MATHEMATISCHES FORSCHUNGSINSTUTUT OBERWOLFACH

Tagungsbericht 15/1985

Gewöhnliche Differentialgleichungen

24.3. bis 30.3.1985

Die zehnte Tagung über gewöhnliche Differentialgleichungen stand unter der Leitung von H.W.Knobloch (Würzburg), J.Mawhin (Louvain), R.Reißig (Bochum) und K.Schmitt (Salt Lake City). Mit 51 Teilnehmern (darunter 31 ausländischen) und 40 Vorträgen war sie die größte der bisher veranstalteten zehn Tagungen.

Eine quantitative Übersicht über die früheren Tagungen ergibt:

1.Tagung, 13.-19.10.1968
33 Teilnehmer (5 ausländ.)
31 Vorträge

2.Tagung, 3.-9.5.1970
22 Teilnehmer (6 ausländ.)
16 Vorträge

3.Tagung, 19.-25.3.1972
30 Teilnehmer (13 ausländ.)
20 Vorträge

4.Tagung, 17.-23.3.1974
34 Teilnehmer (16 ausländ.)
24 Vorträge

5.Tagung, 16.-22.3.1975
38 Teilnehmer (22 ausländ.)
32 Vorträge

6.Tagung, 27.3.-2.4.1977
32 Teilnehmer (15 ausländ.)
27 Vorträge

7.Tagung, 1.-7.4.1979
44 Teilnehmer (25 ausländ.)
33 Vorträge

8.Tagung, 22.-28.3.1981
41 Teilnehmer (21 ausländ.)
34 Vorträge

9.Tagung, 27.3.-2.4.1983
27 Teilnehmer (12 ausländ.)
24 Vorträge

Allerdings ist im Laufe der Zeit nicht nur die Zahl der Teilnehmer und der Vorträge größer geworden, sondern es hat die

Qualität der Referate erheblich zugenommen, und die periodisch stattfindende Tagung hat unter den Fachkollegen (insbesondere im Ausland) an Bedeutung gewonnen.

Das Schwerpunktthema war diesmal die Anwendung funktionalanalytischer Methoden; daneben befaßten sich Vorträge mit der Verzweigungstheorie, mit Rand- und Eigenwertproblemen, abstrakten Differentialgleichungen und allgemeiner Lösungstheorie sowie mit periodischen Lösungen spezieller Differentialgleichungen, auch solcher mit Verzögerung. Zu zwei Vorträgen über Computer-Graphiken wurden z.T. recht interessante und sehenswerte Lichtbilder und Filme vorgeführt.

Für die nächste Tagung ist als Schwerpunktthema die Theorie linearer Systeme (im Reellen und im Komplexen) vorgesehen.

Vortragsauszüge

S. AHMAD : On a System of Nonautonomous Differential Equations

We consider the Lotka-Volterra equations

$$u'(t) = u(t)[a(t) - b(t)u(t) - c(t)v(t)]$$
$$v'(t) = v(t)[d(t) - e(t)u(t) - f(t)v(t)]$$

where the functions $a(t),\ldots,f(t)$ are continuous for t in $[0,\infty)$, and bounded above and below by positive constants.
Let $a_M = \sup_{t\geq 0} a(t)$, $a_L = \inf_{t\geq 0} a(t)$, and let $b_M,\ldots,f_L$ be defined analogously. If $a_L/c_M > d_M/f_L$ and $d_L/e_M > a_M/b_L$ then there exists a solution $(u_o(t),v_o(t))$ satisfying

$$0 < \frac{a_L f_L - c_M d_M}{b_M f_L - c_M e_L} \leq u_o(t) \leq \frac{a_M f_M - c_L d_L}{b_L f_M - c_L e_M}$$
$$0 < \frac{b_L d_L - e_M a_M}{b_L f_M - c_L e_M} \leq v_o(t) \leq \frac{b_M d_M - e_L a_L}{b_M f_L - c_M e_L}$$

for all $t \geq 0$. It can be shown that if $(u(t),v(t))$ is any solution with $u(0) > 0$, $v(0) > 0$ then $u(t)-u_o(t) \to 0$, $v(t)-v_o(t) \to 0$ as $t \to \infty$.

B. AULBACH : On the analyticity of center manifolds

It is known that center manifolds of analytic systems in general are non-analytic. This is true even in the simplest cases where center manifolds are associated with a simple eigenvalue zero or a nontrivial pair of purely imaginary eigenvalues. Nevertheless, it can be shown that in these two cases the equation on any center manifold can be treated by usual analytical technique. This seeming paradox can be understood on the basis of the close relation between center manifold theory and classical concepts such as the Ljapunov-Schmidt procedure or normal forms for analytical systems.

