MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1986

Kombinatorik

16.1. bis 25.1.1986

Die Tagung fand unter der Leitung von Herrn Deuber (Bielefeld), Herrn Jungnickel (Giessen) und Herrn Jackson (Waterloo) statt.

Sie hatte zum Ziel einen Überblick über das Gesamtgebiet der Kombinatorik zu geben, die sich in der Gefahr befindet, in eine Anzahl spezialisierter Einzeldisziplinen zu zerfallen. Die Teilnehmer sollten daher insbesondere dazu Gelegenheit haben, sich über die Arbeit von Kollegen auf verwandten Spezialgebieten zu informieren und im Gespräch Probleme zu diskutieren.

Zu diesem Zweck wurde ein international ausgewogener Teilnehmerkreis eingeladen. Wesentliche Gebiete der Kombinatorik (mit Ausnahme der reinen Graphentheorie) waren vertreten:
- Codierungstheorie
- Designtheorie
- Kombinatorische Optimierung
- Kombinatorische Polytope
- Matroidtheorie
- Ramsey- und Partitionstheorie
- Zähltheorie.

Auch zu den Anwendungen wurden enge Bezüge hergestellt. Die Vorträge wurden bewußt nicht zu Teilgebieten zusammengefaßt, um einen möglichst breiten Gedankenaustausch zu ermöglichen, der auch weitgehend erreicht wurde.
Die Vorträge und die angeregten informellen Gespräche wie die Resonanz die die Problemsitzung gefunden hat, zeigen, daß das Ziel der Tagung erfüllt werden konnte. Zu erwähnen sind neben dem allgemein ansprechen-

den Niveau der Vorträge einige länger ausstehende theoretische Fortschritte, die vorgestellt wurden. Ein Proceedings-Band, der als Sondernummer von "Discrete Mathematics" erscheinen wird, soll einen großen Teil dieser Ergebnisse enthalten. Diese Arbeiten sollen bis zum 1. September 1986 bei einem der Veranstalter eingereicht werden.

Vortragsauszüge

R. AHLSWEDE: The maximal sizes of code pairs with specified Hamming distance

The pair $(A,B); A,B \subset \{1,2,\dots,\alpha\}^n$; is called an (n,δ)-system (or constant distance code pair with parameters n,δ), if for the Hamming Distance Function d,

(1) $d(a,b) = \delta$ for all $a \in A, b \in B$.

Let $S_\alpha(n,\delta)$ denote the set of those systems. We consider the function

(2) $M_\alpha(n,\delta) \triangleq \max\{|A||B| : (A,B) \in S_\alpha(n,\delta)\}$

and conjecture the following:

For $n = 1,2,\dots,0 \leq \delta \leq n$

(a) $$M_2(n,\delta) = \max_{d_1+d_2=\delta} (2!2)^{d_1} \binom{n-2d_1}{d_2}$$

(b) $$M_3(n,\delta) = \max_{2\ell+d_2=\delta} (3!3)^{\ell} \binom{n-3\ell}{d} 2^d$$

(c) $$M_\alpha(n,\delta) = \max_{d_1+d_2=\delta} \left(\left\lfloor\frac{\alpha}{2}\right\rfloor \left\lceil\frac{\alpha}{2}\right\rceil\right)^{d_1} \binom{n-d_1}{d_2} (\alpha-1)^{d_2}$$

for all $\alpha \geq 4$.

Presently we have a proof in the cases $\alpha = 2,4,5$.

M. AIGNER: Vertex-Sets that Meet all Maximal Cliques

Let G be a graph on n vertices without isolated points. $S \subseteq V$ is called a cut-set if S nontrivially meets all maximal cliques. Let $f(G) = \min(|S| : S \text{ cut-set})$, $f(n) = \max_G f(G)$. The talk addresses the function $f(G)$ and in particular, the class of graphs for which $f(G) \leq \frac{n}{2}$. Sample results:

(1) $n - C\sqrt{n} \log n \leq f(n) \leq n-\sqrt{n}$ (n large)

(2) G triangulated $\Rightarrow f(G) \leq \frac{n}{2}$ (conjectured by T. Gallai)

(3) G cotriangulated $\Rightarrow f(G) \leq \frac{n}{2}$

(4) G bipartite or cobipartite $\Rightarrow f(G) \leq \frac{n}{2}$

(5) G comparability graph $\Rightarrow f(G) \leq \frac{n}{2}$

(6) Perfect graphs G do not satisfy in general $f(G) \leq \frac{n}{2}$.

Problem: Do incomparability graphs G satisfy $f(G) \leq \frac{n}{2}$?

