MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 14/1985

Surgery and L-Theory

17.3. bis 23.3.1985

Die Tagung fand unter Leitung der Herren A. Bak (Bielefeld), M. Kreck (Mainz) und A. Ranicki (Edinburgh) statt.

Es wurden Anwendungen der Surgery-Theorie und der algebraischen L-Theorie in verschiedenen Bereichen der Topologie behandelt, wie Klassifikation von Mannigfaltigkeiten und Poincaré-Räumen, G-Mannigfaltigkeiten, die Theorie der höherdimensionalen Knoten, Gruppenoperationen auf Sphären.

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Vortragsauszüge

Bruce WILLIAMS. Geometric Applications of Higher und Lower Algebraic K-Theory

(joint work with Bill Dwyer and Larry Taylor)

Notation. h.f. $(X \longrightarrow Y)$ = homotopy fiber over the base point in Y

M^n = closed topological manifold

$\tilde{S}(M) = \text{h.f.}((G/_{Top})^M \to L_n^h(\mathbb{Z}\pi))$, $\pi = \pi_1 M$

= simplical set of block, topological structure on M

$S(M)$ = " " " fibered, " " " M

= h.fiber $(BTop \longrightarrow BS)$ over M, where

Top = simplical category of closed manifolds, and

S = simplical category of finite CW-complexes.

Stabilize by crossing with $\mathbb{R}^i$

$$\begin{array}{ccl} S(M) & \longrightarrow & S^b(M\times\mathbb{R}^i) = \text{bounded, fibered structures on } M\times\mathbb{R}^i \\ \downarrow & & \downarrow \alpha(i) \\ \tilde{S}(M) & \longrightarrow & \tilde{S}^b(M\times\mathbb{R}^i) = \text{" , block " " "} \end{array}$$

Theorem 1. $S^b(M\times\mathbb{R}^\infty) \xrightarrow{\sim} \tilde{S}^b(M\times\mathbb{R}^\infty)$.

Ideas of Hatcher, Anderson-Hsiang, and Madsen-Rothenberg yield the following results:

Theorem 2.

(a) There exists a (homotopy) diagram

$$\begin{array}{ccccccc} H(M) & & H^b(M\times\mathbb{R}^1) & & & & H^b(M\times\mathbb{R}^i) \\ \downarrow & & \downarrow & & & & \downarrow \\ S(M) & \longrightarrow & S^b(M\times\mathbb{R}^1) & \longrightarrow & \cdots & \longrightarrow & S^b(M\times\mathbb{R}^i) \end{array}$$

where $H^b(M\times\mathbb{R}^i) = S^b(M\times\mathbb{R}^i\times I \text{ rel } M\times\mathbb{R}^i\times 0)$ = s. set of bounded h-bordisms on $M\times\mathbb{R}^i$.

(b) $\Omega H^b(M\times\mathbb{R}^i) \simeq H^b(M\times I\times\mathbb{R}^{i-1})$

(c) There exists an action of $\mathbb{Z}/2$ on $\pi_*(H^b(M\times I\times\mathbb{R}^{i-1}))$ such that the following diagram commutes

$$\begin{array}{ccccc}
\pi_j(H^b(M\times\mathbb{R}^i)) & \longrightarrow & \pi_j(S^b(M\times\mathbb{R}^i)) & \xrightarrow{\partial} & \pi_{j-1}(H^b(M\times\mathbb{R}^{i-1})) \\
(b)\ \downarrow\wr & & & & \downarrow \times I \\
\pi_{j-1}(H^b(M\times I\times\mathbb{R}^{i-1})) & & \xrightarrow{\quad 1-(-1)^i T \quad} & & \pi_{j-1}(H^b(M\times I\times\mathbb{R}^{i-1}))
\end{array}$$

where T = generator of $\mathbb{Z}/2$.

(d) For $j \le i$, $\pi_j(H^b(M\times\mathbb{R}^i)) \simeq Wh_{1+j-i}(\pi)$.

(e) Consider the homotopy group exact couple for the tower of fibrations in (a). Then the derived exact couple yields exact sequences

$$\cdots \longrightarrow \pi_j(\tilde{S}^b(M^n\times\mathbb{R}^i)) \longrightarrow \pi_j(\tilde{S}^b(M^n\times\mathbb{R}^{i+1})) \longrightarrow H^{n+j}(\mathbb{Z}/2, Wh_{1-i}(\pi))$$

(related to work of Anderson-Pedersen, Hambleton-Madsen).

Consider the commutative diagram

$$\begin{array}{ccc}
S(M) & \xrightarrow{\ \beta\alpha\ } & S^b(M\times\mathbb{R}^\infty) = \tilde{S}^b(M\times\mathbb{R}^\infty) \\
& {\scriptstyle\alpha}\searrow \qquad \nearrow{\scriptstyle\beta} & \\
& \tilde{S}(M) &
\end{array}$$

<u>Conjecture</u> h.f. $(\alpha\beta)$ $\simeq$ $S^\infty_+ \wedge_{\mathbb{Z}/2} \underline{H(M)}$
thru the Igusa stable range

where $H(M) = \lim_k H(M\times I^k)$ where $H(M)$ becomes an ∞-loop space $\underline{H(M)}$ via $H(M) \simeq \Omega^i H^b(M\times\mathbb{R}^i) = \Omega^i \lim_k H^b(M\times I^k\times\mathbb{R}^i)$.

This conjecture can be broken into two parts:

<u>High conjecture</u> h.f.(α) $\simeq$ $S^\infty_+ \wedge_{\mathbb{Z}/2} \underline{H(M)}^{high}$.
thru the Igusa stable range

where $\underline{H(M)}^{high} \Leftarrow (-1)$ connected cover of $\underline{H(M)} \simeq \Omega\, Wh(M)$ (in the sense of Waldhausen).