R.E. BARNHILL : Surfaces / Mathematical Representation and Computer Graphics Display

Surfaces arise in many modern applications such as the design of automobiles and aircraft and the representation of physical and physiological phenomena. Modern technology provides new possibilities for the visual display of curves and surfaces by means of computer graphics. The mathematical questions center on what kinds of methods can both fit the given application and utilize the available graphics devices. For example, most applications require the use of smooth surfaces and efficient rendering requires the use of local approximations. We give a survey of criteria for choosing surface methods and of the methods themselves, illustrated by pictures in various media.

H. CARRILLO CALVET : The uniform validity of the averaging method over infinite intervals of time

The considered equation is $\dot{x} = \varepsilon f(t,x,\varepsilon)$. It is known that for exponentially stable solutions of the averaged equation, the average approximation as $\varepsilon \to 0$ is uniformly valid over the interval $[0,\infty)$. The main point is to remark that this holds also true for stable under persistent disturbances solutions of the averaged equation and that there are similar results that hold for orbital stability under persistent disturbances.

W.A. COPPEL : Stratification of continuous maps of an interval

The talk described recent work done in collaboration with L.Block (Florida). The notion of turbulence was defined for a continuous map of an interval into the real line, and its relation with periodic and homoclinic points studied. Strongly simple orbits were also defined and it was shown, in particular, that they represent periodic orbits with minimum entropy. Further results were given for unimodal maps with negative Schwarzian, sharpening results of Block and Hart.

ST.R. DUNBAR : A Branching Poisson Random Walk and a Nonlinear Hyperbolic Equation

I consider a branching Poisson random walk, that is, a random process in which a particle moves at a constant speed c along the line reversing its direction at random times given by a Poisson process. The particle also splits into $K \geq 2$ daughter particles with probability b_K , at a random time T_s, where $P\{T_s \in ds\} = b\, e^{-bs}\, ds$. Then at time t, there are $m(t)$ particles at positions $X_1(t)$, ... , $X_n(t)$. Let $f(s) = 1$ for $s > 0$, $= 0$ for $s < 0$, $F(s) = b_2 s^2 + b_3 s^3 + \ldots$, and $u(x,t) = E\left(\prod_{i=1}^{m(t)} f(x + X_i(t))\right)$. Then by a renewal argument, one can show that $u(x,t)$ satisfies

$$u_{tt} + [(2a+2b)u - 2b\, F(u)]_t = c^2 u_{xx} + (3b^2-2ab)\, u + (b^2+2ab)\, F(u) .$$

Special cases are related to diffusively coupled non-linear oscillators.

K.P. HADELER : Hyperbolic traveling fronts

At this meeting St.Dunbar has presented a model for a population of particles with Poisson motion and branching. Forming in the sense of McKean the probability describing the location of the most advanced particle he arrives at a differential equation of the general form

$$\varepsilon^2\, u_{tt} + g(u)\, u_t = (k(u)\, u_x)_x + f(u)$$

(in his paper k = const., g and f are polynomials). For this equation the same reduction trick for the traveling front problem can be applied as in the parabolic case (K.P.Hadeler 1981). It leads to a rather complete description of the possible speeds of fronts and shocks.

W.N. EVERITT : A note on linear ordinary quasi-differential equations

The theory of differential equations is largely concerned with properties of solutions of individual, or classes of, equations. This paper is given over to the converse problem - that of seeking properties of functions which require them to be, in some respect, solutions of a differential equation, and to determining all possible such differential equations.

From this point of view this paper discusses only linear ordinary quasi-differential equations of the second order. However, the methods can be extended tp quasi-differential equations of general order.
The results will appear shortly in a forthcoming volume of Proceedings A (Mathematics) of the Royal Society of Edinburgh.

D. FLOCKERZI : Bifurcation of higher dimensional tori

For a one-parameter family of ordinary differential equations possessing an invariant k-torus we give conditions that are sufficient for a Hopf-type bifurcation to an invariant (k+1)-torus. Then we show by means of an example that a supercritical bifurcation of an invariant 2-torus into an invariant 3-torus prevailing in the case of nonresonance may be replaced by a transcritical bifurcation into a pinched invariant 3-torus in the case of resonance. The connections of these bifurcation phenomena with the properties of the spectrum of the underlying invariant 2-torus are discussed.

G. FREILING : On the behaviour of eigenfunction expansions in the complex domain

We consider two-point boundary value problems which can be transformed to the form

$$(1) \quad l(y) = y^{(n)} + \sum_{i=2}^{n} f_i(z)\, y^{(n-i)} = \lambda y$$

$$(2) \quad U_i(y) = 0 \;, \quad 1 \leq i \leq n \;,$$

with splitting boundary conditions.
Using the detailed estimates for the eigenfunctions φ_k of (1)-(2) we give necessary conditions for the pointwise convergence and for the L_2-convergence of eigenfunction expansion of the form

$$(3) \quad \sum_{k=1}^{\infty} a_k \varphi_k(z) \;, \quad a_k \in \mathbb{C} \;.$$

In contrary to the "regular" case, we obtain in the "irregular" case that (3) behaves like a power series. The "natural" domain of convergence of (3) is a polygon $P_{m,n} \subset \mathbb{C}$ with n or m+1 vertices.

