MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 2/1986

Modelltheorie

12.1. bis 18.1.1986

Die Modelltheorie-Tagung 1986 wurde von den Herren G. Cherlin (New Brunswick) und A. Prestel (Konstanz) geleitet.

Die beiden Schwerpunkte dieser Arbeitstagung waren "Modelltheorie der algebraischen Gruppen" und "Modelltheorie der Körper". Das erstgenannte Gebiet wurde in ausführlichen Vorträgen systematisch behandelt. Das zweite Gebiet war bereits Gegenstand früherer Modelltheorie-Tagungen; daran anknüpfend wurden neue Forschungsergebnisse vorgestellt.

Neben diesem Schwerpunkt wurden aber auch - wie auf früheren Modelltheorie-Tagungen - andere Themen angesprochen. So wurden modelltheoretische Methoden in der semialgebraischen Geometrie in einem Vortrag über das "L-adische Spektrum" verwendet. Die Frage nach der "existentiellen Äquivalenz geordneter abelscher Gruppen", die für die Modelltheorie bewerteter Körper interessant ist, wurde beantwortet. Auch über exponentielle Algebra wurde gesprochen, und zum Abschluß wurden zur Abwechslung einmal keine Quantoren , sondern die Inversenfunktion in Gruppen eliminiert.

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Vortragsauszüge

I MODEL THEORY OF ALGEBRAIC GROUPS

W.D.Geyer:

Überblick über die klassische Theorie der algebraischen Gruppen

Zur Information der mit Gruppen, die in der Nähe von algebraischen Gruppen "liegen" oder mit ihnen in engerer Beziehung stehen, arbeitenden Logiker wurde zu Beginn dieser Tagung, deren Vormittagssitzungen gruppentheoretischen Themen (Leitung: Cherlin) gewidmet ist, ein Vortrag gewünscht, der einen Überblick über Hauptresultate der klassischen Theorie algebraischer Gruppen gibt und einige Resultate, die mit kommenden Vorträgen in enger Beziehung stehen, mit Beweisen versieht. Folgende Themen wurden im Vortrag "behandelt": Grundlegende Definitionen, lineare Gruppen und abelsche Varietäten, Lie-Algebren und Komplikationen in Char p , Erzeugung einer Gruppe durch eine Familie irreduzibler Untervarietäten, Jordan-Zerlegung, Äquivalenz der Tori und ihrer Charaktergruppen, k-rationale Theorie der Tori (split-anisotrop), Hauptsätze über auflösbare zusammenhängende Gruppen (Fixpunktsatz, Lie-Kolchin, Struktursatz) und nilpotente Gruppen, Hauptsätze über Borel-Gruppen und parabolische Untergruppen, Existenz k-rationaler Tori und Dichtheit von G(k), reduktives Radikal spaltet in Char 0 ab, reduktive und halbeinfache Gruppen, Klassifikation der einfachen Gruppen mod. Isogenie (über alg.abg.Körper) durch Wurzelsystem und Dynkin-Diagramm.

G.Cherlin:

Mekler's Groups

Mekler has used an encoding procedure similar to Ershov's version of the Mal'cev correspondence to encode arbitrary structures in nilpotent groups of class 2 and odd prime exponent p . This would suggest that various projects in the model theory of groups, such as the classification of ω-stable groups, are inherently unfeasible. A closer examination of the noise in Mekler's communication channel leads to a more nuanced assessment.

B. Poizat:

Types Génériques Sur un Groupe Stable

On fait apparaître les types génériques au-dessus d'un groupe stable G en le faisant opérer par translation sur ses types; on observe que le stabilisateur d'un type est (infiniment) définissable et qu'il y a des types pour lesquels ce stabilisateur est d'indice borné: ce sont les types génériques.

Autre définition possible: on constate que tout type peut être translaté au voisinage d'un générique, si bien qu'un générique ne satisfait que des formules φ dont un nombre fini de translaté recouvrent le groupe ($G = a_1 \varphi \vee \ldots \vee a_n \varphi$).

Une telle formule sera dite générique, et les types génériques sont précisément ceux qui ne satisfont que des formules génériques.