A. ANDREWS: Partition Problems: Zeilberger, Propp, Carter, Sagan, et al.

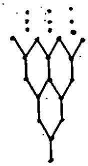
(1) D. Zeilberger asked at the Colloque de Combinatoire Énumérative - V.Q.A.M. 1985 for a proof of the Rogers-Ramanujan identities without use of Jacobi's triple product identity. We give such a proof as follows: Let D_1 be the sequence of polynomials defined by $D_o = D_1 = 1$, and $D_n = D_{n-1} + q^{n-1}D_{n-2}$ for $n > 1$. Let

$$G_n = \prod_{\substack{o<j<n \\ j\equiv 1,4 (\mathrm{mod} 5)}} (1-q^j)^{-1} .$$

Clearly $P_n = D_n/G_n$ is a polynomial in q, and $\lim_{n\to\infty} P_n=1$ is equivalent to the first Rogers-Ramanujan identity. We show

how to combine Schur's representation of D_n with Watson's q-analog of Whipple's theorem to obtain explicit formulae for the P_n. These formulae clearly reveal that the desired limit holds.

(2) J. Propp (U. of Cal.-Berkeley) has recently extended the concept of Ferrers graphs to other lattices besides the rectangular lattice. In many instances the analog of the ordinary partition function has a modular form as generating function. For example, if the hexagonal lattice H is used:



the related generating function found by Propp is

$$\prod_{n=1}^{\infty} \frac{(1+q^{2n-1})}{(1-q^{2n})} = \frac{1}{1-q-q^3+q^6+q^{10}-\dots}$$

(3) B. Sagan communicated the following problem of R. Carter to me: Prove

$$\sum_{i,j=0}^{d} \frac{(-1)^{i+j}\,(x+1)!^2}{(x+1-d+i)!\,(x+1-d+j)!} \sum_{k=\max(i+j-d,0)}^{\min(i,j)} \frac{[(i+j-k)!]^2}{(i-k)!\,(j-k)!} \binom{x+i+j-d}{k} \binom{z-x}{i+j-k} \binom{z-i-j+k}{d-i-j+k}$$

$$= (d!)^2 \binom{x}{d}\binom{x+1}{d}\binom{z+1}{d} .$$

The sum on k is a balanced ${}_4F_3$ hypergeometric series. A transformation of Whipple yielding a new inner sum as a ${}_4F_3$ reduces all three summations to classical sums yielding the desired product of binomial coefficients.

J. BECK: Combinatorial game theory

Our object is to study a large class of combinatorial games including the well-known particular games tic-tac-toe, five-in-a-row and Shannon's switching game. We shall investigate a mysterious and exciting analogy betwen the "Evolution" of some random structures and some combinatorial games. Most of the proofs are based on the so-called "Weight Function Method".

L.T. BILLERA: Piecewise polynomial functions on simplicial complexes

For a d-dimensional simplicial complex $\Delta \subset \mathbb{R}^d$, we define the set of C^r piecewise polynomials on Δ to be

$C^r(\Delta) = F:\Delta \to \mathbb{R} | \; F|_\sigma \in \mathbb{R}[y_1,\ldots,y_d], \forall$ maximal $\sigma \in \Delta$,

F has continuous partial derivatives of all orders $\leqq r\}$

The main problem considered is to find the dimension and a basis for the $\mathbb{R}$-vector space $C^r_m(\Delta) = \{F \in C^r(\Delta) \mid \deg F|_\sigma \leqq m \;\forall\; \sigma\}$.

First, viewing $C^r(\Delta)$ is an $\mathbb{R}$-algebra (with pointwise multiplication) we show that $C^0(\Delta) \approx A_{\Delta/<x_1+\ldots+x_{n-1}>}$ where A_Δ is the face ring of Δ , when $n = |\Delta_0| = f_0$ and Δ_i = set of i-faces of Δ . This leads to proof that $C^0(\Delta)$ is a free R-module, $R = \mathbb{R}[y_1,\ldots,y_d]$, an easy construction for a basis from shellable Δ and a proof that $\dim_{\mathbb{R}} C^0_m(\Delta) = \sum_{j=0}^{d} f_j \binom{m-1}{j}$, $f_j = |\Delta_j|$. By considering localizations and making use of the Quallen-Suslin Theorem, we show for $d=2$ that $C^0(\Delta)$ is free iff Δ is a 2-manifold and $C^1(\Delta)$ is free if Δ is a 2-manifold. Using homological methods, we show, finally, that there is a polynomial $p(x_1,y_1,x_2,y_2,\ldots,x_n,y_n)$ so that if $v_i = (x_i,y_i)$ are the vertex coordinates of Δ and $p(x,y) \neq 0$ then

$\dim C_m^1(\Delta) = \binom{m+z}{z} f_2 - 2(m+1)f_1^o + 3f_o^o$ for a 2-disk Δ, so suggested by Sturm, and conversely we conjecture $p \neq 0$ is a polynomial.