P. HABETS : Periodic solutions for vector second order differential equations

In this report, we present an existence result for the periodic boundary value problem

$$x'' + \frac{d}{dt} \operatorname{grad} f(x(t)) + g(t,x(t)) = 0 ,$$

$$x(0) = x(2\pi) , \quad x'(0) = x'(2\pi) ,$$

with $x \in \mathbb{R}^n$. To this end, we write

(1) $g(t,x) = Q(t,x)\, x + h(t,x)$

where $Q(t,x)$ is a symmetric matrix and $h(t,x)$ is sublinear. Such a decomposition is not unique and we assume it can be chosen such that for some symmetric matrices A , B , $\gamma(t)$, $\Gamma(t)$ and for x large enough, one has

$$A \lneqq \gamma(t) \leqq Q(t,x) \leqq \Gamma(t) \lneqq B .$$

Further we suppose the eigenvalues a_k and b_k of the matrices A and B are such that for some positive integers N_k , either $N_k = 0$ and $b_k \leqq 0$ or $(N_k-1)^2 \leqq a_k \leqq b_k \leqq N_k^2$.

We describe at last several conditions on $g(t,x)$, each of which implies we can write $g(t,x)$ as in (1) with a matrix $Q(t,x)$ that satisfies the above assumptions.

TH. HAGEMANN : Periodische Lösungen bei linearen Funktional-Differentialgleichungen mit unendlicher Verzögerung

Sei B ein Phasenraum von Funktionen φ: $(-\infty, 0] \rightarrow \mathbb{C}^n$ mit "fading memory" (vgl. Hale & Kato, 1978). Für die periodische FDG

(1) $\dot{x}(t) = L(t,x_t) + h(t)$ mit $h \in L^1_{loc}(\mathbb{R},\mathbb{C}^n)$, $L : \mathbb{R} \times B \rightarrow \mathbb{C}^n$ stetig, $L(t,\cdot)$ linear, $L(\cdot,\varphi)$ und h ω-periodisch, sowie $x_t(\theta) = x(t+\theta)$ für $\theta \leqq 0$, läßt sich mit einem Ergebnis über Randwertprobleme ein zur Fredholm-Alternative analoges Kriterium für die Existenz ω-periodischer Lösungen angeben; unter der Zusatzbedingung, daß der Raum der stetigen Funktionen mit kompaktem Träger dicht in B ist, gilt sogar:

(1) hat ω-periodische Lösungen gdw. $\int_0^{\omega} y(s)\, h(s)\, ds = 0$ für jede ω-periodische Lösung y der zu (1) adjungierten Gleichung.

Beide Aussagen erhält S.Murakami (1984) durch Einschränken von (1) auf einen endlich-dimensionalen Eigenraum des homogenen Lösungsoperators von (1).

Für den Spezialfall $B = C_\gamma = \{\varphi | \varphi$ stetig , $\lim_{\theta \to -\infty} e^{-\gamma\theta} \varphi(\theta), \gamma < 0,$ existiert in $\mathbb{C}^n\}$, vgl. Hagemann & Naito (Proc.Equadiff 82) , gilt die Zusatzbedingung <u>nicht</u>; <u>trotzdem</u> läßt sich die obige Äquivalenz zeigen.

O. HÁJEK - S. ELAYDI : <u>Dichotomy in Nonlinear Systems</u>

A linear ODE system in $\mathbb{R}^n$, $\dot{x} = A(t)\, x$, is said to have an exponential dichotomy over $\mathbb{R}^1$ if there exist constants $\alpha, \sigma > 0$ and a linear projection $P : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$|X_t\, P\, X_s^{-1}| \leq \alpha e^{-\sigma(t-s)} \text{ for } s \leq t ,$$

$$|X_t\, (I - P)\, X_s^{-1}| \leq \alpha e^{-\sigma(s-t)} \text{ for } s \geq t$$

where X_t denotes the fundamental matrix solution.

We present versions of this concept applying to nonlinear ODE systems: a local one applicable to weakly nonlinear systems, and a global one.

S. INVERNIZZI : <u>Periodic solutions for the forced Duffing equation with jumping nonlinearity</u>

Let $f : [0,T] \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function such that the limits (as $s \to \pm\infty$) $r_\pm = \liminf s^{-1} f(t,s)$, $s_\pm = \limsup s^{-1} f(t,s)$ exist finite. For c real, let $R_c = [r_+ - c^2/4, s_+ - c^2/4] \times [r_- - c^2/4, s_- - c^2/4]$. Let $A_1 = \{(\mu,\nu) \in \mathbb{R}^2 |\ \mu\nu > 0$, and $(T/\pi)\sqrt{\mu\nu} / (\sqrt{\mu} + \sqrt{\nu}) \notin Z$ when $\mu, \nu > 0\}$.

