MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 4/1985

Mengenlehre

13.1. bis 19.1.1985

Die Tagung fand unter der Leitung von Herrn Menachem Magidor (Jerusalem) und Herrn Ernst-Jochen Thiele (Berlin) statt, parallel zu einer Tagung über Boolesche Algebren.

Es wurden nahezu alle Bereiche der Mengenlehre berührt; Hauptgegenstand der Vorträge und Diskussionen waren Ultrafilter auf ω und auf größeren Kardinalzahlen und die Theorie großer Kardinalzahlen. Als die wichtigsten Resultate könnte man die von Magidor (über eine weitreichende und in einer Richtung maximale Erweiterung von Martins Axiom), von Foreman (über die Existenz nicht regulärer Ultrafilter) und von Woodin (über durch Klassenforcing erzwungene elementare Einbettungen) nennen.

Die Zusammenarbeit mit der Paralleltagung über Boolesche Algebren war äußerst harmonisch und erfolgreich; die Zeitpläne der Tagungen wurden sorgfältig aufeinander abgestimmt, und es wurden 7 Vorträge aus der Tagung über Boolesche Algebren und 4 aus der Tagung über Mengenlehre in das Programm beider Tagungen aufgenommen; darüber hinaus sind, soweit zeitlich möglich, viele der anderen Vorträge von Teilnehmern beider Tagungen besucht worden (in den folgenden Vortragsauszügen sind die Vorträge über Boolesche Algebren nicht mitaufgeführt, die 4 in das gemeinsame Programm aufgenommenen Vorträge aus der Mengenlehre sind durch einen Stern gekennzeichnet).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

S. BEN-DAVID:

Combinatorial properties of successors of singulars

The relative strength of L-like properties of cardinals for successors of strong limit singular cardinals was investigated. R. Jensen proved that $\square_\lambda$ implies $\langle\aleph_1, \aleph_0\rangle \to \langle\lambda^+, \lambda\rangle$. In a joint work with M. Magidor assuming consistency of a κ^+-supercompact cardinal κ, the author shows that the converse implication does not hold by constructing a model in which $\langle\aleph_1, \aleph_0\rangle \to \langle\aleph_{\omega+1}, \aleph_\omega\rangle$ holds true and $\square_\omega$ fails.

R. Jensen showed that $\square_\lambda$ implies the existence of a non reflecting stationary subset of λ^+. He formulated a weaker principle $\square_\lambda^*$ (equivalent to the existence of a λ^+ special Aronszajn tree). Assuming the consistency of infinitely many supercompact cardinals in a joint work with S. Shelah the author shows that $\square_\lambda^*$ does not imply the existence of such a stationary subset of λ^+.

A. BLASS *

Near coherence of filters

The principle of near coherence of filters (NCF) asserts that, for any two filters F_1 and F_2 on ω, each containing all cofinite sets, there exists a finite-to-one $f : \omega \to \omega$ such that $f(F_1) \cup f(F_2)$ generates a proper filter. S. Shelah recently proved that NCF is consistent relative to ZFC. NCF is equivalent to each of the following: (1) For every two non-principal ultrafilter on ω there is a finite-to-one $f : \omega \to \omega$ mapping them to the same ultrafilter. (2) every non-principal ultrafilter on ω has a finite-to-one image generated by fewer than α sets.

(d is the minimal cardinal of a dominating family in ${}^{\omega}\omega$).

(3) In the algebra of bounded linear operators on the complex infinite-dimensinal separable Hilbert space, the ideal of compact operators is not the sum of two smaller ideals. (4) The Stone-Čech remainder $\beta[0,\infty) - [0,\infty)$, known to be an indecomposable continuum, has only one composant, etc. Among the consequences of NCF are: (1) There are no Q-points. (2) Every non-principal ultrafilter on ω maps finite-to-one to a P-point. (3) All countably-indexed ultrapowers of ω have cofinality d.

