

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6/1985

Kombinatorik geordneter Mengen

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Im Mittelpunkt des Interesses stand die Theorie endlicher geordneter Mengen. Die Themen der Vorträge lassen sich schwerpunktmäßig gliedern in:

- Extremal- und Optimierungsprobleme geordneter Mengen (lin. Erweiterungen, scheduling, sorting);
- Struktur und Darstellung (endlicher) geordneter Mengen (Klassifikation, Diagramme);
- Topologische Methoden in der Ordnungstheorie;
- Anwendungen.

Zusätzlich fand eine "Problemsitzung" statt, auf der jeweils mit einer kurzen Einführung offene Probleme vorgestellt wurden.

Vortragsauszüge

W.T. TROTTER: Extremal Problems for Ordered Sets

Extremal Problems for Ordered Sets constitute a relatively new, but promising area of research in combinatorial mathematics. In this talk, we will survey recent results in the area concentrating on results which have not yet been published. In particular, we will outline the proof of the fact that the dimension of an ordered set is bounded as a function of the maximum degree in the comparability graph. Although the exact values of the best possible function are not known, we can prove: $k^2 / \log k \leq f(k) \leq k^2$.

D. KELLY: Free \aleph -lattices on some ordered sets

(This research was done jointly with George Grätzer). Let us call an ordered set P slender if P does not contain $\underline{1} + \underline{1} + \underline{1}$, $\underline{2} + \underline{3}$, or $\underline{1} + \underline{5}$. In an \aleph -lattice, all sets X with $0 < |X| < \aleph$ have joins and meets. (Here \aleph is a fixed infinite regular cardinal.) The results we describe below were first proved for $\aleph = \aleph_0$ by I. Rival and R. Wille [J. Reine Angew. Math. 310 (1979), 56-80]. We described the free \aleph -lattice on the ordered set $H = \begin{matrix} & & & & \\ & & & & \times \\ & & & & \\ & & & & \times \\ & & & & \end{matrix}$ in Order 1 (1984),

47-65. We shall show that $F_{\aleph}(P)$, the free \aleph -lattice on the ordered set P , can be embedded in $F_{\aleph}(H)$ whenever P is linearly indecomposable and slender. Consequently, an ordered set P is slender iff $F_{\aleph}(P)$ does not contain $F_{\aleph}(3)$ as an \aleph -sublattice.

J.KAHN and M. SAKS : On the Width of Distributive Lattices

For a finite ordered set P , let $w(P)$ be the width. We prove the following conjecture of Bill Sands: For every $\epsilon > 0$ there is an n that if L is a distributive lattice with $|L| \geq n$ then $w(L)/|L| < \epsilon$.

P. WINKLER : Random Orders

Fix positive integers k and n and let $P_k(n)$ be the (partial) order obtained by intersecting k random linear orderings of an n -element set. What does $P_k(n)$ look like for fixed k and large n ?

R. CANFIELD : P-recursireness of ménage and related numbers

The derangement numbers count the permutations σ avoiding a repetition in any column of the array

$$\begin{matrix} & 1 & 2 & \dots & n \\ \sigma(1) & \sigma(2) & \dots & \sigma(n), \end{matrix}$$

while the straight ménage numbers count those σ avoiding column repetitions in the three-line array

$$\begin{matrix} & 1 & 2 & \dots & n \\ & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \dots & \sigma(n) \end{matrix}$$

There is a generalization to "ménage-r numbers", and a problem is to decide whether these new sequences, like the classical derangement and ménage numbers, satisfy finite recursions with polynomial coefficients. We discuss this problem and some related issues.

H.A. KIERSTEAD : Inequalities for the Greedy Dimension of Ordered Sets

Every linear extension $L: [x_1 < x_2 < \dots < x_m]$ of an ordered set P on m points arises from the simple algorithm: For each i with $0 < i < m$, choose x_{i+1} as a minimal element of $P - \{x_j : j \leq i\}$. A linear extension is said to be greedy, if we also require that x_{i+1} covers x_i in P whenever possible. The greedy dimension of an ordered set is the minimum number of greedy linear extension of P whose intersection is P . We shall develop several inequalities bounding the greedy dimension of P as a function of other parameters of P . If A is an antichain in P and $|P - A| \geq 2$, we show that the greedy dimension of P does not exceed $|P - A|$. If the width of $P - A$ is n and $n \geq 2$, we show that the greedy dimension of P does not exceed $n^2 + n$. If A is the set of minimal elements of P , then this inequality can be strengthened to $2n - 1$. If A is the set of maximal elements, then the inequality can be further strengthened to $n + 1$. Examples are presented to show that each of these inequalities is best possible.

L.H. HARPER : On The Asymptotic Rota Conjecture

Let P be a (finite) poset with rank function r , and let $R(P) = \frac{\text{max size of an antichain}}{\text{max size of a rank}}$. In 1928 Sperner showed that $R(B_n) = 1$, B_n being the lattice of subsets of an n -set. In 1967 Rota asked if $R(\Pi_n) = 1$, Π_n being the lattice of partitions of an n -set. Canfield (1979) showed $R(\Pi_n) > 1$ for $n >$ Avogadro's number (which Kleitman & Sha have recently lowered to 3.4×10^6 .) However the asymptotic behaviour of $R(\Pi_n)$ has remained a mystery. Based upon the result of Graham-Harper that $R(\Pi_n) = R(P_n)$, P_n being the unordered partitions of n weighted by $W(\tau) = \prod_{r \geq 1} \frac{n!}{\tau_r! (r!)^{\tau_r}}$

and the assumption that P_n may be approximated by a Gaussian process & have made a rough calculation which indicates that $R(\Pi_n) \rightarrow 1.69\dots$ as $n \rightarrow \infty$.