<u>Theorem</u>. Let $c \in \mathbb{R}$ and $e \in L^1 = L^1(0,T)$ be arbitrary. Assume the nonresonance conditions $R_0 \subseteq A_1$, $R_c \subseteq A_1$. The periodic BVP

$$u'' + c\, u' + f(t,u) = e(t) \quad \text{a.e. on } (0,T) ,$$

$$u(0) = u(T) , \quad u'(0) = u'(T) ,$$

has at least one solution u (of class C^1 with u' AC). The assumption $R_c \subseteq A_1$ is unnecessary when $0 < r_+ = r_- \leq s_+ = s_- < (2\pi/T)^2$.

(a joint work with Pavel Drábek)

F. KAPPEL : <u>A Galerkin type approximation scheme for delay equations</u>

Some years ago W.Schappacher and this author developed a scheme where solutions of delay equations are approximated by solutions of ordinary differential equations in the C-space setting. It turned out that the approximating ODEs inherit a lot of qualitative properties from the approximated delay equation. In the talk this is demonstrated for existence of periodic solutions and occurence of Hopf bifurcations. The simple structure of the approximating ODEs allows a detailed and explicit investigation of the behavior of the spectrum of the linearized equation with respect to variations of a parameter. It is proved that whenever a Hopf bifurcation occurs for the delay equation then the same is true for the approximating ODEs. Moreover a study of the nonlinear equation using Krasnoselskij's compression theorem for cones proves the existence of an unbounded (with respect to the parameter) branch of periodic solutions also for the approximating ODEs. We also give some numerical results. The results presented here are joint work with K.Schmitt.

H. KIELHÖFER : <u>Interaction of Stationary and Periodic Bifurcation at Multiple Eigenvalues</u>

For an evolution equation of parabolic type $\frac{du}{dt} + G(\lambda,u) = 0$ depending on a real parameter λ we consider Hopf bifurcation with an eigenvalue zero. We prove the following result: Let $G(\lambda,0) = 0$ and $G_u(\lambda,0) = A(\lambda)$. If the crossing number $\chi(0,\lambda_0)$ of $A(\lambda)$ through 0 at λ_0 is odd then stationary bifurcation occurs at λ_0. If $\chi(0,\lambda_0)$ is even, if the crossing number of $A(\lambda)$ through some simple eigenvalue $i\kappa_0$ is nonzero , and if for all eigenvalues on the positive imaginary axis which are in resonance with $i\kappa_0$ the greatest common divisor is greater than one, then stationary or periodic bifurcation takes place at $\lambda = \lambda_0$.

K. KIRCHGÄSSNER : <u>Solitary Waves under Surface Tension</u>

In a joint undertaking Ch.Amick and myself have solved the long standing problem of the existence of gravity driven solitary surface waves when surface tension is present. The proof works for values of the Bord-number $b > 1/3$, i.e. for not too small surface tension. The case $b \in (0,1/3)$ is still open.

The lecture gives an introduction into the underlying free boundary value problem, its formulation as a "dynamical system" in an infinite-dimensional space. Reduction to an ODE yields a second order nonlinear equation, which contains the full information of all possible solutions of small amplitude. Existence and uniqueness - up to phase shifts - follow.

U. KIRCHGRABER : On the Geometry of Multi-Step-Methods

The purpose of this talk is to provide a dynamical systems approach to the theory of multi-step methods for ordinary differential equations based on Invariant Manifold Theory. This permits to associate a one-step method to each strongly stable multi-step method such that both methods produce the same data provided the associated one-step method is used to start the multi-step method. If the latter condition is dropped there is still an asymptotic relationship.

J. KURZWEIL : Differential Relations

Let $J \subset \mathbb{R}$ be an interval, $X = \mathbb{R}^n$, $u : J \to X$, $s \in J$. The contingent derivative $D^{\bullet}u(s)$ is the set of such $w \in X$ that $w = \lim_{k \to \infty} [u(t_k) - u(s)] (t_k - s)^{-1}$ for a suitable sequence $\{t_k\}$, $t_k \in J$, $t_k \neq s$, $t_k \to s$ for $k \to \infty$. Let $F(t,v)$ be compact convex for $t \in \mathbb{R}$, $v \in X$ and let the right hand side of (1) $\dot{x} \in F(t,x)$ fulfil the standard conditions. Then
(A) Such set $N \subset \mathbb{R}$ of measure zero exists that for every solution u of (1) with domain T and every $s \in T \setminus N$ we have $\emptyset \neq D^{\bullet}u(s) \subset F(s,u(s))$. In particular, if $F(t,v) = \{f(t,v)\}$ for $t \in \mathbb{R}$, $v \in X$ so that (1) may be given the form $\dot{x} = f(t,x)$, we have $\dot{u}(s) = f(s,u(s))$.
(B) Solutions of (1) depend on initial conditions in a strengthened way.
These results are discussed and extended to the case that X is a Banach space or a linear topological space of a suitable type.

