MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1985

Kommutative Algebra und algebraische Geometrie

12.5. bis 18.5.1985

Die Tagung stand unter der Leitung von E. Kunz (Regensburg), H.-J. Nastold (Münster) und L. Szpiro (Paris).

Es war das Ziel der Tagung, Probleme und neuere Ergebnisse aus dem Bereich der kommutativen Algebra und algebraischen Geometrie darzustellen. Mit Vorrang sollten Fragen diskutiert werden, die beiden Gebieten gemeinsam entspringen.

Folgende Themen fanden besondere Aufmerksamkeit: Schnittmultiplizitäten über lokalen Ringen, Hilbertfunktion, Vektorbündel projektiver Varietäten, Kurvensingularitäten.

Das Interesse an der Tagung zeigt sich nicht zuletzt in der großen Zahl ausländischer Gäste. U.a. kamen 6 Teilnehmer aus Frankreich, 13 aus Nord- und Südamerika und je 2 aus England und Italien.

Vortragsauszüge

M. BRODMANN

Bounds for cohomology of projective varieties

Let $X = \mathrm{Proj}(A) \overset{2}{\hookrightarrow} \mathbb{P}^d_k = \mathrm{Proj}(S)$ be a projective scheme over an algebraically closed field k, $A = k \oplus A_1 \oplus \cdots$ being a graded homomorphic image of $S := k[z_o,\dots,z_d]$. Let $\mathcal{F}$ be a coherent sheaf over X. Our goal is to give bounds on the <u>cohomological Hilbert functions</u> $n \mapsto h^i(\mathcal{F}(n)) := \dim_k H^i(\mathcal{F}(n)) = \dim_k H^i(X,\mathcal{F}(n)) = h^i(n)$

Let $L \subseteq A_1$ be a k-space of positive dimension. For $f \in L - \{0\}$ we write H_f for the hyperplace section Proj(A/fA) defined by f and consider the <u>linear system</u> $\mathcal{H} = \{H_f | f \in L - \{0\}\}$. dim L - 1 is the dimension dim $\mathcal{H}$ of $\mathcal{H}$. We want to assume that $\mathcal{H}$ is <u>general with respect to</u> $\mathcal{F}$ e.g. that Ass $(\mathcal{F}) \cap H_f = \emptyset$ for all $H_f \in \mathcal{H}$.

Putting $r^i(n) = \max\{\dim \ker[f : H^i(\mathcal{F}(n)) \to H^i(\mathcal{F}(n+1))] | f \in L - \{0\}\}$

$s^i(n) = \max\{\dim \operatorname{coker}[f : H^i(\mathcal{F}(n)) \to H^i(\mathcal{F}(n+1))] | f \in L - \{0\}\}$

and denoting least integral parts by $[\![\cdot]\!]$ we have

<u>Proposition 1.</u> (i) $r^i(n) < h^i(n) \Rightarrow h^i(n) - h^i(n+1) \leq r^i(n) - [\![\frac{\dim H}{r^i(n)+1}]\!]$.

(i) $s^i(n) < h^i(n+1) \Rightarrow h^i(n+1) - h^i(n) \leq s^i(n) - [\![\frac{\dim H}{s^i(n)+1}]\!]$.

Put $\mathcal{B} := \{s : \mathbb{Z} \to \mathbb{Z}_{\geq 0}\}$. For a function $s \in \mathcal{B}$ we define the define the <u>left</u> - resp. the <u>right-vanishing order by</u> $\nu(s) := \inf\{n \in \mathbb{Z} | s(n+1) \neq 0\}$, $\mu(s) := \sup\{n \in \mathbb{Z} | s(n-1) \neq 0\}$. Moreover we put $\mathcal{B}^- = \{s \in \mathcal{B} | \nu(s) > -\infty\}$, $\mathcal{B}^+ = \{s \in \mathcal{B} | \mu(s) < \infty\}$, $\mathcal{B}^0 = \mathcal{B}^- \cap \mathcal{B}^+$ and

$$c^+ = \begin{cases} 0, \text{ if } c = +\infty \\ \max\{0,c\} \text{ otherwise} \end{cases}, \text{ for } c \in \mathbb{R} \cup \{\pm\infty\}.$$

Now, for $N \in \mathbb{Z}_{\geq 0}$ we define two operators $\mathcal{B}^+ \xrightarrow{T_N} \mathcal{B}^+$, $\mathcal{B}^- \xrightarrow{U_N} \mathcal{B}^-$ by $T_n s(n) := [\sum_{m>n} s(m) - (\nu(s)-n)^+ N]^+$, $(s \in \mathcal{B}^+)$. $U_N s(n) = [\sum_{m \leq n} s(m) - (n-\mu(s)+1)^+ N]^+$, $(s \in \mathcal{B}^-)$.

<u>Proposition 2.</u> (a) Let $i > 0$, $s \in \mathcal{B}^+$, $h^{i-1}(\mathcal{F} \restriction H(n)) \leq s(n), \forall n \in \mathbb{Z}, \forall H \in \mathcal{H}$.

Then $h^i(\mathcal{F}(n)) \leq T_{\dim(\mathcal{H})}, s(n)$, $\forall n \in \mathbb{Z}$.

(b) Let $i \geq 0$, $s, h^i \in \mathcal{B}^-$, $h^i(\mathcal{F} \restriction H(n)) = \leq s(n), \forall n \in \mathbb{Z}$, $\forall H \in \mathcal{H}$.

Then $h^i(\mathcal{F}(n)) \leq U_{\dim(\mathcal{H})}, s(n)$.

In the sequel let $\mathcal{E}$ be a locally free sheaf of rank $r > 0$ over $\mathbb{P}^d_k$.

By (2) we may prove

Proposition 3. Let $i,j,p,q \in \mathbb{Z}_{\geq 0}$ such that $1 \leq p \leq q \leq d, 0 \leq j \leq q$, $j \leq i \leq j + d - q$. Let $\mathbb{P}^p = P \subseteq \mathbb{P}^d$ and assume that there is a function $s \in \mathfrak{B}$ such that for each $\mathbb{P}^q = Q \subseteq \mathbb{P}^d$ with $P \subseteq Q$ it holds $h^j(\mathcal{E} \restriction Q(n)) \leq s(n) \, (n \in \mathbb{Z})$. Then it holds

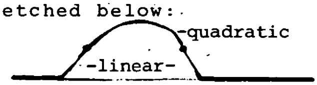
$$h^i(\mathcal{E}(n)) \leq \underbrace{T_{d-p-1} \circ T_{d-p-2} \circ \ldots \circ T_{d-p-i+j}}_{i-j \text{ factors}} \circ \underbrace{U_{d-p-i+j-1} \circ \ldots \circ U_{q-p}}_{d-q-(i-j) \text{ factors}} s(n), \quad (n \in \mathbb{Z})$$

whenever one of the following four hypotheses is satisfied:

a) $p < q$ and $s \in \mathfrak{B}^0$, b) $i = j$ and $s \in \mathfrak{B}^-$, c) $i = j + d - q$ and $s \in \mathfrak{B}^+$, d) $s \equiv 0$.

Now, let $(a_1, \ldots, a_r) \in \mathbb{Z}^r$ be the generic splitting type of $\mathcal{E}$, let $\sigma = a_1 - a_r$ and let $c_1, c_2 \in \mathbb{Z}$ be the first two Chern classes of $\mathcal{E}$. If $cl = 2$ the Riemann-Roch theorem for bundles asserts the existence of a function $s_{c_1,c_2,\sigma}(n) \in \mathfrak{B}^0$ (depending on the parameter) c_1, c_2, σ (and r) such that $h^1(\mathcal{E}(n)) \leq s_{c_1,c_2,\sigma}(n)$ $(n \in \mathbb{Z})$.

Thereby the graph of $s_{c_1,c_2,\sigma}$ has the shape sketched below:



Now, using (3) one gets in the general case

Proposition 4. $0 < i < d \Rightarrow h^i(\mathcal{E}(n)) \leq \underbrace{T_{d-2} \circ \ldots \circ T_{d-i}}_{i-1 \text{ factors}} \circ \underbrace{U_{d-i-1} \circ \ldots \circ U_1}_{d-i-1 \text{ factors}} s_{c_1,c_2,\sigma}(n)$.

This improves a similar type of bounds given (for fixed) n by Elencwajg-Förster in 1980.

