

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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MATHEMATICAL METHODS IN CELESTIAL MECHANICS

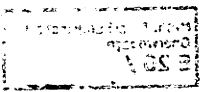
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This conference dedicated to mathematical aspects in celestial mechanics was conducted by U. KIRCHGRABER (Zürich) and H. RUESSMANN (Mainz), (V. SZEBEHELY (Austin) could not attend the meeting). With 31 participants this was quite a small meeting compared to the recent standards of the Institute. On the other hand the scientists being from 12 different countries the audience reflected the international interest in the subject very well.

The hectic developments in the field during the sixties and seventies which were mainly due to spectacular space vehicle experiments have been replaced by sound and more quiet research. These new activities are due to three sources at least: 1) The availability of more and more powerful computers. To give an idea of the possibilities it may be worthwhile to mention that it is now within the range of super computers to predict the behaviour of the solar system for a period of 10^8 years, as discussed by MILANI. 2) Due to a large number of space missions which have been carried out during the last two decades an immense amount of data on the solar system is available. This new information has stimulated extensive theoretical work. As an example we mention the studies on the planetary rings, as presented by BORDERIES. 3) The remarkable progress in general dynamical system theory already has and will continue to have some strong impact on celestial mechanics. To illustrate this point we mention the use of singularity theory to study families of periodic solutions close to the Lagrange point L_4 in the restricted three body problem, as described by VAN DER MEER.

In addition to these main streamlines quite a variety of specific topics have been covered at this meeting, in connection with many new results and new techniques. We mention SIMO's report on the work of Lazoutkin on standard maps, the use of Piccard-Fuchs theory to handle some nasty algebraic problems in local bifurcation theory by SANDERS and CUSHMAN, the simplifying approach of GALGANI and GIORGILLI to the Nekhoroshev theorem, just to mention a few.

Review papers by EKELAND (on variational methods to construct periodic solutions for convex Hamiltonian systems) and MURDOCK (on the spin/orbit resonance problem) proved to be most enjoyable.



Abstracts

E. BELBRUNO (presented by J. LLIBRE)

ON THE PERIODIC ORBITS OF THE CIRCULAR SITNIKOV PROBLEM

We consider the circular Sitnikov problem as a subsystem of the circular spatial restricted three body problem with the two primaries of equal mass. Using elliptic functions we give the explicit solutions for the orbits of the Sitnikov problem. We also study the variation of the period along the family of periodic orbits of the Sitnikov problem.

N. BORDERIES

DYNAMICAL PROBLEMS IN PLANETARY RINGS

The purpose of this talk is to present a formalism that P. Goldreich, S. Tremaine and myself have devised to study the dynamics of planetary rings. This formalism is based on a representation of the streamlines in a ring where the flow of particles is not Keplerian. As a matter of fact, planetary rings are perturbed by satellites which border them or which act at the locations of Lindblad resonances. This formalism allows to study a wide variety of problems such as non-linear density waves, dynamics of eccentric and narrow rings, confinement of narrow rings, sharp edges.

I. EKELAND

PERIODIC SOLUTIONS IN THE LARGE FOR STRONGLY NON-LINEAR HAMILTONIAN SYSTEMS

A new, variational, method has been developed for finding periodic solutions of strongly non-linear Hamiltonian systems. It is a global method (although it can also be used to yield perturbation results) which, however, requires the Hamiltonian to be convex in all variables. In this presentation I will explain the method, survey some results, and discuss their significance for celestial mechanics.

D. FLOCKERZI

RESONANCE AND BIFURCATION OF HIGHER DIMENSIONAL TORI

By means of an example it is shown that a supercritical bifurcation of an invariant 2-torus into an invariant 3-torus prevailing in the case of non-resonance may be replaced by a transcritical bifurcation of a pinched invariant 3-torus in the case of resonance. Depending on the parameter constellation there is still another route of bifurcation: 2-torus \rightsquigarrow pinched 3-torus \rightsquigarrow full 3-torus. The connections of these bifurcation phenomena with the properties of the spectrum of the underlying 2-torus are discussed.

L. GALGANI and A. GIORGILLI

RIGOROUS ESTIMATES IN HAMILTONIAN PERTURBATION THEORY

We consider a nearly integrable Hamiltonian system with Hamiltonian

$H(p,q) = h(p) + \epsilon f(p,q)$ where p, q are action-angle variables. Normalizing the Hamiltonian to order r we obtain an estimate of the radius of convergence of the series expansion of the transformed Hamiltonian with respect to ϵ . In addition we provide an error bound of the truncated Hamiltonian of order r . Moreover the relation with the Nekhoroshev theorem is exposed.

B. GARFINKEL

ON THE BROWN CONJECTURES IN THE THEORY OF THE TROJAN ASTEROIDS

E.W. Brown conjectured (1911) that the family of the long periodic orbits in the Trojan case of the restricted problem of three bodies terminates in an asymptotic orbit passing through the Lagrangian point L_3 at $t = \pm\infty$. In 1977 the author showed that such an orbit deviates from L_3 by the epicyclic term $mg(\pm\infty)$. It is shown here that

$$g(\pm\infty) \neq 0,$$

so that the Brown conjecture regarding L_3 is false.

Contrary to what Brown believed, there is an entire family of *homoclinic* orbits, doubly asymptotic to short periodic orbits around L_3 . In the complex z -plane of the Poincare eccentric variables, the latter orbits are circles of radius mR , with R bounded away from zero. The kinematics of the homoclinic family is investigated here in some detail.

J.D. HADJIDEMETRIOU

A HYPERBOLIC TWIST MAPPING MODEL FOR THE STUDY OF ASTEROID ORBITS

An algebraic mapping is found which contains all the properties of the dynamical system which describes the motion of an asteroid in the Sun-Jupiter system, in the restricted circular 3-body problem, near the 3:1 resonance. The mapping is constructed by starting from the corresponding mapping of the two body problem Sun-asteroid, in a rotating frame (twist mapping) and then by including the main effect of Jupiter, which is obtained analytically to be the generation of instability (hyperbolic mapping). The product of these two mappings is a hyperbolic twist mapping which gives invariant curves close to the actual case. This latter mapping depends on the energy, as a parameter. The properties of this mapping are studied in connection with the evolution of an asteroid orbit which passes through the 3:1 resonance due to energy dissipation.

D.C. HEGGIE

BIFURCATION AT COMPLEX INSTABILITY IN HAMILTONIAN SYSTEMS

In a Hamiltonian problem with *two* degrees of freedom, a family of periodic orbits may change from stable to unstable only if $\lambda = \pm 1$ (where λ is a characteristic multiplier), and generally a bifurcation of a new family of periodic orbits occurs. In *three* degrees of freedom stability may be lost in a different way, called 'complex instability'. This paper describes normal forms valid near complex instability, and at

this level of approximation describes the simplest structure (a two-parameter family of two-dimensional tori) which bifurcates from the original family. The approximate theory also provides a local description of the stable and unstable manifolds of the original periodic orbits just after instability has set in. The theory can be illustrated by numerical work on the planar general three-body problem, and on symplectic four-dimensional mappings.

J. HENRARD

FROM THE CIRCULAR TO THE ELLIPTIC RESTRICTED PROBLEM IN CASE OF RESONANCE

In the case of resonance the circular (and planar) restricted problem can be reduced to a one degree of freedom problem by averaging. In such a model the Kirkwood gaps in the system Sun-Jupiter cannot be explained.

Anne LEMAITRE and the author have considered the effect of the dissipation of a protosolar nebula to explain the formation of the gaps.

J. WISDOM has shown first numerically and recently by perturbative methods that the effect of the eccentricity of Jupiter can be enough to produce the 3/1 gap.

We shall comment on this perturbative method, apply it to the 2/1 resonance, extend it to the small eccentricity regime in order to show that this mechanism does not seem to be able to produce the 2/1 gap.

M. IRIGOYEN

SUR LES EXPOSANTS CARACTERISTIQUES ASSOCIES AUX SOLUTIONS HOMOGRAPHIQUES DU PROBLEME DES TROIS CORPS

Le choix de variables normalisées pour l'étude du problème plan des trois corps permet de représenter les solutions homographiques par des points fixes du flot associé. Au voisinage de ces points fixes, l'allure du flot est déterminée par les exposants caractéristiques correspondants. Siegel a ainsi déterminé les six exposants μ_1 associés à un système de variables q^* et p^* normalisées respectivement par $t^{2/3}$ et $t^{-1/3}$.

On peut aussi proposer une normalisation des variables q et p , respectivement par le rayon d'inertie r et par $r^{-1/2}$, en généralisant au problème plan le choix de McGehee pour le problème rectiligne. Le calcul direct des six exposants λ_1 associés à ce choix montre que, aussi bien aux points d'Euler qu'aux points de Langrange, les relations suivantes sont vérifiées:

$$\lambda_1 = -1.5v_1 \cdot \mu_1 \quad i \in \{1, 2, \dots, 5\}$$

$$\lambda_6 = -0.5v_1 \cdot \mu_6$$

où l'exposant λ_6 est associé à la direction propre du moment angulaire.