M. BOFFA:

The system NF

1. In ZFC there is a formula $\varphi(x)$ which makes any cardinal number ζ discernible from 2^{ζ}:

$$ZFC \vdash (\forall\zeta)(\varphi(\zeta) \leftrightarrow \neg\varphi(2^{\zeta})).$$

This seems to require the axiom of choice.

Question 1: Is there some model of ZF containing a cardinal ζ indiscernible from 2^{ζ}?

A positive answer would imply the consistency of NF.

2. Let $M(\zeta)$ denote the natural model (of the theory of types) $\langle X, PX, PPX, \ldots\rangle$ where $|X| = \zeta$. In ZF, the "ambiguity hypothesis" $M(\zeta) \equiv M(2^{\zeta})$ (which entails the consistency of NF) implies $h(\zeta) = \infty$ where

$$h(\zeta) = \sup\{n \in \mathbb{N} \mid \zeta \text{ has the form } \underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2^{d}}}}}}}_{n}\}.$$

This hypothesis is always false in ZFC (where $h(\zeta) < \infty$).

Question 2: Is the ambiguity hypothesis consistent with ZF?

Even the following weaker question seems open:

Question 3: Is $h(\zeta) = \infty$ consistent with ZF?

N. BRUNNER:

Topological spaces in permutation models

Sets and spaces of urelements in permutation models (where the axiom of choice fails) are characterized by their inherent topological properties. This is especially interesting for the Mostowski model, where the set U of urelements carries some nontrivial topological structure (since it is linearly ordered). In this model examples of such results are the following:

(i) A Hausdorff space X is a continuous image of a Dedekind finite subset of U^{ω} (U with its order topology) if and only if every infinite subset of X contains an infinite compact subset.

(ii) A compact Hausdorff space X is a continuous image of a compact subset of $\omega \times oU$ (oU is the order compactification) if and only if all dense subsets of X are open and the set of isolated points is finite.

L. BUKOVSKY:

On models changing cofinality:

The author and K. Namba have introduced two notions of forcing Perf(κ) and Nm(κ) respectively, for κ an uncountable regular cardinal. The effect of both forcing notions is such that in the corresponding generic extension, κ is cofinal with ω_o and no new subset of ω_o is added. Moreover, the forcing notion Perf(κ) is minimal. The minimality of Nm(κ) was not known. The affirmative answer is given by the general theorem on models.

<u>Theorem:</u> Let $M \subseteq N$ be an extension of models of ZFC such that $^{\omega}\lambda \cap N \subset M$. If $f,g \in {}^{\omega}(\lambda^+) \cap N$, sup f = sup g = λ^+, then $M[f] = M[g]$.

R. CHUAQUI:

<u>Metamathematics of impredicative class theories</u>

By impredicative theories of classes are ment set theories with the impredicative axiom of class specification:

$$\exists A \forall x \; (x \in A \longleftrightarrow \varphi \wedge \exists U (x \in U))$$

where φ is any formula where A does not occur free.

The theory usually called of Kelley-Morse, which is comparable to Zermelo-Fraenkel theory, presents many interesting metamathematical problems. Some results by Corrada, Ihoda and Reinhart on the problem of the definition of types of well orderings and of the strength of different forms of the axiom of choice were presented.

The addition of reflection principles to this theory increases its strength very much. The well-known Bernays reflection principle implies the existence of weakly compact cardinals. The existence of most known large cardinals can be proved from higher order reflection principles introduced by Marshall and described in the lecture.

H.D. DONDER:

Ultrafilters on ω_n

Theorem: Assume GCH and "there is no inner model with a measurable cardinal". Then every uniform ultrafilter on ω_n is regular, $n < \omega_o$.

For the proof of this theorem we consider two-cardinal versions of the weak Chang conjecture and the transversal hypothesis.

M. FOREMAN:

Non regular ultrafilters.

Theorem: If μ is a regular cardinal and $\kappa > \mu$ is huge then there is a partial ordering $\mathbb{P}$ such that

$$V^{P} \models \text{"}\mu^{+} \text{ carries a } (\mu,\mu^{+})\text{-non-regular ultrafilter".}$$

We prove this theorem using the notion of a layered ideal.

