Math. Forschungsinstitut Oberwolfach
E 20 /01963

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 24/1985

Finite Geometries

26.5 bis 1.6.1985

This year's conference on Finite Geometries was held under the direction of F. Buekenhout (Bruxelles), D. R. Hughes (London) and H. Lüneburg (Kaiserslautern). The main topics of the conference were designs, finite projective and affine planes (especially translation planes), combinatorial properties of finite geometries, geometric aspects of graphs and of finite permutation groups, finite buildings, and interactions between these subjects. The most spectacular new result, which was reported on by Prof. Doyen, is L. Teirlinck's theorem establishing that there are plenty of t-designs for all t ; all previously known examples had $t \leq 6$.

gefördert durch DFG Deutsche Forschungsgemeinschaft
©

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

E. Bannai, T. Ito: Distance-regular graphs with fixed valency I, II.

We proved the following results. The proofs are obtained by algebraic methods, studying the eigenvalues and the multiplicities of the adjacency matrix of the graph. We also use Ivanov's diameter bound. Let Γ be a distance-regular graph, and let

$$\left\{\begin{array}{ccccc} * & 1 \;---\; 1 & * \;---\; * & k-a-1 \;---\; k-a-1 & c_d \\ 0 & a \;---\; a & * \;---\; * & a \;---\; a & a_d \\ k & \underbrace{k-a-1 \;---\; k-a-1}_{r} & \underbrace{* \;---\; *}_{t} & \underbrace{1 \;---\; 1}_{s} & * \end{array}\right\}$$

the intersection array of Γ.

Theorem 1 If the graph Γ is bipartite $(k \geq 3)$. Then $t < f_1(k)$.

Theorem 2 For any distance-regular graph, $r < f_2(k, t)$.

Main Theorem (Thm 1 + Thm 2) For each fixed valency $k \geq 3$, there are only finitely many bipartite distance-regular graphs of valency k.

The following result has been almost proved (modulo minor details).

"Theorem 3" (Generalization of Thm 1 for arbitrary distance-regular graphs) For any distance-regular graph, $t < f_4(k)$. (Consequently $d < f(k)$ for any distance-regular graph.)

For small valencies, the proof of Thm 3 is completely finished. Consequently, DRG of $k = 3$ are classified - also by Biggs, Boshier, Shawe-Taylor by combinatorical methods just before our algebraic methods are completed, and for $k = 4$, $d < f(4)$ - at present, $f(4)$ is rather big for practical use but we expect it is easy to deal with the remaining finite cases to determine all DRG of valency 4.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

©

A. Beutelspacher: Embedding of finite linear spaces in projective planes.

Theorem 1. Let S be a finite linear space of order n and denote by $n + 1 - a$ the minimal line size of S. If $4n > 6a^4 + 9a^3 + 19a^2 + 9a + 3$, then S is embeddable in a projective plane of order n. A linear space is said to be H-semiaffine, if for any point p outside a line L, the number of lines through p which do not intersect L, is an element of H.

Theorem 2. Let S be a finite proper $\{0, 1, s\}$-semiaffine linear space of order n. If $s \geq 3$, then S is the complement of a set of type $\{0, 1, s\}$ in a projective plane of order n.

J. Bierbrauer (with A. Brandis): Ramsey numbers for trees.

Let $\mathcal{T}_n$ be the set of trees with n edges (and $n + 1$ vertices). We modify the concept of a diagonal Ramsey number by introducing:

$r(\mathcal{T}_n, k) = \min \{ \mu \mid$ whenever the edges of the complete graph K_μ on vertices are partitioned into k components, then one of the k subgraphs contains a connected component on more than n vertices $\}$.

Then $r(T, k) \geq r(\mathcal{T}_n, k)$ for all $T \in \mathcal{T}_n$.

Counting arguments and various constructions using latin squares, nets, resolvable block designs of index one, and resolvable linear spaces yield upper and lower bounds:

$$r(\mathcal{T}_n, k) > 2 \left[\frac{n}{2}\right] \left[\frac{k+1}{2}\right]$$

$$r(\mathcal{T}_n, k) \leq k(n-1) + 1 \quad \text{for } k \equiv 0(n)$$

$$r(\mathcal{T}_n, n) \leq n(n-1) \quad \text{for } n > 2$$

$$r(\mathcal{T}_n, k) \leq k(n-1) + 2 \quad \text{for } k \equiv 1\ (n)$$

Lemma If $r(\mathcal{T}_n, k) > \mu$ and if there is a set of $n - 1$ MOLS of order μ, then $r(\mathcal{T}_n, k + \mu) > n \cdot \mu$.

In suitable cases the numbers can be determined:

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

$$r(\mathcal{T}_5, 3) = 10 \ , \ r(\mathcal{T}_6, 3) = 13 \ , \ r(\mathcal{T}_8, 3) = 17$$

$$r(\mathcal{T}_4, 6 + \frac{16(4^{i+1} - 1)}{3}) = 4^{i+3} + 1$$

$$r(\mathcal{T}_4, 10 + \frac{28(4^{i+1} - 1)}{3}) = 28 \cdot 4^{i+1} + 1$$

$$r(\mathcal{T}_6, 3 + \frac{12(6^{i+1} - 1)}{5}) = 12 \cdot 6^{i+1} + 1 \qquad (i \geq 0) .$$

Lemma If F is a forest with n edges, without isolated vertices,

then $r(F, k) > [\sqrt{n}]\,[\frac{k+1}{2}]$ $(k, n \geq 2)$

A certain Steiner triple system on 19 points is used to determine the Ramsey numbers for the path with 3 edges for $3^k > 3$ colours.

$$r(P_3, k) = \begin{cases} 2k + 2 & k \equiv 1 \pmod 3 \\ 2k + 1 & k \equiv 0, 2 \pmod 3, \ k \neq 3 \\ 2k = 6 & k = 3 \ . \end{cases}$$

A. Blokhuis (with H. A. Wilbrink): Note on a Theorem of Bruen & Thas, Segre & Korchmâros.

The following theorem generalizes the characterization of exterior lines of a conic by Segre & Korchmâros (and by Bruen and Thas for even characteristic).