<u>Low Theorem</u> 3. h.f.$(\beta) \simeq S^\infty_+ \wedge_{\mathbb{Z}/2} \underline{H(M)}^{low}$

where

$$\underline{H(M)}^{low} = \underline{H(M)}/\underline{H(M)}^{high} \quad \text{(as } \infty\text{-loop space)}$$

$$\simeq Wh(\pi)/Wh(\pi)^{high} \quad \text{(Anderson-Hsiang Theory)}$$

J.C. HAUSMANN. About surgery on Poincaré spaces

Let P be a Poincaré space of formal dimension n, with $\partial P = \partial_- P \cup \partial_+ P$. Let $\eta(P,\partial_+ P)$ be the set of bordism classes of "normal map of degree one" $f : (\bar{P},\partial_-\bar{P},\partial_+\bar{P}) \longrightarrow (P,\partial_- P,\partial_+ P)$ such that $f|\partial_-\bar{P} : \partial_-\bar{P} \longrightarrow \partial_- P$ is a homotopy equivalence. ($\bar{P}$ a Poincaré space). The theory of surgery on manifolds can be extended as follows:

Theorem A. There exists a map $s : \eta(P,\partial_+ P) \longrightarrow L_n(\pi(P),\pi(\partial_+ P),\omega^P)$ (the Wall groups) such that

1) If $\partial_+ P = \emptyset$ or $n \geq h$, then $s(\alpha) = 0$ iff α contains a homotopy equivale
2) s is additive
3) if α is represented by a classical surgery problem $f : M \longrightarrow P$, then $s(f) = \sigma(\alpha)$, the Wall surgery obstruction for f.

Theorem B. If $n \geq 6$, the map s of Theorem A is characterized by 1), 2) + 3).

This is a joint work with P. Vogel and will appear in a book: "The geometry of Poincaré-spaces".

Rainhard SCHULTZ. An infinite exact sequence in equivariant surgery

Let X be a compact manifold and $S_k(X)$ the set of homotopy structures on $D^k \times X$ that are standard on the boundary. The infinitely long exact surgery sequence of Sullivan and Wall provides a means for understanding $S_k(X)$ in terms of two more accessible objects: The homotopy groups of the space of continuous functions from $X \cup \{pt.\}$ to F/O and the Wall surgery obstruction groups $L_*(\pi_1 X)$. If a finite group acts smoothly on X then one consider the corresponding set $S_k(X;G)$ of G-equivariant homotopy structures on X that are standard on the boundary. Under certain restrictions, results of Dovermann and Rothenberg yield a corresponding exact sequence involving $S_k(X)$, suitably defined equivariant homotopy groups of the function space, and equivariant surgery groups $I_*^G(X)$ that are periodic mod 4. However, this sequence stops after finitely many terms because of the basic underlying restrictions. The principal result in this work is an infinite extension of the Dovermann-Rothenberg sequence in some cases.

<u>Theorem</u>. Let G be an odd order cyclic group. Assume all fixed point sets in X are 1-connected. Then there is an infinite long exact sequence extending the Dovermann-Rothenberg sequence, with a first oder approximation $D^{1-}S_k(X)$ replacing $S_k(X)$ after the Dovermann-Rothenberg sequence terminates.

Related but more complicated results hold for arbitrary groups of odd order.

F. CLAUWENS K_2^{top} <u>of rings of power series and applications to L-theory</u>

In my earlier work (see: Aarhus 1978 conference) a $\sigma(P) \in L_{2p}(\mathbb{Z}[t][\pi,P])$ popped up that determines the surgery obstruction of $id_p \times f$ from the obstruction of f. So try to compute $L_nA[t]$ where $A = \mathbb{Z}[\pi,P]$. Using local/complete exact sequence one needs

1) $L_nA\frac{1}{2}[t]$ which is isomorphic to $L_nA\frac{1}{2}$,
2) $L_n\widehat{A[t]}$ where one can use $L_n^h\widehat{A[t]} \simeq L_nA[t]/(\mathbb{Z})$,
3) $L_n\widehat{A[t]}\frac{1}{2}$ where one splits $\hat{A}[\frac{1}{2}]$ in matrix rings over divison rings D and uses the fact that a symmetric element in the convergent power series $D\{t\}$ over D is equivalent to a polynomial.

But all this gives you $L_n^?A[t]$ with some funny decoration at best; so understanding of $K_1A[t]$ is needed.
Look at the case $A = \mathbb{Z}[\pi]$ π abelian 2-group; then the loc/complete exact sequence gives

$$K_1A[t]/K_1A = \text{coker}\left\{\frac{K_2\hat{A}\{t\}}{K_2\hat{A}} \longrightarrow \frac{K_2\hat{A}\{t\}(\frac{1}{2})}{K_2\hat{A}\frac{1}{2}}\right\}$$

In fact one only needs $K_2^{top}\hat{A}\{t\} = \varprojlim_n K_2\,\hat{A}\{t\}/2^n$ instead of $K_2\hat{A}\{t\}$.
Write $R = \tilde{A}\{t\}$. Let I = kernel of augmentation $\hat{A} \to \hat{\mathbb{Z}}\{t\}$.
To compute $K_2(R,I)$ use Maasen/Shensha presentation

generators	relations
$\langle a,b\rangle_{a\in I, b\in R \text{ or } a\in R, b\in I}$	$\langle a,b\rangle = -\langle b,a\rangle$ $\langle a,b_1\rangle + \langle a,b_2\rangle = \langle a,b_1+b_2-ab_1b_2\rangle$ $\langle a,b_1b_2\rangle = \langle ab_1,b_2\rangle + \langle ab_2,b_1\rangle$

Problem that second relation nonlinear; to correct that use that R is a λ-ring.

i.e.

$$\lambda^i : R \longrightarrow R \text{ such that } \begin{cases} \lambda^i(x+y) = \sum_{j+h=i} \lambda^j(x)\lambda^h(y) \\ \lambda^i(x\cdot y) = \text{universal polyn. in } \lambda^{\sim}(x),\lambda^{\sim}(y) \\ \lambda^i \circ \lambda^j(x) = \quad '' \quad '' \quad '' \quad \lambda^{\sim}(x) \end{cases}$$

There is a "universal" λ ring U such that for any $r \in R$ there is unique λ ring map $U \longrightarrow R$ mapping s_1 to r. $U = \mathbb{Z}(s_1, s_2 \ldots)$

Every element of U defines an operation on λ rings.

e.g. adams operations F^i $\quad F^i(r) = (-1)^{i-1} \sum_{j+h=i} j\lambda^j(r)\lambda^h(-r)$

$F^p r = r^p \bmod p$. F^i ring homomorphism; $F^i \circ F^j = F^{ij}$

If R has no $\mathbb{Z}$ torsion then F operations determine λ-structure.

Define new operations $\vartheta^n(r) \underset{\text{def}}{=} \frac{1}{n} \sum_{\text{fin}} F^h(r^{n/h})$ has meaning in $U \otimes \mathbb{Q}$ but lies in U.

$$\eta^m(a,b) \underset{\text{def}}{=} \sum_{\substack{\ell | m \\ \ell \neq m}} \eta^\ell(a, a^{m/\ell - 1})\, F^\ell\, \vartheta^m(b); \quad \eta^1(a,b) = b.$$

Furthermore there is $F^i : \Omega_R \longrightarrow \Omega_R$ (Kahler differentials) such that $iF^i \delta r = \delta F^i r$ $\quad F^i$ additive $\quad F^i(rw) = F^i(r)F^i(w)$.