A. BJÖRNER: Combinatorics of f-vectors and Betti numbers

Let $\beta = (\beta_0, \beta_1, \ldots)$ and $f = (f_0, f_1, \ldots)$ be two ultimately vanishing sequences of nonnegative integers. We show that the following are equivalent:

(a) β is the Betti sequence (i.e., $\beta_i = \dim_k \tilde{H}_i(C,k)$) and f is the f-vector (i.e., face-count vector) of some finite simplicial complex,

(b) let $\chi_{k-1} = \sum_{j \geq k} (-1)^{j-k}(f_j - \beta_j), k \geq 0$; then

(i) $\chi_{-1} = 1$, (Euler-Poincaré formula)

(ii) $\partial_k(\chi_k + \beta_k) \leq \chi_{k-1}$, all $k \geq 1$.

Above, $\partial_k(n)$ denotes the number of k-element subsets of the in reverse lexicographic order n first $(k+1)$-subsets of $\mathbb{N}$, for which there is a wellknown explicit formula (cascade form). Also, we show that the following are equivalent:

(a) β is the Betti sequence of some simplicial complex on at most $n+1$ vertices,

(b) β is the f-vector of a Sperner family (antichain) in the Boolean algebra $B_n - \{\emptyset\}$.

In both results, Betti numbers could be computed over a field of arbitrary characteristic.

(Joint work with Gil Kalai, Jerusalem).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

D.M. BRESSOUD: Problems and Results on Tournamented Statistics

Let $M(a_1,\ldots,a_n)$ be the set of words in an n-letter alphabet with a_i appearances ot the letter i . For $\omega \in M(a_1,\ldots,a_n)$, we define

$$z(\omega) = \sum_{i<j} MAJ(\omega_{ij}) \ , \ \omega_{ij} \text{ the 2-letter subword in } i \text{ and } j.$$

This z-statistic has the same generating function as the inversion number or the major index. Can this fact be proven bijectively?

If we specify a permutation $\sigma \in S_n$ and restrict our words to those where the subword consisting of the last occurrence of each letter is σ , then the restricted generating function is

$$(1) \qquad q^{\sum_{i<j} a_j \chi((i,j)\in I_\sigma)} \frac{(q)_{a_1+\ldots+a_n}}{(q)_{a_1-1}\ldots(q)_{a_n-1}} \prod \frac{1}{1-q^{a_{\sigma(1)}+\ldots+a_{\sigma(i)}}} .$$

Can $M(a_1,\ldots,a_n)$ be partitioned in some natural way into subsets indexed by $\sigma \in S_n$ such that (1) is the generating function for the inversion number restricted to the appropriate subset? A solution to this problem would be useful in extending the proof of the q-Dyson theorem to other root systems.

Given a tournament T , we define

$$MAJ_T(\omega) = \sum_{(i,j)\in T} MAJ_{ij}(\omega_{ij}) \ , \text{ where an inversion is } j$$

followed by i .

The generating function for MAJ_T is $\begin{bmatrix} a_1+\ldots+a_n \\ a_1,\ldots a_n \end{bmatrix} F_T\,(q^{a_1},\ldots,q^{a_n})$

where $F_T(x_1) = F_T(x_1,x_2) = 1$ and

$$F_T(x_1,\ldots,x_n) = \sum_r \frac{(1-x_r)}{(1-x_1\ldots x_n)} \prod_{(r,i)\in T} x_i \, F_{T-r}(x_1,\ldots,\hat{x}_r,\ldots,x_n) .$$

Some properties of F_T are given. What more can be said about it?

W. DEUBER: Recent aspects of Ramsey Theory

Starting from Ramsey's theorem in Graph theory we introduce "Graphic systems on projective and affine spaces over GF(q)". It has been shown recently by Frankl, Graham and Rödl after preliminary work by Deuber, Prömel, Rothschild, Voigt that there is a full q-analog of Ramsey theory for graphs.
For several years partitions with arbitrary many classes were studied in Ramsey theory. In fact Prömel and Voigt gave an axiomatic treatment, incorporating the present state of the art. Some highlights are indicated here.

M. DEZA (joint work with K. Fukuda): On bouquets of Matroids and Orientation

The notion of squashed geometry introduced by Deza and Frankl is a common generalization of matroids and permutation geometries. We study different axiomatizations for squashed geometries. Some new classes of squashed geometries, including bouquets of graphic matroids are given. We introduce a notion of orientability of squashed geometries, which arises naturally in our examples. Finally, some future research problems are discussed.