J. GRIGGS : k-color Sperner theorems

Results will be discussed concerning families F of subsets of an n -set S with the property that with respect to some k -coloring of S there do not exist $A, B \in F$ such that $A \not\subseteq B$ and $B - A$ is monochromatic. Let $f(n,k)$ denote the maximum value of $|F|$ over all such F . By Sperner's theorem and by the two-part Sperner theorem it follows that $f(n,1)$ and $f(n,2)$ equals $\binom{n}{n/2}$. It will be shown here for arbitrary k that $\lim_{n \rightarrow \infty} [f(n,k) / \binom{n}{n/2}]$ exists, call it d_k . For $k \geq 3$ $d_3 > 1.036$. For $k \rightarrow \infty$ $d_k \sim \sqrt{\pi k / 4 \ln k}$. Related results and conjectures will be discussed. This is joint work with A. Odlyzko and T. Shearer and includes independent results of Z. Füredi.

K. ENGEL : Extremal problems in cubical lattices and chain products

Let C_n be the lattice of all faces of an n -dimensional cube, ordered by inclusion, and E_k^n be a product of n copies of the chain $(0 < 1 < \dots < k-1)$. A subset (or family) F of these lattices satisfies the Sperner, intersection, or union condition if $x \not\subseteq y$, $x \wedge y \neq$ minimal element, resp. $x \vee y \neq$ maximal element holds for all $x, y \in F$. We consider families satisfying these conditions in all 7 possible combinations. For C_n we determine all vertices of the convex closure of the set of profiles of such families. Here $\underline{f} = (f_0, \dots, f_n)$ is the profile of F if f_i equals the number of rank i elements. For E_k^n we (together with H.-D.O.F. Gronau) determine the maximum size of such families. The cases of intersecting Sperner families (satisfying the union condition) remain unsolved for even n and $k < 2n$. Last but not least we (together with P. Frankl) determine asymptotically the maximum size of an intersecting family of rank ℓ elements in E_k^n , where $\ell = \lfloor \lambda n \rfloor$, λ fixed, and $n \rightarrow \infty$.

Z. FÜREDI : Finite families of sets and semilattices

Let k, t be positive integers and let \underline{F} be a family of k -element sets. Improving earlier results of Erdős (1967), Erdős and Kleitman (1968) and Erdős and Rado (1962) we obtained the following Theorem There exists a subfamily $\underline{F}^* \subseteq \underline{F}$ with the following properties:

- i) $|\underline{F}^*| \geq C(k, t) \cdot |\underline{F}|$ where C depends only on k and t ,
- ii) \underline{F}^* is k -partite (i.e. There exist V_1, V_2, \dots, V_k such that $|F \cap V_i| = 1$ holds for all $F \in \underline{F}^*$, $1 \leq i \leq k$).
- iii) Every pairwise intersection in \underline{F}^* is a kernel of a t -star in \underline{F}^* (i.e. for all $F_1, F_2 \in \underline{F}^*$ there are $F_3, \dots, F_t \in \underline{F}^*$ such that $F_1 \cap F_j = F_1 \cap F_2$ holds for all $1 \leq i < j \leq t$.)
- iv) There exists a family \underline{M} on the elements $\{1, 2, \dots, k\}$ such that \underline{M} is isomorphic to $M(F, \underline{F}^*)$ for all $F \in \underline{F}^*$, where $M(F, \underline{F}^*) = \{F \cap H : H \in \underline{F}^*\}$.
- v) \underline{M} is a semilattice (i.e., $M, M' \in \underline{M}$ implies $M \cap M' \in \underline{M}$).

This theorem was conjectured by P. Frankl. Several applications were presented.

A. BEUTELSPACHER : Embedding of geometric lattices in projective geometries

A planarspace (geometric lattice of dimension 3, matroid of rank 4) consists of points, lines and planes such that the following axioms hold :

- 1) Any two points are on exactly one line;
- 2) planes are subspaces, there are at least two planes;
- 3) any three non-collinear points are on exactly one plane.

The following theorem will be discussed:

Theorem. Let \underline{S} be a finite planar space in which any two planes meet in a line. Suppose furthermore that \underline{S} is not a degenerate projective space. Then

(a) For any point p , the quotient \underline{S}/p is a projective plane of the same order n .

(b) If the number of points of \underline{S} is at least n^3 , then \underline{S} is embedable in $PG(3, n)$, the 3 dimensional projective space of order n .

B. MONJARDET : Posets and similarity coefficients between linear orders

There exist two classical agreement coefficients between two linear orders: Kendall's τ and Spearman's ρ . The Daniels inequality, and -equivalently- the Guilbaud Formula provides a (not simple) relation between τ and ρ . We prove a third -equivalent- relation between two metrics defined as the set of all linear orders (on a finite set). This proof is obtained by considering several parameters that can be defined on an arbitrary poset.

This point of view allows also

- 1) to give an ordinal interpretation for ρ - τ , allowing to get its extremum values (and the corresponding pair of linear orders)
- 2) to show that the coefficient $\rho(L, L')$ is an isotone mapping of the partial order $L \wedge L'$.
- 3) to raise other problems.

D. SCHWEIGERT : On heuristics for optimal matchings

While drawing, the pen of a plotter moves very often lifted up. The plotter problem is to minimize "wasted pen movements". One of the many approaches is to find an optimal matching. Exact algorithms are known with $O(n^3)$, but these are useless for problems with thousands of points. A heuristic algorithm is developed for the optimization of graphic data, a problem which is related to the plotter problem.