W.F. LANGFORD : Multiparameter Bifurcation in the Taylor-Couette Problem

(Joint work with M.Golubitsky)
Consider the Taylor-Couette problem in the presence of $O(2) \times SO(2)$ symmetry. There exists a curve in the 3-dimensional parameter space along which the linearized problem has eigenvalues $\{0 , i\omega , - i\omega\}$. However, the symmetrics force

all 3 of these eigenvalues to be double, thus yielding a 6-dimensional centre subspace. The Liapunov-Schmidt procedure reduces the problem to a 6-dimensional equivariant bifurcation equation, which possesses a much richer bifurcation structure than the standard $\{0, i\omega, -i\omega\}$ case. Computations currently in progress will determine which of the many possible bifurcation diagrams are actually present in the physical situation.

R. LEMMERT : Gewöhnliche Differentialgleichungen in lokal konvexen Räumen

Es sei E ein lokal konvexer Hausdorff-Raum. Für $f : [0,1] \times E \to E$ wird das Cauchy-Problem $u'(t) = f(t,u(t))$ $(t \in [0,1])$, $u(0) = u_o$ betrachtet. Es werden Bedingungen dissipativer Art angegeben, so daß Monotonie- bzw. Eindeutigkeitssätze gelten. Die angegebenen Sätze sind im Fall $f(t,u) = A u$ mit linear stetigem A bestmöglich.

N.G. LLOYD : Small amplitude limit cycles

Consider systems $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$ in which P and Q are polynomials. The famous sixteenth problem of Hilbert is to estimate the maximum possible number of limit cycles in term of the degree n of P and Q. One way in which this question can be approached is to first consider small amplitude limit cycles: it is supposed that the origin, say, is a fine focus, and perturbation of the coefficients are sought so as to maximize the number of limit cycles which bifurcate out of the origin. In this talk, results concerning this maximum number will be described; these relate to

(i) systems in which P and Q are cubic polynomials;

(ii) systems in which P and Q contain linear terms and terms of degree n only;

(iii) systems which are derived from second order equations of Liénard type.

J. MATIAK : Periodic Solutions of Certain Delay Differential Equations

For delay differential equations of the type

$$\dot{x}(t) = y(t) + \varepsilon f(x(t)) \quad , \quad \dot{y}(t) = - g(x(t-1))$$

the existence of periodic solutions is shown by deriving a-priori-bounds for solutions with a given initial function and then applying the Schauder fixed point theorem to the shift operator associated with the above system.
Because these bounds are given explicitly it is possible to drop some conditions on the asymptotic growth of the functions f and g which are needed by Crafton and Nussbaum. Moreover it is possible to estimate how the period of a periodic solution depends on the parameter ε.

G.H. MEISTERS : Polynomial Flows / Applications and Questions

In order that an n-dimensional ($n \geq 2$) nonlinear C^1 autonomous system $\dot{x} = X(x)$, $x \in \mathbb{R}^n$, have a local flow $x = \varphi(t,x_0)$ which is (for fixed t) polynomial in $x_0 \in \mathbb{R}^n$ (i.e., components in components), where $x_0 = \varphi(0,x_0)$, it is NECESSARY (but not sufficient) that $X(x) = a\,x + \partial\, H(x)$, where $a \in \mathbb{R}$, $\partial = (\partial_1, \ldots, \partial_n)$, and $H(x)$ is an $n \times n$ matrix of polynomials in $x_1, \ldots, x_n$ such that $H^T = -H$; and also the flow must be COMPLETE. When $n = 2$ such flows can be completely classified into six inequivalent types (up to conjugation $P \circ \varphi_t \circ P^{-1}$ by a polynomial automorphism P of $\mathbb{R}^2$); see H.Bass & G.Meisters (1985). One application: The question (studied by Markus, Yamabe, Hartman, Olech and others in the 1950's) of the global asymptotic stability of a stationary point of a two-dimensional system whose Jacobian matrix J satisfies $\det J > 0$ and $\operatorname{tr} J < 0$, can be answered in the affirmative for polynomial flows. Open questions concern finding further necessary conditions for the vector field X in order that its flow be polynomial in initial conditions x_0.