M. GITIK:

Changing cofinalities and the nonstationary ideal

It was presented a κ-c.c. iteration of Prikry type forcing notions. Using this it is possible to define forcing notions for changing cofinalities without adding new bounded sets.

A model with $NS_{\kappa} \wedge S$ saturated is constructed for inaccessible κ and a stationary subset S of κ such that $S \cap \{\alpha < \kappa \mid cf(\alpha) = \beta\}$ is stationary for every $\beta > \kappa$. Some other consistency results can be obtained by this method.

A. HAJNAL:

Embedding graphs into colored graphs

This is a joint work with P. Komjàth. The partition relations used below are defined e.g. in a work of P. Erdös, A. Hajnal and L. Pósa (1973).

Theorem 1: $\forall\lambda\forall\kappa \geq \omega$ regular $\forall H\ K_\kappa \not\subset H \Rightarrow \exists G\ K_\kappa \not\subset G \wedge G \rightarrowtail (H)^1_\lambda$.

Theorem 2: The following are consistent with ZFC

a) $\exists H\ |H| = \aleph_1 \wedge \forall G\ G \rightarrowtail (H)^2_2$.

b) $\exists H\ |H| = \aleph_1 \wedge K_3 \not\subset H \wedge \forall G\ G \rightarrowtail (H)^2_\omega \Rightarrow K_{o} \subset G$.

P. KOEPKE:

Short core models

A (short) D-mouse is a structure $M = J_\delta[F]$, so that

a) F is an end-extension of the filter sequence D;
b) otp (dom(F)) < min (dom(F));
c) F-D is a sequence of measures in M;
d) the iterated ultrapowers of M with respect to F-D are transitive;
e) if κ = min(dom(F)-dom(D)) exists then M is the Σ_1-closure of $\kappa \cup \{\gamma\}$ in M, for some $\gamma \in M$.

The set $K[D] = \bigcup\{M \mid M$ is a D-mouse$\}$ is called a (short) coremodel if $K[D] \models D$ is a sequence of measures.

Assume that for some $\alpha \in On$ there is no inner model with α measurables. Then:

(1) every coremodel is an "L-like" inner model;

(2) the family of coremodels satisfies natural inclusion and embedding properties;

(3) an embedding

$$\pi : K[U]_{\Sigma_\omega} \to K[U]$$

with critical point $>$ sup(dom(U)) induces a "new" measure;

(4) there is a maximal coremodel satisfying a weak covering theorem.

<u>Theorem:</u> If λ is a Jonsson cardinal, $\omega < cf(\lambda) < \lambda$, then there is an inner model with $cf(\lambda)$ measurable cardinals.

J.P. LEVINSKI:

<u>Some principles related to the conjecture of Chang</u>

Three properties related to the conjecture of Chang were considered: the weak conjecture of Chang (WCC), the transversal hypothesis (TH) and the uniform norm property (UNP). They were classified according to their consistency strength.

Letting $A < B$ mean that A is "consistency-wise" strictly weaker than B, the author showed that

$$WCC < UNP < \neg TH < CC.$$

The implieations CC $\to \neg$TH $\to$ UNP $\to$ WCC are either known or not hard to establish. The author tried to give an idea of the "space" between two consecutive properties.

The main methods used were forcing and constructibility.

A. LOUVEAU:

σ-ideals of closed sets

This is a joint work with A.S. Kechris and H. Woodin. The authors studied descriptive set theoretical properties of σ-ideals of closed sets in compact metric spaces. Characterizations of the possible complexity of σ-ideals and the possible complexity of bases for σ-ideals, among $\Sigma^1_1 \cup \Pi^1_1$ σ-ideals, were given. The obtained classification seems to be related to other structural properties of these objects.

M. MAGIDOR: *

Martin's maximum

A natural generalization of Martins axiom is Martin's maximum (MM): If P is forcing notion which does not destroy stationary subsets of ω_1, and $\langle D_\alpha \mid \alpha < \omega_1 \rangle$ is a family of dense subsets of P, then there exists a filter $G \subset P$ such that for $\alpha < \omega$, $G \cap D_\alpha \neq \emptyset$.