W. POGUNTKE : Level schedules by interchanging matched subsets

A well-known result of T.C.Hu (1961) is the basis for the little progress that has been made on this problem: schedule unit time jobs subject to precedence constraints on identical parallel machines to minimize the maximum completion time. In the case that the precedence constraints form an inforest - that is, each job has at most one successor - any "level" schedule is optimal. We present and study a procedure to transform a given schedule for a set of jobs with arbitrary precedence constraints into one

which is more like a level schedule - without increasing the completion time. This is a particularly interesting since, an arbitrary level schedule need in general not be optimal at all, and, indeed, there are even simple settings in which no level schedule can be optimal.

(This is joint work with Ivan Rival).

O. PRETZEL : The diagram of an ordered set

A k -orientation of a graph is an orientation in which every cycle has at least k edges oriented each way. Every orientation determines an additive function, its signature, that gives an integer value to every circuit.

We present an algebraic proof, using this signature, that M_k has no 2 -orientation. This suggests that there should be an "obstruction" theory of k -orientability. As a first step in this direction we give a Theorem that any additive function on circuits, taking only admissible values is the signature of some orientation.

If time permits we shall discuss the relation between the signature and long circuits and "pushing down" maximal vertices.

D. DUFFUS : Gap and Selection Properties for Ordered Sets

An order variety is a class K of ordered set isomorphism types closed under the formation of retracts and direct products. Investigation of conditions sufficient to insure membership in particular varieties has led to the formulation of two types of properties: gap and selection properties. In fact, an ordered set belongs to CH , the variety generated by the class of isomorphism types of chains, if and only if each of its gaps is preserved by some chain and there is an order preserving selection function. Recent work of Duffus and Pouzet has concerned this question: Is each finite dimensional lattice a member of CH ? It is known that all finite dimensional distributive lattices and all 2-dimensional modular lattices belong to CH . Moreover, by a characterization of gaps "preservable" on chains, they prove that all lattices of finite breadth have their gaps so preserved.

M. POUZET : Retracts of Posets and Graphs: The Metric Approach

We consider a generalization of the notion of metric space where the distance take values in a complete ordered semi-group V having an involution. The notions of balls, contraction (or non-expansive) maps and isometries have a natural interpretation. As in the case of metric spaces, the notions of absolute retracts, injective spaces and hyperconvex spaces are equivalents. Also if V satisfies a distributivity property then V is hyperconvex and the hyperconvex spaces are exactly the retracts of powers of V ; moreover every space has an injective envelope. Finally it is shown that the bounded hyperconvex sets have the fixed point property for contraction maps. For particular choices of V this includes the results of N. Aronszajn, P. Panitchpakdi, B. Banaschewski, G. Bruns A. Quilliot, I. Rival, R. Nowakowski on absolute retracts of metric spaces, posets and graphs; the Tarski fixed point theorem and also the recent result of R. Sine and P. Soardi on the fixed point property for some ℓ^∞ subspaces. The results presented here have been obtained in part with D. Misane.

M. SEKANINA : Categorical approach to combinatorics of ordered sets

A. Joyal in his paper "Une theorie combinatoire des series formelles," Advances of Math. 42 (1981), 1-82 described a method joining intuitive combinatorics of the theory of enumeration with a functorial formalism in the category of the sets. In dealing with a generalization to the category of the ordered sets one of the main questions is to find a suitable addition for ordered sets. Two natural candidates are cardinal and ordinal sums. Especially for cardinal sum many principal aspects of Joyal's paper remain valid. There are presented other examples of addition of ordered sets, which can be useful in the studied generalization.

M.K. BENNETT : The Convexity Lattice of a Poset

I wish to report on joint work with Garrett Birkhoff in which we investigate the lattice $Co(P)$ of convex subsets of a general

partially ordered set P . In particular we determine the conditions under which $Co(P)$ and $Co(Q)$ are isomorphic; and give necessary and sufficient conditions on a lattice L so that L is isomorphic to $Co(P)$ for some P .

G. GIERZ : Essential Completions

Let L be a distributive lattice and let $B(L)$ be the maximal essential extension of L . Banachewski and Bruns have shown that $B(L)$ is a complete Boolean algebra. Let $g(L)$ be the smallest complete sublattice of $B(L)$ containing L . We call $g(L)$ the essential completion of L . The Zariski topology on L is the smallest topology for which solution sets of lattice equations $p(x) = q(x)$ are closed.

Theorem. (i) $g(L)$ is complete, meet-continuous, and join-continuous.

(ii) If $g(L)$ is complete, meet-continuous and join-continuous, then $g(L) = L$.

(iii) $g(L)$ is completely distributive $\Leftrightarrow L$ is Hausdorff in the Zariski topology.

(iv) If L is an open sublattice of a completely distributive lattice M (equipped with the interval topology) and if the closure \bar{L} of L in M is connected, then $g(L) = \bar{L}$.

The theorem implies that the essential completion of an open hypercube $]0,1[^n$ is the closed cube $[0,1]^n$.

J.D. LAWSON : Infinite antichains in semilattices

The notion of a continuous semilattice together with its inherent topology is introduced and the existence of countably infinite antichains converging to an upper bound is discussed. In the most free situation the antichain generates (as a complete semilattice) a copy of $2^{\mathbb{N}}$; if the given continuous semilattice has no such copy, then it has compact-finite breadth, i.e. every compact inf may be replaced by a finite one. A semilattice Δ is introduced isomorphic to the free semilattice on countably many

generators e_i modulo the relations that the meet of e_i and e_j is less than e_k for i less than j less than k . Any continuous semilattice containing an infinite antichain converging to a larger element contains a continuous semilattice embedding of DELTA.

