße 21  
8000 München 2

Professor E. Johnson  
IBM Research Center  
P.O. Box 218  
Yorktown Heights, N.Y. 10598  
USA

Dr. A. Frank  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor Dr. P. Kall  
Institut für Operations Research  
Universität Zürich  
Weinbergstr. 59  
CH-8006 Zürich  
Schweiz

Professor M.D. Grigoriadis  
Rutgers University  
Dept. of Computer Science  
Hill Center  
New Brunswick, NJ 08903  
USA

Professor Dr. H. König  
Mathematisches Institut  
Universität des Saarlandes  
6600 Saarbrücken

Professor Dr. B. Korte  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor T.M. Liebling  
Dept. de Mathematiques  
Ecole Polytechnique  
Federale de Lausanne  
Av. de Cour 61  
CH-1000 Lausanne  
Schweiz

Professor V. Kovacevic-Vujicic  
Milavana Marinkovica 5  
11040 Belgrade  
JUGOSLAWIEN

Professor L. Lovász  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor Dr. W. Krabs  
Tech. Hochschule Darmstadt  
Fachbereich Mathematik  
Schloßgartenstr. 7  
6100 Darmstadt

Professor T. Maçnanti  
M.I.T.  
50 Memorial Drive, Room E53-357  
Cambridge, MA 02139  
USA

Dr. C. Kredler  
Technische Universität München  
Institut für Statistik und  
Unternehmensforschung  
Arcisstraße 21  
8000 München 1

Professor O.L. Mangasarian  
University of Wisconsin  
Computer Science Department  
1210 W. Dayton Str.  
Madison, WI 53706  
USA

Professor C. Lemaréchal  
IMRIA  
Domaine de Voluceau-Rocquencourt  
Boite Postale 105  
F-78150 Le Chesnay  
Frankreich

Professor L. McLinden  
Dept. of Mathematics  
University of Illinois  
1409 West Green Street  
Urbana, Illinois 61801  
USA

Professor Dr. F. Lempio  
Lehrstuhl f. Angewandte Mathematik  
Universität Bayreuth  
Opfernstr. 22  
Postfach 3008  
8580 Bayreuth

Professor J.J. More  
Division of Applied Mathematics  
Argonne Nat. Labs  
9700 S. Cass Avenue  
Argonne, IL 60439  
USA



Professor Dr. W. Oettli  
Universität Mannheim  
Lehrstuhl für Mathematik VII  
Schloß  
6800 Mannheim

Dr. B. Riedmüller  
Technische Universität München  
Institut für Statistik und  
Unternehmensforschung  
Arcisstraße 21  
8000 München

Professor Dr. D. Pallaschke  
Institut für Statistik u.  
Mathematische Wirtschaftstheorie  
Universität Karlsruhe  
Kollegium am Schloß, Bau IV  
7500 Karlsruhe

Professor Dr. K. Ritter  
Institut für Statistik und  
Unternehmensforschung  
Technische Universität München  
Arcisstr. 21  
8000 München 2

Professor M.J.D. Powell  
University of Cambridge  
Department of Applied Mathematics  
and Theoretical Physics  
Silver Street  
Cambridge CB3 9EW  
ENGLAND

Professor S.M. Robinson  
Dept. of Industrial Engineering  
University of Wisconsin  
Madison, WI 53706  
USA

Professor A. Prekopa  
Computer and Automation Institute  
Hungarian Academy of Sciences  
MTA Sztaki  
Kende u. 13-17  
1111 Budapest  
UNGARN

Professor Dr. K. Schittkowski  
Universität Würzburg  
Institut für Angewandte Mathematik  
und Statistik  
Am Hubland  
9700 Würzburg

Professor W.R. Pulleyblank  
Dept. of Combinatorics & Optimization  
University of Waterloo  
Waterloo, Ontario N2L 3G1  
KANADA

Dr. R. Schrader  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor Dr. Claus Richter  
Abteilung Mathematik/Rechentechnik  
Ingenieurhochschule Köthen  
Bernburger Straße 52-57  
DDR-4370 Köthen

Professor A. Schrijver  
Department of Econometrics  
Tilburg University  
P.O. Box 90153  
NL-5000 LE Tilburg  
Niederlande

Professor D. F. Shanno  
University of Arizona  
College of Business and  
Public Administration  
MIS Department  
Tucson, AZ 85721  
USA

Professor L. E. Trotter  
Département de Mathématiques  
Ecole Polytechnique Fédérale  
de Lausanne  
CH-1015 Lausanne-Ecublens  
SCHWEIZ

Professor Dr. J. Stoer  
Institut für Angewandte Mathematik  
Am Hubland  
9700 Würzburg

Professor D. de Werra  
Département de Mathématiques  
Ecole Polytechnique Fédérale  
de Lausanne  
CH-1015 Lausanne-Ecublens  
SCHWEIZ

Professor P. Tapia  
Math. Sciences Dept.  
Rice University  
Houston, TX 77001  
USA

Professor Minyi Yue  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Dr. E. Tardos  
Institut für Operations Research  
Universität Bonn  
Massestr. 2  
5300 Bonn 1

Professor Dr. J. Zowe  
Mathematisches Institut  
Universität Bayreuth  
Postfach 3008  
8580 Bayreuth

Professor Dr. G. Tinhofer  
Technische Universität München  
Institut für Statistik und  
Unternehmensforschung  
Arcisstr. 21  
8000 München 2