Now define $K_2^{top}(R,I) \longrightarrow I\Omega_R^{top}/\delta I^2$

$$\underset{a \in I, b \in R}{\langle a,b \rangle} \longrightarrow \Sigma_{m=1}^{\infty} \vartheta^m(a) F^m \,\delta\, \Sigma_{n=1}^{\infty} \eta^n(a,b).$$

That is well defined, and has very small kernel & cokernel in the cases to which it is applied.

I. HAMBLETON. Surgery Obstructions on Closed Manifolds

Let π be a finite group and $w : \pi \to \mathbb{Z}/2$ a homomorphism. ∂f $\xi \searrow B\pi$ is a line bundle with $w_1 = w$ denote by $B\pi^w$ the Thomspace of ξ. The problem of computing the surgery obstruction $\lambda(M^n \xrightarrow{(f,\hat{f})} N^n) \in L_n^s(\mathbb{Z}\pi, w)$ for any normal map of closed topological manifolds, with $\pi_1 N = \pi$, $w_1 N = w$, is equivalent to the evaluation of two sequences of natural homomorphisms:

$$\partial_i : H_i(\pi; \mathbb{Z}_{(2)}) \longrightarrow L_i^s(\mathbb{Z}\pi, w)_{(2)}$$

$$K_i : H_i(\pi, \mathbb{Z}/2) \longrightarrow L_{i+2}^s(\mathbb{Z}\pi, w)_{(2)}.$$

If we evaluate instead in $L'_*(\mathbb{Z}\pi,w)$ where $'$ denotes torsions allowed in $SK_1(\mathbb{Z}\pi) \oplus \{\pm\pi^{ab}\} \subseteq K_1(\mathbb{Z}\pi)_{/\pm 1}$, then it is known that $\partial_i = 0$ for $i > 0$ and ∂_o is the ordinary signature.

In the talk we described how the K_i's are determined by another sequence of homomorphisms

$$\beta_n : \Omega_n(B\pi^w;\mathbb{Z}/2) \longrightarrow L'_{n+2}(\mathbb{Z}\pi,w)$$

obtained by product with the twisted Kervaire problem. Let $R = \mathbb{Z}[\frac{1+\sqrt{5}}{2}]$ with involution $\frac{1+\sqrt{5}}{2} \longrightarrow \frac{1-\sqrt{5}}{2}$, then there is a transfer map

$L_i^{\tilde{Y}}(R\pi,w) \xrightarrow{\text{trf.}} L'_{i+2}(\mathbb{Z}\pi,w)$ where $\tilde{Y} = \mathrm{Im}\{SK_1(R\pi) \oplus R^* \oplus \{\pi^{ab}\} \to K_1(R\pi)/(\pm 1)\}$.

<u>Theorem</u> 1. There is a natural factorization

$$\begin{array}{ccc} \Omega_n(B\pi^w;\mathbb{Z}/2) & \xrightarrow{\beta_n} & L'_{n+2}(\mathbb{Z}\pi,w) \\ & {\scriptstyle\tilde{\beta}_n}\searrow \quad \nearrow{\scriptstyle \text{trf.}} & \\ & L_n^{\tilde{Y}}(R\pi,w) & \end{array}$$

We then showed how to compute the groups $L_n^{\tilde{Y}}(R\pi,w)$ and the maps $\tilde{\beta}_n$ and trf.

<u>Theorem</u> 2. There is a natural factorization of the maps K_i through

$$\text{trf} : L_i^{\tilde{Y}}(R\pi,w) \longrightarrow L'_{i+2}(\mathbb{Z}\pi,w).$$

C. STARK. <u>Surgery on Seifert fibered spaces over 3-orbifolds</u>

A conjecture sometimes attributed to A. Borel holds that any homotopy equivalence $M^n \longrightarrow N^n$ of closed, aspherical manifolds ought to be deformable to a homeomorphism, at least for large enough n's. This conjecture seems especially probable for aspherical manifolds with a great deal of symmetry, and we know (Conner-Raymond) a lot about such a manifold if it possesses an effective action of a compact Lie group. This Lie group must be a torus T^n; if $(T^n ; M^{n+2})$ is an effective torus action on a closed aspherical manifold, then we know that $S_{Top}(M \times D^k,\partial) = 0$ for $n+k \geq 4$ ($\partial M \neq \emptyset$ has also been dealt with), by work of Farrell-Hsiang, Stark, and Nicas-Stark. This talk dealt with the codimension-3 problem, (T^n,M^{n+3}), for which $T^n\backslash M^{n+3}$ is a 3-orbifold in the sense of Thurston.

Theorem. Suppose (T^n, M^{n+3}) is an effective action on an orientable, closed aspherical manifold such that all the isotropy groups T^n_X have odd order and either

(a) $T^n \setminus M^{n+3}$ is a Haken hyperbolic 3-orbifold,

(b) $T^n \setminus M^{n+3}$ is a Seifert fibered 3-orbifold, or

(c) $T^n \setminus M^{n+3}$ is a Haken orbifold made from hyperbolic and Seifert fibered subspaces. If $n+k \geq 3$ then $S_{Top}(M \times D^k, \partial) = 0$.

The proof in case (a) relies on a root-closure proposition to see that Cappell's UNil groups vanish here.

Jim DAVIS. Swan Modules and Swan Formations

Let π be a finite group. The Swan subgroup $T_m(\pi)$ of $L_n(\mathbb{Z}\pi)$ is defined analogously to the Swan subgroup $T(\pi)$ of $\tilde{K}_0(\mathbb{Z}\pi)$. The point is that the Swan subgroups of L-theory often play the same role in closed manifold problems as the Swan subgroup of K-theory plays in the theory of finite complexes. $T_m(\pi)$ is the subgroup of $L_n(\mathbb{Z}\pi)$ represented by surgery obstructions of degree one normal maps $f: M \longrightarrow X$ such that $K_*(f) \otimes \mathbb{Z}/_{|\pi|} = 0$ and π acts trivially on $K_*(f)$.

Two examples of the above philosophy are discussed: (1) free actions of finite groups on spheres - here the k-invariant of a spherical space form is relating to the vanishing of an element (depending on k) in $T^h_n(\pi)$. (2) Smooth semifree actions of π on S^n. Here the homology of the fixed set is related to the vanishing of an element in $T^h_m(\pi)$. (1) and (2) are joint work with Shmuel Weinberger.