Dans le cas des groupes algébriques, ces types sont bien les "points génériques" des géomètres; si bien qu'on a réussi à définir sous la seule hypothèse de stabilité, l'analogue des "ensembles constructibles denses pour la Topologie de Zariski". -

On ne sait pas trop s'il est possible, au moins dans le cas des groupes de rang de Morley fini, de récuperer des analogues d'autres outils algébriques. On observera enfin que tout ce qu'on sait sur les groupes stables, malgré la diversité des résultats obtenus, se ramène à des manipulations de génériques.

G. Cherlin:

Connected Solvable Groups

Let G be a connected, solvable, ω-stable group of finite rank. Results of Zil'ber and Nesin show that it resembles an algebraic group.

Theorem: Let $\bar{G}$ act faithfully and irreducibly on the abelian group A. Let $B \triangleleft \bar{G}$ be abelian and minimal normal. Assume $(\bar{G},A)$ is ω-stable of finite rank. Then the image of B in End A

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generates a field Φ, A is a finite dimensional Φ-space, and $\bar{G} \leq GL(A)$.

Corollary 1: [Zil'ber]. If G (as above) is nonnilpotent then it interprets a field.

Corollary 2: [Nesin]. G' is nilpotent.

G. Cherlin:

Bad Groups

Theorem [Nesin]. In a rank 3 ω-stable simple group G with abelian Borel subgroups all finite subgroups are commutative of odd order. In particular there is no involution. For the proof, one shows that the geometry formed in a counterexample by taking all cosets of the Borel subgroups as lines would be a projective geometry of dimension 3 over a field F which is algebraically closed, yet admits a definite quadratic form.

A. Pillay:

Topologically ω-stable Groups

We consider "first order topological structures", that is L-structures M for some L, such that M has an "explicitly definable" topology, usually assumed Hausdorff and not discrete. For any such M, Th(M) is unstable (in fact has the strict order property). We impose a number of conditions on the definable sets in such M and draw some consequences.

Assumption I. Every definable set is a Boolean combination of definable closed sets.

Assumption II. For every closed definable set $X \subset M$, $\dim(X) < \infty$; where for closed definable X we define $\dim(X) \geq \alpha + 1$ if $\exists$ closed definable $Y \subseteq X$ with no interior in X and $\dim(Y) \geq \alpha$.

Assumption III. Every definable X is a finite disjoint union of clopen (in X) definably connected definable subsets of X.

Any model of Th($\mathbb{R}$, +, ·) or Th($\mathbb{Q}_p$,+,·) or more generally any structure interpretable in such a model, satisfies (I) and (II). (III) fails for ($\mathbb{Q}_p$,+,·) but holds for ($\mathbb{R}$,+,·).

Our conclusions:

(1) If M satisfies (I) then every definable X has interior in its closure, in particular, if X is "homogeneous" then X is locally closed.

(2) If every model of T satisfies (I),(II),(III), then T has prime models over all sets.

(3) If M satisfies (I),(II),(III) and also has a definable topological group structure, then M has an infinite definable abelian subgroup. In particular, if M is 1-dimensional and definably connected then M is abelian.

J. Baldwin:

Free Subgroups of ω-stable Groups

Call an ω-stable group standard if there exists a set A of formulas such that i) every formula is equivalent to a Boolean combination of formulas in A, ii) if $\varphi_1, \varphi_2 \in A$ and $\varphi_1(M) \subsetneq \varphi_2(M)$ then $R_M(\varphi_1) < R_M(\varphi_2)$.

Theorem. If G is a saturated standard group G contains a copy of the free group on two generators in the variety generated by G .

U. Hruschowski:

Groups in Superstable Theories

Let p,q be regular types, let $\phi(x,y,a_1,\ldots,a_n, b_1,\ldots,b_m)$ be a formula witnessing non-orthogonality between them, and assume n is as small as possible. Then $n \leq 3$, and one of the following classical situations occurs: the projective line over a field (n=3), the affine line over a field (n=2), or a group with a regular action on a set (n=1).
This has the following corollary: if G is a simple stable group with a definable subgroup H such that G/H is regular, then $G \cong PGL_2(K)$ for some algebraically closed field K .