Let A, with cardinality q, and B with card $q + 1$ be disjoint sets of points in $PG(2, q)$, such that each line containing a point of A, also contains a point of B. Then B is a line.

Proof. If B is not a line then there is a line ℓ disjoint from B. Identify $PG(2, q)\backslash\ell \cong AG(2, q)$ with $GF(q^2)$. Then all points of A are zeroes of $f(x) = \sum_{b \in B} (x - b)^{q-1}$, contradiction.

A. E. Brouwer: Characterizations of Grassmann graphs.

We generalize Numata's results to arbitrary diameter and discuss the problem of finding all graphs that are locally $GQ(s, t)$ and have μ-graphs $K_{t+1\,,\,t+1}$.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©

A. A. Bruen (with A. Blokhuis and R. Silverman): M.D.S. codes, arcs and the problem of B. Segre.

Let C be a code of length n over an alphabet A of size Q. So C is just a collection of code words x of length n over A, a code word being any n-tuple over A. Let $2 \le k \le n$. We impose the following condition. Condition 1: No 2 words in C agree in as many as k positions. It follows that $|C| \le q^k$. If $|C| = q^k$, C is called an M.D.S. code and has minimum distance $d = n - k + 1$. For given q, k we want to maximize d and, so, n. This leads to the main problem.

Problem: For given k, q what is the maximum value of n? And what is the structure of C in the optimal case? One can show the following result.

Theorem $n \le q + k - 1$.

We examine the case of equality. For $k = 2$, $n = q + 1$, the code C yields an affine plane of order q, and vice versa. For $k = 3$, $n = q + 2$, C is equivalent to an affine plane π of order q with an elaborate system of hyperovals: the only known example occurs when π is desarguesian. The case $k = 4$, $n = q + 3$ probably cannot occur: it is only known that $36 \mid q$. The linear version of the main problem goes as follows. Let X be a k-dimensional subspace of $V(n, q)$. Choose any k basis vectors for x arranged in the form of a $k \times n$ matrix B over $GF(q)$. Since B has rank k, some k columns of B are linearly independent. The analogue of Condition 1 is Condition 2: Every set of k columns of B is linearly independent. The main problem can now be rephrased in several different ways. For example, columns of B yield an arc in $\Sigma = PG(k-1, q)$ and the problem of the title asks for the size of the largest arc Y in Σ and the structure in the optimal case. It is conjectured that $|Y| \le q + 1$ when $n + 2 \le q + 1$. We discuss recent results on this and obtain an analogue for q even of a result of J. A. Thas and the late B. Segre for q odd.

P. J. Cameron: Stirling numbers and affine equivalence.

If $F_n(G)$ is the number of orbits of the permutation group G on n-tuples of distinct points, and $F_n^*(G)$ the number of orbits on all n-tuples, then $F_n^*(G) = \sum_{k=1}^{n} S(n, k) F_k(G)$, where $S(n, k)$ is the Stirling number of the

Gefördert durch DFG Deutsche Forschungsgemeinschaft

©

second kind. This result has a linear analogue: if $\phi_n(G)$ is the number of orbits of the linear group G on linearly independent n-tuples, then $F_n^*(G) = \sum_{k=0}^{n} [{n \atop k}]_q \, \phi_k(G)$, where $[{n \atop k}]_q$ is the Gaussian or q-binomial coefficient. Combining these results leads to a formula for the number of n-tuples of distinct points of affine space over $GF(q)$, up to affine equivalence; the number is $\sum_{k=1}^{n} s(n, k)\, F(k-1, q)$, where $s(n, k)$ is the Stirling number of the first kind, and $F(n, q)$ the number of vector subspaces of $GF(q)^n$.

P. J. Cameron: Groups generated by transvection subgroups.

McLaughlin's determination of groups generated by full transvection subgroups has been extended, by J. I. Hall and me, to infinite-dimensional vector spaces. Our result is formally very similar to McLaughlin's, but in the infinite-dimensional case there are many strange examples. The proof requires an extension to infinite dimensional spaces of a result of Lefèvre-Percsy which determines all point sets in a projective space which meet any line L in none, one, all but one, or all points of L.

F. de Clerck: Translation partial geometries.

Let $S = (P, B, I)$ be a proper partial geometry with parameters t, s, α $(1 < \alpha < \min(s, t))$. If L is a line of S, then $L^{\perp}$ is the set of lines concurrent to L. More generally if A is a subset of the lineset then $A^{\perp}$ is the intersection of all $L^{\perp} (L \in A)$. The span of a pair of lines $\{L, M\}$ is defined to be $\{L, M\}^{\perp\perp} = \{U \in B \,||\, U \in N^{\perp} \,\forall N \in \{L, M\}^{\perp}\}$. If L and M are two non-concurrent lines then one can prove that $\{L, M\}^{\perp\perp}$ is a partial spread (i.e. a set of pairwise nonconcurrent lines) and $|\{L, M\}^{\perp\perp}| \le s+1$. If equality holds then $\{L, M\}$ is called α-regular. If V is a spread (i.e. a maximal set of $(st+\alpha)/\alpha$ pairwise nonconcurrent lines) then V is called normal iff every pair $\{L, M\} \subset V$ is α-regular and $\{L, M\}^{\perp\perp} \subset V$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

If G is an automorphism group of S, then S is called a translation partial geometry with translation group G, provided

(1) G acts regular on the points of S
(2) $t = \alpha(s+2)$
(3) every line orbit of G is a normal spread.