E. PEDERSEN. Finiteness obstruction of nilpotent spaces

Given a nilpotent group π, let $N(\pi) \subseteq \tilde{K}_0(\mathbb{Z}\pi)$ be the set of finiteness obstructions realized by nilpotent complexes. Mislin proves that $N(\pi) = 0$ if π is infinite and in general Mislin-Naradarajan prove that $N(\pi) \subseteq D(\pi)$. Consider $\pi = x\pi_p$ a finite nilpotent group. Then $\mathbb{Q}\pi$ decomposes $\mathbb{Q}\pi = \mathbb{Q}\pi_p \times \mathbb{Q}\pi/\varepsilon$. Let $N_p(\pi)$ be the image of

$$K_1(\mathbb{Q}\pi_p) \subset K_1(\mathbb{Q}\pi_p) \times K_1(\mathbb{Q}\pi/\varepsilon) = K_1(\mathbb{Q}\pi) \xrightarrow{\partial^{(p)}} K_o(\mathbb{Z}\pi)$$

where $\partial^{(p)}$ is associated with the square

$$\begin{array}{ccc} \mathbb{Z}\pi & \longrightarrow & \mathbb{Z}_{(p)}\pi \\ \downarrow & & \downarrow \\ \mathbb{Z}_{\frac{1}{p}}\pi & \longrightarrow & \mathbb{Q}\pi . \end{array}$$

Then $N(\pi) = \Sigma N_p(\pi) = T(\pi) + \sum_{p \mid |\pi|} N_p(\pi)$ where $T(\pi)$ is the image of the Swan homomorphism. This proves that $N(\pi)$ is a group, and $N(\pi) \subseteq D(\pi)$ follows as an easy corollary, thus giving a new proof of Mislin-Naradarajan's result.

Sören ILLMAN. A product formula for equivariant Whitehead torsion. Geometric applications.

Let G and P denote compact Lie groups, and let $f: X \longrightarrow Y$ be a G-homotopy equivalence between finite G-CW complexes and let $h:X' \longrightarrow Y'$ be a P-homotopy equivalence between finite P-CW complexes. We give a formula for the equivariant Whitehead torsion $\tau(f \times h) \in Wh_{G\times P}(X \times X')$ of the $(G \times P)$-homotopy equivalence

$$f \times h : X \times X' \longrightarrow Y \times Y'$$

in terms of the equivariant Whitehead torsions of f and h, and various Euler characteristics derived from the G-space X and the P-space X'.

In the case of a finite group G we obtain as a corollary of the product formula the geometric result given below. We let V be any complex unitary representation space of G, and by $S(V)$ we denote the unit sphere in V, with the induced G-action.

Theorem. Let $f:X \longrightarrow Y$ be a G-homotopy equivalence between finite G-CW complexes, where G is a finite group. Then

$$f \times id_{S(V)}: X \times S(V) \longrightarrow Y \times S(V)$$

is a simple $(G \times G)$-homotopy equivalence, and hence also a simple G-homotopy equivalence when $X \times S(V)$ and $Y \times S(V)$ have diagonal G-action.

This result does not hold for arbitrary compact Lie groups. We show in this connection, by a simple example, that equivariant Whitehead torsion for compact Lie groups is not determined by restricting to all finite subgroups.

Eva BAYER Stably hyperbolic ε-hermitian forms

Let R be a commutative ring with 1, and let L be a R-module of finite type. Let $h : L \times L \longrightarrow R$ be an ε-hermitian form. We shall say that h is unimodular if $ad(h) : L \longrightarrow L^* = Hom_R(L,R)$ is an isomorphism. Let N be a reflexive R-module, then we associate to N the hyperbolic form $H(N)$. A form (L,h) is said to be stably hyperbolic if there exist reflexive modules N_1 and N_2 such that $(L,h) \boxplus H(N_1) \simeq H(N_2)$.

Theorem (E.B. - Neal Stoltzfus)

Let R be an S-order in a product of number fields with an involution $r \mapsto \bar{r}$. Let (L,h) be a stably hyperbolic ε-hermitian form, $\varepsilon = \pm 1$. Then (L,h) is hyperbolic (except possibly in an exceptional case of "rank" 2).

This theorem has the following application in knot theory:

Corollary (E.B. - Neal Stoltzfus)

Let $\Sigma^{2q-1} \subset S^{2q+1}$, $q > 1$, be a simple knot such that the knot module is annihilated by a square free polynomial λ. Assume that Σ^{2q-1} is stably doubly sliced. Then Σ^{2q-1} is doubly sliced (this is an application of the above with $R = \mathbb{Z}[t,t^{-1}]/(\lambda)$). On the other hand, there exist stably isomorphic hyperbolic forms which are not isomorphic:

Proposition

Let $E = \mathbb{Q}(\sqrt{d})$ with d a negative integer. Set $\overline{(a+b\sqrt{d})} = a - b\sqrt{d}$. Then there exists an order R of E and an invertible R-ideal I such that $H(I) \boxplus H(D) \simeq H(R) \boxplus H(D)$ (where D is the ring of integers of E) but $H(I) \not\simeq H(R)$. In fact, it is possible to choose I such that $I \oplus I^{-1} \not\simeq R \oplus R$. As a consequence of this, we obtain:

Corollary

For every integer $n \geq 3$, there exist knots Σ_1^n, Σ_2^n, and Σ^n such that $\Sigma_1^n \# \Sigma^n$ and $\Sigma_2^n \# \Sigma^n$ are isotopic but Σ_1^n and Σ_2^n are not isotopic.

L. VASERSTEIN. Quadratic forms over rings of continuous functions

Let $B = \mathbb{R}$ (the reals) or $\mathbb{C}$ (the complex numbers), X a topological space, $A = B^X$ (the ring of continuous functions $X \longrightarrow B$), $S_n A = \{\alpha = \alpha^* \in GL_n A\}$.

Definition. α equivalent β if $\exists \gamma \in GL_n A$ such that $\gamma^* \alpha\gamma = \beta$.

Definition. α homotopic β if $\exists d \in S_n B^{(X \times [0,1])}$ such that

$$d\big|_{X \times \{0\}} = \alpha \quad \text{and} \quad d\big|_{X \times \{1\}} = \beta.$$

Theorem 1. α homotopic $\beta \Rightarrow \alpha$ equivalent β,

Theorem 2. α equivalent $\beta \Rightarrow \alpha$ stably homotopic β.