Ch. Berline:

Locally Finite Simple Stable Groups

Cal M_c a group which admits the chain condition on centralizers; linear groups and stable groups are M_c-groups. The aim of the lecture is to sketch a proof for the following result of S. Thomas: Every locally finite simple M_c-group is a Chevalley group or a twisted Chevalley group over a locally finite field. A consequence being that every locally finite simple stable group is an algebraic group over the algebraic closure $\bar{\mathbb{F}}_p$ of an $\mathbb{F}_p$.

To derive the consequence one proves that in a Chevalley or a twisted Chevally group G over a field k, k is interpretable. Thus G stable implies k stable and, since k is locally finite, it is algebraically closed; then it suffices to remark that there are no twisted Chevalley groups over $\bar{\mathbb{F}}_p$.

L. van den Dries:

Constructible Groups

Let K be an algebraically closed field of characteristic 0, and let G be a constructible group over K, that is, a constructible subset of some algebraic variety $/K$ equipped with a constructible group operation. Then there is a unique structure of K-algebraic variety on G making G, together with the given group structure, an algebraic group G^a such that the identity map $x \mapsto x: G \to G^a$ is a constructible isomorphism. This result follows from Weil's theorem on group chunks. In characteristic $p > 0$ there is a somewhat weaker result: replace "algebraic group" by "quasi-algebraic group".

S. Thomas:

Homogeneous Geometries

Recently David Evans has discovered an elementary proof of the following

Theorem. (Cherlin, Mills, Neumann, Evans)
An infinite nondegenerate homogeneous locally finite geometry is an affine or projective geometry over a finite field.

The main ideas of his proof were sketched. Then the basic theory of association schemes was developed, and an application to strongly regular graphs was given as an application.

II MODEL THEORY OF FIELDS AND OTHER TOPICS

L. van den Dries:

Dimension Theory for Fields

Let T be the theory of a field K enriched with extra constants and predicates (to single out on K , say, an ordering or valuation). Under certain conditions on T , satisfied for example when K is a local field of characteristic 0 (or the (completion of the) algebraic closure of such a field) one assigns to each definable set $S \subset K^m$ its dimension $\dim S =_{def}$ Krull dimension of Zariski closure of S in $\bar{K}^m$, where $\bar{K}$ = algebraic closure of K . This "dimension" has the following properties:

1) If $f: S \to K^n$ is a definable map, then $\dim f(S) \leq \dim S$.
   (So dimension is invariant under definable bijections)

2) If $A \subset K^m \times K^n = K^{m+n}$ is a definable relation and $A_x = \{y \in K^n : (x,y) \in A\} \subset K^n$ for each $x \in K^m$, then for each $d \in \{-\infty, 0, \ldots, m\}$ the set $A(d) =_{def} \{x \in K^m : \dim A_x = d\}$ is definable, and $\dim(\bigcup_{x \in A(d)} \{x\} \times A_x) = \dim A(d) + d$.

Moreover, in the examples mentioned (where we also have a "definable" strong topology), a nonempty definable set $S \subset K^m$ has the following properties:

3) S contains a nonempty definable subset of the same dimension which is strongly open in S's Zariski closure in $\bar{K}^m$.

4) $\dim(\mathrm{cl}\, S \setminus S) < \dim S$, where $\mathrm{cl}\, S$ is the strong closure of S in K^m .

5) the image of S under a suitable coordinate projection $K^m \to K^d$, where $d = \dim S$, contains a strongly open subset of K^d.

For $\mathbb{Q}_p$ this is joint work with P. Scowcroft.