Generalizing results on generalized quadrangles we prove that G is a translation group of a translation partial geometry iff G is a group of order $(s+1)^3$ with a set T of $t+1$ subgroups A_i, $i \in J = \{0, 1, \ldots, t\}$ of order $s+1$ satisfying the following conditions

(1) $A_i \cap A_j = \{1\}$ for all i,j $i \neq j$.
(2) for any pair $\{i, j\} \subset J$, there exists a subset $V(i, j)$ of J, $|V(i, j)| = \alpha + 1$, $1 < \alpha < s$, $i, j \in V(i, j)$ such that $A_iA_j = A_kA_l$ $\forall k, l \in V(i, j)$ $(k \neq l)$.
(3) $A_iA_j \cap A_m = \{1\}$ $\forall m \in J-V(i, j)$.

Moreover we prove that the existence of a translation partial geometry is equivalent to the existence of a class of an α-uniform $(n-1)$-spreads in $PG(3n-1, q)$.

P. van den Cruyce: Action of a subgroup A_5 of $PSL(2, q)$ on $PG(2, q)$.
We study the action of a subgroup A_5 of $PSL(2, q)$ on the projective plane $PG(2, q)$, with q odd. In particular, we determine the linear structure induced by the lines of $PG(2, q)$ on the orbits of length 6, 10 and 15 of A_5.

U. Dempwolff: Large cyclic groups in linear groups and translation planes.
Let V be a finite dimensional $GF(q)$-space, $q = p^f$ and $R \leq GL(V)$ such that $V = V_1 \oplus V_2$ is a decomposition into R-spaces such that V_1 is irreducible and R is trivial on V_2. We call such a group 1-irreducible. Irreducible subgroups G of $GL(V)$ are discussed which are generated by 1-irreducible groups R of prime order. A general structure result is given and in particular G is determined if $\dim V \leq 2n$, where $\dim V_1 = n$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

This determines also the groups $X = \langle R_1, R_2 \rangle$, where $R_i (i = 1, 2)$ is 1-irreducible of prime order. These results are further applied to translation planes $\rho = (V(n, q), \pi)$ such that there is a partition $\pi = \Delta \cup \Gamma$, $|\Delta| = q + 1$ and a subgroup $G \leq Aut(\rho)$ which is transitive on Γ and fixes Δ.

J. Doyen: Is there a non-trivial t-design without repeated blocks for $t > 6$?

The following recent remarkable result of Luc Teirlinck was discussed: for every $v \equiv t \pmod{(t + 1)!^{2t+1}}$, there exists a $t - (v, t+1, \lambda)$ design with $\lambda = (t+1)!^{2t+1}$ without repeated blocks. Moreover, for every such v, the set of all $(t+1)$-subsets of a v-set can be partitioned into pairwise disjoint designs having the above parameters. The proof is by induction on t.

D. M. Evans: Homogeneous geometries.

For our purposes, a geometry will consist of a non-empty set together with a closure operation on that set such that the empty set and singletons are closed, and the exchange condition is satisfied. The geometry is degenerate if every subset is closed, and is locally finite if the closure of a finite subset if finite. The geometry is homogeneous if in the automorphism group of the geometry the pointwise stabiliser of any finite dimensional closed subset is transitive on the complement of that subset. I shall sketch a proof (using techniques from finite geometry and coherent configurations) that an infinite, non-degenerate, locally finite, homogeneous geometry is a projective or affine geometry over a finite field. Our methods in fact show that a finite homogeneous geometry (with at least 4 points on a plane) of sufficiently large dimension is a (possibly truncated) projective or affine geometry. Previous proofs of this result have relied on the classification of finite simple groups.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Th. Grundhöfer: Finite and compact disconnected planes.

The projective plane over the p-adic numbers can be written as an inverse limit of finite Hjelmslev planes. More generally, we have Theorem 1: A projective plane P is a compact disconnected plane iff $P = \varprojlim P_n$ with finite incidence structures P_n. Theorem 2 (B. Artmann): Every finite projective plane is a continuous epimorphic image of some compact disconnected plane. Theorem 3 (R. Rink): There are compact disconnected translation planes admitting continuous epimorphisms onto all finite translation planes of fixed order. Theorem 4: There are compact disconnected planes over distributive quasifields not admitting any continuous epimorphism onto a finite projective plane. The planes of Theorem 4 cannot be written as inverse limits of finite Hjelmslev planes or Klingenberg planes.

Ch. Hering: A remark on a theorem of T. G. Ostrom.

Let E be a Klein 4-group contained in the linear translation complement G of a translation plane $\mathfrak{A}$ of finite odd order. Assume that all involutions in E are Baer involutions. By a theorem of Ostrom (Arch. Math. 36 (1981), p. 21), the dimension of $\mathfrak{A}$ over its kernel is divisible by 4 and also, if a and b are two different involutions in E, then b induces a Baer involution on the fixed point subplane of a.
If G does not contain any Klein 4-groups of the type described above, then G has cyclic or quaternion Sylow 2-subgroups or G contains involutory homologies, in which case the subgroup generated by perspectivities in G will provide much information about $\mathfrak{A}$ and G. Therefore it seems important to know if such groups can exist at all. In joint work with H. J. Schaeffer an example was constructed to decide this question. This is a translation plane of order 81 with a translation complement of order 128.

A. Herzer: A synthetic construction of affine chain-geometries.

For r a prime consider $\Pi_0 = PG(r, q)$ as subgeometry of $\Pi = PG(r, q^r)$, namely as fix-structure of the collineation σ given by $(x_0,\dots, x_r) \longrightarrow (x_0^q,\dots, x_r^q)$. A hyperplane H of Π_0 gives Π, Π_0

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

affine structures A, A_o . Let $P_1, \ldots, P_r$ be a spanning set of points of H with $P_i^{\sigma} = P_{i+1}$, indices mod r. We denote the normal rational curves of Π by V_1^r . Through r + 3 points of Π in general position goes exactly one V_1^r . Lemma 1: For any 3 non collinear points Q_1, Q_2, Q_3 of A_o the points $Q_1, Q_2, Q_3, P_1, \ldots, P_r$ are in general position. (Here r prime is crucial). Lemma 2: The trace of the V_1^r through $Q_1, Q_2, Q_3, P_1, \ldots, P_r$ in Π_o is a V_1^r wholly contained in A_o . Theorem: Given the points Q_1, Q_2, Q_3 of A_o we define as chain through these points 1) $L \cup \{\infty\}$ if Q_1, Q_2, Q_3 are contained in the line L of A_o , 2) the trace of V_1^r through $Q_1, Q_2, Q_3, P_1, \ldots, P_r$ in A_o , if Q_1, Q_2, Q_3 are not collinear. Then we have constructed $A(GF(q), GF(q^r))$ in the sense of Benz in a „synthetic" way.