P.H.EDELMAN : Convex geometries and some dimension problems

Associated with a notion of abstract convexity is the class of meet-distributive lattices. We examine the structure of the set of all meet-distributive lattices with a fixed labeled set of join-irreducibles. This set has a natural partial ordering on it which forms a lattice. Using these ideas we define the notion of convex dimension for a meet-distributive lattice and discuss how it relates to standard notions of dimension.

J.W.WALKER : Shellability of Hypogeometric Lattices

We study a new class of lattices, to be called hypogeometric lattices, which includes the class of geometric lattices. Two other examples are the poset of independent sets of a matroid, with a greatest element adjoined, and the dual of the lattice of affine subspaces of a finite dimensional vector space. Hypogeometric lattices can be shown to be shellable. These results imply two lemmas used by Lusztig in representation theory.

D.HIGGS : (A) Nicely graded lattices.

Or(B) Lattices of antichains and related subsets in finite posets.

(A) An interpolation antichain in a poset P is an antichain A in P such that if a, b are in A and $a \leq y, y \geq x, x \leq b$ then $x \leq c \leq y$ for some c in A . P is nicely graded if it is graded and, for each interpolation antichain A in P , the elements of A are all on the same level. Various results and questions about nicely graded lattices will be discussed.

(B) It is known that the set of maximal antichains and the set of minimal cutsets in a finite poset P each form lattices under the

appropriate order and that these two lattices intersect in the distributive lattice of cross-cuts of P . The possibility of finding a single lattice of suitable subsets of P containing the above two lattices will be discussed, along with some related questions.

K.B. BOGART : Subalgebras of the Incidence Algebra and Generating Functions

Doubilet, Rota and Stanley introduced the idea of studying generating functions by representing various important algebras of generating functions as subalgebras of the incidence $I(P)$ algebra of an appropriate partially ordered set P . In particular, they studied the standard algebra, the algebra of functions constant on isomorphism classes of intervals. Although the incidence algebra $I(P \oplus Q)$ of a direct sum of partially ordered sets is the tensor product $I(P_1) \otimes I(P_2)$ of the incidence algebras of the components, this does not carry over to the standard algebra. We introduce a slightly larger subalgebra $A(P)$ of the incidence algebra with the property that $A(P \oplus Q) = A(P) \otimes A(Q)$. In all the cases of algebras of generating functions Doubilet, Stanley, and Rota studied, $A(P)$ is the algebra of generating functions. Further, the tensor factorization of $A(P)$ carries over to infinite direct sums. This tensor factorization provides a bridge which lets us unify the Bender-Goldman theory of prefabs with the incidence algebra approach to generating functions.

H.-J. BANDELDT : Modular graphs and ordered sets

A modular graph is a graph in which for every triple u, v, w of vertices there exists a vertex geodesically between each pair of u, v, w . Among modular graphs are all median graphs, absolute retracts of bipartite graphs, and the covering graphs of discrete modular lattices. One can characterize those discrete ordered sets with 0 which have modular covering graphs. The ordered sets in question are modular multilattices (sensu Benado) satisfying two additional conditions.

D.B. WEST : Semiantichains, Unichain Coverings, and Related Parameters

We review recent results concerning semiantichains, unichain coverings, and related problems in direct products of posets. Semiantichains are subsets of a product in which distinct elements are incomparable if equal in either coordinate. Unichains are chains in which one coordinate is fixed. Since semiantichains and unichains intersect at most once, any unichain covering uses at least as many chains as the size of any semiantichain. It is conjectured that every direct product has a unichain covering and a semiantichain of the same size. West and Tovey proved a special case of this. We prove another special case not covered by their result and suggest a common generalization. In addition, we introduce new parameters generalizing unichain coverings, semiantichains, and chain coverings to arbitrary posets without product structure, and we obtain several results about these using a network flow model.

D.T. KLEITMAN : Some Extremal Problems Concerning Ordered Sets

Some recent (and not so recent) results are discussed:

- (1) (with Sha J. Chang). We describe a class of antichains larger than any rank in the lattice of partitions of a finite set (of size at least 4×10^6) under refinement ordering, and give an intuitive picture of what is happening and a calculus for computing some.
- (2) (with C. Greene). We give a characterization of maximum sized chains in the lattice of partitions of n under the majorization ordering; this yields the maximal length of such a chain, (essentially $\frac{2}{3}\sqrt{2} \cdot n^{3/2}$) and other related results.
- (3) (with J. Chang). We find bounds of $\prod_j \binom{n}{j}^{\binom{n}{j}}$ (upper) and $\prod_j \binom{n}{j}!$ (lower) on the number of linear extensions of 2^n . Open problem! Which is closer to the answer?
- (4) The maximum number of meet-irreducibles given n join-irreducibles is shown to be between $\binom{n}{n/2} (1 + cn^{-1})$ and $\binom{n}{n/2} \cdot (1 + cn^{-1/2})$; which is right? (the first probably is.)
- (5) (with M. Saks) We describe strengthenings of the LYM inequality

ty for sets on 2^n .

(6) The largest family of elements of 2^n that is 2-intersection and 2-union free has size $\binom{n}{n/2} (1 + (1/n))$ for odd n and $\binom{n}{n/2} + 1$ for even n .