M. Jarden:

Undecidability and Transcendence Probabilities of Sentences

Let K be an infinite field, finitely generated over its prime field. Denote the absolute Galois group of K by $G(K)$. Fix a positive integer e and let μ be the normalized Haar measure of $G(K)^e$. For each $\underline{\sigma} \in G(K)^e$ let $\tilde{K}(\underline{\sigma})$ be the fixed field of $\underline{\sigma}$ in the algebraic closure $\tilde{K}$ of K. Now denote the elementary language of fields with constant variables for the elements of K by $\mathcal{L}(\text{ring},K)$. For each sentence Θ of $\mathcal{L}(\text{ring},K)$ let $\text{Truth}(\Theta) = \{\underline{\sigma} \in G(K)^e \mid \tilde{K}(\underline{\sigma}) \models \Theta\}$. Then $\text{prob}(\Theta) = \mu(\text{Truth}(\Theta))$ is the probability of Θ to be true among the fields $\tilde{K}(\underline{\sigma})$. It is known that $\text{prob}(\Theta)$ is rational and computable. In particular, the theory $T(K,e) = \{\Theta \mid \text{prob}(\Theta) = 1\}$ is decidable.

S. Shelah suggested to extend the language $\mathcal{L}(\text{ring},K)$ to a language $\mathcal{L}(\text{ring},K,\underline{\Sigma})$ by adding e unary function symbols $\Sigma_1,\dots,\Sigma_e$. The structures for the extended language we consider are $\langle\tilde{K},\underline{\sigma}\rangle$, where $\sigma_i \in G(K)$ interprets Σ_i, $i=1,\dots,e$. Let $T(K,e,\underline{\Sigma})$ be the set of all sentences in $\mathcal{L}(\text{ring},K,\underline{\Sigma})$ for which $\text{prob}(\Theta) = 1$.

Theorem: Suppose that $e \geq 2$. a) $T(K,e,\underline{\Sigma})$ is undecidable. Moreover, with $N=\langle\mathbb{N},+,\cdot\rangle$, $Th(N)$ is interpretable in $T(K,e,\underline{\Sigma})$.

b) For each sentence Θ of $\mathcal{L}(\text{ring},K,\underline{\Sigma})$, $\text{prob}(\Theta)$ is a definable real number (i.e., there exists a formula $\Psi(x,y)$ of N such that $N \models \Psi(x,y)$ if and only if $r < x/y$).

c) For each definable real r, $0 \leq r \leq 1$, there exists a sentence Θ of $\mathcal{L}(\text{ring},K,\underline{\Sigma})$ such that $\text{prob}(\Theta) = r$.

Corollary: There exist sentences Θ of $\mathcal{L}(\text{ring},K,\underline{\Sigma})$ with transcendental probabilities.

Problem: Is $T(K,1,\underline{\Sigma})$ decidable?

Z. Chatzidakis:

<u>Profinite Groups Having IP</u>

<u>Definition</u>. A profinite group G has IP iff every diagram

$$\begin{array}{ccc} & & G \\ & & \downarrow \\ B & \twoheadrightarrow & A \end{array}$$

where B is isomorphic to a finite continuous quotient of G, can be completed by an epi : $G \twoheadrightarrow B$ to a commuting diagram.

We give some results on the algebraic structure of such groups. These results are obtained using techniques of model theory.

Yu.L. Ershov:

<u>RRC-Fields With Small Absolute Galois Group</u>

For a profinite group G let $N(G)_n \rightleftharpoons \cap\{N \mid N$ is an open normal subgroup of G and $|G:N| \leq n\}$; $G_n \rightleftharpoons G/N(G)_n$. G is <u>small</u> iff G_n is finite for every $n \in \omega$; an involutory (i-)group $\mathfrak{G} = \langle G, G^o, I_G \rangle$ is <u>small</u> iff G is small and $X_G = I_G/G$ is finite.

For a field F let X_F be the space of orders of F; $G(F)$ be the absolute Galois group $(G(\tilde{F}/F))$; $\mathfrak{G}(F)$ be the absolute Galois (i-)group.

<u>Proposition</u>. a) If X_F is infinite then $G(F)_2$ is infinite;
b) If F is SAP-field and $|X_F| \geq n$, then $|G(F)_2| \geq 2^n$.

<u>Corollary</u>. If $G(F)$ is small then $\mathfrak{G}(F)$ is small.
<u>Remark</u>. There is an i-group $\mathfrak{G} = \langle G, G^o, I_G \rangle$ such that G is finitely generated (so small) but X_G is infinite.