Y. Hiramine: Some classes of translation planes.

We present three classes of translation planes.

(1) A class of translation planes of order q^2 with kernel GF(q) admitting linear autotopism groups of order q ; This class includes the Hall planes, the planes constructed by Narayana Rao-Satyanarayana and the planes constructed by Cohen-Ganley.

(2) A class of translation planes of order q^3 with kernel GF(q) , $q \equiv 1 \pmod 2$, admitting linear autotopism groups with orbits of length 2, $q^3 - 1$ on ℓ_∞ ; This class includes the planes constructed by Suetake and therefore includes the Hering plane of order 27.

D. R. Hughes (with N. Singhi): Partitions and schemes in graphs.

An A-partition is a generalisation of the concept of an association scheme, where A is an arbitrary square matrix. If A is an adjacency matrix of a graph Γ , this leads to a (unique minimal) decomposition of Γ into "local schemes" and to the determination of the eigenvalues of Γ . (An interesting corollary is a simple proof of Block's Lemma for incidence structures.)

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Z. Janko: A new biplane of order 9.

The following result will be presented. Let B be a biplane of order 9 $(k = 11)$ which possesses an automorphism group of order 6. Then B is either known or is isomorphic to a new biplane B_0. The biplane B_0 is self-dual and its full automorphism group H is isomorphic to $Z_2 \times A_4$. The group H has exactly five line (point) orbits on B_0. In addition, B_0 has exactly 44 ovals and the rank of its incidence matrix over $GF(3)$ is 26.

D. Jungnickel: On a theorem of Ganley.

Theorem 1: Let G be an abelian group of order n^2, n even. Then there exists an n-subset D of G satisfying (i) $N = 2D$ is a subgroup of order n of G; (ii) N contains all involutions of G; (iii) D is a system of coset representations of N if and only if $G \cong \mathbb{Z}_4^k$. Example: Take $D = \{0, 1\}^k$ in $\mathbb{Z}_4^k$. Theorem 2: Let D be a relative difference set with parameters $(n, n, n, 1)$ in an abelian group G, where n is even. Then, assuming $0 \in D$, D satisfies the conditions of Thm. 1. Corollary (Ganley's theorem): A relative difference set with parameters $(n, n, n, 1)$, n even, in an abelian group G exists iff n is a power of 2 and $G \cong \mathbb{Z}_4^k$. Remark: note that the examples given above are not relative difference sets, so Thm. 1 is stronger than the corollary.

E. Köhler: The non-existence of some t-designs.

Observation: Simple t-designs $S_\lambda(t, k, v)$ with the following parameters do not exist: $S_7(5, 9, 19)$; $S_4(12, 14, 29)$; $S_3(13, 16, 32)$; $S_5(25, 28, 56)$; $S_8(28, 30, 60)$. Proof: compute the intersection numbers belonging to these parameters using the Mendelsohn-equations.

W. Lempken: The maximal subgroups of J_4.

The following result on the subgroup-structure of the finite simple group J_4 of order $2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11^3 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 43$, which was discovered by Z. Janko in 1975, has been proved. Theorem: Let M be a

Gefördert durch DFG Deutsche Forschungsgemeinschaft

maximal subgroup of J_4 . Then one of the following holds: (I) M is a 2-local of J_4 and isomorphic to one of $Ex(2^{1+12}) * \hat{6}$ Aut (M_{22}) , Spec $(2^{3+12}) \setminus (\Sigma_5 \times L_3(2))$, $E(2^{11}) \cdot M_{24}$, or $E(2^{10}) \cdot L_5(2)$. (II) M is a p-Sylow-normalizer with $p \in \{11, 29, 37, 43\}$. (III) M is isomorphic to $PGL_2(23)$, $P\Gamma L_2(32)$, $P\Sigma U_3(11)$. (IV) M is isomorphic to one of a) A_5^* , A_6^* , A_7, A_8, $L_3(2)$, $L_3(4)^*$; b) $L_2(11)^*$, M_{11}, M_{22}^*, $L_2(23)$; c) $U_3(3)$, where $M \cong X^*$ iff $X \trianglelefteq M \leq$ Aut (X) . Conversely, any subgroup of type (I), (II) or (III) is a maximal subgroup of J_4 .

Remarks. 1) It is still not known if there exist subgroups isomorphic to $U_3(3)$ within J_4 ; nevertheless there is strong evidence for the existence of such subgroups. 2) It seems very likely that case (IV, b) can be omitted in the list of the theorem.

R. A. Liebler: A representation theoretic approach to finite geometries of spherical type.

Generic rings and their geometrically significant subrings are used to study finite geometries of spherical type involving generalized quadrangles with parameters s, t . Aside from buildings and thin cases, the only possible types are F_4 and $B_3 = C_3$. If $s \geq 2$, $t \geq 2$ are the parameters of a generalized quadrangle now known to exist then $s = t = 2$ for type F_4 and s, t are powers of the same prime for type flat $B_3 = C_3$.

S. S. Magliveras: An infinite family of t-designs.

Procedures for constructing t-designs with prescribed automorphism groups are presented. These methods have resulted in the construction of many new, simple t-designs with $t \leq 6$. General questions, conjectures, and current problems are also discussed.