From now on, let X be integral of dimension > 1 and put

$t(X) := \min\{ t > 1 \mid H^t_{\mathfrak{m}_{X,x}}(\mathcal{O}_{X,x}) \neq 0;\ \forall x \in X, \text{ closed}\}$

$e(X) := \sum_{\substack{x \in X \\ \text{closed}}} h^1_{\mathfrak{m}_{X,x}}(\mathcal{O}_{X,x}) \ (< \infty)$,

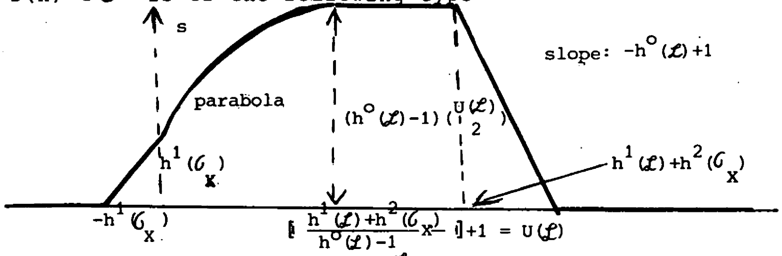
where $H^i_{\mathfrak{m}_{X,x}}$ denotes local cohomology in x and $h^i_{\mathfrak{m}_{X,x}}$ its length. Then

Proposition 5. $n < 0 \Rightarrow e(X) \le h^1(\mathcal{O}_X(n)) \le \max\{e(X), h^1(\mathcal{O}_X)+n(t(X)+1)\}$

Corollary 6. Let X be a complete irreducible variety of dimension > 1 and let $\mathcal{L}$ be a very ample invertible sheaf on X. Then

$$h^1(\mathcal{L}^n) = e(X) \quad \text{for } \forall n \le \left[1 - \frac{h^1(\mathcal{O}_X)-e(X)}{t(X)-1}\right]$$

Corollary 7. Let X be a complete normal surface and let $\mathcal{L}$ be a very ample invertible sheaf over X. Then $h^1(\mathcal{L}^n) \le s(n)$, where $s(n) \in \mathfrak{B}^{\circ}$ is of the following type



Question 8. What holds if $\mathcal{L}$ only is ample?

W. BRUNS

Length formulas for the local cohomology of exterior powers

(joint work with U. Vetter)

We report on joint work with Udo Vetter concerning problems related to the following theorem of Angeniol and Giusti: Let R be a local ring, $f : R^m \to R^n$ an injective map, grade $I = n-m+1$, wher $I = I_m(f)$, and $\lambda(R/I) < \infty$ (λ = length), then C = Coker f^* and R/I have the same length. Let M = Coker f, r = rank M (= n-m). Then a complex $\mathcal{D}_k$ is built by splicing complexes $\mathcal{C}_k$ and $\mathcal{C}_{r-k}^*$ via a duality $\bigwedge^k M \to (\bigwedge^{r-k} M)^*$, $\mathcal{C}_k$ resolving $\bigwedge^k M$ if grade $I \ge k$ (Lebelt). It turns out that the Euler-Poincaré characteristic $\chi(\mathcal{D}_k)$ vanishes for $r \ge d = \dim R$. We of course assume $\lambda(R/I) < \infty$. In case $r = d$ = grade I this can be translated into length formulas of the type $\lambda(H_{m}^{r-k}(\overset{k}{\wedge} M)) = \lambda(H_{m}^{r-k+1}(\overset{k}{\wedge} M))$, or, dually, the corresponding formulas for Ext. Specializing M to the module of differentials of a complete intersection with isolated singularity, we obtain length formulas of Greuel, Naruki. In the case, in which I has its maximal possible grade, that is grade I = n - m + 1, all the modules $\overset{d}{\wedge} M$, R/I, C, $S^2(C),\ldots,S^d(C),H_{m}^{o}(\overset{d}{\wedge} M),\ldots,H_{m}^{d-1}(M)$ $Ext^1(M,R),\ldots,Ext^d(\overset{d}{\wedge} M,R)$ have the same length.

R.-O. BUCHWEITZ

Maximal Cohen-Macanlay modules over Gorenstein rings

Recently much attention has been focused on the search of local rings which have only finitely many isomorphism classes of indecomposable maximal Cohen-Macaulay modules (MCM's for short). [An R-module M is maximal Cohen-Macaulay iff $depth_m M = \dim R$; $m \subseteq R$ the maximal ideal].

This talk adressed the following questions:

- Why should one study MCM's?
- How to decide for a given one whether it contains a free summand and what the number of non-free indecomposable summands is.
- What is the minimal rank of an indecomposable non-free MCM?

Theorem 1. Let R be a local Gorenstein ring. Then the following categories are equivalent:

(i) AFC(R): the homotopy category of acyclic free complexes of R-modules

(ii) MCM(R): the Auslander-Reiten category of MCM's whose objects are all MCM's and whose morphisms are given by $\underline{Hom}_R(M_1,M_2) =$

$\mathrm{Hom}_R(M_1,M_2)/\{\varphi : M_1 \to M_2 / \varphi$ factor over a free module$\}$

(iii) $D^b(R)/D^b_{perf}(R)$: the derived category of R-modules modulo the subcategory of perfect complexes.

Corollary. - As (i) and (iii) are naturally triangulated categories MCM(R) is triangulated.

- Every R-module M admits a presentation

$$0 \to U \to M \to N \to 0$$

with U of finite proj. dimension and M an MCM (U & M are uniquely determined up to isomorphism.

As a consequence of theorem 1 one might define stable Ext-modules as

$$\underline{\mathrm{Ext}}^i_R(X^\cdot, Y^\cdot) := \mathrm{Hom}_{D^b(R)/D^b_{perf(R)}}(X^\cdot, T^i, Y^\cdot),\ i \in \mathbb{Z},\ x,y \in D^b(R).$$

Example: Let $R = P/_f$, P a regular local ring containing the field $k \cong P/_{m_P}$, $V = {}^m/_{m^2}$. Then $\underline{\mathrm{Ext}}^\cdot_R(k,k) = \mathrm{Ext}^\cdot_R(k,k)[\sigma^{-1}]$, where σ is the unique generator in $\mathrm{Ext}^2_R(k,k)$. It follows that

$$\underline{\mathrm{Ext}}^0_R(k,k) = \frac{T^\cdot(({}^m/_{m^2})^*)}{(v^x \otimes v^x - Q_{\bar f}(v^*)\sigma)} \otimes_k \frac{k[\sigma][\sigma^{-1}]}{(v^* \in {}^m/_{m^2}{}^*)},$$

where $Q_{\bar f} : V^* \to k$ is the quadratic form defined by the class of f in $m^2/_{m^3} = S_2V$ (w.l.o.g. $f \in m^2$).

In particular $\underline{\mathrm{Ext}}^0_R(k,k) = \mathrm{Cliff}^+(Q_{\bar f})$, the even Clifford-algebra of the quadratic form $Q_{\bar f}$.

As an application one obtains the following result on ranks of non-free MCM's over a Gorenstein ring of multiplicity 2:

Theorem 2. Let l be the index of $Q_{\bar f}$. Then $2^{l-2} \mid \mathrm{rk}\, M$ for every non-free MCM M.

Remarks. 1. Theorem 1 holds also for non-commutative, local Go-

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renstein rings. For $R = \wedge^{\bullet} V$, the exterior algebra, it is due to Bernstein-Beilinson-Gelfand-Gelfand. In this case <u>MCM</u>(R) is also equivalent to $D^b(\mathbb{P}(V^*))$, the derived category of coherent sheaves on projective space.

2. The methods draw heavily on work of M. Auslander (with Bridger and I. Reiten).

3. In case of a complete local ring over $\mathbb{C}$, Theorem 2 is a consequence of H. Knörrers periodicity theorem for MCM's over hypersurfaces.

4. Theorem 2 can be sharpened by studying the Clifford-algebra more explicitely (whose structure is well known, of course). Theorem 2 is best possible for $k = \bar{k}$.

E.D. DAVIS

<u>Projective embeddings of certain rational surfaces</u>

(with A.V. Geramita)

Fix Z, a reduced 0-dimensional subscheme of $\mathbb{P} = \mathbb{P}^2(k)$, $k = \bar{k}$, and let S be the smooth rational surface obtained by blowing up $\mathbb{P}$ with center Z. Let I(Z) be the homogeneous ideal of Z. For $d \geq \alpha = \min\{t \mid I(z)_t \neq 0\}$, let $S_d = \mathrm{Proj}(k[I(Z)_d])$ and let $\varphi_d : S \to S_d$ be the rational correspondence induced by inclusion of function fields. <u>Well known:</u> If Z is the scheme-theoretic base locus of $I(Z)_d$, then φ_d is a morphism and φ_{d+1} is an isomorphism; I(Z) is generated by forms of degree $\leq \tau + 1$, where τ is the least degree for which requiring passage through Z imposes card(Z) independent linear conditions on the linear system of curves in $\mathbb{P}$ of that degree. So φ_d is an isomorphism for $d > \tau + 1$.

A special case of the considerations in § 4 of our paper with P. Maroscia [Bull.Sc.Math.(2) 108 (1984) 143-185] shows that for any 0-dimensional subscheme X of $\mathbb{P}$: $\dim_k I(X)_d - \dim_k I(X)_{d-1} = d \Rightarrow$ either some line in $\mathbb{P}$ contains a subscheme of X of degree $d+1$, or $I(X)$ is generated by forms of degree $\leq d$. Applying this result to $X = Z + \{p\}$, $p \in S$ - appropriately, interpreted as a 0-dimensional subscheme of $\mathbb{P}$ - we prove:

Theorem. $\pi_{\tau+1}$ is an isomorphism $\Leftrightarrow$ no line contains more then τ points of Z.