J.W. NEUBERGER : Differential Equations of Steepest Descent

Suppose that each of H and K is a real Hilbert space and F is a function from H to K which has a locally Lipschitz derivative. Define $\Phi : H \to R$ so that $\Phi(x) = \|F(x)\|^2/2$, $x \in H$.
Lemma. If $x \in H$ there is a unique function $z: [0,\infty) \to H$ so that (1) $z(0) = x$, $z'(t) = -\operatorname{grad} \Phi\,(z(t))$, $t \geq 0$.
Theorem. Suppose $\Omega \subset H$ and $C > 0$ so that $\|\operatorname{grad} \Phi(x)\| \geq C\|F(x)\|$, $x \in \Omega$. If $x \in \Omega$, z satisfied (1) and $R(z) \subset \Omega$, then $u = \lim_{t\to\infty} x(t)$ exists and satisfies $F(u) = 0$.
Applications of this theorem (and generalizations of it) to ordinary and partial differential equations are given.

H.O. PEITGEN: Newton Flows and Julia Sets

Die Erklärung der Natur von physikalischen Phasen und deren kritisches Verhalten (Phasenübergänge) ist das Hauptproblem der statistischen Thermodynamik. Zwei grundlegende, allerdings völlig verschiedene Ansätze sind in der Yang-Lee-Theorie bzw. der Renormierungstheorie niedergelegt. Die Yang-Lee-Theorie beschreibt eine abstrakte physikalische Phase als maximales Analytizitätsgebiet der freien Energie, aufgefaßt in einer komplexen Temperaturebene. Die Ränder dieser Gebiete werden durch die Nullstellen der Zustandssumme im thermodynamischen Limes gegeben und beschreiben den Locus der Phasenübergänge. Für spezielle Modelle läßt sich die Zustandssumme exakt renormieren. Damit läßt sich ein verblüffender Zusammenhang herstellen, der die Theorie der Phasenübergänge in eine völlig neue Perspektive stellt: Der gesuchte Locus der Phasenübergänge ist die Julia-Menge der entsprechenden Renormierungstransformation. In diesem Sinne verbindet die Theorie komplexer dynamischer Systeme zwei grundlegende Theorien der statistischen Physik. Das Studium der Parameterabhängigkeit der Modelle führt auf Anwendungen von Sullivan's Klassifikationssatz. Computergraphische Experimente, die auf dieser Grundlage erstellt werden, ergeben die Existenz einer Mandelbrot-Menge und weisen auf deren universelle Bedeutung hin.

R. RAUTMANN : Eine Klasse asymptotisch stabiler autonomer Systeme und ihre Anwendung

Es sei $Q_\lambda = \{x \mid a_i(\lambda^{-1}) \leq x_i \leq a_i(\lambda)$ für $\lambda > 1\}$ eine Schar n-dimensionaler Quader, die im positiven Kegel $\mathbb{R}^n_\alpha = \{x \mid \alpha_i < x_i\}$ des $\mathbb{R}^n$ durch die stetigen, streng monoton wachsenden Funktionen $a_i: (0,\infty) \to (\alpha_i, \infty)$ festgelegt sind, $n \geq 2$. Jeder Quader $\overset{\circ}{Q}_\lambda$ enthalte den Punkt $(1,\ldots,1)$, ihre Vereinigung sei $\mathbb{R}^n_\alpha$. Wir setzen dazu $\alpha_i \in [0,1)$, $a_i(1) = 1$ sowie $a_i(\lambda) \to \alpha_i$ für $\lambda \to 0$, $a_i(\lambda) \to \infty$ für $\lambda \to \infty$ voraus. Schließlich sei P_i die Projektion des $\mathbb{R}^n$ auf die Koordinatenebene $x_i = 0$. Für n jeweils auf $P_i\,\mathbb{R}^n_\alpha$ stetige, lokal Lipschitz-stetige Funktionen V_i gelte (1) $V_i(P_i x) \in (a_i(\lambda^{-1}), a_i(\lambda))$ auf jedem Quader Q_λ. Dann hat das autonome System (2) $\frac{d}{dt} x_i = [V_i(P_i x) - x_i] g_i(x)$ in $\mathbb{R}^n_\alpha$ mit beliebigen, auf $\mathbb{R}^n_\alpha$ stetigen und lokal Lipschitz-stetigen Funktionen $g_i(x) > 0$ den einzigen stationären Punkt $(1,\ldots,1)$. Jede Lösung von (2) in $\mathbb{R}^n_\alpha$ existiert global, ist asymptotisch stabil und hat die Grenzmenge $\{(1,\ldots,1)\}$. Die durch (1) und (2) charakterisierte Klasse autonomer Systeme enthält z.B. alle Systeme der Gestalt $\frac{d}{dt} x_i = [\prod_{j\neq i} x_j^{\alpha_{ij}q_j} - x_i^{q_i}] \cdot \prod_j x_j^{\beta_{ij}}$ mit $\alpha_{ij} \in (-1,1)$, $\sum_{j\neq i} \alpha_{ij} \in (0,1)$ für beliebige Exponenten $q_i > 0$, $\beta_{ij} \in \mathbb{R}$. Quasimonotone Systeme dieser Art führen zu günstigen Normschranken für spezielle nichtlineare Reaktions-Diffusionsprozesse.