A finite system of generators $\sigma_1, \ldots, \sigma_n$ for a small i-group $\mathfrak{G}$ is called <u>special</u> iff $X_G = (\{\sigma_1, \ldots, \sigma_n\} \cap I_G)^G$.

Let RRC_n $(n \in \omega)$ be the class of all RRC-fields F of zero characteristic such that $\mathfrak{G}(F)$ has a special system of generators of <u>n</u> elements.

<u>Theorem</u>. The class RRC_n has a decidable elementary theory for all <u>n</u> $\in \omega$.
<u>Remark</u>. Let RRC_n^* be the class of all RRC-fields F of zero characteristic such that $G(F)$ has a system of generators of <u>n</u>

elements; then $RRC_n \subseteq RRC_n^* \subseteq RRC_{2n}$.

For a proof of the theorem we find out some special system of axioms for the elementary theory of every RRC-field F with small G(F).

F. Delon:

Ideals and Types Over Separably Closed Fields

It is known that the theory of separably closed fields of fixed characteristic and degree of imperfection is complete and stable. But no description of the types had been given previously. We give such a description in terms of ideals of polynomial rings in infinitely many variables. This allows us to determine the generic type, to describe forking and to give natural notions of rank, to show that d.o.p. holds, and not f.c.p. and that, in the case of a finite degree of imperfection, the expansion of the language via finitely many constants admits elimination of imaginaries.

M. Fried:

L-Series on a Galois Stratification

The key tool for the elimination of quantifiers for the theory of all finite field extensions of $\mathbb{F}_q$ is the non-regular Cebotarev analogue. L-series provide a (primitive recursive) precise geometric tool for measuring the truth of a statement over all finite extensions of $\mathbb{F}_q$. These L-series are "nearly" rational.

The theory of all unramified extensions of a p-adic completion of a number field generalizes the finite field theory. For this, too, there is a (geometrically precise) primitive recursive elimination of quantifiers. The L-series analogues here are more complicated, and a new concept, invariance, suggests a p-adic version of the Cebotarev theorem. These ideas are based on papers of Denef and Meuser.

K. Schmidt-Göttsch:

Bounds and Definability Over Fields

Let K be a field, R a finitely generated K-algebra, M a finitely generated R-module.

A theorem of v.d.Dries says that the statement "M is a projective R-module" is elementary in the data determining R and M (these data consist of finitely many field elements). For R being just a polynomial ring "projective" is the same as "free". We show that in the general case freeness is not an elementary property which implies that being isomorphic is not elementary.

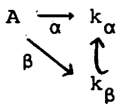
In an algebraic-geometric context we show that rationality is an elementary property for curves over arbitrary fields and - using Castelnuovo's criterion - for surfaces over algebraically closed fields.

The proofs use the relationship between objects defined in the usual way over an internal field and the corresponding internal objects. The (non-)definability statements translate into statements about the (non-)existence of bounds for these constructions over arbitrary fields.

L. Bröcker:

On the L-adic Spectrum

Let L be a disconnected local field of char. 0 . Thus $L \supset \mathbb{Q}_p$ for a prime p . A field k is called formally L-adic if k admits a place $\varphi: k \to L$ such that $\varphi(k)$ is dense in L .
k is called L-adically closed, if k is formally L-adic and admits no formally L-adic proper algebraic extension. Let $\mathcal{A}$ be the category of all k-algebras, where k is formally L-adic.
Fix an embedding $k \to k_\varphi$ into an L-adic closure. Then for $A \in \mathcal{A}$ we define L-space(A) := {homomorphisms $\alpha: A \to k_\alpha$}/ ~ where k_α is L-adically closed, α extends φ and the equivalence relation ~ is generated by all triangles



We provide L-spec(A) with the topology, which is generated by all $B_n(a) := \{\alpha \in \text{L-spec}(A) \mid \alpha(a) \in k_\alpha^{\times n}\}$, $a \in A$, $n \in \mathbb{N}$.

Theorem: For $B \in \mathcal{A}$ suppose $A \xrightarrow{f} B$ is étale. Then f^*:L-spec(B) $\to$ L-spec(A) is a local homeomorphism.