And specializing to the case $d = \tau + 1$ gives:

Corollary. Suppose: $\mathrm{card}(Z) = d(d+1)/2$; no curve of degree $d-1$ contains Z; no line contains d points of Z. Then π_d embeds S in $\mathbb{P}^d$ with degree $d(d-1)/2$.

Remark. Putting $d = 3$ in this result gives the well known fact that blowing up a general set of 6 points of $\mathbb{P}$ produces the rational cubic surface in $\mathbb{P}^3$. Putting $d = 4$ generalizes the well known fact that blowing up a general set of 10 points of $\mathbb{P}$ produces a "Bordiga Surface" in $\mathbb{P}^4$.

Question. Does this result give all cases in which π_d is an isomorphism? (Yes, if $\dim_k I(Z)_d \leq 5$). What about this question for "general" Z?

W. DECKER

On the uniqueness of the Horrocks-Mumford-bundle

(joint with F.O. Schreyer)

We prove the

Theorem. Let $\mathfrak{F}$ be a stable rank-2 vector bundle on $\mathbb{P}_4 = \mathbb{P}(V)$

(over $\mathbb{C}$) with Chern-classes $c_1 = -1$, $c_2 = 4$. Assume $H^2(\mathbb{P}_4, \mathcal{F}(-1)) = 0$. Then there exists $T \in \mathbb{P}GL(V)$ with $\mathcal{F} \cong T^*\mathcal{F}_{HM}$.

Here $\mathcal{F}_{HM}$ is the Horrocks-Mumford-bundle. This bundle, discovered in 1973, is still essentially the only known indecomposable rank-2 vectorbundle on $\mathbb{P}_4$.

The proof of the theorem is based on a detailed study of the variety S of unstable planes for $\mathcal{F}$. This can be described as follows. Via monads $\mathcal{F}$ "corresponds" to a 5×2-matrix A with entries in $\Lambda^2 V$ satisfying certain conditions. Then $S = (G \wedge X)$ where G is the Grassmanian $G = G(2,V^*) \subset \mathbb{P}(\Lambda^2 V^*)$ and X the determinantal variety $X = \{\langle a \rangle \in \mathbb{P}(\Lambda^2 V) \mid \text{rank } (a(a^{ij})) \leq 1\}$, $(a^{ij}) = A$. For the Horrocks-Mumford-bundle Barth-Hulek - Moore have shown that S_{HM} is just a copy of Shioda's modular surface for elliptic curves with level 5-structure.

E.G. EVANS

Remarks on Syzygies of finite projective dimension

Let R be a regular local ring containing a field. By mimicing Gröbner's proof of Hilbert's syzygy theorem and using Griffith's construction of Cohen Macaulay modules of the expected projective dimension we showed:

Theorem. Let M be a k-th syzygy, $m \in M - \mathfrak{m}M$ and $I = \{f(m) \mid f \in \operatorname{Hom}(M,R)\}$. Then the height of I is at least k.

Proof. Suppose not. Then one has

$$\dots \quad R^{n_k} \xrightarrow{(a_{ij})} R^{n_{k-1}} \to \dots \to R^{n_0} \to N \to 0 \quad \text{exact.}$$
$$R^{n_k} \searrow M \nearrow R^{n_{k-1}}, \qquad e_1 \longmapsto m$$

Let m be the first generator of M. Thus the entries of the first row of (a_{ij}) are in I. Let the height of I be $1 < k$. Let H be a Cohen Macaulay module over R/I with $pd_R H = 1$. (This is possible by Griffith.) Then $Tor^k(H,N) = 0$ by projective dimension of H, but $Tor^k(H,N) \neq 0$ from the resolution of N since $H \otimes R e_1 \longmapsto o$ in $R^{n_{k-1}}$ and cannot come from $R^{n_{k+1}} \otimes H$ since $\mathfrak{m} H \neq H$.

<u>Remark.</u> In the non regular case if M (and hence N) have finite projective dimension, Griffith gives that $Tor^j(H,I) = 0$ if $pd\ I < \infty$ and $j > 1$.

<u>Corollary 1.</u> Let R be a local ring containing a field and M a k-th syzygy of rank $< k$. Then M is free.

<u>Corollary 2.</u> Let R be a local ring containing a field and M be a k-th syzygy of rank $= k$ and M not free. Then M is an image of the minimal k-th syzygy of $Ext^{n-1}(M^*,R)$.

H. FLENNER

<u>Babylonian tower theorems on the punctured spectrum</u>

We prove the following

<u>Theorem.</u> Let $\dots \to R_{n+1} \to R_n \to \dots \to R_o$ be a tower of local rings, which are regular, i.e. $R_{n-1} = R_n/t_n R_n$ for some $t_R \neq 0$ in R_n. Suppose we are given vector bundles $\mathcal{E}_n$ on the punctured spectrum X_n of R_n such that $\mathcal{E}_n$ extends $\mathcal{E}_{n-1}$, i.e. $\mathcal{E}_n|_{X_{n-1}} \cong \mathcal{E}_{n-1}$. Then $\mathcal{E}_n$ is trivial for all n.

This solves a conjecture of Horrocks. In special cases this result has been shown by Horrocks and Evans-Griffiths. In the projective case the corresponding question was previously known by the work of Barth-Vande Ven, E. Saito, Tyurin. The main idea consists in applying formal deformation theory. A similar result holds for locally complete intersections instead of bundles.

H. FLENNER

The infinitesimal Torelli problem for zero sets of sections of vector bundles

The classical Torelli theorem for curves says that a smooth compact curve C of genus ≥ 1 is uniquely determined by its Jacobian. The Jacobian is given by the position of the integral lattice in $H^1(X,\mathbb{C})$ which has a Hodge decomposition $H^1(\mathcal{O}_C) \oplus H^0(\Omega_C)$. More generally, for a projective manifold X of higher dimension one has the Hodge decomposition of $H^*(X,\mathbb{C})$ and there arises the question whether for a given class $\mathcal{C}$ of manifolds which admits a module space, the map from $\mathcal{C}$ into the space of Hodge structures is injective. In this talk we consider the problem whether it is at least locally injective. By a result of Griffiths this holds if the canonical map

$$H^1(X,\Theta_X) \xrightarrow{\alpha} \bigoplus_{p,q} \mathrm{Hom}_{\mathbb{C}}(H^{pq},H^{p-1,q+1})$$

is an injection where $H^{pq} = H^q(\Omega_X^p)$. We will say then, that the infinitesimal Torelli theorem holds for X. We have shown

<u>Theorem.</u> Suppose there is an exact sequence $0 \to \mathcal{G} \to \mathcal{F} \to \Omega_X^1 \to 0$ and that the following two conditions are satisfied for some p in the range $1 \leq p \leq d$:

(a) The pairing $H^0(S^{d-1}\mathcal{G}^v \otimes \omega_X) \otimes H^0(S^{d-p}\mathcal{G}^v \otimes \omega_X) \to H^0(S^{d-p}\mathcal{G}^v \otimes \omega_X^2)$ is surjective.

(b) $H^{j+1}(S^d \mathcal{G} \otimes \overset{d-1-j}{\wedge} \mathcal{F} \otimes \omega_X^{-1}) = o$ for $0 \leq j \leq d-2$.

Then α above is injective.

As an application we obtain the infinitesimal Torelli theorem for arbitrary smooth complete intersections in $\mathbb{P}^n$ with the only exception of surfaces of degree 3 in $\mathbb{P}^3$ and intersection of two quadrics of dimension ≥ 2. For the case $\omega_X \geq o$ resp. hypersurfaces this has been shown by Peters and Usni resp. Griffiths. Moreover we get that for a sufficiently ample bundle $\mathcal{E}$ on a projective manifold and for a section $s \in H^o(\mathcal{E})$ the infinitesimal Torelli theorem holds for $X := \{s = o\}$. This generalizes a result of M. Green.

H.-B. FOXBY

Algebras of finite flat dimension

Let $\varphi : A \to B$ be a morphism of local rings, and assume that B has propoerty $\mathcal{P}$. Does A have property $\mathcal{P}$? ($\mathcal{P}$ = regular, Gorenstein, CM = Cohen-Macaulay,...). if B is flat as an A-module the answer is yes for many $\mathcal{P}$. If $B = A/\mathfrak{a}$ where $\mathfrak{a}$ is an ideal of finite projective, then the answer is known to be yes for some $\mathcal{P}$. In general, let fd denote flat (Tor) dimension.

Theorem. (i) if $fd_A B < \infty$ and B is Gorenstein, then A is Gorenstein.

(ii) if M is a f.g. B-module with $fd_A M < \infty$, and if there exists an A-module C with $depth_A C = \dim A$ (and this is the case if A contains a field; Hochster), then $\dim_A A - depth_A A \leq \dim_B M - depth\ M$.

(iii) if M is a f.g. Cohen-Macaulay B-module of dimension n and

$fd_A M < \infty$ and if A is Cohen-Macaulay of dimension d, then $\mu^d_A(A)$ divides $\mu^n_B(M)$ (Bass numbers).

(iv) if $fd_A B < \infty$, then

$\text{emdim } A - \text{depth}_A A \leq \text{emdim } B - \text{depth}_B B$ where emdim is the embedding dimension.

The number $\dim A - \text{depth}_A A$ is the Cohen-Macaulay defect, while $\text{emdim } A - \text{depth}_A A$ is a regularity defect.

This is joint work with L. Avramov [Københavns Universitetets Matematicke Institut, Preprint Series No. 2, 1985]. There are a few results in the other direction: A has $\mathcal{P}$ $\Rightarrow$ B has $\mathcal{P}$; L. Avramov and S. Halperin [ibid].

W. FULTON

<u>Characteristic classes of direct image bundles</u>

(joint with R. MacPherson)

For a covering $f : X \to Y$ (finite, unramified) of algebraic varieties or topological spaces, and a vector bundle E on X, we give a formula for Chern classes of $f_* E$ in term of Chern classes of E and the geometry of f. Special cases were known in topology, particularly, when f is the covering $B_G \to B_{G'}$, corresponding to a subgroup G of finite index in G'.

If $s_1 = c_1$, $s_2 = c_1^2 - 2c_2, \ldots, s_n$ is the k^{th} Newton polynomial, then

$$L_k(s_n(f_x E) - f_y s_n E) = 0,$$

where L_k is the product of all primes that divide $L/k! \prod_p p^{[\frac{k}{p-1}]}$.

If M_k is defined by

$$M_k = \prod_{(p-1)/k} p^{1+\mathrm{ord}_p k}$$

and E is a bundle on X such that $M_k c_n f_* E = 0$ for all k, then $M_k c_n f_* E = 0$ for all k. (In characteristic p, the prime p can be omitted from L_k and M_k).

The general formula involves multiplicative transfers $\wedge_f : A^*X \to A^*Y$, which can be described on a general cycle z on X by (locally) intersecting the n image of z, n = deg(f). In the topological setting, the formula appears in CR. Acad. Sci. (1984).

A. V. GERAMITA

<u>Hilbert funktions of 0-dimensional subschemes of $\mathbb{P}^2$ and the geometry of rational surfaces</u>

(joint work with E. D. Davis and also on recent work of B. Harbourne)

Let $\bar{X} = \{P_1,\ldots,P_s\} \subseteq \mathbb{P}^2$ be s distinct points and let $P_i \leftrightarrow \mathfrak{g}_i \subseteq R = k[x_0,x_1,x_2]$. If $\alpha_1,\ldots,\alpha_s$ are non-negative integers, $I = \mathfrak{g}_1^{\alpha_1} \cap \ldots \cap \mathfrak{g}_s^{\alpha_s}$ and $A = R/I = \oplus A_i$, the problem is to find the "expected" Hilbert function, $H(A,t) = \dim_k A_t$, of A. (Well known if all the $\alpha_i = 1$).

Since for $t \gg 0$, $H(A,t) = e(A)$ and $e(A) = \Sigma \frac{\alpha_i(\alpha_i+1)}{2}$ we know the eventual value of the Hilbert function. If we let $\tau(A)$ denote the least integer t for which $H(A,t) = e(A)$, then

<u>Proposition 1.</u> $\tau(A) \le (\Sigma \alpha_i) - 1$; equality $\Leftrightarrow \{P_1,\ldots,P_s\}$ lie on a line (This is also true in $\mathbb{P}^n$).

<u>Proposition 2.</u> [D-G-Queen's Papers in Pure and Applied Math., Vol. 67 - "The Curves seminars at Queen's - Vol. III"]. It is

possible to completely describe the Hilbert function when $P_1,\dots,P_s$ lie on a line, $\alpha_1,\dots,\alpha_s$ are arbitrary. The Hilbert function is independent of the position of $P_1,\dots,P_s$ on the line.

Proposition 3. (B. Harbourne). There is an algorithm for finding the Hilbert function of A as above when $P_1,\dots,P_n$ lie on an irreducible cone. Again, the Hilbert function is independent of the position of the points chosen on the cone.

The approach to 1) and 2) by [DG] is through an analysis of the ring structure on A, while the approach by Harbourne consists of identifying $\dim_k I_d$ with $\mathcal{L}^o(X,\mathcal{L}(F_d))$ where X is the rational surface obtained by blowing up $\mathbb{P}^2$ at $\{P_1,\dots,P_s\}$ and F_d is the divisor $dE_o - \sum_{i=1} \alpha_i E$ ($E_i (i > o)$ the preimages of P_i under the blow-up and E_o the proper transform of a general line. The work of Harbourne will appear in Proc. of Vancouver Conference in Alg. Geo. (1984).

G.-M. GREUEL

Simple singularities and maximal Cohen-Macaulay modules

(Report on joint work with R.-O. Buchweitz, F.-O. Schreyer)

After Arnold, a convergent power series $f \in \mathbb{C}\{x_o,\dots,x_n\}$ is called simple if for any $F \in \mathbb{C}\{x_o,\dots,x_n,t_1,\dots,t_p\}$ such that $F_o = f$ the set of isomorphism classes of $\mathbb{C}\{x_o,\dots,x_n\}/f_t$ is finite for $t \in \mathbb{C}^p$, sufficiently near to O, (here $f_t(x) = F(x,t)$). Arnold showed that the simple singularities are exactly the so-called A-D-E singularities (up to isomorphism) in the case of isolated singularities. For arbitrary f, allowing higher dimensional singularities, his classification shows that there are two more singularities, namely A_∞ : $f(x_o,\dots,x_n) = x_1^2 +\dots+ x_n^2$, D_∞ : $f(x_o,\dots,x_n) = x_o x_1^2 + x_2^2 +\dots+ x_n^2$. Recently it was discovered that these singularities have - besides many other characterizations - characterisations through the set $M(\mathbb{C}\{x\}/f)$ = {isomorph classes of indecomposable finitely generated max. Cohen Macaulay modules over $\mathbb{C}\{x\}/f\}$. In dimension 2 (i.e. n = 2) this follows from work of Herzog, Artin, Verdier and in dimension 1 it was proved by Greuel and Knörrer. Knörrer moreover showed among other things that in any dimension $M(\mathbb{C}\{x\}/f)$ is a finite set for f simple, with an isolated singularity. The aim of the talk was to give a hint to the proof of the converse. Together with Knörrers result we get.

<u>Theorem.</u> $R = \mathbb{C}\{x_o,\dots,x_n\}/(f)$, $f \in m^2$, $n = \dim R \geq 1$.

a) M(R) finite ⇔ R (resp. f) has an isolated A-D-E singularity.

b) M(R) countably infinite ⇔ f of type A_∞ or D_∞.

c) The following are equivalent:

i) f simple

ii) f of type A-D-E (including A_∞, D_∞)

iii) each maximal Cohen Macaulay module M is simple (i.e.) for any deform of M over a finite dimensional base, there are only finitely many isomorphy classes of maximal Cohen-Macaulay modules)

iv) M(R) is finite or countably infinitely.

(M(R) = set of isomorphy classes of indecomposable infinitely generated maximal Cohen-Macaulay modules over R)

R. HARTSHORNE

Possible spectra of stable rank 2 vector bundles on $\mathbb{P}^3$

(joint work with A.P. Rao)

Let $\mathcal{E}$ be a stable rank 2 vector bundle on $\mathbb{P}^3$ with $c_1 = o$ and c_2 given. We recall the spectrum $X = \{k_1 \leq \ldots \leq k_{c_2}\}$, $k_i \in \mathbb{Z}$ of $\mathcal{E}$, first introduced by Barth and Elencwajg. It has the following properties)where $\mathcal{K}$ denotes the sheaf $\oplus \mathcal{O}_{\mathbb{P}^1}(k_i)$ on $\mathbb{P}^1$)

1. For each $l \leq -1$, $h^1(\mathcal{E}(l)) = h^o(\mathbb{P}^1, \mathcal{K}(l+1))$
2. For each $l \geq -3$, $h^2(\mathcal{E}(l)) = h'(\mathbb{P}^1, \mathcal{K}(l+1))$
3. X is symmetric about o, i.e. $\{-k_i\} = \{k_i\}$
4. It has no gaps, i.e. every integer between $\min\{k_i\}$ and $\max\{k_i\}$ occurs (at least once) in the spectrum
5. if $1 \leq k_o < k = \max\{k_i\}$ is such that k_o occurs only once in X, then every $k_o \leq k_i \leq k$ occur only once.

Problem 1. Which sequences of integers $k_1 \leq \ldots \leq k_{c_2}$ satisfying conditions 3. - 5. above can actually occur as the spectrum of a stable rank 2 bundle with the given c_2? (We expect the above conditions will not be sufficient.)

This problem was studied by Barth in his paper "some experimental data..." where he listed possible invariants of bundles, but did not always settle the question of existence. Furthermore, if we let $M_{\mathcal{E}} = \bigoplus_{l \in \mathbb{Z}} H^1(\mathcal{E}(l))$, considered as a finite length, graded S-module, $S = k[x_o, x_1, x_2, x_3]$, then Barth's tables were constructed under the "hypothesis" that M should always be generated by elements of degree ≤ -1, which was true in all the cases he knew.