Das für (2) erhaltene Resultat gilt auch für die Systeme der allgemeineren Form (3) $\frac{d}{dt} x_i = [f_i \circ V_i(P_i x) - f_i(x_i)]\, g_i(x)$ in $\mathbb{R}^n_\alpha$ mit zusätzlichen, streng monoton wachsenden, stetigen und lokal Lipschitz-stetigen Funktionen $f_i\colon (0,\infty) \to (0,\infty)$.

H. RÜSSMANN : On the behaviour of differential equations near a singular point

In the theory of critical cases only the cases with one and two purely imaginary eigenvalues allow a complete discussion of the stability behaviour of the singular point in question. More than two purely imaginary eigenvalues make a complete discussion difficult or even impossible. But in the case of Hamiltonian systems with two degrees of freedom and four nonzero purely imaginary eigenvalues a rather complete discussion seems to be possible. In this direction two new theorems are established.

K.P. RYBAKOWSKI : Persistence in models with diffusion

This report concerns some joint work with S.Dunbar and K.Schmitt. In some earlier work, Butler McGehee and Waltman proved an abstract lemma about (two-sided) flows on a locally compact metric space which, as a corollary, yields persistence of a large class of population models described by ordinary differential equations. We generalize this lemma to (one-sided) semiflows on arbitrary metric spaces. In this setting the lemma can be applied to show persistence of models with diffusion. We discuss such a result and illustrate it with numerical examples in the case of a one-predator two-prey Volterra-Lotka model with diffusion.

R. SCHAAF : The periods of solutions of some planar Hamiltonian systems

Consider a system $\dot{x} = - g(y)$, $\dot{y} = f(x)$, for which (0,0) is a center: $f(0) = g(0) = 0$; $f'(0)$, $g'(0) > 0$. Let $p(a)$ be the period of the solution (x,y) with $(x(0),y(0)) = (a,0)$, $a \in]0, a^+[$. Then the following two sets of assumptions on both $h = f$, $h = g$ imply that p is monotonic on $]a,a^+[$:

(A) (i) For all x where $h'(x) \geqq 0$: $(3h'h''' - 5(h'')^2)(x) < 0$

(ii) If $h'(x) = 0$ then $h(x)\, h''(x) < 0$.

(B) $(3h'h''' - 5(h'')^2)(x) > 0$ for all $x \in D(h)$.

(A) $\Rightarrow$ $p' > 0$ on $]0,a^+[$, (B) $\Rightarrow$ $p' < 0$ on $]0,a^+[$.

Condition (A) is e.g. satisfied by all polynomials h of degree $\geqq 2$ which only have real, simple zeroes, and for $h \circ f$, where f has negative Schwarzian and $f' > 0$ (as e^x). Thus the above result applies e.g. to the transformed Lotka-Volterra-system.

(B) is valid for all inverse functions of above polynomials h taken on an interval where $h' > 0$.

W. SCHAPPACHER : Nonlinear Boundary Value Problems in Banach Spaces

We consider the Cauchy problem $\frac{d}{dt}\, x(t) = A\,(x(t) + g(t,x(t))$, $t > t_o$; $x(t_o) = x_o$ in a Banach space X. Such problems arise, for instance, in the context of delay equations, feedback boundary control problems. Our main task is to investigate questions concerning (i) well-posedness and (ii) the asymptotic behavior of solutions.

J. SCHEURLE : Chaotic solutions of systems with almost-periodic forcing

The chaotic behaviour of certain dynamical systems can be explained by the presence of homoclinic or heteroclinic orbits. Consider, for instance, a diffeomorphism F on a two-dimensional manifold. It is well-known that if F has a transversal homoclinic point, then there is a Cantor-like set Λ near it which is invariant under a certain iterate of F, and the corresponding flow on Λ is topologically conjugated to the Bernoulli shift on a finite number of symbols. Examples arise as period maps of periodic systems of differential equations. The socalled Melnikov function provides a sufficient condition for the existence of a transversal homoclinic point of the period map for a two-dimensional system that results from adding a small periodic forcing term to an autonomous system which has a saddle point and a corresponding homoclinic orbit. - In this lecture the more general case of an almost-periodic forcing term is considered. We shall construct solutions which in general evolve even more irregularly than those which are starting from Λ in the periodic case.