Therefore, as for the real spectrum, L-spec(A) can be provided with a sheaf $\mathcal{N}$ of Nash-functions, (L-spec(A), $\mathcal{N}$) admits an universal property among the L-adically closed ringed spaces. Moreover, one has the following

Idempotency: $(\mathcal{U}, \mathcal{N}|\mathcal{U}) \simeq (\text{L-spec}(N), \mathcal{N}(N))$ for $N = \mathcal{N}(\mathcal{U})$ and $\mathcal{U}$ open and compact in L-spec(A).

(joint work with J. Schinke)

C. Ward Henson:

Ritt's Factorization Theorem for Simple Exponential Sums and its Generalization to Exponential Polynomials

In 1927 Ritt published a factorization theorem for the ring of simple exponential sums, i.e. functions of the form $\Sigma\alpha_j e^{\beta_j z}$ with $\{\alpha_j, \beta_j\} \subseteq \mathbb{C}$. With essentially the same proof, this is also valid for the ring A_n of "logicians" exponential polynomials - this is the E-ring of functions on $\mathbb{C}^n$ containing constants, projections and closed under $+,\cdot$ and exponentiation. (These are the functions defined by terms of the language of $(\mathbb{C}, +, \cdot, \exp)$ in which the variables are among $z_1, \ldots, z_n$).

Theorem: Each $f \in A_n$ ($\neq 0$, not a unit) is a product $p_1(e^{t_1}) \cdot \ldots \cdot p_k(e^{t_k}) \cdot g_1 \cdot \ldots \cdot g_m$ where $g_1, \ldots, g_m$ are primes in A_n, $t_1, \ldots, t_k$ are elements of A_n such that each t_i, t_j pair is independent over $\mathbb{Q}$, $i \neq j$, and $p_1(w), \ldots, p_k(w)$ are polynomials over $\mathbb{C}$. This factorization is unique, up to order and mult. by units.

In the lecture this result was discussed, especially the methods of proof and other uses to which they can be put.

F.-V. Kuhlmann:

Defectless Extensions of Valued Fields

The notion "defectless" can be defined for arbitrary field extensions $(L,v)|(K,v)$ using the notion "valuation basis" (both generalizing the well known valuation-theoretical notions). Both were already studied by Baur and Delon under the name "separated pair" and "separated basis". Consider the properties

(A) $(L,v)|(K,v)$ defectless

(C) For every immediate maximally complete extension (M_K,v) of (K,v) included in a valued extension field of (L,v), L and M_K are linearly disjoint over K

(M) There is a cardinal κ_o s.t. for every cardinal $\kappa \geq \kappa_o$ a κ-saturated elementary extension of (K,v) includes an immediate maximally complete extension of (K,v)

Theorem: (C) $\wedge$ (M) $\Rightarrow$ (A) ((A)$\Rightarrow$(C) is easy to show)

Using this theorem it is shown that there exist algebraically maximal fields (K,v) which are not algebraically complete and violate (M). These fields can't be existentially closed in any maximally complete extension (M_K,v) though we have $v(M_K) = v(K)$ and $\bar{M}_K = \bar{K}$. Hence looking for an analogue in characteristic $p>0$ to the AX-KOCHEN-ERSHOV-theorem, it is not enough to replace the condition "henselian" by "algebraically maximal".

Questions: Does it work with "algebraically complete" or "(M)"?
Does $(K,v) = \mathbb{F}_p((t))$ satisfy (M)?

V.Weispfenning:

Existential Equivalence of Ordered Abelian Groups With Parameters

In 1963, Gurevich proved that any two nontrivial ordered abelian groups ($\mathcal{O}$-groups) satisfy the same existential sentences. Let now G,H be $\mathcal{O}$-groups with a common $\mathcal{O}$-subgroup G_o. We determine, when G and H are existentially equivalent over G_o.