The origin of the present work was to decide whether Barth's hypothesis was always true, and to complete his work by considering also the question of existence. The result is a complete classification of possible spectra and possible degrees of generators for all bundles with $c_2 \leq 8$. We give here the table only for $c_2 = 7$, which is the first case where there is a case in which M needs a generator in degree ≥ 0:

stable rank 2 bundles on $\mathbb{P}^3$ with $c_1 = 0,\ c_2 = 7.$

Spectrum	Generators for M	Existence
0^7	-1^1	$\mathcal{E}(1) \leftrightarrow$ 8 skew lines
$-1,0^5,1$	$-2,-1^3$	$\mathcal{E}(2) \leftrightarrow E_4 \cup E_8$ (E_d elliptic curve deg d)
	$-2,-1^4$	$\mathcal{E}(2) \leftrightarrow E_3 \cup E_8$
$-1^2,0^3,1^2$	-2^2	$\mathcal{E}(2) \leftrightarrow E_3 \cup E_4 \cup E_7$
	$-2^2,-1$	$\mathcal{E}(2) \leftrightarrow E_3 \cup E_3 \cup E_5$
	$-2^2,-1^2$	(more complicated)
$-1^3,0,1^3$	$-2^3,0$	(*)
$-2,-1,0^3,1,2$	$-3,-1^2$	( )
$-2,-1^2,0,1^2,2$	-3	$\Upsilon(2,2,4,4)$
	$-3,-2$	( )
$-3,-2,-1,0,1,2,3$	-4	$\Upsilon(1,4,4,7)$

Constructions. There are four kinds of constructions

I. "Serre" to construct $\mathcal{E}$ from a curve $Y \subseteq \mathbb{P}^3$ with $\omega_Y \cong \mathcal{O}_Y(l)$ for some l. (First 5 curves of table)

II. Apply I. to a disjoint union of curves obtained as sections of vector bundles previously constructed. Cases ( ) in table

III. Bundle associated to a principal module $M = S/(f_1,f_2,f_3,f_4)$ where $e_i = \deg f_i$ and $e_1 \leq e_2 \leq e_3 \leq e_4$ and $e_1 + e_4 = e_2 + e_3$. This is explained in Rao's paper J. Algebra 86 (1984) 23-34. Denoted Υ in table.

IV. "Ferrand". This is used to construct the new bundle (*). Let X be a rational quartic curve in $\mathbb{P}^3$. Take a surjective map $u : \mathcal{I}_X \to \omega_X(2) \to 0$. Then $\ker u = \mathcal{I}_Y$ defines a multiplicity 2 structure Y on X, with $\omega_Y \cong \mathcal{I}_Y(-2)$. Then apply I to get (*). It is the fact that $H^1(\mathcal{I}_X(1)) = k$ which gives the generator of M_ε in degree 0.

M. HOCHSTER

Intersection theory and multiplicities: easy answers and hard questions

The first part of the talk gave a very short proof of a lemma in a recent paper of Angéniol and Giusti which asserts the following: If $(R,\mathfrak{m})$ is a Cohen-Macaulay local ring of dimension $r - s + 1$, $X = (x_{ij})$ is a matrix of size $r \times s$ over R, $r \geq s$, such that the ideal $I_s(X)$ of $s \times s$ minors of X is $\mathfrak{m}$-primary, and M = Coker X, then $l(M) = l(R/I_s(X))$, where l denotes length. The idea is to view $I_s(X)$, M as arising by applying $R \otimes$ to generic set-up over a regular ring T. Then $l(R/I_s(X)) - l(M)$ can be reinterpreted as an intersection multiplicity over T of R with a module N of dimension dim T - dim R - 1, so that by Serre'results, the multiplicity must vanish. This proof is intended as a demonstration of the power of multiplicity theory.

The second part of the talk dealt with a recent result of W. Smoke which asserts the following: Let R be a Noetherian ring, M an R-module with m generators. Suppose $x_1,\dots,x_r$ is an R-sequence, $r \geq 2$, and $(x_1,\dots,x_r)\, M = 0$. Then there is an exakt sequence

$$0 \to M \to M_1 \to R/_{(x_1,\dots,x_r)} \to 0$$

where M_1 has at most $m - (r-1)$ generators if $m \geq r$ and M_1 is cyclic if $2 \leq m-1 \leq r$.

A corollary is that for any Cohen-Macaulay ring R, the Grothendieck group $\mathcal{K}(R)$ of modules of finite length and finite projective dimension is generated by the classes [R/J] of cyclic R-modules of finite projective dimension. In particular, if M is a module with five generators over $R = K[[x,y,u,v]]/(xu-yv)$ such that pd M = 3, $\chi(M,R/(x,y)) = -1$ (and $l(M) = 15$) and x_1,x_2,x_3 is any system of parameters for R contained in $\mathrm{Ann}_R M$, one may extend M first by $R/(x_1,x_2,x_3)$ and then by $R/(x_1^2,x_2^2,x_3^2)$ to obtain a cyclic module M_1 of finite projective dimension such that $\chi(M_1,R/(x,y)) = -1$. (see Dutta-Hochster-McLaughlin, Inventiones Math., 1985 for the construction of M). The third part of the talk dealt with the question, for which local rings $(R,\mathfrak{m})$ is it true that whenever I,J are primary to $\mathfrak{m}$, then $\mathrm{Tor}_1(R/I,R/J)$, i.e. $I \cap J/IJ$, is not zero? This is easily seen to be true if R is regular (but using rigidity) and less easily if R is a hypersurface (W. Brown, S. Dutta). The rest apparently is not known even for R = T/f,g) where T is regular and f,g is a regular sequence. It was pointed out that the result is false for one dimensional Gorenstein domains (e.g. C. Hunecke observes one may take $R = k[t^9,t^{11},t^{13},t^{15},t^{17},t^{19},t^{21},t^{25}]$, $I = (t^{13},t^{15},t^{19})$, $J = (t^9,t^{11},t^{17},t^{21},t^{23})$: $I \cap J = IJ$, $I + J = \mathfrak{m}$, $I^2 \subset IJ$.

The question was raised, if R is n-dimensional regular, must $\mathrm{Tor}_1(R/I,R/J)$ have at least n generators? This would imply the result for $R = T/(f_1,\dots,f_m)$ if T is regular, dim T = n, m < n, whether $f_1,\dots,f_m$ is a regular sequence or not.

G. HORROCKS

Unstable invariants of bundles on the punctured spectrum of a local ring

Define algebraic equivalence for X-bundles, X the punctured spectrum of a regular local ring, to be the equivalence relation generated by confluence. A confluence is obtained by embedding X as a hyperplane section of a punctured spectrum Y and taking on Y a sheaf locally free except on a 1-dimensional subscheme (as a subscheme of the closed spectrum). The restriction of this sheaf to X is said to be a confluence of its localisations at the components of the exceptional subscheme. There are no stable (i.e. presented by taking direct sum with a trivial bundle) invariants for this equivalence. The simplest unstable invariant occur for bundles whose rank equals d-1, d being the dimension of the defining local ring for A. In the case when $\mathbb{C}$ is the residue-class field this is the characteristic map $\eta \in \mathcal{H}_{2d-2}(U(d-1)) \cong \mathbb{Z}/(d-1)!$ In the general case this can be defined in terms of Chern class on $\mathbb{C}P^{d-1}$ for bundles which lift from projective space. It is the formal Euler characteristic calculated on $\mathbb{C}P^{d}$ from the Riemann-Roch Theorem and then multiplied by (d-1)! (the formal characteristic exists only in $\mathbb{Q}$). For arbitrary X-bundles this can be defined by extending over the blown up closed point using the fact that we are assuming rank is d - 1. The resulting group under algebraic equivalence is cyclic. Provided it can be shown that the cokernel of the matrix $(x_1, x_2, \ldots, x_d^{d-1})$ is a rank d - 1 bundle algebraically equivalent to the trivial bundle the order is (d-1)! The invariant $\eta/2$ has an alternative discription when d is odd.

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viz as the length mod 2 of the cohomology group $H^{(d-1)/2}(X, \wedge^{(d-1)/2} E)$.

C. HUNEKE

Algebras with small divisor class group

(joint work with B. Ulrich)

Say a normal local Cohen-Macaulay ring (R,m) has small divisor class group if cl(R) = $\mathbb{R}[\omega_R]$ where ω_R = canonical module of R.

Then we first recall a theorem:

Theorem 1. Let $S = k[X_o,\dots,X_n]$ $I \subseteq S$ homogeneous, R = S/I C-M and normal with minimal homog. resolution,

$$0 \to \bigoplus_{i=1}^{n_i} S(-n_{qi}) \dots \to \bigoplus_{i=1}^{n_i} S(-n,i) \to S \to R \to 0$$

If $\min\{n_{qi}\} > (q-1)\max\{n,i\}$ and if $\hat{R}$ is rigid, then $\hat{R}$ (and hence R) has small divisor class group.

We apply this to the case if R is the linkage class of a complete intersection.

Theorem 2. Let S,R be as in Theorem 1, and further assume R is in the linkage class of a complete intersection. Then $\max\{n_{qi}\} > (q-1)\min\{n,i\}$.

Corollary. If R and ω_R are generated by forms of the same degree in the situation of Theorem 2 then R has small divisor class group.

B. ULRICH

The singular locus of algebras in the linkage class of a complete intersection

(joint work with C. Huneke)

Theorem. Let $S = k[X_1,\dots,X_n]$, $I \subseteq S$, R = S/I. Assume that I is in the linkage class of a complete intersection, but not a complete intersection.

a) If R is Gorenstein, then R is smoothable in codimension 6, but not in codimension 7.

b) If R is not Gorenstein, then R is smoothable in codimension 3, but not in codimension 4.

For the proof we introduce the notion of "universal linkage" to reduce the problem to ideals doubly linked to regular ideals.

F. ISCHEBECK

The behavior of Pic, K_o, etc. under subintegral ring extensions

Let $A \subset B$ be a subintegral extensions of reduced rings, then the following propositions hold.

1. $FA \to FB$ is surjective, where F is any one of the functors K_o, Pic, NK_o, NPic (etc.)
2. The map $\{C \text{ ring} / A \subseteq C \subseteq B\} \to \{\text{subgroups of } FA\}$ $C \mapsto \text{Ker}(FA \to FC)$ is injective for $F = NK_o$ and $F = N\,\text{Pic}$. If further $UA = UB$ the same holds for $F = K_o$ and $F = \text{Pic}$.
3. If B/A (quotient of additive groups) is an S-torsion group ($S \subset \mathbb{Z}$ multiplicative), so is $\text{Ker}(FA \to FB)$ for $F = NK_o$ and $F = N\,\text{Pic}$.
4. If B/A has no S-torsion, so has not $\text{Ker}(N\,\text{Pic}\,A \to N\,\text{Pic}\,B)$.

K.H. KIYEK

Simple curve singularities in arbitrary characteristic

(Work done with G. Steinke)

Let $k = \overline{k}$ be of arbitrary characteristic. $F \in k[[X,Y]]$ defines a

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simple curve singularity if F has no multiple factors, $2 \leq O(F) \leq 3$ and for the reduced total quadratic transform F of F also $2 \leq O(F') \leq 3$.

Theorem 1. F has one of the following forms:

A_{2m-1} $Y^2 + YX^m, m \geq 1$

A_{2m} $Y^2 + X^{2m+1} + G$, where $G = 0$ if $Ch(k) \neq 2$, $G = 0$ or
$G = XY^{m+i}$, $1 \leq i \leq m-1$, if $Ch(k) = 2$

D_{2m} $XY^2 + YX^m$ $m \geq 2$,

D_{2m+1} $X(Y^2+X^{2m-1}) + G$, where $G = 0$ if $Ch(k) \neq 2$, $G = 0$ or
$G = XY^{m+i}$, $1 \leq i \leq m-1$, if $Ch(k) = 2$

E_6 $Y^3 + X^4$ in every characteristic, in addition
$Y^3 + X^4 + X^2 \quad Y$ $Ch(k) = 2$,
$Y^3 + X^4 + X^2Y^2$ $Ch(k) = 3$,

E_7 $Y(Y^2+X^3)$, in addition
$Y(Y^2+X^3) + X^2Y^2$ $Ch(k) = 3$

E_8 $Y^3 + X^5$;
in $Ch(k) = 3$ also
$Y^3 + X^5 + X^2Y^2$, $Y^3 + X^5 + X^3Y^2$
in $Ch(k) = 5$ also
$Y^3 + X^5 + X^nY$.

In $Ch(k) = 0$ Greuel und Knörrer showed, that simple curve singularities are characterized by the fact that the isomorphism classes of indecomposable torsion free A-moduls, $A = k\,[X,Y]]/(F)$ reduced curve singularity are finite. This is true in the general case also.

Furthermore the Auslander-Reiten quiver of simple curve singularities can be calculated following Dieterich and Wiedemann.

If $Ch(k)\ 2 \neq 2$, the surface singularity in a double cover of the plane of a simple curve singularity is a rational double point

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of type $A_n, \ldots, E_8$.

H. LINDEL

Unimodular elements in projective modules over Laurent polynomial extensions

We show the following theorem answering a question of Bass and Murthy. (Joint work with S.M. Bhatwadekar and R.A. Rao):

Let P be projective module over a Laurent extension $R = A[X_1, \ldots, X_n, Y_1^{\pm 1}, \ldots, Y_m^{\pm 1}]$, where A is noetherian of finite Krull dimension d and rank (P) $\geq$ d + 1. Then P has a unimodular element.

As a main tool serves a Critenon for the existence of unimodular elements in modules over positively graded rings:

Let M be a f.g. module over a positively graded ring $R = \bigoplus_{i \geq o} R_i$. Assume there exists a $q \in M$ that is unimodular in M_{1+J} and M_{1+R^+}, I the Quillen ideal of M in R_o. Then M contains a unimodular element.

The case d = 1, rank P is treated separately by means of this theorem. In case when rank P $\geq$ max(3,d+1) the proof uses deeply split automorphism for patching of unimodular elements over localisations and, moreover, a result of Suslin.

W. LÜTKEBOHMERT

Uniformization of abelian varieties

Let K be a field with a non-trivial, non-Archimedean valuation assumed to be complete and algebraically closed. Let C/K be a smooth curve, projective, genus of $C = g$; A/K be an abelian variety of dimension g.

Stable Reduction Theorem. There exists a formal analytic structure $\pi : C \to \tilde{C}$, such that $\tilde{C}$ has at most ordinary double points as singularities.

Semi-Abelian Reduction Theorem. There exists an open analytic subgroup $\bar{A} \subset A$ with the following properties

(a) $\bar{A}$ is a connected formal analytic group with semi-abelian reduction.

(b) $\bar{A}$ is an extension of a proper formal analytic group B having abelian reduction, by an affinoid torus $\bar{T}$.

(c) If X is a formal analytic, $\overset{\circ}{K}$-smooth variety and if $\varphi : X \to A$ is a rigid morphism, such that $\varphi(X) \cap \bar{A} \neq \emptyset$, then $\varphi(X) \subset \bar{A}$, if X is connected ($\overset{\circ}{K}$ = valuation ring of K).

Uniformization Theorem. Let T be the affine torus extending $\bar{T}$ and containing $\bar{T}$ as subgroup of units. Then

(a) $\hat{A} := T \times \bar{A}/\{(s^{-1},s); s \in \bar{T}\}$ exists as an analytic quotient; $\hat{A}$ is an extension of $B = \bar{A}/\bar{T}$ by the affine torus T.

(b) The open immersion $\bar{A} \hookrightarrow A$ extends uniquely to a surjective covering map $p : \hat{A} \to A$.

(c) $\Gamma = \ker p$ is a discrete subgroup in $\hat{A}$, it is free of rank equal to the dimension of the torus part $\bar{T}$ of $\bar{A}$.

(d) $H^1_{rig}(\hat{A},\mathbb{Z}) = 0$, $H^1_{rig}(\bar{A},\mathbb{Z}) = 0$, $H^1_{rig}(A,\mathbb{Z}) = \mathbb{Z}^{\operatorname{rank} T}$.

The uniformization of abelian varieties was first attacked by Raynaud (Nice 1970) where the valuation of the ground field is discrete. This is a joint work with S. Bosch (Münster) (cf. Math. Ann. 270 (1985) and Inventiones math. 78 (1984). M. van der Put gave also a proof of the Stable Red. Theorem.

L. MORET-BAILLY

A purity theorem for families of curves

Theorem. Let S be a regular locally Noetherian scheme, $U \subseteq S$ an open subset such that $\mathrm{codim}_S(S-U) \geq 2$. Let $X_U \xrightarrow{f_U} U$ be a U-curve (i.e. a proper smooth morphism whose fibres are geometrically connected curves of genus ≥ 1). Then there exists a unique (up to unique isomorphism) S-curve $X \xrightarrow{f} S$ extending X_U.

The genus 1 case is treated separately. For genus ≥ 2 one first reduces to the case where $S = \mathrm{spec}\, R$, R local complete regular of dimension 2 with algebraically closed residue field. Then there exists a finite sequence of blowing ups of points $\tilde{S} \to S$, and an $\tilde{S}$-curve $\tilde{X}$ extending X_U, and one finally shows that $\tilde{X}$ is "constant" on the exceptional divisor, using in particular the fact that a generically smooth stable curve of genus ≥ 2 over $\mathbb{P}^1$ with ≤ 2 singular fibres is constant. Cf. C.R. Acad. Sci. Paris 300, n° 14 (1985), pp. 489-492.

C. PESKINE

Splitting of the normal bundle of a smooth projective curve

Theorem. (Ellingsrud, Gruson, -,). Let S be a smooth surface in $\mathbb{P}^n_{\mathbb{C}}$, and C a curve on S.

(i) If the normal bundle of C in S is a direct summand of the normal bundle of C in $\mathbb{P}^n$, then some multiple of the divisor C on S is numerically equivalent to a hypersurface section of S.