Theorem: Con(ZFC + ∃supercompact cardinal) → Con(MM).

Some conclusions from MM:

(a) Every stationary subset S of a regular cardinal $\lambda > \omega_1$ such that for $\alpha \in S$ $cf(\alpha) = \omega$, has a stationary initial segment (actually an initial segment which contains a club).

(b) For every regular cardinal $\lambda > \omega_1$ holds $\lambda^\omega = \lambda$. Hence in particular $2^{\aleph_0} = \aleph_2$ and the singular cardinal hypothesis holds.

(c) The closed unbounded filter on ω_1 is ω_2-saturated.

M. SREBRNY:

Measure and category at higher levels

Theorem 1: Assume $\boldsymbol{\Delta}^1_2$ Determinacy. If all $\boldsymbol{\Sigma}^1_4$ sets are Lebesgue measurable then all $\boldsymbol{\Sigma}^1_4$ sets have the property of Baire.

Theorem 2: (with A. Krawczyk) Assume $(\forall a \subseteq \omega)$ ($a^{\#}$ exists) and $D^{\dagger}$ does not exist. If all $\boldsymbol{\Sigma}^1_3$ sets are Lebesgue measurable then all $\boldsymbol{\Sigma}^1_3$ sets have the property of Baire.

Relevant results in the area: (1) J. Raisonnier and J. Stern have recently proved in ZFC that if all $\boldsymbol{\Sigma}^1_2$ sets are Lebesgue measurable then all $\boldsymbol{\Sigma}^1_2$ sets have the property of Baire. (2) A.S. Kechris has proved from $\boldsymbol{\Delta}^1_2$-Determinacy that all $\boldsymbol{\Sigma}^1_3$ sets are Lebesgue measurable and have the property of Baire. (3) W.H. Woodin then showed from $\boldsymbol{\Delta}^1_2$-Determinacy plus all $\boldsymbol{\Sigma}^1_4$ well-orderings of sets of reals are countable that all $\boldsymbol{\Sigma}^1_4$ sets are Lebesgue measurable and have the property of Baire.

L.J. STANLEY:

Constructing simplified gap-2 morasses form ordinary ones: towards a forcing principle for gap-2

A simplified $(\omega_1, 2)$ morass is constructed from countable approximations, using an ordinary $(\omega_1, 2)$ morass. The construction prefigures that which will (hopefully) prove that a yet-to-be-formulated forcing principle is a consequence of gap-2 morasses.

F.D. TALL: *

Some set theoretic problems arising from topology

In connection with the use of interated forcing plus reflection to prove the consistency of universal statements, one wants to know what properties are preserved by what kinds of forcing over what models. A new method of proving such preservation lemmas, based on results of Alan Dow, has proved useful in work on problems related to the Normal Moore Space conjecture. We discuss what is known and its applications and raise several questions.

References:

A. Dow, Some linked subsets of posets, preprint.
F.D. Tall, Countably paracompact Moore spaces are metrizable in the Cohen model, Top. Proc., to appear.

D. VELLEMAN:

Simplified higher gap morasses

This talk was a progress report on research to find a simplified version of gap 2 morasses. A proposed definition of simplified gap 2 morass was deseribed and several theorems and conjectures coneerning the existence of these structures and their applications were presented.

W.H. WOODIN: *

More on changing cofinalities

Theorem: Assume there is an unbounded class of supercompact cardinals (less will suffice). Then there is a class partial order, $\mathbb{P}$, and a generic elementary embedding:

$$j : V \to V^{\mathbb{P}}$$

Further if $G \subseteq \mathbb{P}$ is generic over V then, for every λ, $G \cap V_\lambda$ is set generic over V. Finally if $S \subseteq P(V_\lambda)$ is stationary then there is a condition a_S in $\mathbb{P}$ such that

$$a_S \Vdash j"V_\lambda \in j(S).$$

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft