MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1985

Unendlichdimensionale Lie-Algebren und Gruppen

21.4. bis 27.4.1985

An der unter der Leitung von V. Kac (Cambridge, MA) und P. Slodowy (Bonn) stattfindenden Tagung nahmen 48 Mathematiker und Physiker aus 14 Ländern teil. Inhaltlich lassen sich die 24 gehaltenen Vorträge grob den folgenden Themenbereichen zuordnen:

1) Theorie der den Kac-Moody-Algebren zugeordneten Gruppen und ihrer homogenen Räume
2) Klassifikation graduierter Liealgebren und ihrer Automorphismen
3) Beziehungen zur algebraischen Geometrie und zur Theorie der Singularitäten
4) Konstruktion und Untersuchung fundamentaler Darstellungen der affinen Kac-Moody-Algebren, der Virasoro-Algebra und anderer Liealgebren von Abbildungen
5) Beziehungen zu physikalischen Modellen (Quantenfeldtheorie, stat. Physik)
6) Beziehungen zur Theorie der einfachen endlichen Gruppen
7) Vollständig integrable Hamilton'sche Systeme.

Dabei stehen diese Bereiche keineswegs disjunkt da, sondern es gibt die stärksten Wechselwirkungen (z. B. zwischen 1) und 3), 1) und 7), vor allem aber zwischen den Gebieten 4), 5), 6), 7)). Gerade diese Wechselwirkungen dürften verantwortlich sein für das in den letzten Jahren stark angestiegene Interesse an dem ganzen Themenkomplex sowie auch für das Interesse, auf das diese Tagung stieß. Während die meisten Vorträge über neuere Resultate berichteten, wurden von einigen Sprechern Überblicke gegeben über die wesentlichen Entwicklungen in einigen der genannten Themenbereiche. Für Einzelheiten vergleiche man die Vortragsauszüge.

Vortragsauszüge

V. G. KAC:

Introduction to infinite-dimensional groups

This talk gives an introduction to Kac-Moody groups. Their construction is explained via integrable representations of Kac-Moody Lie algebras. Among other things, the generalized Plücker relations and the connection to KdV-type equations are mentioned. Finally a proof is given of our theorem (joint work with D. Peterson) about the closure of the orbit of a highest weight vector. From this we deduce our result about conjugacy of Cartan subalgebras in a Kac-Moody algebra.

C. B. THORN:

Introduction to Dual String Models

The classical and quantum dynamics of the relativistic string was reviewed. The role of the Virasoro algebra in the covariant operator formalism was explained. The potential problem of "ghosts" was described, and the No-ghost Theorem of Goddard and Thorn was proved. It was further shown how the ghost elimination mechanism in dual models provides a new derivation of the Kac formula for the determinant of the contravariant form.

I. DOLGACHEV:

Infinite Weyl groups in algebraic geometry

The root systems of type $E_n (n \geq 6)$ can be realized in the space of 2-homology of certain algebraic surfaces. The group of automorphisms of some of these surfaces can be represented in the Weyl group of the corresponding system. For example, $n = 7,8,9,10$ correspond to surfaces obtained by blowing up 7 (resp. 8 , resp. 9 , resp. 10) points in the projective plane $\mathbb{P}^2$. If, moreover, these points are realized as a complete intersection of two cubics $(n = 9)$ or ten nodes of a rational sextic $(n = 10)$, the automorphism group is isomorphic to the level-2-congruence subgroup $W(2)$ of W (under certain assumptions on general position of the points). The latter group is the smallest normal subgroup containing the element
$w_0 \in W(E_8) \subset W(E_n)$, $n = 9,10$.

K. SAITO:

Regular systems of weights and associated root systems

Let us consider the rational function

$$\chi(T) := T^{-h}(T^h-T^a)(T^h-T^b)(T^h-T^c)/(T^a-1)(T^b-1)(T^c-1)$$

associated to a system of integers (called the weights) h,a,b,c with $h > a,b,c > 0$. This system is called regular if $\chi(T)$ does not have a pole except at the origin. Then there exist $\mu := (h-a)(h-b)(h-c)/abc$ integers $\varepsilon =: m_1 < m_2 \leq \dots \leq m_{\mu-1} < m_\mu := h-\varepsilon$, called exponents (here $\varepsilon := a+b+c-h$) such that $\chi(T) = \sum_{i=1}^{\mu} T^{m_i}$. The system of weights a,b,c,h is regular iff there exists a weighted homogeneous polynomial $f(x,y,z)$ in three variables of degree h with $\deg x = a$, $\deg y = b$, $\deg z = c$, such that the hypersurface $X_o \subset \mathbb{C}^3$ defined by $f = 0$ has an isolated singular point at 0. It is well known that $X_o - \{0\}$ is a quotient of $\mathbb{C}^2 - \{0\}$ by a finite subgroup of $SU(2,\mathbb{C})$ for $\varepsilon = 1$, a quotient of $\mathbb{C}^2$ by a Heisenberg group for $\varepsilon = 0$, and a quotient of $\tilde{\mathbb{H}} = \{(u,v) \in \mathbb{C}^2 \mid \mathrm{Im}(u/v) > 0\}$ by a "binary" Fuchsian group of the first kind for $\varepsilon < 0$. The set R of vanishing cycles in the middle homology group $H_2(X_1,\mathbb{Z})$ of the Milnor fiber $X_1 = f^{-1}(1)$ satisfies a system of axioms for a "generalized" root system with respect to the intersection form I on $H_2(X_1,\mathbb{Z})$. In case $\varepsilon = 1$, I is negative definite so that R is finite of type A_ℓ, D_ℓ, or E_ℓ. In case $\varepsilon = 0$, I is negative semi-definite and R is an extended affine root system of type $E_\ell^{(1,1)}$ for $\ell = 6,7,8$. For the first 14 cases of $\varepsilon = -1$, I is indefinite with two-dimensional maximal positive subspaces. For those cases one can define a Dynkin diagram for the root system. Let c be a product of the reflections corresponding to the vertices of the Dynkin diagram (such that reflections belonging to vertices connected like •===• follow each other in the product). We call c a Coxeter element. One of the most remarkable features of these root systems is the following:

1) <u>A Coxeter element</u> c <u>is of finite order</u> h; <u>the eigenvalues of</u> c <u>are given by</u> $\exp(2\pi\sqrt{-1}\, m_i/h)$, $i = 1,\dots,\ell$.

2) <u>Let</u> $\Phi_h(X)$ <u>be the cyclotomic polynomial for the</u> h-th <u>primitive roots of unity</u>. <u>Then</u> $R \cap \mathrm{Image}\,\Phi_h(c) = \emptyset$ <u>for</u> $\varepsilon = 1$ <u>or</u> $\varepsilon = -1$ (<u>if one of the</u> 14 <u>first cases</u>) <u>and</u> $R \cap \mathrm{Image}(c-1) = \emptyset$ <u>for</u> $\varepsilon = 0$.

3) For $\varepsilon = 0$, let $\tilde{W}$ be the central extension of the Weyl group W (generated by the reflections of R) defined via a hyperbolic extension. Then the hyperbolic Coxeter element $\tilde{c}$ (defined similarly as c but in the hyperbolic extension) is quasi-unipotent and $\tilde{c}^h$ generates the center of the extension $\tilde{W}$.

These properties of Coxeter elements will be used strongly for the construction of flat invariants for the Weyl group.

D. PETERSON:

Heisenberg groups and basic representations

We generalize results of Lepowsky-Wilson, Kac-Kazhdan-Lepowsky-Wilson and Frenkel-Kac on the action of the so-called "homogeneous" and "principal" Heisenberg subalgebras on the basic module V.

Let $\underline{g}$ be a finite-dimensional simple Lie algebra over $\mathbb{C}$ and let G be the corresponding connected simply-connected algebraic group. Let $\tilde{G} = \mathrm{Map}(\mathbb{C}^*, G)$ and $\tilde{\underline{g}} = \mathrm{Map}(\mathbb{C}^*, \underline{g})$ be the corresponding loop group and loop algebra, and let $\sigma : \hat{G} \to \tilde{G}$ and $d\sigma : \hat{\underline{g}} \to \tilde{\underline{g}}$ be the corresponding central Kac-Moody extensions. Let $\mathbb{H}$ be the variety of Cartan subalgebras of $\underline{g}$ and $s \in \mathrm{Map}(\mathbb{C}^*, \mathbb{H})$. Put $\tilde{\underline{s}} = \{x \in \tilde{\underline{g}} \mid x(t) \in s(t) \text{ for all } t \in \mathbb{C}^*\}$, $\hat{\underline{s}} = (d\sigma)^{-1}(\tilde{\underline{s}})$, $\hat{\underline{s}}_o = \mathrm{center}(\hat{\underline{s}})$, $\tilde{S} = \{g \in \tilde{G} \mid g(t) \in \exp s(t) \text{ for all } t \in \mathbb{C}^*\}$, $\hat{S} = \sigma^{-1}(\tilde{S})$, and $\hat{S}_o = \{g \in \hat{S} \mid \mathrm{Ad}(g) = \mathrm{id} \text{ on } \hat{\underline{s}}_o\}$.

Theorem (V. Kac, D. Peterson): Let $\underline{g}$ be of type A, D or E. Then the basic module is irreducible under the algebra of operators generated by $\hat{S}$ and $\hat{\underline{s}}$.

Two new methods play a key role in the proof: use of the asymptotics of V, $\hat{\underline{s}}$ and $\hat{S}$, and a commutator formula for elements of $\hat{S}_o$. Using the theorem, we construct vertex operators specifying the action of $\hat{\underline{g}}$ on V.

Two problems pose themselves for further study:

1. Find the commutation relations in $\hat{S}$.

2. Explain my observation that the number of conjugacy classes of the Weyl group satisfying $\det(I-w) = \det A$, where A is the Cartan matrix of $\underline{g}$ of type A, D or E, equals the number of orbits of $\mathrm{Aut}(\mathcal{D})$ on the set of vertices of $\mathcal{D}$ where $\mathcal{D}$ is the extended Dynkin diagram of $\underline{g}$.

K. MISRA:

Constructions of higher level representations of affine Lie algebras

One of the main features of the representation theory of affine Lie algebras is the existence of explicit constructions of some nontrivial representations. A representation of an affine Lie algebra is said to be of level $k \in \mathbb{C}$ if the central element (normalized suitably) acts as multiplication by k. Level one representations of affine Lie algebras have been studied by several researchers including Frenkel, Kac, Kazhdan, Lepowsky, Misra, Peterson, Segal and Wilson.

In this talk we will be mostly interested about constructions of higher level representations of some affine Lie algebras. In 1981, Lepowsky and Wilson introduced a new family of algebras associated with the representations of affine Lie algebras. They used these algebras to construct some higher level representations of $A_1^{(1)}$. In 1984, Misra used these algebras to give the constructions of some higher level representations of $A_n^{(1)}$ $(n > 1)$. Misra also gave explicit constructions of the level one representations of some symplectic affine Lie algebras. More recently, Lepowsky and Wilson have given explicit constructions of all irreducible representations of integral level of the affine Lie algebra $A_1^{(1)}$ in the principal picture. Also Lepowsky and Primc have given constructions of these representations of $A_1^{(1)}$ in the homogeneous picture.

J. TITS:

Kac-Moody groups over rings

Let $\mathcal{f} = (\Lambda, (\alpha_i)_{1 \leq i \leq \ell}, (h_i)_{1 \leq i \leq \ell})$, where Λ is a free abelian group of finite rank, (α_i) a system of points in Λ and (h_i) a system of points in the $\mathbb{Z}$-dual Λ^* of Λ, indexed by the same finite set $\{1,\ldots,\ell\}$. If $A = (\alpha_j(h_i))$ is a Cartan matrix, the system $\mathcal{f}$ determines a reductive group G over $\mathbb{C}$ (here, Λ is the character group of a maximal torus of G, (α_i) a basis of the corresponding root system and h_i the coroot associated with α_i) - and all reductive groups over $\mathbb{C}$ are obtained once in that way. More generally to such an $\mathcal{f}$ is associated a Chevalley group-scheme over $\mathbb{Z}$, hence a group functor $\mathfrak{G}_{\mathcal{f}}$ over the category $\mathfrak{R}$ of all rings, the above group G being $\mathfrak{G}_{\mathcal{f}}(\mathbb{C})$. In this lecture (an updated version of a lecture held on the same topic at the OW-Tagung n° 26 in 1982) was considered the problem of associating a group-functor $\mathfrak{G}_{\mathfrak{D}}$ over $\mathfrak{R}$ to any data $\mathfrak{D}$, assuming only that A is a <u>generalized Cartan Matrix</u>. Four ways of approaching the question have been discussed. (I) Define $\mathfrak{G}_{\mathfrak{D}}(R)$ as $\mathrm{Hom}_{cont}(\mathfrak{H}_{\mathfrak{D}}, R)$, where $\mathfrak{H}_{\mathfrak{D}}$ is a suitable topological Hopf algebra and R is given the discrete topology. A likely candidate for $\mathfrak{H}_{\mathfrak{D}}$ was described but several properties remain to be checked. (II) Define $\mathfrak{G}_{\mathfrak{D}}(R)$ by generators and relations. This works well when R is a field (one gets $\mathfrak{G}_{\mathfrak{D}}(R)$ as amalgamated product of suitably defined "minimal" parabolic subgroups containing a given Borel subgroup, and the normalizer of a "maximal torus" $\mathrm{Hom}(\Lambda, R^{\times})$), and $\mathfrak{G}_{\mathfrak{D}}$ is then easily extended to principal ideal domains (PID), but the method does not seem to be well applicable to more general rings.

(III) <u>Relate</u> $\mathfrak{G}_{\mathfrak{D}}$ <u>to more classical objects</u>. So far, this has been done only in the affine case. Assume A irreducible and semi-definite. For simplicity, assume further that Λ^* is generated by the h_i's and that $\Lambda \otimes \mathbb{Q} = \sum \mathbb{Q}\alpha_i$. Then, one can define a group-scheme $\underline{G}_{\mathfrak{D}}$ over $\mathbb{Z}[t,t^{-1}]$ such that, for a PID R, the functor $\mathfrak{G}_{\mathfrak{D}}$ obtained by method (II) is given by $\mathfrak{G}_{\mathfrak{D}}(R) = \underline{G}_{\mathfrak{D}}(R((t)))$. (For more details about this and further references, see Springer Lecture Notes Nr. 1111, pp. 191 - 223, in particular appendix 2.) (IV) This last equality was proved by using a suitable <u>axiom system</u> which can also be used to define $\mathfrak{G}_{\mathfrak{D}}$, at least in the affine case.

<u>Remark</u>. Both methods (I) and (II) require the choice of a suitable $\mathbb{Z}$-form of the universal enveloping algebra of the Kac-Moody Lie algebra over $\mathbb{C}$ associated to $\mathfrak{D}$; there is a natural choice for that $\mathbb{Z}$-form (already used by H. Garland in the affine case) but it may not be unique.

O. MATHIEU:

<u>Classification of simple graded Lie algebras</u>

In 1968 V. Kac showed that any complex simple graded Lie algebra $\underline{g} = \oplus g_n$ of finite growth, which is generated by $\underline{g}_{-1} \oplus g_0 \oplus g_1$ and such that the g_0-module g_{-1} is simple, is isomorphic to either a simple finite-dimensional Lie algebra, or an affine Lie algebra, or a Lie algebra of Cartan type. More general, Kac conjectured that any simple graded Lie algebra of finite growth is isomorphic to one of the algebras just mentioned or to the algebra W or W_1. The speaker recently proved a part of this conjecture:

Theorem: Let g be a simple graded Lie algebra of growth ≤ 1. Then g is isomorphic to either a simple finite-dimensional Lie algebra, or an affine Lie algebra, or W, or W_1.

B. GRIESS:

Finite Subgroups of $E_8(\mathbb{C})$; Code Loops

I. Arjeh Cohen and I have begun to classify finite subgroups of $G = E_8(\mathbb{C})$. Such a classification exists for a number of proper Lie subgroups of G. We have some preliminary results about G. For example, the alternating group of degree n lies in G if and only if $n \leq 10$. The situation is not understood even for the family $L_2(p)$, p a prime! If $L_2(p)$ embeds in G, $p \leq 61$. The case $p = 61$ is especially interesting because of Kostant's theory about elements of certain finite orders in simple Lie groups. We have a great deal of information about a possible $L_2(61)$ subgroup of G and we hope to decide whether $L_2(61)$ is a subgroup of G.

II. A code loop is a finite Moufang loop $\mathfrak{L}$ with a subgroup $\mathfrak{Z} \cong \mathbb{Z}_2$ of the center such that there exists an isomorphism $\alpha : \bar{\mathfrak{L}} = \mathfrak{L}/\mathfrak{Z} \to C$, C a doubly even binary code, with the properties

$$x^2 = \frac{1}{4}|\alpha(\bar{x})| \in \mathfrak{Z}$$

$$(yx)^{-1}(xy) = \frac{1}{4}|\alpha(\bar{x})| + \frac{1}{4}|\alpha(\bar{y})| + \frac{1}{4}|\alpha(\overline{x+y})| \in \mathfrak{Z}$$

$$(xy\cdot z)^{-1}(x\cdot yz) = \sum_{i,j,k \in \mathbb{F}_2} \frac{1}{4}|\alpha(i\bar{x}+j\bar{y}+k\bar{z})| \in \mathfrak{Z} .$$

A basic theory of code loops is discussed (see my paper "Code Loops", to appear in J. Alg.). A special case of this was constructed by Richard Parker (for C the Golay code) and that loop was used by John Conway to give an efficient construction of the monster and its nonassociative algebra. Further applications of code loops to studies of nonassociative algebras seem likely.

A. MEURMAN:

A moonshine module for the monster

Let F_1 be the Fischer-Griess "monster" group. Conway and Norton have conjectured that there is a sequence of representations of F_1, $(W_n)_{n \geq -1}$, such that, if for $m \in F_1$ we set

$$T_m(q) = \sum_{n \geq -1} \operatorname{tr} m|_{W_n} q^n \quad , \quad q = e^{2\pi i\tau} \quad , \quad \operatorname{Im} \tau > 0 \quad ,$$

then T_m generates a genus 0 function field corresponding to a discrete group Γ_m containing $\Gamma_o(N)$ where $N = |m|h$ and $h|(|m|,24)$. In particular

$$T_1(q) = q^{-1} + 0 + 196884q + \ldots = j(q) - 744 \quad ,$$

the modular invariant. In joint work with Frenkel and Lepowsky, I have constructed a natural representation $\bigoplus_{n \geq -1} V_n$ of F_1 with many of the properties of $\bigoplus_{n \geq -1} W_n$, for example $\sum_n (\dim V_n)q^n = j(q) - 744$. This construction is based on the vertex operator representation of affine Lie algebras of ADE-type obtained by Lepowsky, Wilson, Frenkel, Kac. To state our main result, let B be the commutative nonassociative algebra of dimension 196884 used by Griess to construct $F_1 = \operatorname{Aut}(B)$. Define an "affinization" $\hat{B} = B \otimes \mathbb{C}[t,t^{-1}] \oplus \mathbb{C}c$ by $(x \otimes t^m)(y \otimes t^n) = xy \otimes t^{m+n} + \langle x,y\rangle m^2 \delta_{m+n,o} c$, $c\,\hat{B} = \hat{B}\,c = 0$, $x,y \in B, m,n \in \mathbb{Z}$.

Theorem: There is a sequence $(V_n)_{n \geq -1}$ of F_1-modules, and for $V = \bigoplus_{n \geq -1} V_n$, a linear map $\pi : \hat{B} \to \operatorname{End}(V)$ such that

(i) the action of F_1 and $\hat{B}$ on V are compatible

(ii) π is a "representation" of $\hat{B}$ in the sense that

(a) $$\pi((x \otimes t^m)(y \otimes t^n)) = \frac{1}{2}([\pi(x \otimes t^{m+1}),\pi(y \otimes t^{n-1})] + [\pi(y \otimes t^{n+1}),\pi(x \otimes t^{m-1})])$$

(b) $\pi(x \otimes t^n)$ is homogeneous of degree n

(c) $\pi(c) = 1$

(iii) we have $\sum_{n \geq -1} (\dim V_n)q^n = j(q) - 744$.

D. OLIVE:

Fermion representations of Virasoro and Kac-Moody algebras

The Virasoro algebra occurs naturally in two dimensional physical systems which are local and scale invariant. The critical indices describing phase transitions are essentially weights of the algebra and are controlled by the value of its central term which either exceeds unity or equals $1 - \frac{6}{m(m+1)}$, $m \in \mathbb{Z}$. Any highest weight representation of a Kac-Moody algebra yields a unitary representation of the Virasoro algebra with rational but typically non-integral central term. A coset variant of this construction involving symplectic groups yields the above discrete series. Representations of the Kac-Moody algebra bilinear in fermion fields were considered and the necessary and sufficient condition that the constructed Virasoro algebra generators collapse to a form quadratic in fermions was established. The criterion involved symmetric spaces and guaranteed the finite reducibility of the Kac-Moody algebra representation.

A. PRESSLEY:

A geometric approach to representations of loop groups

The irreducible highest weight representations of the Lie group LG of smooth maps $S^1 \to G$, G a compact Lie group, are realized on the space of sections of holomorphic line bundles on LG/T. The latter space is an infinite dimensional complex manifold which has many interesting geometrical properties. All non-trivial such representations are projective, more precisely, they are representations of central extensions $\widetilde{LG}$ of LG by the circle. The extensions themselves are constructed by differential geometric methods. The complex line bundles on LG/T are classified by the characters λ of the torus $\tilde{T} = S^1 \times T \subset \widetilde{LG}$, i.e. by the weights of $\tilde{T}$, and each has a unique holomorphic structure. The space $\Gamma(L_\lambda)$ of holomorphic sections is non-zero iff $-\lambda$ is dominant, and then $\Gamma(L_\lambda)$ is the representation of LG of lowest weight λ. The proof uses the geometry of LG/T and some (finite dimensional) complex analysis, but requires no detailed knowledge of the structure of the Lie algebra of LG. (Much of the work described is due to Graeme Segal.)

A. McDANIEL:

Representations of $sl(n,\mathbb{C})$ and the Toda lattice

The Toda lattice equations may be written as a Lax equation $\frac{dA}{dt} = [A,B]$ where $A,B \in sl(n,\mathbb{C}) \otimes \mathbb{C}[h,h^{-1}]$. When A and B are represented as matrices by using the classical representation of $sl(n,\mathbb{C})$, a linearization recipe of van Moerbeke and Mumford linearizes the Toda lattice flow on the Jacobi variety of the representation dependent spectral curve of A . This linearization recipe can be extended to the matrix Lax equations obtained using any finite dimensional representation of $sl(n,\mathbb{C})$. Then the linearization of the Toda lattice flow is independent of the representation in the following sense. The various spectral curves are related by algebraic correspondences inducing homomorphisms between their respective Jacobi varieties. The flow obtained using any higher dimensional representation is the homomorphic image of the linear flow on the Jacobi variety coming from the classical representation. So the various linearizations all lead to essentially the same flow on the same abelian variety.

T. MIWA:

Infinite dimensional Lie algebras and Soliton equations - a survey of work by Date, Jimbo, Kashiwara, Miwa

Consider Hirota's bilinear equation $P(D)\tau(x) \cdot \tau(x) = 0$, where $x = (x_1,x_2,\ldots)$, $D = (D_1,D_2,\ldots)$, P an even polynomial, e. g. $P(D) = $ $= D_1^4 + 3D_2^2 - 4D_1D_3$ for the KP-equation. Soliton solutions are of the form $\tau(x) = \sum_{J \subset \{1,\ldots,N\}} e^{\xi_J}$, where $\xi_j = c_j + \eta_j \cdot x$, $P(\eta_j) = 0$ and

$$\xi_J = \sum_{j \in J} \xi_j - \sum_{j,j' \in J, j < j'} \frac{P(\eta_j - \eta_{j'})}{P(\eta_j + \eta_{j'})} .$$

Complete integrability for the KP-equation means that for all $N \in \mathbb{N}$ a τ like above is a solution. For arbitrary $P(D)$, $N = 0,1,2$ only give solutions. Complete integrability also implies the existence of infinitely many equations $P_i(D)\tau(x) \cdot \tau(x) = 0$ solved by the above N-soliton. For the KP-equation $\eta_j = (p_j - q_j, p_j^2 - q_j^2, \ldots)$ and $\tau(x) = e^{\sum \log c_j X(p_j,q_j)} \cdot 1$, where the vertex operator $X(p,q)$ is given by

$$X(p,q) = e^{\xi(x,p)-\xi(x,q)} \cdot e^{-\xi(\tilde{\partial},p^{-1}) + \xi(\tilde{\partial},q^{-1})}$$

with $\xi(x,k) = \sum_{n=1}^{\infty} k^n x_n$. The $X(p,q)$ span a Lie algebra. This can be explained as the coincidende of the following two bilinear identities

(1) $$\sum_{n \in \mathbb{Z}} \psi_n | v> \otimes \psi_n^* | v> = 0$$

(2) $$\oint \frac{dk}{2\pi i} w(x,k) w^*(x',k) = 0 ,$$

where (1) characterizes the orbit of highest weight vectors of $gl(\infty)$ and (2) characterizes the KP-hierarchy.

M. JIMBO:

A q-difference analogue of $U(\mathfrak{g})$ and the Yang-Baxter equation

The Yang-Baxter equation is one for the unknown matrix $R(u) \in End(V \otimes V)$:

(YB) $$R^{12}(u) R^{13}(u+v) R^{23}(v) = R^{23}(v) R^{13}(u+v) R^{12}(u) \quad \text{in} \quad End(V \otimes V \otimes V)$$

where $R^{12}(u) = R(u) \otimes I$, etc. This equation is important in the quantum theory of integrable systems. One is interested in solutions containing a parameter $\hbar$ such that

(CL) $$R(u,\hbar) = 1 + \hbar r(u) + \ldots \qquad \text{as} \quad \hbar \to 0 .$$

The term $r(u)$ now satisfies the "classical" YB equation

(CYB) $$[r^{12}(u), r^{13}(u+v)] + [r^{13}(u+v), r^{23}(v)] + [r^{12}(u), r^{23}(v)] = 0 .$$

Now $r(u)$ in this form can be regarded as taking its value in $\mathfrak{g} \otimes \mathfrak{g}$, $\mathfrak{g}$ being an abstract Lie algebra. If $\mathfrak{g}$ is simple, finite dimensional over $\mathbb{C}$, solutions to (CYB) have been classified by Belavin-Drinfel'd.

Problem: Given $r(u)$, construct $R(u)$ satisfying (YB) and (CL) !
It is shown that by introducing a q-difference analogue of $U(\mathfrak{g})$, (YB) reduces to a linear equation for $R(u)$. For $\mathfrak{g} = sl(2)$ the algebra looks like this:

(*) $$[h,e] = 2e , \quad [h,f] = -2f , \quad [e,f] = \frac{\sinh(2\hbar h)}{\sinh(2\hbar)} .$$

In this simplest case, the solution $R(u)$ is constructed explicitely with the aid of (*) .

G. ROUSSEAU:

Automorphisms of finite order of affine Kac-Moody algebras and affine symmetric spaces

The classification of affine symmetric spaces of non-trivial type (given by M. Berger in 1957) is equivalent to that of pairs of commuting involutions of a simple Lie algebra $\mathring{\underline{g}}$. If we try more generally to classify pairs $(\mathring{\sigma},\mathring{\tau})$ of automorphisms of finite order of $\mathring{\underline{g}}$ modulo conjugation, a first step is the classification (modulo conjugation) of (some) automorphisms τ of finite order and of first kind (i.e. stabilizing each conjugacy class of Borel subalgebras) of the affine Kac-Moody algebra $\underline{g} = \hat{L}(\mathring{\underline{g}},\mathring{\sigma})$. This classification (initiated by Levstein for involutions) has been almost finished by Jean Bausch. The results look like those of Kac for automorphisms of $\mathring{\underline{g}}$ except that there is still no straightforward way to compute $\underline{g}^{\tau}$. The key point is the structure of $\underline{g}^{\tau}$: its center $\underline{z}$ is one dimensional, its derived algebra $(\underline{g}^{\tau})'$ one codimensional and $(\underline{g}^{\tau})'/\underline{z}$ is the product of a finite number of affine adjoint Kac-Moody algebras and an algebra with only imaginary roots (which gives all the troubles).

A. FIALOWSKI:

Classification of graded Lie algebras on two generators

Let $\underline{g} = \bigoplus_{i=1}^{\infty} \underline{g}_i$ be a graded Lie algebra over a field k of characteristic 0 . Assume $\dim \underline{g}_i = 1$ for all $i (\underline{g}_i = ke_i)$ and that $\underline{g}$ is generated by two generators (the minimal possible number under this condition). Three examples are well known: 1) L_1 the Lie algebra of polynomial vector fields on the line vanishing at the origin as well as their first derivatives, 2) $\underline{n}_+ (A_1^{(1)})$, 3) $\underline{n}_+ (A_2^{(2)})$, where $A_1^{(1)}$ and $A_2^{(2)}$ are the rank-2 affine Kac-Moody Lie algebras. We define two other algebras:

$\underline{m}_1$ - determined by $[e_1,e_i] = e_{i+1}$ for $i > 1$ and $[e_i,e_j] = 0$ for $i,j > 1$,

$\underline{m}_2$ - determined by $[e_i,e_j] = 0$ for $i,j > 2$ and $[e_i,e_j] = e_{i+j}$ for $i = 1,2, j > i$.

Theorem 1: Let $\underline{g}$ be a Lie algebra as above generated by e_1 and e_2. Then

a) if $[e_1,e_4] \neq 0$, then $\underline{g} \cong L_1$ or $\underline{g} \cong \underline{m}_2$,

b) if $[e_2,e_3] = 0$, then $\underline{g} \cong \underline{n}_+(A_2^{(2)})$ or $\underline{g} \cong \underline{m}_1$,

c) if $[e_1,e_4] = 0$, then $\underline{g} \cong \underline{m}_1$, or $\underline{g}$ is isomorphic to a Lie algebra of the next family $\underline{g}(\lambda_8,\lambda_{12},\lambda_{16},\ldots)$, where $\lambda_{4i} \in \mathbb{P}^1(k)$.

In $\underline{g}(\lambda_8,\lambda_{12},\lambda_{16},\ldots)$ we have $[e_i,e_j] = 0$ if either i and j are even and $\neq 2$ or i is even, $i \neq 2$, and j is odd. Furthermore, $[e_1,e_{4k-1}] = \alpha_{4k}e_{4k}$, $[e_2,e_{4k-2}] = \beta_{4k}e_{4k}$ for $k = 3,4,5,\ldots$, where (α_{4k},β_{4k}) are the homogeneous coordinates of λ_{4k}. The remaining commutators can be uniquely calculated from these formulas.

Theorem 2: The algebras L_1, $\underline{n}_+(A_1^{(1)})$, $\underline{n}_+(A_2^{(2)})$, and $\underline{m}_2$ have two relations of weights 5 and 7; the algebra $\underline{m}_1$ has independent relations of weight $2k+1$, $k \geq 2$; in $\underline{g}(\lambda_8,\lambda_{12},\ldots)$ there are relations between the generators of weights $2k+1$, $k \geq 2$, and $4k$, $k \geq 2$.

Corollary: Consider the relations of weight 5 and 7 in an arbitrary mentioned graded Lie algebra $\underline{g}$. For most of the relations $\underline{g}$ is finite dimensional or $\dim \underline{g}_i$ grows exponentially with i. The exceptional cases are the ones mentioned above.

J. MICKELSSON:

Step algebras and their applications

Let $\underline{g}$ be a complex Lie algebra and $\underline{k}$ a reductive subalgebra in $\underline{g}$. Define $S'(\underline{g},\underline{k}) = \{u \in \mathcal{U}(\underline{g}) \mid \underline{k}_+u \subset \mathcal{U}(\underline{g})\underline{k}_+\}$ for a fixed triangular decomposition $\underline{k} = \underline{k}_- \oplus \underline{h} \oplus \underline{k}_+$ corresponding to a choice $\underline{h}$ of a Cartan subalgebra in $\underline{k}$ and a choice $\Phi^+ \subset \Phi$ of positive roots for the pair $(\underline{h},\underline{k})$. Set $S(\underline{g},\underline{k}) = S'(\underline{g},\underline{k})/S' \cap \mathcal{U}(\underline{g})\underline{k}_+$. The step algebra is a subalgebra $S_o(\underline{g},\underline{k}) \subset S(\underline{g},\underline{k})$ generated by $\underline{h}$ and elements $s_1,s_2,\ldots$ which are of first order in a basis $\{t_1,t_2,\ldots\}$ of $\underline{g} \ominus \underline{k}$ and are represented by vectors of minimal possible degree in $\mathcal{U}(\underline{g})$. Let $\Lambda^+ \subset \underline{h}^*$ be the set of dominant integral weights and fix the lexicographical ordering in Λ^+ with respect to $\langle\lambda,\alpha_1\rangle,\ldots,\langle\lambda,\alpha_\ell\rangle$ ($\alpha_1,\ldots,\alpha_\ell$ simple roots), $\lambda \in \Lambda^+$. All irreducible $\underline{k}$-finite $\underline{g}$-modules V can be characterized by the action of $S_o(\underline{g},\underline{k})^{\underline{h}}$ on the minimal $\underline{k}$-type $V_\lambda^+ \subset V^+ = \{v \in V \mid \underline{k}_+v = 0\}$. Application to Kac-Moody algebras: Let $\underline{g} = A_1$, $\hat{\underline{g}} = A_1^{(1)}$. The $\underline{g}$-finite irreducible $\hat{\underline{g}}$-modules can be classified by the following data, when $\dim(\lambda) \geq 3$ for the minimal $\underline{g}$-type λ : 1) the minimal $\underline{g}$-type λ , 2) an irreducible representation of an infinite dimensional Heisenberg algebra acting on V_λ^+ (this algebra arises from the action of $S_o(\hat{\underline{g}},\underline{g})^{\underline{h}}$ on V_λ^+ ; $\underline{h} \subset \underline{g}$ Cartan subalgebra).

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B. KUPERSHMIDT:

Differential Lie algebras and KdV-type equations

Let $\partial : K \to K$ be a differential ring, $D(K)$ the Lie algebra of "vector fields" on K : $[X,Y] = XY'-X'Y$, where $(\cdot)' = \partial(\cdot)$. The natural Hamiltonian matrix B on $C_u = K[u^{(n)}]$ associated to $D(K)^*$ is defined as $XB(Y) \equiv u[X,Y]$ (mod Im ∂). Thus $B = u\partial + \partial u$. Let $\omega(X,Y) = XY^{(3)}$, where $(\cdot)^{(k)} = \partial^k(\cdot)$. Then ω is a (generalized) 2-cocycle on $D(K)$. Let $b = b_\omega$ such that $Xb_\omega(Y) \equiv \omega(X,Y)$ (mod Im ∂) . Then $B^2 = u\partial + \partial u - \partial 3/2$ is a Hamiltonian matrix, i.e. the associated Poisson bracket $\{H,F\} = \frac{\delta F}{\delta u} B(\frac{\delta H}{\delta u})$ satisfies the Jacobi identity mod Im ∂ . The KdV-equation is connected with $D(K)$ in this way: $u_t = B^2(\delta H_1/\delta u) = B^1(\delta H_2/\delta u)$, $H_1 = u^2$, $H_2 = u^3 + u_x^2$, $B^1 = \partial/2$ being the b_ν for the trivial 2-cocycle $\nu(X,Y) = \frac{1}{2} XY$! This bi-Hamiltonian definition $B^2(\delta H_n/\delta u) = B^1(\delta H_{n+1}/\delta u)$, $B^1(\delta H_o/\delta u) = 0$ can be iterated to the whole KdV-hierarchy. The procedure can be applied to other differential Lie algebras and superalgebras, in particular to the following Lie superalgebra $\hat{\underline{g}}$:

Let $K = K_{\bar{0}} \oplus K_{\bar{1}}$ be a differential commutative superalgebra, $\underline{g}$ a finite dimensional Lie algebra over $k = \operatorname{Ker} \partial|K$. Consider

$$\left[\begin{pmatrix} X_1 \\ \dfrac{f_1 \otimes a_1}{\gamma_1 \otimes b_1} \\ \alpha_1 \end{pmatrix}, \begin{pmatrix} X_2 \\ \dfrac{f_2 \otimes a_2}{\gamma_2 \otimes b_2} \\ \alpha_2 \end{pmatrix}\right] =$$

$$= \begin{pmatrix} X_1X_2' - X_1'X_2 - 2\alpha_1\alpha_2 \\ \dfrac{f_1f_2 \otimes [a_1,a_2] + X_1f_2' \otimes a_2 + \gamma_1\alpha_2 \otimes b_1 - X_2f_1' \otimes a_1 - \gamma_2\alpha_1 \otimes b_2}{(X_1\gamma_2' + \frac{1}{2}X_1'\gamma_2) \otimes b_2 + f_1\gamma_2 \otimes [a_1,b_2] + f_1'\alpha_2 \otimes a_1 - (1 \leftrightarrow 2)} \\ (X_1\alpha_2' - \frac{1}{2}X_1'\alpha_2) - (1 \leftrightarrow 2) \end{pmatrix}$$

where $X_i, f_i \in \underline{K}_{\bar{0}}$, $\gamma_i, \alpha_i \in \underline{K}_{\bar{1}}$, $a_i, b_i \in \underline{g}$. There is a 2-cocycle $\omega_1(1,2) = X_1''X_2 + 4\alpha_1''\alpha_2$ on $\hat{\underline{g}}$; if $\underline{g}$ has an invariant form (,) , there is another one $\omega_2(1,2) = f_1'f_2(a_1,a_2) + \ldots$. If (,) is nondegenerate the corresponding super KdV-system is bi-superhamiltonian and integrable:

$$\begin{aligned} u_t &= \partial(3u^2 - u_{xx} + \underline{p}^t\underline{p} + \underline{\sigma}^t\underline{\sigma}_x + 3\varphi\varphi_x) \\ \underline{p}_t &= 2u\underline{p} - [\underline{\sigma},\underline{\sigma}] + 2\varphi_x\underline{\sigma} \\ \underline{\sigma}_t &= u_x\underline{\sigma} + 2u\underline{\sigma}_x + 2[\underline{p},\underline{\sigma}] - 2\varphi_x\underline{p} \\ \varphi_t &= 3\varphi u_x + 6u\varphi_x - 4\varphi_{xxx} - 2(\underline{p},\underline{\sigma})_x \quad , \end{aligned}$$

here $u, \underline{p} \in \underline{g} \otimes \underline{K}_{\bar{0}}$ are even, $\underline{\sigma} \in \underline{g} \otimes \underline{K}_{\bar{1}}$, φ are odd. For $\underline{\sigma}, \varphi = 0$, the remaining even part loses the interaction.

K. UENO / H. YAMADA:

The super-Graßmann-hierarchy, the super-KP-hierarchy, and $gl(\infty|\infty)$

The ordinay KP-hierarchy is naturally interpreted, through the Graßmann equation, as a dynamical system on the "Universal Graßmann Manifold" (UGM), first introduced by M. Sato. Motivated by this framework, we introduce a supersymmetric extension of the KP-hierarchy (SKP for short).

Let $\underline{S}$ be a superalgebra of superfields, which is a tensor product of the Berezin algebra $\underline{K}$ of infinitely many variables, say $x, t_2, t_4, \ldots$; $s, s_1, s_3, \ldots$, and the constant Graßmann algebra $\underline{A}$. Let $\theta = \partial_s + s \cdot \partial_x$, $\theta_{2j-1} = \partial_{j_{2j-1}} + \sum_{k=1}^{\infty} s_{2k-1}\partial_{t_{2j+2k-2}}$, $\theta_{2j} = \partial_{t_{2j}}$. These are super-derivations acting on $\underline{S}$. Further, $\underline{S}((\theta^{-1}))$ and $\underline{S}[\theta]$ are the super-analogues of the ring of micro-differential operators and the ring of differential operators. Then the SKP is, by definition, the following system of nonlinear super-differential equations:

$$(-)^{j}\dot{\theta}_{2j}B_{2k} - (-)^{k}\dot{\theta}_{2k}B_{2j} + [B_{2k},B_{2j}] = 0$$

$$(-)^{k}\dot{\theta}_{2k}B_{2j-1} + (-)^{j}\dot{\theta}_{2j-1}B_{2k} + (B_{2j-1}B_{2k} - B^{*}_{2k}B_{2j-1}) = 0$$

$$(-)^{j-1}\dot{\theta}_{2j-1}B_{2k-1} - (-)^{k}\dot{\theta}_{2k-1}B_{2j-1} + B^{*}_{2k-1}B_{2j-1} + B^{*}_{2j-1}B_{2k-1} + 2B_{2(j+k-1)} = 0$$

where $*$ is the canonical involution on $\underline{S}((\theta^{-1}))$, and the B_j are in $\underline{S}[\theta](j)^{\text{monic}}$. The SKP-hierarchy is integrated through the super-Graßmann-equation: Let $\Phi = \exp(s\Lambda + x\Lambda^2 + \sum t_{2j}\Gamma^{2j} + \sum s_{2j-1}\Gamma^{2j-1})$, where Λ is a shift matrix of size $\mathbb{Z}$ and Γ is a canonical matrix which anti-commutes with Λ . Let ξ be a $(\mathbb{Z},\mathbb{N}^{o})$-frame over the constant Graßmann algebra $\underline{A}$, which is thought of as a point of the super UGM, and let ξ_o be the reference point. We regard the following super-Graßmann-equation:

$$(\ldots,w_{3/2},w_1,w_{1/2},1)\,{}^{t}\xi_o\Phi\xi = 0 \quad .$$

For a generic ξ , this equation actually can be solved. For a solution, we put $w = \sum_{j=o}^{\infty} w_{j/2}\theta^{-j}$. Then w satisfies

$$\dot{\theta}_{2j}w = (-)^{j}(B_{2j}w - w\theta^{2j})$$

$$\dot{\theta}_{2j-1}w = (-)^{j-1}(B_{2j-1}w - w^{*}\theta^{2j-1})$$

which just corresponds to the linearization for SKP. In fact, taking the integrability condition, one gets the SKP. The SKP is a nonlinear super evolution equation, and its super time evolution is interpreted as a dynamical motion on the super UGM : $\xi \mapsto \Phi\xi$.

Finally, we mention a super-symmetric extension of $gl(\infty)$, which is denoted by $gl(\infty|\infty)$. This Lie superalgebra contains many important Lie supersubalgebras, e.g. the Neveu-Schwarz algebra, the Ramond algebra, which are the super analogues of the celebrated Virasoro algebra, and it also contains the super Kac-Moody algebras. We hope that the link between $gl(\infty/\infty)$ and SKP will be revealed in the near future.

W. NAHM:

Euclidean Kac-Moody algebras in quantum field theory

Euclidean Kac-Moody algebras appear naturally in quantum field theories describing maps from a two-dimensional space to symmetric spaces or tori, if these theories are conformally invariant. This also applies to string theories.

The simplest example is a scalar field $\varphi : S^1 \to T^k$, $T^k = \mathbb{R}^k/\Lambda$, (for fixed time) with action $\int (\dot\varphi^2 - \varphi'^2)dxdt$. If the φ_n are the Fourier coefficients of φ , the eigenstates of the Hamiltonian are given by $\Psi = p(\varphi_n)\Psi_o$, where p is a polynomial in the φ_n and $\Psi_o = \sum_{n \geq o} \exp(-\frac{n}{2}|\varphi_n|^2)$. Let $(T_f\psi)(\varphi) \sim \Psi(\varphi+f)$. Then $V_f^{(\pm)} = \exp(\sum_n \frac{n}{2}\varphi_{\pm n}f_{\pm n})T_f$ yields two commuting families of vertex operators for $f(x) \to \lambda\theta(x-x_o)$, $\lambda \in \Lambda$, $x_o \in S^1$. One finds the Wilson operator product expansion corresponding to the conformal dimension $\lambda^2/2$ for such an operator. The scale of λ is given by Planck's constant. If T^k is a Cartan subgroup of an ADE-type Lie group one obtaines the corresponding Kac-Moody algebra as

$$V_\lambda(x)V_\mu(y) \sim (x-y)^{-1}V_{\lambda+\mu}(x) \qquad \text{if} \qquad \lambda\cdot\mu = -1 \ , \quad \text{etc.}$$

For $\lambda^2 = 1$, the V_λ are Fermion fields, for $\lambda^2 = 4$ on the Leech torus one obtains the Norton algebra by choosing the leading non-trivial operator on the right-hand-side of the operator product expansion

$$V_\lambda(x)V_\mu(y) \sim \sum_k (x-y)^{-\lambda^2/2-\mu^2/2+d_k} O_k(x) \quad .$$

The Z-algebras used to construct higher level representations of Kac-Moody algebras can be interpreted in an analogous way.

H. JAKOBSEN:

A new class ov unitary highest weight representations of $su(n,1)^x$

We present here joint work with V. Kac. Let Δ be an affine root system and $\pi = \{\alpha_o, \alpha_1, \ldots, \alpha_\ell\}$ a standard set of simple roots such that $\mathring{\pi} = \{\alpha_1, \ldots, \alpha_\ell\}$ defines a simple finite dimensional Lie algebra $\mathring{\underline{g}}$ over $\mathbb{C}$. There is a natural definition of a set of positive roots Δ^+ : $(\alpha \in \Delta^+, \beta \in \Delta^+, \alpha+\beta \in \Delta) \Rightarrow$

$\Rightarrow \alpha+\beta \in \Delta^+$, and α or $-\alpha$ but not both belong to Δ^+ , for all $\alpha \in \Delta$. The Borel subalgebra $B(\Delta^+)$ of Δ^+ is then $B(\Delta^+) = \sum_{\alpha \in \Delta^+ \cup \{0\}} \underline{g}^\alpha$ ($\underline{g}$ the affine Lie algebra corresponding to Δ). Furthermore, there are natural definitions of parabolics, of compatible anti-linear anti-involutions ω , and of generalized highest weight modules $M(\lambda)$. We classify the set of $W \times \{\pm 1\}$ - conjugacy classes of Δ^+'s . They are parametrized by the subsets of π . Further, besides the integrable case, $M(\lambda)$ is unitarizable only if: a) (Elementary) $\underline{g} = \underline{g} \otimes \mathbb{C}[t,t^{-1}] \oplus \mathbb{C}c$, $\underline{g}$ corresponds to a hermitian symmetric space, $c \to 0$, and $M(\lambda)$ is constructed from a finite number of unitarizable modules of $\underline{g}$, b) (Exceptional) $\underline{g} = su(n,1)$, $c \to 0$, and $\lambda(a \otimes h) = -\int_0^{2\pi} a(\theta)d\theta$ for $a \in \mathbb{C}[t,t^{-1}]$, $d\theta$ a finite measure on S^1 , and $n = 1$; for $n > 1$ there is a similar construction. Also, this generalizes to the group of maps $X \to su(n,1)$ for X compact Hausdorff and for Radon measures on X .

M. WAKIMOTO:

Hermitian representations of extended affine Lie algebras

The concept of extended affine root systems was introduced by K. Saito in close connection to elliptic singularities, and it naturally posed the problem of existence and construction of corresponding Lie algebras. In this talk I describe a method to realize such Lie algebras and to count the multiplicities of their imaginary roots. I also give a series of Hermitian representations for the Lie algebra $A_1^{(1,1)}$, $A_1^{(1,1)*}$, and $A_2^{(1,1)}$ in analogy with the Jakobsen-Kac discrete series representations of the affine Lie algebra $A_1^{(1)}$.

P. SLODOWY:

An adjoint quotient for Kac-Moody groups and singularities

In this talk we give a survey of the relations between deformations of certain isolated surface singularities (simple, simply elliptic, cusps) and corresponding Kac-Moody groups. This correspondence is established via the second homology lattice of the Milnor fiber of the singularity, which also serves as the coroot lattice of the corresponding group. To realize a semi-universal deformation of the singularity one has to construct a satisfactory adjoint quotient for the corresponding group G . We sketch the construction of a conjugation invariant map $\chi : G \to \hat{T}/W$ of G onto a "partial compactification" $\hat{T}/W$ of the quotient T/W of a maximal Torus T of G by the Weyl group. In an analytic setting (restricting T to a subdomain $\mathcal{T}$ of T) this space had been constructed by Looijenga, and he had also identified it with the base space U of a semiuniversal deformation $X \to U$ of the corresponding singularity. The analysis of the fibers of χ reveals that a big part of the total space X can be embedded into G, in such a way that

$$\begin{array}{ccc} X & \dashrightarrow & G \\ \downarrow & & \downarrow \\ U \cong \hat{\mathcal{T}}/W & \subset & \hat{T}/W \end{array}$$

commutes.

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