(ii) There is a constant d_o (depending only of the embedding of S in P^n) such that the converse of (i) holds when $d^o C \geq d_o$.

<u>Corollary.</u> For a smooth connected surface S in $\mathbb{P}^n_{\mathbb{C}}$, the following conditions are equivalent:

(i) Pic S = Num S and a hyperplane section is not divisible in Pic S.

(ii) Any curve C on S such that the normal bundle of C is a direct summand of the normal bundle of C in $\mathbb{P}^n$ is linearly equivalent to a hypersurface section of S.

<u>Remain.</u> Complete intersection surfaces and their projections (isomorphic) as well as rational surfaces containing a line verify the conditions of the corollary.

<u>Key Lemma.</u> Let S_2 denote the dimension 2 scheme defined by the square of the ideal of S. Then (i) the image of Pic S_2 in Num S is $\mathbb{Z}$. (ii) the cokernels of the maps Pic $S_2 \to$ Pic S and Pic $S_2 \to$ Num S are isomorphic and torsion free.

<u>Recall.</u> Theorem (Griffths, Harris, Hulek). If S is a complete intersection with non-negative canonical line bundle then a curve C on S is a section of S by a hypersurface if and only if any first order deformation of S contains a first order deformation of C.

<u>Theorem</u> (Flexor). Let A be a "reasonable" local Gorenstein

ring of dim 3 with an isolated singularity (containing $\mathbb{C}$). If an effective divisor D can be lifted to any 1st order deformation of A, then O is set theoretically defined by one equation.

P. ROBERTS

Intersection multiplicities over local rings

Let M and N be finitely generated modules over a local ring A, and suppose that $M \otimes_A N$ is an A-module of finite length. Serre defined the intersection multiplicity $\chi(M,N) = \sum_{i \geq 0}$ length $(\mathrm{Tor}_i(M,N))$ if A is a regular local ring, and if A is also equicharacteristic, he showed that

1. If dim M + dim N < dim A, then $\chi(M,N) = 0$
2. If dim M + dim N = dim A, then $\chi(M,N) > 0$

He conjectured that this is true for any regular local ring. Since then, it has been asked, whether these two statements hold for an arbitrary local ring, if M has finite projective dimension. One attempt to prove this was to constructing a sequence of invariants of F. where F. is a finite free resolution of M, say $ch_i(F.)$, and a sequence of invariants of a module N, say $\tau_i(N)$, so that $\tau_i(N) = 0$ for i > dim N and $\Sigma\, ch$ F.)$\tau_i(N) = \chi(F. \otimes N) = \chi(M,N)$. If it is also true that $ch_i(F) = 0$ for i < codim M, the vanishing statement (1.) would hold. The local Chern characters of Baum-Fulton-MacPherson do satisfy the first of these conditions, but Dutta, Hochster and MacPherson constructed an example where M has codimension 3 but $ch_2(F.) \neq 0$. Nevertheless, the last condition holds if A is regular, proving the first part of the conjecture of

Serre and somewhat longer arguments, using the multiplicativity and commutativity of the local Chern characters, show that this holds also if A is a complete intersection of if the singular torus has dimension ≤ 1. These two properties can be proven by blowing up to reduce to the case where the complex F. is replaced by one with a filtration of complexes of the sort $\mathcal{O}_Y(-D) \hookrightarrow \mathcal{O}_Y$, where D is a Cartier divisor on a not necessarily affine scheme Y; in this case they can be shown by fairly simple computations. with divisors.

J. SALLY

Local sandwiches

Let $(R, \mathfrak{m})$ be a 2-dimensional regular local ring and let $(S, \mathfrak{n})$ be a 2-dimensional normal local ring which birationally dominates R. It follows from Zariski's Main Theorem and a result of Lipman that

(*) S is a spot over R i.e., S is the localization at a prime ideal of a finitely generated ring over R.

[(*) is also a Corollary of a result of Heinzer, Huneke, Sally which gives criteria in a more general situation for one local ring to be a spot over another.]

Since S is a spot over R, Lipman has proved that S is a rational singularity. The aim here is to try to understand these rational singularities by applying algebraic techniques. Here are two such results proved by using facts about the analytic spread of the ideal $\mathfrak{m}S$.

(1) If S is a UFD, then S is regular.

(2) Assume $R/\mathfrak{m}$ is infinite. Then $\exists$ a finite quadratic transform of R which dominates S.

R.Y. SHARP

Lengths of certain generalized fractions

Let $(A,\mathfrak{m})$ be a local ring of dimension $d > 0$. Let $x_1,\ldots,x_d$ be a fixed system of parameters (s.o.p.) for A. Set

$$U_{d+1} = \{(u_1,\ldots,u_d,1) : u_1,\ldots,u_d \text{ form a s.o.p. for } A\},$$

a triangular subset of A^{d+1}. The module of generalized fractions $U_{d+1}^{-d-1}A$ is the union of its cyclic submodules $A(1/(x_1^{n_1},\ldots,x_d^{n_d},1))$ (where $n_1,\ldots,n_d \in \mathbb{N}$), all of which have finite length; also $U_{d+1}^{-d-1}A \cong H_{\mathfrak{m}}^d(A)$. Furthermore, $H_{\mathfrak{m}}^d(A)$ may be viewed as a direct limit of the modules $A/(\Sigma_{i=1}^d Ax_i^{n_i})$. Lech's limit formula for the multiplicity $e(x_1,\ldots,x_d)$ states that

$$e(x_1,\ldots,x_d) = \lim_{\{\min n_1,\ldots,n_d\}\to\infty} \frac{l(A/\Sigma_{i=1}^d Ax_i^{n_i})}{n_1\ldots n_d}.$$

This talk was concerned with the following two questions.

Question 1. Does there exist $g \in \mathbb{Q}[X_1,\ldots,X_d]$, of total degree d, having homogeneous component of degree d equal to $e(x_1,\ldots,x_d)X_1\ldots X_d$, such that, for all sufficiently large $n_1,\ldots,n_d$, $l(A/\Sigma_{i=1}^d Ax_i^{n_i}) = g(n_1,\ldots,n_d)$?

Question 2. Does there exist $h \in \mathbb{Q}[X_1,\ldots,X_d]$ such that, for all sufficiently large $n_1,\ldots,n_d$, $l(A(x_1^{n_1},\ldots,x_d^{n_d},1))) = h(n_1,\ldots,n_d)$?

Although Question 1 has an affirmative answer in the special case when A is a generalized Cohen-Macaulay local ring, it does not have an affirmative answer in general: a 2-dimensional counterexample was given. Question 2 has an affirmative answer in the cases where $d = 1$, $d = 2$, or A is a generalized Cohen-Macaulay local ring, but is open in general.

A. SIMIS

Multiplicities of almost perfect ideals

This work is a natural continuation of the work by Peskine-Szpiro (1974), Herzog-Kühl (1984), Huneke-Miller (1984) in which one looks for "closed formulae" for the multiplicity of a ring S/I ($S := k[X_1,\ldots,X_n]$, I : homog. ideal). Let

$$0 \to \bigoplus_{k_p=1}^{b_p} S(-d_{pk_p}) \to \ldots \to \bigoplus_{k_1=1}^{b_1} S(-d_{1k_1}) \to S \to S/I \to 0$$

be a minimal (homogeneous) resolution of S/I on S-module. Let $g := \mathrm{ht}(I)$. Then $p - g$ is the perfection deviation of S/I. We prove

Proposition. Assume the resolution of S/I is "almost-pure", i.e., $d_{i1} = d_{i2} = \ldots = d_{ik_i}$ $(=: d_i)$ for $i = 2,\ldots,p$.
If S/I has perfect deviation 1 then

$$e(S/I) = \frac{1}{(p-1)!}\left(\prod_{i=2}^{p} d_i - \sum_{k=1}\prod_{i=2}^{p}(d_i - d_{1k})\right).$$

Remark. This formula yields some (lax) bounds for $\nu(I)$ (# generators of I).

One of the interests of such formulae is in that one could try to find reasonable bounds for the degrees of the generators of I in terms of the multiplicity.

There are various examples of curves in $\mathbb{P}^3$ (even reduced & irreducible) that obey the above format.

J. STROOKER

On a criterion of Bourbaki

It is shown that the five conditions in Bourbaki's Critère des suites complètement sécantes are equivalent for a module which is complete in the topology defined by the ideal in question (Bourbaki, Algègre Homologique). This follow from our results in a talk on this Tagung in 1983. Since Bourbaki's weakest condition is seen to coincide with our "pre-regular". Bourbaki's result then follows from our approach. Connections with Cohen-Macaulayness on the monomial conjecture are sketched. To end with a

Theorem. Let A be a local Noetherian ring, the following are equivalent:

1) There is a balanced big Cohen-Macaulay-module for A;

2) $E - dh\ M + T - cod\ h\ M = \dim A$ for some A-module M

(Here $E - dh\ M = \inf_i (Ext^i_A(k,M) \neq 0$, $T - cod\ h\ M = \inf_i Tor^A_i(k,M) \neq 0$, k = residue class field of A.)

3) For some system of parameters x, $H_*(x,A)$ has the property (*) below.