F. Mazzocca: Some remarks on blocking-sets.

A blocking-set preserving bijection between the points of two finite affine or projective planes is proved to be a collineation. Consequently, the blocking-set preserving permutation group on the points of an affine or projective plane is precisely the collineation group of the plane. In general, this property is not true in an arbitrary linear space. Finally, the same problem is investigated for h-blocking-sets in a projective space over a Galois field.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

A. Neumaier: K-geometries and Buildings.

Call a geometry Γ projectively closed if Γ is a connected partial linear space such that every triangle is contained in some (possibly degenerate) projective subplane of Γ. If K is a graph we say that a subgeometry Σ of Γ is a K-set if there is a bijection $\pi : \Sigma \longrightarrow K$ which preserves distances (measured in the incidence graphs). A path $a_0, a_1, \ldots, a_i$ in the incidence graph of Γ is called short if $d(a_0, a_{i-1}) = d(a_1, a_i) = i - 1$. A K-geometry is a projectively closed geometry such that every short path is in some K-set. Theorem. (i) Points and lines of the shadow geometry of a building with respect to any variety form a K-geometry, where K is the corresponding Coxeter graph. (ii) If Γ is a K-geometry all of whose lines are thick, and if K is the Coxeter graph of type $A_{n,1}$, $B_{n,1}$, $B_{n,n}$ or $G_{2,1}^{(n)}$ then Γ is a projective space, a polar space, a dual polar space, or a generalized polygon. Conjecture: If K is a Coxeter graph and Γ is a K-geometry all of whose lines are thick then Γ is the point-line geometry of a building.

S. Norton: The Monstrous Monogram and the Projective Plane.

If one considers the incidence graph of the projective plane of order 3, then the group by considering the 26 nodes as involutary generators which commute unless the nodes are joined (when their product has order 3) has a subgroup of index 2 (consisting of the even words) which has the Monster as a quotient group. Using this one can obtain subgraphs and quotient graphs for many subgroups of the Monster, which in many cases can be proved to yield presentations (using certain extra relations) by means of coset enumeration. These presentations are all of the „fabulous" type, and one can ask whether the Monster is fabulous.

D. Olanda: On {1, 3}-semiaffine planes.

A {1, 3}- semiaffine plane is a linear space with the property that through any point outside a line ℓ there are exactly 1 or 3 lines which do not intersect ℓ. All finite {1, 3}-semiaffine planes are characterized. In particular it turns out apart from a finite number of possible exceptions, any such structure is embeddable in a finite projective plane.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

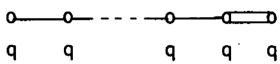
©

T. Oyama: Finite quasifields.

I will give new representations of finite quasifields and construct some quasifields using these representations. Furthermore I will give the way to have new quasifields of order q^4 induced by any quasifield of order q^2.

A. Pasini: Tits' geometries of type C_n.

Let Γ be a residually connected Tit's geometry of rank $n \geq 4$ belonging to the following diagram:



$(1 < q < \infty)$.

Then Γ is a building.

T. Penttila: Tactical Decompositions.

A tactical decomposition of a finite incidence structure is symmetric if (i) the incidence structure has an incidence matrix of rank the number of points, and (ii) the decomposition has the same number of point classes and block classes. A brief description of results concerning symmetric tactical decompositions will be given, with emphasis on decompositions of PG(d, q).

C. Praeger: The Maximal Subgroups of the Finite Symmetric and Alternating Groups.

It follows from the "folklore" and from the Reduction Theorem for primitive permutation groups that a maximal subgroup G of the symmetric group S_n of degree n belongs to one of the following six classes.

1) (stabilizer of a k-set) $S_k \times S_{n-k}$, for some $1 \leq k \leq n$,
2) (stabilizer of a partition) $S_a \text{ wr } S_b$, where $n = ab$, $a < 1$, $b < 1$,
3) (affine) $AGL(d, p)$, where $n = p^d$, p a prime, $d \geq 1$,
) (product) $S_a \text{ wr } S_b$, where $n = a^b$, $a \geq 5$, $b > 1$,

5) (simple diagonal) $T^k . (\mathrm{Out}\ T \times S_k)$, where $n = |T|^{k-1}$, T is a nonabelian simple group, $k > 1$,

6) (almost simple) $T \le G \le \mathrm{Aut}\ T$, where T is a nonabelian simple group, and G is primitive of degree n .

Note that a classification of the groups in class 6 is precisely a classification of all maximal subgroups of all almost simple groups (that is of all groups G such that $T \le G \le \mathrm{Aut}\ T$ for a nonabelian simple group T). We classify which of the groups G in classes 1-5 are maximal in S_n , and also which $G \cap A_n$, for G in classes 1-5, are maximal in A_n . Further, for groups $G = N_{S_n}(T)$ in class 6 , we classify precisely the ones which are not maximal in S_n , and those $N_{A_n}(T)$ which are not maximal in A_n .

This is joint work with Martin Liebeck and Jan Saxl. A section of it has been obtained independently by Michael Aschbacher. The proof relies heavily on factorization theorems for almost simple groups due severally to Christoph Hering, Martin Liebeck, Jan Saxl and myself.

S. Rees: <u>Embeddings for the 2-local geometry for M_{12}</u>.

A 2-local „building-like" geometry can be defined from M_{12} whose points and lines are the groups in the two conjugacy classes of proper subgroups containing a Sylow 2-group. An incident point and line intersect in a Sylow 2-group. Geometrically the points and lines can be recognised as 4-sets and certain 4, 4, 4 partitions of the set of 12 points of the S(5, 6, 12) Steiner system . We get a geometry of 495 points and 495 lines with 3 points on each line and 3 lines on each point.

It is natural (since we look for building-like properties of this geometry) to try to find the geometry as a system of subspaces of a vector space over GF(2) (which will automatically be a module for the group). It is elementary to embed the geometry in 10 and 44 dimensional space. We find the points of the geometry as certain 1-spaces (corresponding to 4, 8 partitions) and lines as 2-spaces in the 10-dimensional module consisting of the set of all even partitions of $\{1,\dots,12\}$. We find points of the geometry as 2-spaces and lines as 1-spaces in the 44-dimensional module consisting of the set of all switching classes of even valency graphs on 12 points which also have an even number of edges (Each line corresponds to the switching class

Gefördert durch DFG Deutsche Forschungsgemeinschaft

of a $K_{4,4,4}$ graph). In both these embeddings a point and a line are incident precisely when the corresponding 1- and 2-spaces are related by inclusion. Using the methods of Ronan and Smith we can find all „good" embeddings of the geometry in vector spaces over GF(2) in which both points and lines appear as 1- or 2-spaces, a point and a line being incident if the corresponding spaces intersect in a 1-space. It seems that every irreducible module for M_{12} over GF(2) supports at least one such embedding.