Cette différence de comportement entre les six exposants traduit le fait que l'on passe des variables de Siegel aux variables de McGehee par un changement dépendant du temps. In n'y avait donc aucune raison *a priori* pour que les exposants se transforment de façon uniforme.

U. KIRCHGRABER

LONG-TERM INTEGRATION OF PERIODIC SYSTEMS

Consider the system $dx/dt = f^0(x)$ and assume its flow $\Phi_{f^0}(t, x)$ to be 1-periodic for all x . Next introduce the perturbed system $dx/dt = f(x, \epsilon)$, $f(x, 0) = f^0(x)$ and its associated time-one map $P(x, \epsilon) = \Phi_f(1, x)$. For each j , arbitrarily large but fixed, P^j tends to the identity I as $\epsilon \rightarrow 0$. On the other hand: $P^{1/\epsilon}$ does not tend to I , as $\epsilon \rightarrow 0$, $1/\epsilon \in \mathbb{N}$, in general. Therefore the knowledge of $P^{1/\epsilon}$ provides insight into the behaviour of $dx/dt = f$ on a time scale which permits the perturbation to develop a significant contribution. Unfortunately the *direct* computation of $P^{1/\epsilon}$ is rapidly more expensive as $\epsilon \rightarrow 0$. We therefore aim at presenting more efficient methods. The *first method* (jointly worked out with G. POSPIECH-WILLERS) is an extrapolation method based on the fact that $P^{1/\epsilon}$ admits an asymptotic expansion with respect to ϵ . It reduces the number of evaluation of P from $1/\epsilon$ to $O(\epsilon^{-1/k})$, $k \in \{2, 3, \dots\}$. The *second device* is based on the Method of Averaging. Borrowing ideas from the Runge-Kutta methods the averaged vector field is expressed in terms of P , without using derivatives of P . Error bounds and efficiency results are presented. Some examples, in particular the application to artificial satellite theory are described.

E.A. LACOMBA

TOPOLOGY OF JACOBI LEVELS IN THE RESTRICTED THREE BODY PROBLEM WITHOUT ZERO VELOCITY CURVES

We consider the topological description of the Jacobi levels in the circular planar restricted 3-body problem, when the mass parameter μ is close to zero and the Jacobi constant C is zero. For $C=0$ there are no zero velocity curves so that it is possible that the infinitesimal body comes from infinity and has collisions with any of the other two bodies. After regularization of such collisions and addition of a 2-torus at infinity through blow up, we show that the Jacobi level is topologically equivalent to the unique orientable $[0,1]$ -bundle over the Klein bottle.

Then we find coordinates making explicit this topology as a cube where some of the faces are identified. We show how hyperbolic or parabolic orbits escaping to (or coming from) infinity are asymptotic to periodic orbits on the torus at infinity.

(Joint work with C. SIMO)

J. LLIBRE

A NOTE ON THE GLOBAL FLOW OF THE TWO BODY ROTATING PROBLEM

The goal of this note is to describe the complete picture of the global behaviour of the solutions of the planar two body rotating problem in each Jacobian level augmented with the collision and infinity manifold.

L. LOSCO

EQUATIONS HAMILTONIENNES DE CONTACT ET PROBLEME RESTREINT DES TROIS CORPS

L'exposé concerne le problème restreint des trois corps, non nécessairement circulaire. Il a pour but d'obtenir des équations très analogues à celles du problème circulaire, pour des axes mobiles en direction et en dimension. Ces équations ne sont plus hamiltoniennes mais hamiltoniennes de contact.

Les équations de contact sont actuelles en Mécanique Analytique. L'espace des phases est de dimension impaire et en coordonnées canoniques, elles s'écrivent:

$$\begin{aligned}dq^1/dt &= \partial K/\partial p_1 & dp_1/dt &= -\partial K/\partial q^1 - p_1 \partial K/\partial s \\ ds/dt &= \sum p_i \partial K/\partial p_i - K\end{aligned}$$

Si $\partial K/\partial s=0$, ce sont les équations hamiltoniennes où s est la variable action (J.Bryant).

Pour le problème circulaire, en axes tournants OXY, les équations ont pour hamiltonien

$$\begin{aligned}H &= \frac{1}{2}\{(P_x + \omega Y)^2 + (P_y - \omega X)^2\} - \frac{1}{2}k(X^2 + Y^2) - \\ &\quad - k\mu\{(X + 1 - \mu)^2 + Y^2\}^{-\frac{1}{2}} - k(1-\mu)\{(X - \mu)^2 + Y^2\}^{-\frac{1}{2}}\end{aligned}$$

Pour le problème non circulaire, en axes tournants OXY d'unité telle que la distance des masses primaires est 1, on a des équations de contact de hamiltonien $K=H+0.5\lambda s$, où ω et λ sont deux fonctions dépendant du mouvement képlérien des primaires.

CH. MARCHAL

STABILITY OF A PERIODIC SOLUTION IN A HAMILTONIAN SYSTEM

Let us consider an *analytic* Hamiltonian system and let us assume that it is equivalent to a two-dimensional area preserving mapping.

The periodic solutions of the Hamiltonian system correspond to the invariant points of the mapping or of some of its powers.

The stability analysis of the invariant points can be done in almost all cases and especially in all cases with a rational rotation angle.

It seems that all elliptic analytic invariant points with an irrational rotation angle are stable (no counter-example is known).

P.J. MESSAGE

DETERMINATION OF THE LIBRATION PERIOD OF A COMET ORBIT

The period between successive perihelion returns of Halley's comet shows, between 1136 and 1835 AD, an oscillation of period rather less than 800 years. The angle equal to 11 times the mean anomaly of the comet, plus twice the difference between the mean longitudes of the comet and the planet Jupiter, shows an oscillation of like period over the same time interval, indicating that we have here a free libration associated with the near 13:2 resonance between the orbital periods of these two bodies. In order to make a theoretical calculation of the period of this libration for a series of finite amplitudes, a transformation to an auxiliary angle was employed to

enable a rapidly convergent Fourier series to be found for quantities in the equations for the perturbations. Periods found ranged up to 772 years.

J. MURDOCK

AVERAGING AND HYPERBOLICITY, WITH SPECIAL REFERENCE TO THE SPIN/ORBIT RESONANCE PROBLEM

Two common systems suitable for averaging are:

$$dr/dt = \varepsilon f(r, \theta, \varepsilon) \quad d\theta/dt = \Omega(r) + \varepsilon g(r, \theta, \varepsilon) \quad r \in \mathbb{R}^n \text{ and } \theta \in \mathbb{R}^m \text{ mod } 2\pi$$

and

$$dx/dt = \varepsilon f(x, t, \varepsilon) \quad x \in \mathbb{R}^n \text{ and } t \in \mathbb{R} \text{ mod } 2\pi.$$

We discuss rigorous qualitative results for these systems. The first system may admit infinitely many resonances; we show that for non Hamiltonian (nearly Hamiltonian) systems only finitely many are "active" (produce qualitative changes in the orbit pattern). Near resonance this type of system can sometimes be rewritten in the second form. For the second form we discuss conditions under which the averaged flow (to first or higher order) and the exact flow are topologically conjugate. The spin/orbit resonance problem provides an illustration of these ideas.

J. PALMORE

LIMITS ON COMPUTING CHAOTIC ORBITS

We analyse the orbit decomposition of uniform binary lattices by four maps in order to show the difficulties of computing chaotic orbits by finite state machines. Two of the maps of the circle are 1) $x \rightsquigarrow 2x \text{ mod } 1$ and 2) $x \rightsquigarrow 3x \text{ mod } 1$. Arnold's linear automorphism of the torus T^2 , the "cat map", is examined and is shown to produce tilings of the torus on some non binary uniform lattices. These tilings are not present on binary lattices and represent hidden symmetries of the map. We also examine a cellular automaton given by the logistic map of the interval. We conclude that very long period cycles are difficult to distinguish from truly chaotic motion when the resolution used to observe the orbit is low.

(Joint work with J. McCauley, Houston, Texas, USA)

E.A. ROTH

ON THE USE OF EQUINOCTIAL ELEMENTS IN SATELLITE THEORIES

A number of terrestrial or planetary artificial satellite projects are requesting near circular or/and near equatorial orbits. It is well known that in these cases the formulation of the equations of variation of parameters (VOP) presents singularities for the classical orbital elements. In order to remove them one possibility is to introduce equinoctial elements

$$\begin{aligned} a, \quad f &= e \cos(\omega + \Omega), & g &= e \sin(\omega + \Omega) \\ h &= \tan(i/2) \cos\Omega, & k &= \tan(i/2) \sin\Omega \end{aligned}$$

The new VOP equations have been presented in the Lagrangian and in the Gaussian form. It has been shown how the most important perturbations namely the effects of the non sphericity of the central body, of a third body, of air drag with refined air

density models, and of the radiation pressure can be treated. Some examples demonstrated the efficacy of the chosen approach, based on classical perturbation theory.