4) For some system of parameters x there exists an x-pre-regular module.

Property (*): The Korsul homology looks as follows: $\neq 0,0,*,\dots,*$ or $*,\dots,*,0,\neq 0$.

In the proof, Bourbaki's Criterion is exploited as well as certain homological identities and inequalities.

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G. VALLA

On the depth of the symmetric algebra of a local ring

Let $(A,\mathfrak{n})$ be a regular local ring, $I \subseteq \mathfrak{n}^2$ an ideal such that $R = A/I$ is a local ring of depth d and maximal ideal $\mathfrak{m} = \mathfrak{n}/I$. If $S_R(\mathfrak{m})$ denotes the symmetric algebra of the R-module $\mathfrak{m}$, then depth $S_R(\mathfrak{m})$ will be the grade of the irrelevant ideal of $S_R(\mathfrak{m})$. We assume further that R is Cohen-Macaulay and then we know that depth $S_R(\mathfrak{m}) \leq d + 1$.

By functional properties of symmetric algebras we have $S_R(\mathfrak{m}) = \bigoplus_{t \geq 0} \mathfrak{n}^t / I\mathfrak{n}^{t-1}$. Further the kernel of the "downgrading" homomorphism $\lambda : S_R(\mathfrak{m})_t \to S_R(\mathfrak{m})$ is Ker $\lambda = gr_{\mathfrak{n}}(I)(-2)$ (here I is coincidered as a filtered module over the ring A with the $\mathfrak{n}$-adic-filtration).

If $k \leq \min(d, \text{depth } gr_{\mathfrak{n}}(I))$ then we can find a set of elements $x_1 = \bar{y}_1, \ldots, x_k = \bar{y}_k \in R$ such that

1) $y_1, \ldots, y_k$ is part of a regular system of parameters in A
2) $y_1, \ldots, y_k$ is a regular sequence mod I
3) $y_1, \ldots, y_k$ is a regular sequence on $gr_{\mathfrak{n}}(I)$

(here $y_i^* = \bar{y}_i \in \mathfrak{n}/\mathfrak{n}^2$).

We say that the elements $x_1, \ldots, x_k$ with the above properties from a "nice" regular sequence in R.

Now we can prove that if $x_1, \ldots, x_k$ is a nice regular sequence in R then $\tilde{x}_1, \tilde{x}_2 - x_1, \ldots, \tilde{x}_k - x_{k-1}$, is a regular sequence in $S_R(\mathfrak{m})$ (here if $x \in \mathfrak{m}$, $\tilde{x}$ denotes the corresponding element in $S_R(\mathfrak{m})_1 = \mathfrak{m}$).

Using this fact we can prove the following result

Theorem. a) depth $gr_{\mathfrak{n}}(I) \leq d + 1$

b) If depth $gr_{\mathfrak{m}}(t) \leq d$, then depth $gr_{\mathfrak{n}}(I)$ = depth $S_R(\mathfrak{m})$

c) depth $S_R(\mathfrak{m}) = d + 1$ if and only if depth $\mathfrak{Gr}_{\mathfrak{n}}(I) = d + 1$ and there exist a nice regular sequence in R, say $x_1,\dots,x_d$ such that $R/(x_1,\dots,x_d)$ has d-strong socle

(here we say, for a positive integer t, that the local ring $(A/\mathfrak{a}, \mathfrak{n}/\mathfrak{a})$ has t-strong socle if $\mathfrak{a}^t : \mathfrak{a} \subseteq \mathfrak{a}$). As a consequence of this result we get the following estimates for depth $S_R(\mathfrak{m})$.

Theorem. a) If I is generated by a inter regular sequence $f_1,\dots,f_n$ such that $\nu_1 = \nu_n(f_1) \geq \nu_2 = \nu_n(f_2) \geq \dots \geq \nu_2 = \nu_n(f_r)$ we have

1) $d \leq$ depth $S_R(\mathfrak{m}) \leq d + 1$

2) depth $S_R(\mathfrak{m}) = d + 1$ if and only if $d \geq \sum_{i=2}^{r} \nu_i - r + 2$

b) If I has a linear resolution then depth $S_R(\mathfrak{m}) = d + 1$.

c) If I is an homogeneous ideal in $k[x_1,\dots,x_n]$ generated by elements of the same degree, $A = k[[x_r,\dots,x_n]]$ and $I = JA$ then $d \leq$ depth $S_R(\mathfrak{m}) = d + i$. Moreover, if $d = 1$, then depth $S_R(\mathfrak{m}) = 2$ if and only if I has a linear resolution.

d) Let I be a perfect codimension 2 ideal. If $M = (a_{ij})$ is an $(r+1) \times r$ matrix given by the minimal resolution $0 \to A^r \xrightarrow{M} A^{n+1} \to I \to 0$, we let $\nu_d = \min_i \nu(a_{ij})$, $\overline{M} = (\overline{a}_{ij} \in \mathfrak{n}^{\nu_j}/\mathfrak{n}^{\nu_j+1})$ and $M_2 = (\overline{a}_{ij} \in \mathfrak{n}/\mathfrak{n}^2)$.

Then we have

1) If rank $\overline{M} = r$ then: $d \leq$ depth $S(\mathfrak{m}) \leq d + 1$

2) If $d = 1$ the following conditions are equivalent

α) depth $S_R(\mathfrak{m}) = 2$

β) rank $M_2 = r$

γ) I has a linear resolution.

W.V. VASCONCELOS

Normal Ries-algebras

1. We address the question: Given an ideal $I = (f_1,\ldots,f_n)$ of $R = \mathbb{Q}[x_1,\ldots,x_n]$, how to decide effectively whether the Ries algebra of I, $A = R[It]$ is normal? The approach used is through the ideal of relations of the algebra A:

$$0 \to T \to R[T_1,\ldots,T_m] \to A \to 0, \quad T_i \to f_i t.$$

is accessed through a package that runs within the Macsyma computer algebra system. (Obtained by the implementation of an algorithm of Buchberger for constructing Gröbner bases. The package used is still quite primitive, but others are in the offing). Normality is checked by the following version of Serre's interion.

Proposition. A is normal $\Leftrightarrow$ (a) (I,J) is an unmixed ideal of $R[T_1,\ldots,T_n]$ and (b) for each prime P of (I,J) the image of J in P/P^2 has rank = ht(P) - 1. Deciding (b) - via the Jacobian criterion - is often possible. (a) is quite tucky since it is equivalent to: Given a module (by its gen's and relations) over a polynomial ring, to decide whether it is torsion free. [This is possible to carry out if a proj. resolution is known and some programs to this effect are emerging.]

One case that is quite nice to deal with is when J is generated by linear forms in the Tj's. Then if condition (a) is known to be valid - (b) has a nice and fast formulation outside of Macsyma entirely. Many examples of prime ideals with normal/or non normal Rees algebras have been found by this method.

G. WÜSTHOLZ

Problems in algebraic geometry arising from transcendence

In the lecture we explained the use of algebraic geometry in the theory of transcendental numbers. To do this we started by explaining the general principles of a transcendence proof by proving the following

Theorem. (Gelfond-Schneider) If $\alpha, \beta \in \bar{\mathbb{Q}}$, $\alpha \neq 1$, $\beta \notin \mathbb{Q}$ then $\gamma = \alpha^{\beta} \notin \bar{\mathbb{Q}}$.

In making the proof of this as general as possible we discussed the following problem which appears there: Let $X \subset \mathbb{P}^N$ be projective, smooth, defined over an algebraically closed field K of char. 0. Then for an affine open set $X' \subset X$ denote by R the coordinate ring of X', by $\mathfrak{D}$ the module of derivation which is supposed to be free and by $\mathfrak{a} \subset \mathfrak{D}$ a free submodule of rank $a \leq \text{rank } \mathfrak{D}$. For an ideal $I \subset R$ and for a fixed set of generators $\Delta_1, \ldots, \Delta_a$ of $\mathfrak{a}$ let

$$\rho(I) = \text{rank } [\Delta_i P \bmod I]_{i=1,\ldots,a} \quad , \quad P \in I$$

$$\rho(\mathfrak{a}) = \min_I \frac{\rho(I)}{\text{rank } I}$$

Then we asked the following problem.

Problem. Suppose that $\mathfrak{a}$ is involutive, $\mathfrak{m} \subset R$ a maximal ideal,

$$\mathfrak{m}^{(T)} = \langle r \in R ; \Delta_1^{t_1} \ldots \Delta_a^{t_a} r \in \mathfrak{m}, \quad 0 \leq t_1 \leq \ldots \leq t_a < T \rangle$$

for integers $T > 0$, where $\langle\ \rangle$ denotes the ideal generated by the elements in parantheses. Then if $F \in \mathfrak{m}^{(T)}$ has degree D and if

$$T^{\rho(\mathfrak{a})} \gg D$$

then $F = 0$.

We can prove this conjecture if corank $\mathfrak{a} = 1$ or in general if $X^{\cdot} = \bar{G}$ is the projective closure of an algebraic group. In this situations we can even take more general ideals $\mathfrak{m}$ of rank $n = \dim G$.

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