W. DEUBER : The uniqueness of the solution for Kruskal's extremal problem

Abstract. The Kruskal problem is the following. Find a collection of i many sets $A_1 \dots A_i$ in $\binom{n}{k}$, such that the l -shadow $\bigcup_{j=1}^i \binom{A_j}{l}$ has minimal cardinality. ($1 \leq l < k < n$, $1 \leq i \leq \binom{n}{k}$).

One solution is given by the first i sets in the antilexicographic ordering on $\binom{n}{k}$.

Theorem. For $i = \binom{Q_1}{k} + \binom{Q_2}{k-1} + \dots + \binom{Q_t}{k-t+1}$, this solution is unique up to renumbering of elements iff

$l \geq t$ or $i = \binom{Q_1+1}{k} - 1$ or $i \leq k+1$.

As an intermediate step an extension of the Kruskal problem is considered, which as a consequence generalizes Kleitman's extension of the Erdős, Ko, Rado theorem.

M.ERNÉ : Clique numbers of graphs

(joint work with P. Erdős and U. Vollert). We study the sets $G(n)$ ($n \in \mathbb{N}$) of all numbers c for which there exists a graph with n vertices and c cliques (i.e. complete subgraphs, not necessarily maximal). This problem is strongly related to the problem of determining the number of antichains in posets. In order to know the sets $G(n)$, it suffices to compute the numbers $g(c) = \min \{ n : c \in G(n) \}$ since c belongs to $G(n)$ iff $g(c) \leq n < c$. Let $b(c)$ denote the number of ones in the binary representation of c and write ld for \log_2 . Then a straightforward computation gives the estimates $ld c + ld \frac{4}{3} \cdot ld b(c) \leq g(c) \leq ld c + \frac{b(c)}{2} + \sqrt{ld c - b(c)}$, while a rather technical proof is necessary in order to obtain the upper bound $ld c + 9(ld c)^{5/6}$ which shows that $c \in G(n)$ whenever $n < c \leq 2^{n-9n^{5/6}}$. On the other hand, the lower bound for $g(c)$ gives $|G(n)| \leq 2^n \cdot n^{-2/5} = o(2^n)$. The numbers in $G(n)$ which are greater than 2^{n-2} can be described explicitly by simple inequalities.

E.HARZHEIM : A generalization of the mapping degree to cartesian products of linearly ordered continua

A linearly ordered continuum C is a non-empty dense linearly ordered set, which has no first and no last element and has no gaps: If I is an initial segment of C with $\emptyset \neq I \neq C$ then I has a last or $C-I$ a first element. In C we have the usual order topology. If C_1, \dots, C_n are linearly ordered continua we call their cartesian product $C = C_1 \times \dots \times C_n$ an n -continuum. It is equipped with the product topology which has as basis the set of all boxes $O_1 \times \dots \times O_n$, where O_v is an open interval of C_v . The question arises which notions and theorems of \mathbb{R}^n can be generalized to n -continua. Here we generalize the notion of mapping degree and winding number to n -continua. Using this concept one can generalize all classical invariance theorems, in particular the separation theorem of Jordan - Brouwer - Alexander (which implies the others) :
If K is a compact subset of C , $f : K \rightarrow C$ continuous and injective then $C-K$ and $C - f(K)$ have the same (cardinal) number of connectivity components.

U. FAIGLE : Identifying ordered sets of height 1

The identification problem for ordered sets asks for the minimum number of calls of a comparability oracle (with respect to pairs of elements) necessary so that a hidden (partial) order on a ground set E can be identified. This talk reports on joint work with G. Turán on this problem. In particular, we present an algorithm which is optimal for height 1 ordered sets within factor 2. The general problem remains open. Our current approach is based on the notion of an essential set and we offer a conjecture which would imply that these methods are not sufficient to even handle the case of bounded height (note, in contrast, that the case of bounded width is easy: the information-theoretic lower bound can be achieved).

M. AIGNER : Lower Bounds for Sorting Problems

Let P be a poset, $|P| = n$. The sorting complexity $C(P)$ is the minimum number of comparisons needed to produce P . Let $P = \left(\frac{T}{A} \right)$

be the ordinal sum of a top-part $T, |T|=t$ and an antichain $A, |A|=n-t$; Result 1: $C(T/A) \geq (n-t) + \lceil \log([n]_t/e(T)) \rceil$, $[n]_t = n(n-1)\dots(n-t+1)$, $e(T)$ = number of linear extensions.

If $P=(T/A/B)$, T top-part, $|T|=t$, B bottom-part, $|B|=b$, A antichain, $|A|=n-t-b$ we have Result 2: $C(T/A/B) \geq (t+b) - |\min T| - |\max B| + \lceil \log \binom{n}{t} \binom{n-t}{b} \rceil$. Let $V_t(n), V_{t-1,t}(n)$ denote the complexities of selecting the t -th element and the $(t-1)$ -th, t -th element, resp..

Let $0 < x < 1$; Conjecture(Backelin): $\lim_{n \rightarrow \infty} (V_{\lfloor xn \rfloor}/n) = 1 + (-x \log x - (1-x) \log(1-x))$. Bounds on this limit are obtained and it is mentioned that $1 - x \log x - (1-x) \log(1-x)$ is a lower bound for $V_{\lfloor xn \rfloor} - 1, \lfloor xn \rfloor/n$, adding strength to the conjecture: single selection \sim double selection.