B. SCHMITT: Sur l'équation du pendule forcé

Les symétries de l'équation du pendule forcé

(1) $x'' + \sin x = b \cos(\omega t)$, $\omega \neq 0$,

sont utilisées, d'abord pour prouver de manière élémentaire que cette équation admet pour b et ω fixés au moins deux solutions $2\pi/\omega$-périodiques paires, ensuite pour construire numériquement des cartes (par exemple pour $\omega = 1$) dans le plan (x,b) qui explicitent en fonction de b les conditions initiales x des solutions paires de (1) qui sont 2π-périodiques, 4π-périodiques, etc ..., ce qui fait apparaître la structure des solutions paires de l'équations.

H.L. SMITH : Periodic competitive and cooperative systems of ordinary differential equations

The existence, uniqueness, stability and geometry of the basin of attraction of periodic solutions of equations of the type indicated in the title will be discussed.

A. VANDERBAUWHEDE : Families of periodic solutions in symmetric systems

We give a modified formulation of some recent results of Golubitsky and Stewart on Hopf bifurcation in symmetric parameter-dependent systems; our approach allows to include a class of systems not considered in the original treatment. When the system has a symmetry group Γ , then the resulting bifurcation equation has symmetry $\Gamma \times S^1$, and one can establish the existence of certain isotropy subgroups Σ of $\Gamma \times S^1$ corresponding to bifurcating branches of periodic solutions with a space-time symmetry related to Σ . We also show how the same approach can be used to prove the existence of branches of periodic solutions with similar symmetries in symmetric systems (not depending on parameters) which are either time-reversible or have a first integral.

P. VOLKMANN: Über die positive Invarianz einer Menge bezüglich der Differentialgleichung $u' = f(t,u)$

Es sei E ein normierter Raum, $M \subseteq D \subseteq \mathbb{R} \times E$, $f: D \to E$, D offen und M in D (relativ) abgeschlossen. In Verallgemeinerung bekannter Resultate werden hinreichende Bedingungen dafür angegeben, daß für Lösungen $u: [\alpha, \omega] \to E$ der Differentialgleichung $u' = f(t,u)$ aus $(\alpha, u(\alpha)) \in M$ stets Graph $u \subseteq M$ folgt. Im Falle $E = \mathbb{R}^n$ und einer stetigen Funktion f sind diese Bedingungen nach Nagumo (1942) auch notwendig.

P. WALTMAN : Persistence in Dynamical Systems

Various concepts of persistence arise naturally in the study of three dimensional differential systems modeling population interactions, particularly those modeling ecological systems. One frequently used is that $\underline{\lim}_{t \to \infty} x_i(t) > 0$ for each component $x_i(t)$ of the solution vector.
The notion is abstracted to a setting of topological dynamics (flows on a locally compact metric space) and investigated in this setting. The basic theorems give a sufficient condition for persistence and a condition under which persistence implies uniform persistence.
(Joint work with B.Butler and H.Freedman, University of Alberta)

J.R. WARD : A semilinear elliptic boundary value problem

Existence of a solution to a semi-linear elliptic boundary value problem is established. The problem satisfies a resonance condition with regard to the first eigenvalue of the linear part. Our results are sharp in more than one sense, and exhibit the need for more regularity in some nonlinear problems than is needed for related linear ones.

M. WILLEM : Necessary and sufficient conditions for the existence of periodic solutions of Hamiltonian systems

Let $H(t,u)$ be a C^1 Hamiltonian T-periodic in t and strictly convex in u. Assume that

$$\overline{\lim_{|u| \to \infty}} \; 2\,H(t,u) \,/\, |u|^2 \leqq c < 2\pi / T$$

Then the boundary value problem

$$J\,\dot{u} + \operatorname{grad} H(t,u) = 0$$
$$u(0) = u(T)$$

is solvable if and only if there is $x \in \mathbb{R}^{2N}$ such that

$$\int_0^T \operatorname{grad} H(t,x)\,dt = 0$$

The proof depends on the dual action. Various extensions to boundary value problems for ordinary and partial differential equations are given.

F. ZANOLIN : Sharp nonresonance conditions for some nonlinear boundary value problems

(a joint paper with P.Onari)
In some recent papers of the authors, contributions to the study of the existence of solutions to nonlinear boundary value problems at resonance of the type

$$L\,x = g(x) + h \quad (L \text{ linear}\ ,\ g \text{ nonlinear})$$

have been given.
For a pair (λ_1,λ_2) of consecutive eigenvalues of the differential operator L, a resonance condition with respect to λ_1 and a nonresonance condition with respect to λ_2 are considered. In the latter case, we extend the classical (sharp) assumption $g(x)/x \leqq k < \lambda_2$ to $g(x)/x \uparrow \lambda_2$, provided not too fastly.
Applications of an abstract result, describing this approach, are given to the periodic boundary value problem for Duffing and Liénard differential delay equations.
For g depending on t, the nonuniform nonresonance case is also considered.

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