(i.e. $\left(\begin{array}{c|cc} \alpha & 0 & \\ \hline 0 & 0 & I_m \\ & I_m & 0 \end{array}\right)$ homotopic $\left(\begin{array}{c|cc} \beta & 0 & \\ \hline 0 & 0 & I_m \\ & I_m & 0 \end{array}\right)$ for some m).

Remark. In general, "stably homotopic" in Theorem 2 cannot be replaced by "homotopic".

Corollary. WB^X is a homotopy type invariant of X.

Theorem 3. $WB^X = K_oB^X = WB_o^X = K_oB_o^X$, where $B_o^X \subset B^X$ is the subring of bounded functions.

Open problem. SK_1B^X is a homotopy type invariant of X. (Done for $X = \mathbb{R}^d$ with $d = 1,2,3$; the case $d = 3$ jointly with Thurston).

Justin R. SMITH. Topological Realization of Chain Complexes

I study the question: Given a group π and a projective $\mathbb{Z}\pi$ chain-complex C_*, does there exist a topological space with fundemental group π whose equivariant chain complex is C_* ?

I present an obstruction theory for the existence of the topological space realizing C_* – the obstruction lie in the cohomology of C_*. The existence of this obstruction theory and basic facts about Eilenberg-MacLane spaces imply that if C_* is a finite dimensional $\mathbb{Q}\pi$-projective chain complex then some suspension of C_* is reducible.

I focus upon the rational case and give a fairly complete solution. I develop an equivariant theory of "minimal modules" for homotopy types.

Neal W. STOLTZFUS. Algebraic Approach to Diffeomorphisms of Surfaces

An algebraic obstruction theory analog of higher dimensional surgery obstruction can be developed for the theory of diffeomorphisms of two-manifolds (surfaces) using the Biderivations of Papakuriakopolous. Let $F(g:1)$ be a surface of genus g with one boundary circle, $\Lambda = \mathbb{Z}\pi_1(F(g:1))$ and k = kernel of the augmentation $\varepsilon : \mathbb{Z}\pi_1 \longrightarrow \mathbb{Z}$, the foundamental ideal. There is a pairing $\lambda : k \times k \longrightarrow \Lambda$ satisfying:

i) λ is $\mathbb{Z}$-linear

ii) $\lambda(\alpha\beta,\gamma) = \lambda(\alpha,\gamma)\varepsilon(\beta) + \alpha\lambda(\beta,\gamma)$
$\lambda(\alpha,\beta\gamma) = \lambda(\alpha,\beta)\varepsilon(\gamma) + \lambda(\alpha,\gamma)\bar{\beta}$ (Biderivation)

iii) Adjoint : $k \longrightarrow Der_\Lambda(k,\Lambda)$ is an isomorphism (unimodularity)

iv) $\lambda(\alpha,\beta) + \overline{\lambda(\beta,\alpha)} = (\alpha-\varepsilon(\alpha))\overline{(\beta-\varepsilon(\beta))}$

v) λ induces a pairing $\lambda_o : k/k^2 \times k/k^2 \longrightarrow \Lambda/k = \mathbb{Z}$ which is the algebraic intersection pairing under the identification $k/k^2 \simeq H_1(F,\mathbb{Z})$.

If f is a diffeomorphism of $F(g:1)$, then f induces a "twisted isometry" $f_\#$ of λ : $\lambda(f_\# x, f_\# y) = f_\# \lambda(x,y)$. For the first application, we prove that $(k,\lambda,f_\#)$ can be used to give a faithful algebraic obstruction to the question of whether a diffeomorphism extends to a diffeomorphism of some handlebody (solid torus of dimension 3).

Theorem. (F,f) extends to a diffeomorphism $\hat{f}$ of some handlebody bounding $F \iff$ there is an ideal $K \subset \Lambda$ satisfying:

i) $\lambda(K,K) \subset K$

ii) $f_\#(K) \subset K$

iii) The image of K in k/k^2 is a subspace of rank g $(= \frac{1}{2}$ rank $k/k^2)$ and self-annihilating under the algebraic intersection pairing on k/k^2.

As a second application we show that, under certain restrictions, we can capture the isotopy class of f, using an easily computeable invariant obtained as follows: Consider the following group extension derived from the universal spin cover $\hat{F}$ of F : $H_1(\hat{F};\mathbb{Z}) \longrightarrow \Sigma_g \longrightarrow H_1(F;\mathbb{Z}/2) \cong (\mathbb{Z}/2)^{2g}$ together with the quotient biderivation $\bar{\lambda}$ on $\mathbb{Z}[\Sigma_{2g}]$. Using Thurston's classification of surface diffeomorphisms into "geometric" pieces, we can show:

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Theorem. If the pseudo-Anosov "pieces" of the Thruston decomposition of f each leave a non-trivial one-form invariant (up to a real factor > 1) then f is isotopic to the identity $\Leftrightarrow f_{\#}$ is an inner auto. of Σ_g.

Frank CONNOLLY. Proper Actions of Virtually Torsion Free Groups on Contractible Complexes

Let Γ be a group with virtual cohomological dimension finite ($\mathrm{vcd}(\Gamma) < \infty$). (This means that for a subgroup Γ_o of finite index in Γ, $K(\Gamma,1)$ is a finite dimensional complex). Serre proved that there is a finite dimensional Γ cw-complex $\xi\Gamma$, on which Γ acts properly (i.e. all isotropy groups are finite), such that $(\xi\Gamma)^H$ is contractible for each finite subgroup H in Γ. $\xi\Gamma$ is determined, up to Γ homotopy type, by this condition, and is a familiar object to topologists. For example, if Γ is an arithmetically defined group in the algebraic group G, then $\xi\Gamma$ is G(R)/Max.Compact.Subgroup; if Γ is chrystallographic, $\xi\Gamma$ is the corresponding euclidean space; if Γ is one of the Coxeter groups studied recently by M. Davis, then $\xi\Gamma$ is the space $U(\Gamma,X)$ studied by Vinberg and later Davis.

Serre's lovely construction of $\xi\Gamma$ leaves unsettled the conditions under which $\xi\Gamma$ can be chosen as a finite Γ cw-complex, when it is dominated by a finite Γ complex, and when the dimension of $\xi\Gamma$ and its fixed sets, will coincide with the dimension which the algebra would predict. Specifically, it is conjectured (by C.T.C. Wall and K. Brown) that it is possible to choose $\xi\Gamma$ so that $\dim(\xi\Gamma)^H = \mathrm{vcd}\, N_\Gamma(H)$ for each finite group H.

It is the goal of the present work to answer these questions.