D.G. SAARI

SYMMETRIES IN THE N-BODY PROBLEM

The general Newtonian N-body problem admits an invariant manifold determined by the intersection of the manifold of constant angular momentum and the manifold of fixed energy. The vector field on this manifold is characterized. From this characterization it is shown how 1) S. Smale's topological characterization of this manifold is partially extended from the coplanar problem to the full three dimensional problem, 2) the restrictions on motion described by Marchal and Saari are extended to "best possible" results for the three-dimensional problem, and 3) C.L. Siegel's solution of the Painleve conjecture is extended from the three body problem to the general N-body problem.

J.A. SANDERS

PICARD-FUCHS TECHNIQUE IN BIFURCATION THEORY

As an example we consider the Josephson equation

$$d^2\varphi/dt^2 + \epsilon(1 + \gamma \cos\varphi)d\varphi/dt + \sin\varphi = \epsilon a \quad \varphi \in S^1$$

as a perturbed mathematical pendulum. Using an averaging procedure we arrive at an equation involving complete elliptic integrals. We derive the Picard-Fuchs equation for these integrals and from this we can obtain existence and stability results of (un)stable periodic solutions, both contractible and winding. This leads to a complete analysis of the bifurcations of the phase portrait in the a - γ -plane.

Of special interest is the occurrence of a flat contact of two bifurcation curves meeting in the point $(a, \gamma) = (3\pi/16, 1)$. Our guess is that this is due to the fact that we work on TS^1 . We believe that flat contact cannot happen for bifurcation curves of two parameter families of planar vector fields.

(Joint work with R. CUSHMAN; to appear in SIAM J. Mathematical Analysis)

C. SIMO

A STUDY OF A MODIFICATION OF THE STANDARD MAPPING

Consider a Hamiltonian flow in the plane (one-degree of freedom) with a hyperbolic point and a saddle connection. Let F_ϵ be an area preserving mapping near the time- ϵ map of the flow: $\|\varphi_\epsilon - F_\epsilon\| = O(\epsilon^2)$. If φ_ϵ is C^∞ and F_ϵ is C^∞ with respect to ϵ , then the distance between the related W^u and W^s under F_ϵ is a C^∞ flat function of ϵ . We would like to have more precise information.

First we consider the standard map in the form

$$(x, y) \mapsto (x', y') = (x + 2\pi\epsilon \sin 2\pi y, y + \epsilon x')$$

It has been proven by V. Lazutkin that the angle of the invariant manifolds of the origin at $y=0.5$ (where a homoclinic point is found for all ϵ) is

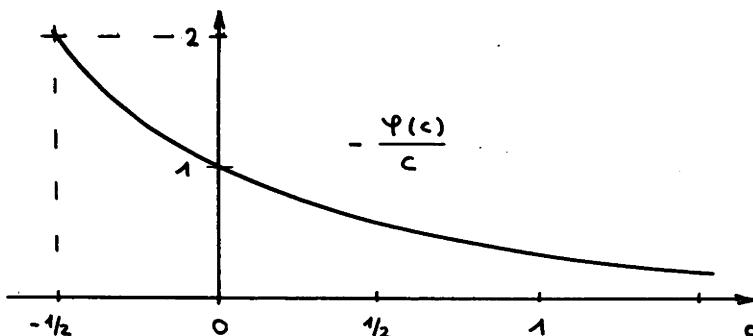
$$1557.5\pi^{-5/2}\epsilon^{-3}\exp(-\pi/(2\epsilon))(1 + O(\epsilon^{1/4-\delta}))$$

with $\delta \in (0, 1/4)$ when $\epsilon \rightarrow 0$.