M. Ronan: Presheaves and Embeddings.
In this talk we considered the special case of an embedding for which points are 1-spaces and lines are 2-spaces of some vector space V . More generally any embedding of a chamber system Δ can be regarded as a presheaf $\mathcal{F}$ on Δ ; in the case above for points p and lines L we have presheaf terms $\mathcal{F}_p$, $\mathcal{F}_L$ and $\mathcal{F}_{p,L}$ for each flag p, L , and we have maps $\mathcal{F}_{p,L} \xrightarrow{\varphi_{p,L}} \mathcal{F}_L$ and $\mathcal{F}_{p,L} \xrightarrow{\varphi_{p,L}} \mathcal{F}_p$.
Defining a boundary operator $\partial : \mathcal{F}_{p,L} \xrightarrow{\varphi_{Lp} - \varphi_{pL}} \mathcal{F}_p \oplus \mathcal{F}_L$, we obtain a chain complex $C_1 \xrightarrow{\partial} C_0$ where $C_1 = \oplus \mathcal{F}_{p,L}$, $C_0 = \oplus \mathcal{F}_p \oplus \mathcal{F}_L$. Theorem: The vector spaces V which admit an embedding $\mathcal{F}$ (which generates V) are precisely the quotients of $H_0(\mathcal{F})$ which admit $\mathcal{F}$. Thus $H_0(\mathcal{F})$ is the universal embedding. Theorem: Suppose we have a set P of points such that every line L meets P in no points or in all but one point p_0 . Given $v_p \in \mathcal{F}_p$ for all $p \in P$ such that if L determines $p_0 \notin P$, then $v_p - v_q \in \mathcal{F}_{p_0}$ $\forall$ p,q on L , then this determines a vector of H^0 (equivalently the dual of H_0) , and all such arise this way. This gives a criterion for determining when $H_0(\mathcal{F}) \neq 0$, and hence when such an $\mathcal{F}$ -embedding exists.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

P. Rowlinson: Cycles in tournaments.

We say that a tournament has property $P_m (m \geq 3)$ if $\exists \ c = c(m) > 0$ such that each arc lies in precisely c m-cycles. We discuss the relations between properties P_3, P_4 and P_5.

J. Saxl: Factorizations of finite simple groups.

A group G is factorizable if $G = AB$ with A, B proper subgroups of G. Such a factorization is maximal if both A, B are maximal in G. In joint work with M. W. Liebeck, Ch. Hering and C. E. Praeger, we determine all maximal factorizations of the finite almost simple groups that give rise to factorizations of the corresponding simple groups. (Here an almost simple group G is a group satisfying $L \lhd G \leq \text{Aut } L$ for some non-abelian simple group L.)

J. J. Seidel: Conference matrices from projective planes of order nine.

From the 7 known affine planes of order 9 we construct 26 nonequivalent conference matrices of order 82.

M. de Soete (with J. A. Thas): Recent results on characterizations of generalized quadrangles.

We introduce the concept of (0, 2)-set in finite generalized quadrangles $S = (P, B, I)$ of order (s, t) i.e. a non-empty subset $K \subset P$ of pairwise non-collinear points such that $|x^{\perp} \cap K| \in \{0, 2\}$, $\forall x \in P \setminus K$. There immediately follows that $|K| = s + 1$ and s is odd. Examples are given in the known models of order (q, q) and $(q - 1, q + 1)$. Using these (0, 2)-sets we obtain characterizations for the generalized quadrangles $T_2^*(0)$ and $Q(4, q)$, q odd. Analogously we consider for generalized quadrangles of order (s, s), s even, $(0, 1, 2, s + 1)$-sets which gives rise to a characterization of $T_2(0)$, q even.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

J. A. Thas: Generalized quadrangles and flocks of cones.

The following construction of generalized quadrangles (GQ) is due to W. M. Kantor. Let G be a group of order s^2t, let $J = \{A, B, \ldots\}$ be a set of $1+t$ subgroups of order s of G, and let $J^* = \{A^*, B^*, \ldots\}$ be a set of $1+t$ subgroups of order st of G with $A \subset A^*$, $B \subset B^*, \ldots$ Define points as (i) the elements of G, (ii) the cosets $A^*g, \ldots$, (iii) a symbol ∞; define lines as (a) the cosets $Ag, \ldots$, (b) the elements $[A], [B], \ldots$. Incidence is defined as follows: a point g of type (i) is incident with the cosets $Ag, Bg, \ldots$; a point A^*g of type (ii) is incident with $[A]$ and with all the cosets Ah contained in it; the point ∞ is incident with all lines $[A], [B], \ldots$ This incidence structure $S(G,J)$ is shown to be a GQ(of order (s, t)) iff (1) $AB \cap C = \{1\}$ for all distinct A, B, C in J and (2) $A^* \cap B = \{1\}$ for all distinct A, B. Now Kantor considers the group $G = \{(\alpha, c, \beta) \parallel c \in F, \alpha, \beta \in F \times F\}$, $F = GF(q)$, with $(\alpha, c, \beta) \cdot (\alpha', c', \beta') = (\alpha+\alpha', c+c'+\beta.\alpha', \beta+\beta')$ and $\beta.\alpha'$ the usual dot product. Let $A_t = \begin{pmatrix} x_t & y_t \\ 0 & z_t \end{pmatrix}$, $t \in F$, with $A_0 = 0$; let $K_t = A_t + A_t^T$; let $A(t) = \{(\alpha, \alpha A_t \alpha^T, \alpha K_t) \parallel \alpha \in F \times F\}$ and let $A(\infty) = \{(0, 0, \beta) \parallel \beta \in F \times F\}$; let $A^*(t) = A(t).C$ with $C = \{(0,c,0) \parallel c \in F\}$. Now put $J = \{A(t) \parallel t \in F \cup \{\infty\}\}$ and $J^* = \{A^*(t) \parallel t \in F \cup \{\infty\}\}$. Then W. M. Kantor showed that for q odd conditions (1) and (2) are satisfied iff $-\det(K_t - K_u)$ is a nonsquare whenever $t \neq u$; S. E. Payne showed that for q even (1) and (2) are satisfied iff $(x_t+x_u)(z_t+z_u)(y_t+y_u)^{-2} \in C_2$, with $C_2 = \{\delta \in F \parallel x^2+x+\delta\}$ is irreducible, whenever $t \neq u$. Using these results they were able to construct new infinite classes of GQ of order (s, t) with $s = t^2$.