Theorem 1. $\xi\Gamma$ is a finitely dominated Γ complex $\Leftrightarrow$ for each finite subgroup H in Γ, N(H) has type FP_∞ and also there are only finitely many conjugacy classes of finite subgroups of Γ. In this case, $\xi\Gamma$ can be chosen as a finite Γ cw-complex $\Leftrightarrow$ an obstruction $\pi(\Gamma)$ in $\oplus \tilde{K}_o(\mathbb{Z}N(H)/H)$ vanishes. The sum is over a complete set of representatives of conjugacy classes of finite subgroups. (A group Γ "has type FP_∞" iff $K(\Gamma,1)$ can be chosen to have finite skeleta).

In order to discuss the dimension conjecture we need notation. Let H be a finite subgroup of Γ. $J_H(\Gamma)$ means the collection of finite subgroups of Γ which properly contain H. $J_H(\Gamma)$ is a poset (via $\subset$) on which N(H)/H acts. Its geometric realization is a Γ space whose homology and equivariant cohomology will be used.

Theorem 2. $\xi\Gamma$ can be chosen so that for each H finite,

$$\dim(\xi\Gamma)^H = \begin{cases} \operatorname{vcd} N(H) & \text{if this is } \neq 2 \\ 3 & \text{if } \operatorname{vcd} N(H) = 2 \end{cases}$$ if and only if the following two conditions hold for each finite subgroup H of Γ:

(a) $\tilde{H}_d(J_H(\Gamma);\mathbb{Z}) = 0$

(b) There is a subgroup Δ of finite index in $N(H)/H$ such that $\tilde{H}^d_\Delta(J_H(\Gamma);B) = 0$. Here $d =$ vcd $N(H)$ or 3, as above.

Corollary. $\xi\Gamma$ can be chosen as in Theorem 2 in case $\Gamma = SP(4,\mathbb{Z})$ or $GL(n,\mathbb{Z})$. (The latter case, $GL_n(\mathbb{Z})$, is known from work of C. Soulé and A. Ash). This result also has striking consequences for virtual Poincaré duality groups).

The proof of Theorem 2 uses the following lemma on projective modules which we believe will be useful to topologists.

Lemma (Projective Module Criterion). Let Γ be a group. Let M be a $\mathbb{Z}\Gamma$ module. Assume $\operatorname{vcd}(\Gamma) < \infty$. Then M is $\mathbb{Z}\Gamma$ projective $\Longleftrightarrow$ M is projective over $\mathbb{Z}H$ for each finite subgroup H and M is $\mathbb{Z}\Delta$ projective for some subgroup Δ of finite index in Γ.

Michael WEISS Products in Surgery

Let A be a ring with involution. The symmetric L-groups of Mishehako-Ranicki are defined to be the algebraic bordism groups of n-dimensional algebraic Poincaré complexes (C,φ), where C is a finitely generated left projective A-module chain complex graded over $\mathbb{Z}$, and φ is an n-dimensional cycle in

$$\operatorname{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_A C)$$

satisfying a nonsingularity condition. Here W is a projection resolution of $\mathbb{Z}$ over $\mathbb{Z}[\mathbb{Z}_2]$. The cycle φ, or its homotopy class $[\varphi] \in H^{-n}(\mathbb{Z}_2; C \otimes_A C)$, should be regarded as a nonsingular symmetric form on the dual chain complex C^{-*}.

Difficulties in computing $L^n(A)$ suggested the following modification. Suppose that A is a group ring $\mathbb{Z}\pi$ with the ω-twisted involution for some $\omega : \pi \longrightarrow \mathbb{Z}_2$. Insted of using

$$\operatorname{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_{\mathbb{Z}\pi} C) \simeq \mathbb{Z} \otimes_{\mathbb{Z}\pi} (\operatorname{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_{\mathbb{Z}} C)),$$

use

$$P \otimes_{\mathbb{Z}} (\mathrm{Hom}_{\mathbb{Z}[Z_2]}(W, C \otimes_{\mathbb{Z}} C)),$$

where P is a projective resolution of $\mathbb{Z}$ over $\mathbb{Z}\pi$. (To make sense of this, let $\mathbb{Z}\pi$ act on $\mathrm{Hom}_{\mathbb{Z}[Z_2]}(W, C \otimes_{\mathbb{Z}} C)$ using the diagonal action on $C \otimes_{\mathbb{Z}} C$). Define $VL^n(\mathbb{Z}\pi)$ to be the bordism group of objects (C,φ), where φ is a nonsingular cycle of dimension n in

$$P \otimes_{\mathbb{Z}\pi} \mathrm{Hom}_{\mathbb{Z}[Z_2]}(W, C \otimes_{\mathbb{Z}} C)).$$

The augmentation $P \longrightarrow \mathbb{Z}$ induces homomorphisms $VL^n(\mathbb{Z}\pi) \longrightarrow L^n(\mathbb{Z}\pi)$. It turns out that $VL^n(\mathbb{Z}\pi)$ has about as many uses as $L^n(\mathbb{Z}\pi)$, especially in product formular for surgery obstructions.

THEOREM. There is a long exact sequence

$$\cdots \longrightarrow L_n(\mathbb{Z}\pi) \longrightarrow VL^n(\mathbb{Z}\pi) \longrightarrow \left[\bigoplus_{m\in\mathbb{Z}} H_{n-m}(\pi;\hat{L}^m(\mathbb{Z}))\right] \longrightarrow L_{n-1}(\mathbb{Z}\pi) \longrightarrow \cdots$$

where

$$\hat{L}^m(\mathbb{Z}) \simeq \left\{ \begin{array}{ll} \mathbb{Z}_8 & m \equiv 0 \\ \mathbb{Z}_2 & m \equiv 1 \\ 0 & m \equiv 2 \\ \mathbb{Z}_2 & m \equiv 3 \end{array} \right\} \bmod 4.$$

REMARK. The definition of $VL^n(\mathbb{Z}\pi)$ uses the fact that $\mathbb{Z}\pi$ is a commutative Hopf algebra.

Wolfgang LÜCK. <u>The transfer in Surgery theory</u>

Given a fibration $F \longrightarrow E \xrightarrow{p} B$ of connected spaces with F a n-dimensional Poincaré complex, we define algebraic transfer homomorphisms p^*. $L_k(\mathbb{Z}[\pi_1(B)]) \longrightarrow L_{k+n}(\mathbb{Z}[\pi_1(E)])$. If F is a mainfold and p a topological bundle, p^* coincides with the geometric transfer of Wall and Quinn assigning to the surgery obstruction of a surgery problem for B the obstruction of the problem for E obtained by the pull-back-construction with p.