I.G. ROSENBERG : Cover graphs via 0 - 1 inequalities

The cover graph $Cov(\underline{P})$ of an order is the symmetric hull of the covering relation and a graph is a diagram if it is the cover graph of some order. Given a graph $G = (P, E)$ fix an orientation of E . For a simple 1-cycle $C = \{V_1 V_2, \dots, V_1 V_1\}$ of G put $\alpha_1 = 1$ if (V_i, V_{i+1}) is in the orientation and $\alpha_i = 0$ otherwise. Further let $a^0 := 1 - a$ and $a^1 := a$. For $x: E \rightarrow \mathbb{Z}_2$ ($\mathbb{Z}_2 = \{0, 1\}$) the cycle inequality of C is

$$2 \leq x(V_1 V_2)^{\alpha_1} + \dots + x(V_1 V_1)^{\alpha_1} \leq 1 - 2 \quad (1_c).$$

There is a bijection between the solutions $x \in \mathbb{Z}_2^E$ of the system (1_c) (over all simple cycles of G) and the discrete orders on P whose cover graph is G . The system (1_c) may be reduced. We solved it for the complete bipartite graph $K_{n,n}$, find an infinite set critical non-diagrams and an infinite set of critically 4-chromatic triangle free diagrams.

I. RIVAL : Greedy Linear Extensions

Loosely speaking a greedy linear extension of a finite ordered set is a linear extension constructed according to the rule "climb as high as you can". It is a natural construction for the jump number problem. In recent work with N. Zaguia we have used the ideas of a "subdiagram" and a "chain interchange algorithm" to estab-

lish new results about the role of greedy linear extensions in the study of the jump number, the greedy dimension, and ensersibility of greedy linear extensions.

M.M.SYSLO : Algorithmic aspects of greedy linear extensions of partially ordered sets

Let P be a partially ordered set (poset). We consider the jump number problem which is to find an optimal linear extension L of P that consists of the least number of chains of P . An arc diagram D of P is an acyclic directed graph in which the poset elements are represented by a subset of arcs of D . Every poset has an optimal linear extension which consists of chains generated by the greedy method. Making use of arc diagrams, we define a strongly greedy chain in P and show that if P has such a chain C then there exists an optimal linear extension of P which begins with C . As a corollary, we obtain the expression for the jump number of N -free posets. Moreover, we identify a class M of posets which properly contains N -free posets and every $P \in M$ has an optimal linear extension which can be generated by the strongly greedy algorithm.

R.H.MÖHRING : On series-parallel and N -free partial orders

(Jointly with M.Habib). We consider two generalizations of series-parallel posets and investigate their complexity with respect to 3 well-known combinatorial (optimization) problems which are polynomially solvable on series-parallel posets: the jump number problem, the isomorphism problem, and the $1/\text{prec}/\sum w_j C_j$ scheduling problem (minimizing the sum of weighted completion times on one machine).

The first generalization is the class of N -free posets, for which the jump number problem is known to be polynomially solvable. We show that both other problems are, however, hard on this class (i.e. isomorphism complete and NP-complete, respectively).

On the other hand, all 3 problems are shown to be polynomially solvable for the second generalization, which consists of all posets obtained by substitution (lexicographic sum) from indecom-

possible posets of fixed size (posets of bounded diameter).

This adds some evidence to our opinion that, with regard to computational complexity, the second class constitutes a much more natural generalization of series-parallel posets.

F.FARMER: Retracts: Topological Methods

Problem: If $\alpha: A \rightarrow X$, when does there exist $r: X \rightarrow A$ so that $r \circ \alpha = \text{id}_A$? A solution to this problem depends on being able to compute the reduced Poincaré polynomials $\tilde{p}(X)$, $\tilde{p}(A)$, and $\tilde{p}(X/A)$ when the spaces are nice and α is an embedding.

If $R \subseteq X \times X$ then $T(X, R)$ is the topological realization of the simplicial complex $\{ \text{Im}(f) : f: \text{fin.lin.ord} \rightarrow rR \}$ where rR is a reflexive relation.

Theorem: If (X, R) is a directed forest then

- (1) $T(X, cR)$ is a homotopy n -sphere for some $n \leq \infty$,
 - (2) $T(X, cstR)$ is a homotopy cluster of spheres,
- where t, s are transitive (resp. symmetric) closure and $cR = (X \times X) - R$. On the class (2) above the elementary result $[\tilde{p}(X) \neq \tilde{p}(A) + \tilde{p}(X/A) \Rightarrow A \text{ is not a retract of } X]$ almost becomes an equivalence. The polynomials $\tilde{p}(X)$, $\tilde{p}(A)$ are easily computable but more work needs to be done to compute $\tilde{p}(X/A)$.

B.VOIGT: Ramsey's theorem and the property of Baire

We introduce the notion of partition category and then have the following

Theorem: For every Baire mapping $\Delta: \mathbb{C}(\binom{\omega}{k}) \rightarrow \{0, \dots, r-1\}$, where $k, r < \omega$, there exists $F \in \mathbb{C}(\binom{\omega}{\omega})$ with $\Delta(F \cdot G) = \Delta(F \cdot H)$.

As corollaries one obtains Ramsey's theorem (J. London Math. Soc. 1930), a theorem for Graham - Rothschild parameter word (Prömel - Voigt, to appear in Trans. AMS), an extension of the Graham - Leeb - Rothschild theorem for vector spaces (Voigt, to appear in Crelle's J.) as well as other results. In particular, we have the following result of Prömel and Voigt:

Theorem For every Baire mapping $\mathbb{Z}^\omega = B_0 \cup \dots \cup B_{r-1}$ (powerset of ω , topologized as Cantorspace) there exist nonempty and mutually

disjoint subsets A_k , $k < \omega$, as well as $i < r$ such that $\bigcup_I A_k \in B_i$ for all nonempty $I \subseteq \omega$.