Next, consider the quadric cone $K : X_0X_1 = X_2^2$ of $PG(3, q)$. Let π_t be the plane $x_tX_0 + z_tX_1 + y_tX_2 + X_3 = 0$, $t \in GF(q)$, and let $|K \cap \pi_t| = C_t$. Then $\{C_t \parallel t \in GF(q)\}$ is a flock of K (i.e. $\bigcup_t C_t = K - \{\text{vertex}\}$) iff the condition of Kantor or Payne is satisfied according as to q is odd or even. In this way new flocks of cones (and possibly new translation planes) arise from the new GQ, and new GQ arise from the known flocks.

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

F. Timmesfeld: Classifications of locally finite classical Tits' chamber-systems.

A chambersystem C of type M (in the sense of Tits) is classical, if all the rank 2 residues are either generalized digons or classical generalized m_{ij}-gons for some $m_{ij} \geq 3$. Such a chambersystem is called a classical Tits' chambersystem. The diagram Δ of C is defined in the obvious way. The following two theorems were discussed. Theorem 1. Suppose C is a classical locally finite Tits chambersystem with transitive automorphism group G and finite chamber-stabilizer. If $|\Delta_i(c)| \geq 6$ for all $i \in I$, then one obtains a complete (local) list for C and G (including the spherical buildings). Theorem 2: Suppose one has the same hypthesis as above and rank (C) = 3, char(C) = 2. Then one obtains a complete (relatively long) list for G and C.

V. D. Tonchev: Self-orthogonal codes and designs. Embedding of designs by automorphisms.

1. Generalizing a concept for self-orthogonal Steiner system due to Assmus, a method for investigating designs by means of self-orthogonal binary codes is introduced. Using this method and the classification of self-orthogonal codes, the uniqueness of the quasi-symmetric and other designs arising from the Witt systems, as well as the classification and the non-existence of certain quasi-symmetric designs is established, including some counter-examples to the "only if"-part of Hamada's conjecture. 2. A symmetric 2-(78, 22, 6) design possessing the Witt system S(3, 6, 22) as a derived design and invariant under a group of order 168 is constructed. As a by-product, the existence of a quasi-symmetric 2-(56, 16, 6) design is established.

T. van Trung: Two infinite families of 2-designs.

By studying the maximal n-arcs in some classes of symmetric designs we prove the existence of the following infinite families of 2-designs:

$$2\text{-}(v = 2^m(2^{2m-s} + 2^{m-s} - 1),\ b = 2^s(2^m + 1)(2^{2m-s} + 2^{m-s} - 1),\ r = 2^m(2^m + 1),$$
$$k = 2^{2m-s},\ \lambda = 2^m),\ 1 \leq s \leq m$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

and

$$2\text{-}(v = (2^m + 1)^h 2^{m-s} - 2^m \; , \; b = (2^m + 1)^{h+1} - 2^{m+s} - 2^s \; ,$$
$$r = (2^m + 1)^h \; , \; k = (2^m + 1)^{h-1} 2^{m-s} \; ,$$
$$\lambda = (2^m + 1)^{h-1}) \; ,$$

where $(2^m + 1)$ is a prime power, $h \geqq 2$ and $1 \leq s \leq m$.

Berichterstatter: Th. Grundhöfer

Gefördert durch DFG Deutsche Forschungsgemeinschaft

©

Tagungsteilnehmer

Prof. E. F. Assmus

Dept. of Mathematics, Bldg. 14
Lehigh University

Bethlehem, PA 18015/U.S.A.

Prof. E. Bannai

Dept. of Mathematics
Ohio State University

C o l u m b u s, Ohio 43210
U.S.A.

Prof. Th. Beth

Institut für Informatik
Fritz-Erler-Str. 1-3

7500 K a r l s r u h e

Prof. A. Beutelspacher

Fachbereich Mathematik
Saarstr. 21

6500 M a i n z

Dr. J. Bierbrauer

Mathematisches Institut
Im Neuenheimer Feld 288

6900 H e i d e l b e r g

Dr. A. Blokhuis

Univ. of Technology
Dept. of Mathematics
P.O. Box 513

NL-5600 MP E i n d h o v e n

Prof. A. Brouwer

Mathematical Centre
Kruislaan 413

NL-1098 AL A m s t e r d a m

Prof. A. Bruen
c/o Prof. U. Ott

Institut für Geometrie
Pockelstr. 14

3300 B r a u n s c h w e i g

Prof. F. Buekenhout

Dept. de Mathematique CP 216
Université de Bruxelles

1050 B r ü s s e l / Belgien

Prof. P. Cameron

Mathematical Institute
Saint Giles 24

O x f o r d / England

Prof. F. de Clerck

Rijksuniversiteit Gent
Seminarie Meetkunde en
Kombinatoriek
Krijgslaan 281

B-9000 G e n t / Belgien

Prof. A. M. Cohen

Mathematical Centre
Kruislaan 413

NL-1098 AL A m s t e r d a m

Gefördert durch DFG Deutsche Forschungsgemeinschaft ©

Dr. P. van den Cruyce

Dept. de Mathematique CP 216
Universitê de Bruxelles

1050 B r ü s s e l / Belgien

Prof. D. Held

Fachbereich Mathematik
Saarstr. 21

6500 M a i n z

Prof. U. Dempwolff

Fachbereich Mathematik
Erwin-Schrödinger-Str.