If $\hat{F}$ is the $\pi_1(E)$-fibre of the $\pi_1(E)$-fibration $\tilde{E} \longrightarrow E \xrightarrow{p} B$, the equivariant transport along paths in B defines a homomorphism $u : \pi_1(B) \longrightarrow [\hat{F},\hat{F}]_{\pi_1(E)}$. Let $S: C(\hat{F})^{n-*} \xrightarrow{\simeq} C(\hat{F})$ be the chain map given by Poincaré duality. Because of $S \simeq S^{n-*}$ we can equip the ring $[C(\hat{F}),C(\hat{F})]_{\mathbb{Z}[\pi_1(E)]}$ with an involution $[f] \to [S \circ f^{n-*} \circ S^{-1}]$. Then u yields a homomorphism of rings with involution $U : \mathbb{Z}[\pi_1(B)] \longrightarrow [C(\hat{F}),C(\hat{F})]_{\mathbb{Z}[\pi_1(E)]}$.

The transfer p^* assigns to an element in $L_o(\mathbb{Z}[\pi_1(B)],\varepsilon)$ represented by an ε-quadratic non-singular form $(\mathbb{Z}[\pi_1(B)]^m,[A])$ for $A \in Gl_m(\mathbb{Z}[\pi_1(B)])$ the bordism class of the ε-quadratic Poincaré complex $(\bigoplus_m C^{n-*}, \Phi \in S_n(\bigoplus_m C^{n-*},\varepsilon))$ in $L_n(\mathbb{Z}[\pi_1(E)],\varepsilon)$ for

$$\Phi_o,\ \bigoplus_m C(\hat{F})^{n-*} \xrightarrow[\oplus S]{} \bigoplus_m C(\hat{F}) \xrightarrow[U(A)]{} \bigoplus_m C(\hat{F})$$

and $\Phi_t = 0$ for $t > 0$. The definition for $L_1 \longrightarrow L_{n+1}$ is similar.

We can compute $p_* \circ p^*$ and $p^* \circ p_*$ for orientable resp. untwisted fibrations. If $G \longrightarrow E \xrightarrow{p} B$ is an orientable fibration with a connected compact Lie group as fibre and $\pi_1(p)$ an isomorphism, then p^* vanishes.

T. tom DIECK. Darstellungsformen und Verschlingungszahlen.

Sei $G = H_o \times H_1$ ein Produkt zyklischer Gruppen H_i ungerader Ordnung. Es gibt eine glatte Operation von G auf einer Sphäre $S^{n(o)+n(1)+1} = X$ mit Fixpunktmengen $X^{H_i} = S^{n(i)}$, $n(i)$ ungerade ≥ 5 mit den Eigenschaften:

(i) Die Verschlingungszahl von X^{H_o} und X^{H_1} in X kann eine vorgegebene Zahl k im Kern des Swan-Homomorphismus $s_G :(\mathbb{Z}/|G|)^* \longrightarrow \tilde{K}_o(\mathbb{Z}G)$ sein.

(ii) Die Einbettungen $X^{H_i} \subset X$ mit trivialem Normalenbündel können beliebig vorgegeben werden. Man kann ein Beispiel für solche X finden, die gerahmt-bordant zu $kS(V) = S(V) + \ldots + S(V)$ k-mal, $S(V)$ Einheitssphäre in einer Darstellung V sind. (Gemeinsam mit P. Löffler.)

Dieses Beispiel ist wichtig für die Untersuchung der Darstellungsformen. Es gilt allgemein für abelsche Gruppen ungerader Ordnung und Darstellungen $V = V_1 \oplus \dots \oplus V_n$ mit $\dim_{\mathbb{C}} V_i \geq 3$ Darstellungsformen $X(V)$, die gerahmt-bordant zu $kS(V)$ sind, wobei k so gewählt werden muß, daß keine Endlichkeitshindernisse auftreten.

Es wurde die prinzipielle Bedeutung dieser Ergebnisse für die Theorie der Darstellungsformen angedeutet.

Matthias KRECK. On the role of duality in the geometry of manifolds

Consider a manifold as a handle body $M = D^n \cup 1$-handles $\cup$ 2-handeles $\cup \dots \cup (k+1)$-handles $\cup \dots \cup D^n$. One way to study the role of duality in the geometry of manifolds is to ask the following question: Take off the handles of index $\geq (k+2)$, how many possibilities exist to close it by adding handles of index $\geq k+2$? I have sketched a theory which can be used to attack this problem. As in the classical surgery approach the theory has to ingredients: A bordism group and some L-group type obstruction groups. The surprising thing is that even if we control only weaker geometric conditions (instead of the homotopy type we control a $(k+1)$-skeleton for some $k < n$), the obstruction groups are ordinary Wall groups as long as $k > \frac{n}{2}$. The reason for this must be that the geometry of manifolds contains a very rich duality structure.

I have demonstrated this theory and given some applications.

For example in the case of the symmetric group S_k results of Taylor and Williams imply that the image of closed manifolds surgery obstructions in dimension $4m + 2$ is at most $\mathbb{Z}_2$ and detected by the Arf-invariant. On the other hand a computation of Kolster implies that the kernel of the map from minimal to maximal form parameter groups has order equal to the number of 2-regular conjugacy classes in S_k. Thus we have a subgroup of this order in $L^{S}_{4m+2}(S_k)$ which leads to fake (i.e. not homotopic to a diffeomorphism) homotopy self equivalence of a $(4m+1)$-dimensional closed oriented manifold M with fundamental group S_k and $S(M) = S_{max}$. One can easily show that manifolds with this property exist.

Claudia TRAVING. Classificatin of some complete intersections

A complete intersection of complex dimension n is the transversal intersection of r nonsingular hyperplanes in $\mathbb{C}P^{n+r}$. If the hyperplanes are given by polynomials of degree $d_1,\ldots,d_r$, then it follows from a remark of Thom, that the r-trupel $(d_1,\ldots,d_r)$ and the complex dimension n determine the diffeomorphism type of the complete intersection. We write $X_n(d) = X_n(d_1,\ldots,d_r)$. $(d_1,\ldots,d_r)$ is called multidegree.
As it can occur that two complete intersections with different multidegrees are diffeomorphic there is the following problem: Which invariants, computed from the dimension and the multidegree determine the diffeomorphism class.