In view of a result of Shelah and Stern it follows that it is consistent with ZF (if ZF is consistent) to assume that this result holds for all finite partitions, although this contradicts the axiom of choice.

G.F. McNULTY : Ordering Words

Two words will be regarded as equivalent if there is a permutation of the alphabet which transforms one of the words into the other. For example, "start" and "order" are equivalent. Let P_n denote the set of all (equivalence classes of) words on the n letter alphabet. For words u and w we write $u \triangleleft w$ to mean that some substitution instance of u is a subword (i.e. a factor) of w . \triangleleft is a partial order on P_n which arises in algebra and logic. (P_n, \triangleleft) has a very intricate structure and it is the purpose of this talk to present what is known about it and to frame some problems concerning it. We say that the word u is n -avoidable provided the complement in P_n of the principal filter above u is infinite; u is said to be avoidable iff u is n -avoidable for some natural number n . There is a nice algorithm (Bean, Ehrenfeucht, and McNulty 1979) for determining whether words are avoidable. It is unknown whether every avoidable word is actually 3-avoidable. It is known that the length 2 word aa is 3-avoidable (Thue 1906), that the complement in P_3 of the principal filter over aa has an infinite antichain (Jezek 1977), and that aa has infinitely many covers in P_3 . Whether similar results hold for arbitrary words is unknown.

R.WILLE : An order-theoretic foundation for similarity measures

(Joint work with Silvia Geist). The similarity between objects are often determined by the sets of their attributes which are elements of a given attribute set M . Then the proposed similarity measures are certain order-preserving maps from $(\mathcal{P}(M)^2, \leq)$ into (\mathbb{R}_0^+, \leq) where the order on $\mathcal{P}(M)^2$ can be defined as follows: $(A, B) \leq (C, D) \Leftrightarrow A \cap B \subseteq C \cap D, A \cap \bar{B} \supseteq C \cap \bar{D}, \bar{A} \cap B \supseteq \bar{C} \cap D, \text{ and } \bar{A} \cap \bar{B} \subseteq \bar{C} \cap \bar{D}$. The structure of $(\mathcal{P}(M)^2, \leq)$ is clarified by the

following

Theorem: $\varphi_A^1(X) := (A, (A \cap X) \cup (\bar{A} \cap \bar{X}))$ and $\varphi_A^2(X) := ((A \cap X) \cup (\bar{A} \cap \bar{X}), A)$ defines isomorphisms $\varphi_A^1: \mathfrak{I}(M) \rightarrow [(A, \bar{A}), (A, A)]$ and

$\varphi_A^2: \mathfrak{I}(M) \rightarrow [(\bar{A}, A), (A, A)]$, respectively;

in particular, $\mathfrak{I}(M)^2 = \bigcup_{A \in \mathfrak{I}(M)} [(A, \bar{A}), (A, A)] = \bigcup_{A \in \mathfrak{I}(M)} [(\bar{A}, A), (A, A)]$

and the order of $\mathfrak{I}(M)^2$ is the transitive hull of the orders of the described intervalls. For characterizations of common similarity measures the following definition is used:

An order-preserving map $\mathcal{G}: (\mathfrak{I}(M)^2, \leq) \rightarrow (\mathbb{R}_0^+, \leq)$ is additive

iff $\mathcal{G} \varphi_A^i(X) + \mathcal{G} \varphi_A^i(Y) = \mathcal{G} \varphi_A^i(X \cup Y)$ for $X \cap Y = \emptyset$ ($i=1,2$).

Theorem: let $\mu: \mathfrak{I}(M) \rightarrow \mathbb{R}_0^+$ be additive and let $\alpha, \beta, \delta, \delta \in \mathbb{R}_0^+$.

If $\mathcal{G}(A, B) := \alpha \mu(A \cap B) + \beta \mu(A \cap \bar{B}) + \delta \mu(\bar{A} \cap B) + \delta \mu(\bar{A} \cap \bar{B})$ then is additive iff $\beta = \delta = 0$.

Probleme

I.RIVAL: Call an order theoretical property a diagram invariant if it is satisfied in every orientation (=diagram) of the covering graph of an ordered set provided it is satisfied in one of them. What are the non-trivial diagram invariants?

P.EDELMAN: For $\sigma \in S_n$ (symmetric group of an n-element set) the inversion set of σ is $I(\sigma) = \{(\sigma(i), \sigma(j)) : i < j, \sigma(i) > \sigma(j)\}$. Order S_n by $\sigma \leq \tau \iff I(\sigma) \subseteq I(\tau)$, the weak ordering of S_n denoted by $WB(S_n)$. Is $WB(S_n)$ Sperner? - decomposable into symmetric chains?

D.KELLY: Are the middle two layers of the diagram of 2^n (n odd) Hamiltonian?