6750 K a i s e r s l a u t e r n

Prof. Ch. Hering

Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

7400 T ü b i n g e n

Prof. J. Doyen

Dept. de Mathematique CP 216
Universitê de Bruxelles

1050 B r ü s s e l / Belgien

Prof. A. Herzer

Fachbereich Mathematik
Saarstr. 21

6500 M a i n z

Dr. D. M. Evans

Mathematical Institute
Saint Giles 24

O x f o r d / England

Prof. Y. Hiramine

Osaka University
Toyonaka

O s a k a / Japan

Dr. Th. Grundhöfer

Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

7400 T ü b i n g e n

Prof. D. R. Hughes

Mathematics
Queen Mary College

L o n d o n E1 4NS / England

Dr. G. Hanssens

Rijksuniversiteit Gent
Seminarie Meetkunde en
Kombinatoriek
Krijgslaan 281
B-9000 G e n t / Belgien

Prof. T. Ito

Dept. of Mathematics
Joetsu Univ. of Education
Joetsu, N i i g a t a 943
Japan

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©

Prof. Z. Janko

Mathematisches Institut
Im Neuenheimer Feld 288

6900 H e i d e l b e r g

Prof. D. Jungnickel

Mathematisches Institut
Arndtstr. 2

6300 G i e s s e n

Prof. E. Köhler

Mathematisches Seminar
Bundesstr. 55

2000 H a m b u r g 13

Prof. Ch. Lefevre-Percsy

Dept. de Mathematique CP 216
Université de Bruxelles

1050 B r ü s s e l / Belgien

Dr. W. Lempken

Fachbereich Mathematik
Saarstr. 21

6500 M a i n z

Prof. H. Lenz

Institut für Mathematik II (FU)
Arnimallee 3

1000 B e r l i n 33

Prof. R. A. Liebler

Dept. of Mathematics
Colorado State University
Fort Collins,
C o l o r a d o 80524 /U.S.A.

Prof. J. H. van Lint

Univ. of Technology
Dept. of Mathematics
P.O. Box 513

NL-5600 MP E i n d h o v e n

Prof. D. Livingstone

Dept. of Mathematics
Univ. of Birmingham

B i r m i n g h a m /England

Prof. H. Lüneburg

Fachbereich Mathematik
Erwin-Schrödinger-Str.

6750 K a i s e r s l a u t e rn

Prof. S. S. Magliveras

Computer Science Dept.
Univ. of Nebraska-Lincoln
Lincoln, N e b r a s k a 68588
U.S.A.

Prof. F. Mazzocca

Dipartimento di Matematica
Via Mezzocannone 8

I-80134 N a p o l i /Italien

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Dr. A. Neumaier

Inst. f. Angewandte Mathematik
Univ. Freiburg
Hermann-Herder-Str. 10

7800 F r e i b u r g

Dr. S. Norton

Dept. of Mathematics
16 Mill Lane

C a m b r i d g e CB2 1SB
England

Prof. D. Olanda

Dipartimento di Matematica
Via Mezzocannone 8

I-80134 N a p o l i /Italien

Prof. T. Oyama

Dept. of Mathematics
Osaka Kyoiku University

Tennoji, O s a k a 543
Japan

Prof. A. Pasini

Universitá di Siena
Dipartimento di Matematica
Via del Capitano, 15

I-53100 S i e n a /Italien

Dr. T. Penttila

Mathematical Institute
Saint Giles 24

O x f o r d / England

Prof. N. Percsy

Faculti des Sciences
Universiti de Mons
Avenue Maistrion

D-7000 M o n s / Belgien

Prof. C. Praeger

Dept. of Mathematics
Univ. of Western Australia

N e d l a n d s W.A. 6009
Australien

Dr. Sarah Rees

Univ. of Illinois
Dept. of Mathematics
P. O. Box 4348

C h i c a g o /IL 60680 U.S.A.

Prof. Marialuisa J. de Resmini

Dipartimento di Matematica
Istituto "G. Castelnuovo"
Universitá di Roma "La Sapienza"

I-00185 R o m / Italien

Prof. M. Ronan

Math. Dept.
Univ. of Illinois
P. O. Box 4348

C h i c a g o, IL 60680 /U.S.A.

Prof. P. Rowlinson

Dept. of Mathematics
University of Stirling

S c o t l a n d FK9 4LA
Großbritannien

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©

Dr. J. Saxl

Dept. of Mathematics
University of Cambridge

C a m b r i d g e / England

Prof. V. D. Tonchev

Inst. of Mathematics
Bulgarian Acad. of Sciences
P.O. Box 373

1090 S o f i a /Bulgarien

Prof. J. J. Seidel

Univ. of Technology
Dept. of Mathematics
P.O. Box 513

NL- 5600 MP E i n d h o v e n

Dr. T. van Trung

Institut f. Angewandte Mathematik
Im Neuenheimer Feld 294

6900 H e i d e l b e r g

Dr. M. de Soete

Rijksuniversiteit Gent
Seminarie Meetkunde en
Kombinatoriek
Krijgslaan 281
B-9000 G e n t / Belgien

Dr. K. Vedder

Mathematisches Institut
Arndtstr. 2

6300 G i e s s e n

Prof. J. A. Thas

Rijksuniversiteit Gent
Seminarie Meetkunde en
Kombinatoriek
Krijgslaan 281

B-9000 G e n t / Belgien

Dr. H. A. Wilbrink

Univ. of Technology
Dept. of Mathematics
P.O. Box 513

NL-5600 MP E i n d h o v e n

Prof. F. Timmesfeld

Mathematisches Institut
Arndtstr. 2

6300 G i e s s e n

Gefördert durch
DFG Deutsche Forschungsgemeinschaft
©