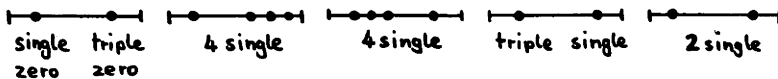
The following modification has been introduced

$$(x, y) \rightarrow (x', y') = (x + 2\pi\epsilon\{\sin 2\pi y + c \sin 4\pi y\}, y + \epsilon x')$$

By a careful computation of the angle at the homoclinic point sitting on $y=0.5$ (which exists independently of ϵ and c , but is not symmetric) using higher order expansions for the local behaviour and numerical continuation, the following expression is found for the angle: $A\epsilon^{-3}R\epsilon\exp(-(d+if)/\epsilon)$, where A, d, f depend only on c . In fact $d+if$ can be expressed as $0.5\pi(1+2c)^{-1/2}(1-\varphi(c)^{1/2})$ and $\varphi(c)$ is numerically fit to the data. It is found that $-\varphi(c)/c$ has the following behaviour:



In particular, for $c < 0$ the angle never changes sign, but for $c > 0$ there is a countable set of values of ϵ at which the angle changes sign. At the values at which the angle is zero $d(W^u, W^s)$ has cubic zeros and when ϵ is changed (to one of the sides) a new pair of homoclinic points appears in a fundamental domain. Later on this pair disappears when these points meet the related homoclinic point as follows:



ϵ decreasing from left to right

J. SCHEURLE

CHAOTIC MOTION IN QUASI PERIODICALLY FORCED SYSTEMS

It is well known that the chaotic behaviour of certain dynamical systems can be explained by the presence of homoclinic or heteroclinic orbits. In this lecture a two-dimensional system is considered which results from adding a small quasi periodic forcing term to an autonomous system which has a hyperbolic saddle point and a corresponding homoclinic orbit. Actually we are even going to consider the

more general case of almost periodic forcing terms. It is shown that the existence of a transversal homoclinic orbit corresponding to the perturbed saddle point leads to a large number of solutions which behave more or less chaotically. The Melnikov function provides a sufficient condition for the occurrence of such chaos. This result generalizes known results for periodically forced systems.

D. STOFFER

REMARKS ON THE CHAOTIC BEHAVIOUR OF NON INVERTIBLE MAPS

- a) Pseudo orbits and shadowing orbits: The shadowing lemma holds for maps which have a so-called snap-back repeller.
- b) The Bernoulli shift map as a subsystem: If the shadowing lemma holds for a map f , it is shown that the shift map is a subsystem of some iterate f^m . More generally, an analogon to Smale's horseshoe is presented for non invertible maps in Hausdorff spaces and it is demonstrated in which way the shift map can be embedded. (With an application to partial differential equations).
- c) Chaos in the sense of Li and Yorke: The shift map itself is Li-Yorke chaotic. Thus a) or b) imply Li-Yorke chaos.

J.C. VAN DER MEER

BIFURCATION OF PERIODIC SOLUTIONS FOR FAMILIES OF HAMILTONIAN SYSTEMS PASSING THROUGH NON SEMISIMPLE 1:-1 RESONANCE AT AN EQUILIBRIUM

For a Hamiltonian system in non semisimple 1:-1 resonance at an equilibrium a description will be given of the bifurcation of periodic orbits near the equilibrium when the resonance is detuned.

The periodic solutions we study are those with period close to the period of the solutions of the semisimple part in the Jordan decomposition of the linearized system at resonance.

The description of the bifurcation is obtained by reducing the general problem to the study of a relatively simple integrable Hamiltonian system by the use of normal form theory, Moser-Weinstein reduction, and the equivariant theory of stability of maps.

It turns out that generically one of two different bifurcations appears, which are both described. One of these bifurcations is found in the restricted problem of three bodies at L_4 when the mass parameter passes through the critical value of Routh.

F. VARADI

TWO-PARAMETER LIE TRANSFORMS

Generalizations of the Lie series perturbation methods for the case of two small parameters are presented.

J. WALDVOGEL

THE THREE BODY PROBLEM WITH TWO SMALL MASSES: A SINGULAR
PERTURBATION APPROACH TO THE PROBLEM OF SATURN'S
CO-ORBITING SATELLITES

The three body problem with masses $m_0, \epsilon m_1, \epsilon m_2$ is considered in the limiting case $\epsilon \rightarrow 0$. By appropriately scaling the coordinates the motion is described by the following two matched approximations: (1) the outer solution consisting of two independent Kepler motions about m_0 , (2) the inner solution satisfying Hill's lunar equation. The discussion of Hill's problem with appropriate boundary conditions at infinity correctly predicts that Saturn's co-orbiting satellites Janus and Epimetheus exchange orbits at the close encounter, whereas the F Ring Shepherds (1980 S26 and S27) do not.

(Joint work with F. SPIRIG, Rorschacherberg, Switzerland)

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