Let $L = \{0,+,-,<\}$ be the language of $\mathcal{O}$-groups, $L' = L \cup \{\underset{n}{\equiv}\}_{n<\omega}$, where $x \underset{n}{\equiv} y \leftrightarrow \exists z(nz=x-y)$. A cc-formula is a conjunction of congruences in L'. With any cc-formula $\varphi(\underline{z}_1,\ldots,\underline{z}_n,\underline{y})$ we associate a new predicate $R_{\varphi(\underline{z}_1,\ldots,\underline{z}_n,\underline{y})}(x_1,\ldots,x_n,\underline{y})$ defined by

$$R_{\varphi(\underline{z}_1,\ldots,\underline{z}_n,\underline{y})}(x_1,\ldots,x_n,\underline{y}) \leftrightarrow \exists \underline{z}_1\ldots\underline{z}_n \left(\bigwedge_{i=1}^{n} 0 < \underline{z}_i < x_i \wedge\right.$$

$\wedge\ \varphi(\underline{z}_1,\ldots,\underline{z}_n,y))$, where $0 < \underline{v} < x$ with $\underline{v}=(v_1,\ldots,v_r)$ stands for $0 < v_1 <\ldots< v_r < x$.

Let L" be L' together with all these predicates.

Theorem 1: Let $0 \neq G,H$ be σ-groups with a common σ-subgroup G_o. Then G and H are existentially equivalent over G_o (as L-structures) iff they are atomically equivalent over G_o as L"-structures.

This is a consequence of the following result.

Theorem 2: The theory of nontrivial σ-groups admits primitive recursive quantifier elimination for positive existential formulas in L".

Corollary 1: Let $0 \neq G \subseteq H$. Then G is existentially closed in H iff $G \underset{\text{pure}}{\subseteq} H$ and for every cc-formula $\varphi(\underline{z}_1,\ldots,\underline{z}_n,\underline{y})$ in L', $a_1,\ldots,a_n,\underline{b}$ in G , if $\exists \underline{z}_1,\ldots,\underline{z}_n \left(\bigwedge_{i=1}^{n} 0 < \underline{z}_i < a_i \wedge \varphi(\underline{z}_1,\ldots,\underline{z}_n,\underline{b})\right)$ holds in H , then it holds in G .

Corollary 2: (R. Transier) Let $0 \neq G \underset{\text{convex}}{\subseteq} H$. Then G is existentially closed in H .

Corollary 3: Let $0 \neq G \subseteq H$ and G dense regular. Then G is existentially closed in H iff $G \underset{\text{pure}}{\subseteq} H$.

Corollary 4: Let $G \subseteq H$, G a $\mathbb{Z}$-group. Then G is existentially closed in H iff G and H have the same smallest positive element.

Corollary 5: Let $H = \prod_{i\in I} G_i$ (Hahn-product over $(I,<)$, all $G_i \neq 0$), $J \subseteq I$ such that $i_o < J$ for some $i_o \in I$. Let $G_i' = G_i$ $(G_i'=0)$ for $i \in I \setminus J$ $(i\in J)$ and put $G = \prod_{i\in I} G_i'$. Then G is existentially closed in H .

These facts can be applied e.g. to prove relative existential closedness for certain Henselian fields.

M. Boffa:

"Elimination of Inverse" in Groups

A group G has EI (Elimination of inverse) if each open formula of $\mathcal{L}_{groups} = \{\cdot, ^{-1}, 1\}$ is equivalent in G to an open formula of $\mathcal{L}_{monoids} = \{\cdot, 1\}$.

Theorem 1: G has EI iff G satisfies a finite disjunction of monoidal identities.

[A monoidal identity is one of the form $\alpha(x,y) = \beta(x,y)$ where $\alpha(x,y)$ and $\beta(x,y)$ are two distinct words over $\{x,y\}$].

Any nilpotent (-by-finite) group has EI (because it already satisfies one monoidal identity). The converse is not true (infinite Burnside groups are counterexamples) but Theorem 1 implies:

Theorem 2: For finitely generated linear groups:
EI $\leftrightarrow$ nilpotent-by-finite.

Questions:

(1) Does Theorem 2 extend to all linear groups?

(2) If a group has EI, does it already satisfy one monoidal identity? (This is true for linear groups).

Berichterstatter: F.-V. Kuhlmann

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