As an application of the classification program of M. Kreck, we can determine the diffeomorphism type of a complete intersection $X_n(d)$ satisfying the condition (*) :

$$(*) \qquad \nu_p(d) \geq \frac{2n+1}{2(p-1)} + 1 \quad \text{for all primes } p \leq \sqrt{n + \frac{5}{4}} + \frac{1}{2}$$

where $\nu_p(d)$ denotes the exponent of p in the prime factor decomposition of the total degree $d = \prod_{i=1}^{r} d_i = \prod_p p^{\nu_p(d)}$.

<u>Theorem</u>. Let be $n \geq 3$, $X_n(d)$ a complete intersection satisfying (*). Then the diffeomorphism type of $X_n(d)$ is determined by

(i) $p_i(\xi(n,d))$ $\qquad i = 1,\ldots,\left[\frac{n}{2}\right]$,

(ii) $d = \prod d_i$,

(iii) $e(X_n(d))$ $\quad$ (e = Eulercharacteristic)

where $\xi(n,d) := -(n+r+1)\, H \oplus H^{d_1} \oplus \ldots \oplus H^{d_r} \in K(\mathbb{C}P^\infty)$ and H denotes the Hopfbundle.

The proof is given as an application of the program of M. Kreck involving as one main argument the mod p Adams-filtration of $X_n(d)$ which is given by $\gamma_p(d)$

The last step of the program, i.e. cancellation, is done by using the results of Libgober, Wood and Browder on the topological structur of complete intersections.

Andrew RANICKI. The algebraic L-theory of generalized free products

The work of Waldhausen on the algebraic K-theory of generalized free products can be extended to obtain algebraic proofs and extensions of Cappell's results on the L-theory of generalized free products. The main ingredient in both K- and L-theory is an existence theorem for "fundamental domains" of chain complexes by an algebraic transversality technique (generalizing the Higman linearization trick) abstracting the existence of fundamental domains for covers of manifolds in which the group of covering translations is a generalized free product.

Given a morphism of rings $f:R \longrightarrow S$ let $f_!:((\text{left})R\text{-modules}) \longrightarrow (S\text{-modules})$; $M \longmapsto f_!M = S \otimes_R M$ $f^!:(S\text{-modules}) \longrightarrow (R\text{-modules})$; $N \longmapsto f^!N = N$ be the usual induction and restriction functors.

Given pure injections of rings $\begin{cases} i_1:B \longrightarrow A_1, i_2:B \longrightarrow A_2 \\ i_1, i_2:B \longrightarrow A \end{cases}$.

Let $R = \begin{cases} A_1 *_B A_2 \\ A*_B[t,t^{-1}] \end{cases}$ be the $\begin{cases} \text{amalgamated free product} \\ \text{HNN extension} \end{cases}$

ring with $\begin{cases} i_1(b) = i_2(b) \in R \\ i_1(b) = t\, i_2(b)t^{-1} \in R \end{cases}$ $(b \in B)$. Denote the inclusion by

$$\begin{cases} j_1:A_1 \longrightarrow R, \quad j_2:A_2 \longrightarrow R, \quad k = j_1 i_1 = j_2 i_2 : B \longrightarrow R \\ j:A \longrightarrow R, \quad K = j i_1 : B \longrightarrow R. \end{cases}$$

A fundamental domain $\begin{cases} (D_1, D_2) \\ D \end{cases}$ for a finite f.g. free R-module chain complex C is defined by finite f.g. free subcomplexes

$\begin{cases} D_1 \subset j_1{}^!C, \quad D_2 \subset j_2{}^!C \\ D \subset j^!C \end{cases}$ (over $\begin{cases} A_1, A_2 \\ A \end{cases}$) such that

$\begin{cases} E = D_1 \cap D_2 \\ E = D \cap tD \end{cases}$ is a finite f.g. free B-module

chain complex, with $\begin{cases} RD_1 + RD_2 = C, \\ RD = C \end{cases}$ so that there

is defined a Mayer-Vietoris exact sequence of R-module chain complexes

$$\begin{cases} 0 \longrightarrow k_!E \xrightarrow{q} j_{1!}D_1 \oplus j_{2!}D_2 \longrightarrow C \longrightarrow 0 \\ 0 \longrightarrow K_!E \xrightarrow{q} j_!D \longrightarrow C \longrightarrow 0 \end{cases} \quad \text{with} \quad \begin{cases} q(E) \subseteq D_1 \oplus D_2 \\ q(E) \subseteq D \oplus tD \end{cases}$$

The existence of fundamental domains for chain complexes was the key step in Waldhausen's (first) proof of the exact sequence of K-groups

$$\begin{cases} K_1(B) \xrightarrow{\begin{pmatrix} i_1 \\ -i_2 \end{pmatrix}} K_1(A_1) \oplus K_1(A_2) \xrightarrow{(j_1 j_2)} K_1(A_1 *_B A_2) \to K_o(B) \oplus \widetilde{Nil} \xrightarrow{\begin{pmatrix} i_1 0 \\ -i_2 0 \end{pmatrix}} K_o(A_1) \oplus K_o(A_2) \xrightarrow{(j_1 j_2)} K_o(A_1 *_B A_2) \\ K_1(B) \xrightarrow{i_1 - i_2} K_1(A) \xrightarrow{j} K_1(A *_B [t,t^{-1}]) \to K_o(B) \oplus \widetilde{Nil} \xrightarrow{(i_1 - i_2 - o)} K_o(A) \xrightarrow{j} K_o(A *_B [t,t^{-1}]) \end{cases}$$

including the previous splitting theorems of Higman, Bass-Heller-Swan, Stallings, Gersten, Casson, Farrell-Hsiang as special cases.

Similarly, the existence of fundamental domains for chain complexes with duality is the key step in the algebraic proof of Cappell's exact sequence of L-groups

$$\begin{cases} \to L'_n(B) \oplus UNil_n \xrightarrow{\begin{pmatrix} i_1 & o \\ -i_2 & o \end{pmatrix}} L_n(A_1) \oplus L_n(A_2) \xrightarrow{(j_1 j_2)} L_n(A_1 *_B A_2) \longrightarrow L'_{n-1}(B) \oplus UNil_n \to \dots \\ \to L'_n(B) \oplus UNil_n \xrightarrow{(i_1 - i_2 - o)} L_n(A) \xrightarrow{j} L_n(A *_B [t,t^{-1}]) \longrightarrow L'_{n-1}(B) \oplus UNil_n \to \dots \end{cases}$$

for an $\begin{cases}\text{amalgamated free product of rings with involution,} \\ \text{HNN extension}\end{cases}$

including the previous splitting theorem of Browder, Wall, Shamson, Farrell-Hsiang, Novikov and Ranicki as special cases. One advantage of the algebraic approach is that it also applies to the symmetric L-groups, which are not topologically realizable.

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