M.AIGNER: (due to Fürstenberg) Call $A \subseteq \{0,1\}^n$ a cylinder set if $A = \{x : x_{i_1} = a_1, \dots, x_{i_k} = a_k\}$ for some $a_1, \dots, a_k \in \{0,1\}$, $i_1, \dots, i_k \in \{1, \dots, n\}$. For cylinder sets A, B define $(\text{supp}(A) = \{i_1, \dots, i_k\})$:

$$A \square B = \begin{cases} A \cap B & \text{if } \text{supp}(A) \cap \text{supp}(B) = \emptyset \\ \emptyset & \text{else.} \end{cases}$$

Let $\mathcal{A} = \bigcup_{i \in I} A_i$, $\mathcal{B} = \bigcup_{j \in J} B_j$ be collections of cylinder sets, and $(\mathcal{A} \square \mathcal{B} = \bigcup_{i,j} (A_i \square B_j))$. Conjecture: $2^n |\mathcal{A} \square \mathcal{B}| \leq |\mathcal{A}| \cdot |\mathcal{B}|$

U.FAIGLE: P a (finite) ordered set, $0 < \delta < 1$; Call $x \in P$ δ -central if $\delta \leq \frac{N(x)}{N} \leq 1 - \delta$ ($N(x) = \#$ of ideals containing x, $N = \#$ of ideals of P).

Is there an efficient algorithm to detect δ -central elements?

K.ENGEL: A representation of a finite poset P is a function $x: P \rightarrow \mathbb{R}$ with $x(p) - x(p') \geq 1$ whenever $p' < p$. Define:

$$\mu_x = \frac{1}{|P|} \sum_{p \in P} x(p), \quad \sigma_x^2 = \frac{1}{|P|} \sum_{p \in P} (x(p) - \mu_x)^2, \quad \text{and} \quad \sigma^2(P) = \inf \{ \sigma_x^2 :$$

x is a representation of P}. A ranked poset P is called rank compressed if this infimum is attained by the rank function r.

Is the lattice of partitions of an n-element set rank compressed?

D.WEST: Does there exist a minimum chain decomposition of $P \times Q$ so that consecutive elements on each chain are unicomparable (i.e. equal in one coordinate)?

M.ERNÉ: P a poset, $M(P)$ the system of principal ideals, $C(P)$ the system of non-empty lower ends generated by chains; Does $C(P) \cong P$ imply $C(P) = M(P)$?

Z.FÜREDI: Is there a chain decomposition of 2^n into $\binom{n}{n/2}$ chains of almost equal length (i.e. $||C_i| - |C_j|| \leq 1$) ?

T.W.TROTTER: Find a least $f(n)$ so that for every poset of width n , P , there is a poset Q of width $f(n)$ s.t. for every 2-coloring of Q there is some monochromatic $P' \subseteq Q$ with $P' \cong P$.

I.ROSENBERG: Characterize the strict infinite orders \langle s.t. $\text{Pol}(\langle)$ (the clone of isotone operations) is a coatom in the poset of locally closed clones.

R.NOWAKOWSKI: Let $x \in 2^n$. A set F is a cutset for x if no element of F is comparable to x and if every maximal chain intersects $F \cup \{x\}$. Find the size of a maximum-sized irredundant cutset for x .

M.SAKS: L a finite distributive lattice of rank $r(L)$, L_k the set of elts. of rank k , $u(x)$ the number of elts. covering x . Is it true that for each $0 < k \leq r(L)$: $\frac{1}{|L_k|} \sum_{x \in L_k} u(x) \geq \left(\frac{1}{|L_{k-1}|} \sum_{y \in L_{k-1}} u(y) \right) - 1$?

W.POGUNTKE: It was observed by M.Pouzet that in each finite ordered set with a proper automorphism there are elts. x, y s.t. $1/3 \leq \text{pr}(x < y) \leq 2/3$. Is it possible to prove an even better estimate? (partial results obtained by G.Häfner).

O.PRETZEL: Call a linear extension of a poset P imperial if it can be obtained as follows: if an initial segment x_1, \dots, x_{i-1} has been selected, choose x_i among the minimal remaining elts. so as to maximize $\max \{j < i: x_j < x_i\}$. Prove results for "imperial dimension" analogous to those for greedy dimension.

R.CANFIELD: Consider the transformation on permutations which maps $\sigma(1)\dots\sigma(n)$ into $\tau = \sigma(i)\sigma(i-1)\dots\sigma(1)\sigma(i+1)\dots\sigma(n)$ where $i = \sigma(1)$ (i.e. flipping an initial segment of length $\sigma(1)$). Wilf has shown that σ is transformed into τ with $\tau(1) = 1$ in $\leq 2^{n-1}$ iterates of the transformation. Prove or disprove $O(n^2)$.

J.KAHN: L, L', M geometric lattices. An embedding from L to L' is an injective, rank- and join-preserving $\varphi: L \rightarrow L'$ with $\varphi(\hat{0}) = \hat{0}$, $\varphi(\hat{1}) = \hat{1}$. M is a minor of L ($M \leq L$) if M embeds in $[F, \hat{1}]$ for some $F \in L$. L is k -connected if there is no partition $A \cup B$ of the atoms of L with $|A|, |B| \geq k$ and $r(A) + r(B) \leq r(L) + k - 1$.

Conj.: for all M there is some $c(M)$ so that if (a) L is a geometric lattice, $L \geq M$, (b) $X \in L$, $r(X) = r(M)$, and (c) $r(L) > c(M)$, L is $c(M)$ -connected, then there is a complement F of X s.t. M embeds in $[F, \hat{1}]$.

M.POUZET: (with E.Milner and F.Galvin) $G = (V, E)$ a countable directed graph. Is there a subset A of V and an ordering \leq on A s.t. for every $B \in A$ B dominates A (with resp. to \leq) iff B dominates V (with resp. to E) ?

D.SVRTAN: Realize \prod_n (the partition lattice of an n -elt. set) in some Euclidean space E^N so that every automorphism of \prod_n will be the restriction of some isometry of E^N .

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