P. FRANKL (joint work with Vojtěch Rödl): Euclidean Ramsey Theory

For a finite set $A \subset R^d$ and $n \geq d$ one defines the (infinite) $|A|$-uniform hypergraph $H(A,n)$ as the collection of those $|A|$-element subsets $\tilde{A} \subset R^n$, such that A and $\tilde{A}$ are congruent (isometric).
A set A is called Ramsey if the chromatic number of $H(A,n)$ (denoted by $\chi(H(A,n))$ tends to infinity as $n \to \infty$.

Equivalently, for every r there exists $n = n_o(A,r)$, so that if the points of R^n are partitioned into r classes, then one of the classes contains a subset $\tilde{A}$ congruent to A.
In 1972 Erdös, Graham, Montgomery, Rothschild, Spencer and Straus proved that the vertex set of every brick is Ramsey and if A is Ramsey then it is spherical (can be embedded into a sphere).
The first open question was obtuse triangles.
Theorem Suppose that A is a non-degenerate simplex, i.e., a set of $d+1$ affinely independent points in R^d. Then for some $\varepsilon=\varepsilon(A) > 0$ we have $\chi(H(A,n)) > (1+\varepsilon)^n$.
A similar result is obtained for bricks. The proofs use extremal set theory.

C.D. GODSIL: Distance Spaces

A distance space (Ω,ρ) consists of a set Ω and a real function ρ on $\Omega\times\Omega$ such that

(a) $\rho(x,y) = \rho(y,x)$

(b) $\rho(y,z) = \rho(x,x)$ iff $y=z$.

Given a polynomial q on $\mathbb{R}$ and x in Ω, we define the zonal polynomial q_x by setting $q_x(y) := q(\rho(x,y))$ for all y in Ω.
We call q_x linear if q is. The real vector space spanned by all products of at most r zonal polynomials is denoted by $\mathrm{Pol}(\Omega,r)$; we also set $\mathrm{Pol}(\Omega)$ equal to $\bigcup_{r\geq 0} \mathrm{Pol}(\Omega,r)$.
If $\Phi\subseteq\Omega$ then $\rho(\Phi) := \{\rho(x,y) \mid x,y \text{ in } \Omega ;\ x\neq y\}$. If Φ is a finite subset of Ω then, for q and h in $\mathrm{Pol}(\Omega)$, $(q,h)_\Phi := \Sigma q(u)h(u)\ (u\in\Phi)$ while (q,h) is the average value of

qh on Ω, with respect to a suitable measure if Ω is infinite.
We call Φ a t-design if $(1,q)_\Phi = (1,q)_\Omega$ for all q in $Pol(\Omega,t)$. Our two basic results are

A : if $|\rho(\Phi)| = d$ then $|\Phi| \leq \dim Pol(\Omega,d)$

B : if Φ is a t-design then $|\Phi| \geq \dim Pol(\Omega,\lfloor\frac{t}{2}\rfloor)$.

(Minor modifications of these results generalize such things as the Frankl-Wilson bounds and the Ray Chaudhuri-Wilson bounds).

C. GREENE: Balanced Shifted Tableaux

We consider tableaux of shifted (but unimodal) shape whose entries are balanced, in a sense introduced earlier by the author and P. Edelman ("Balanced Tableaux", to appear in Advances in Mathematics). This means that the rank of each element in its hook is equal to the height of the hook. If θ denotes a shifted shape (i.e. a unimodal composition of N), let b_θ denote the number of balanced tableaux of shape θ .
The main result (proved by a rather complicated but nonetheless bijective argument) is that $b_\theta = f_{\theta^{**}}$, where f denotes the number of standard tableaux of shape λ and θ^{**} denotes the partition obtained by arranging the parts of θ in nonincreasing order.

M. GRÖTSCHEL: Combinatorial Problems in Data Analysis

Many problems in data analysis can be phrased as follows: Given k binary relations on an n-set, find within a specified class ℓ of binary relations on this set one binary relation which represents the given relation (in a sense that can be made precise) best. We determine the computational complexity of this problem for a natural class of linear and quadratic objective functions and for all those classes ℓ of binary relations which are defined by choosing any subset of the following properties: reflexive, symmetric, anti-symmetric, total and transitive. It turns out that all - for practical purposes - interesting problems arising this way are NP-hard. We have investigated (one of the most important, NP-hard special cases of this class of problems) the so-called clustering problem from the viewpoint of polyhedral combinatorics and determined large classes of facets of the associated polytape. These results (joint work with Yoshiko Wakabayashi) and their use in a cutting plane algorithm will also be reported.

A. HAJNAL: <u>A remark on partition relations for infinite ordinals with an application to a finite problem</u>

This is a joint work of J. Baumgartner and the lecturer.
The details will appear in the Proceedings of the 1985 Arcata conference "Application of Mathematical Logic to Finite Combinatorics".

Th. 1 $CH \Rightarrow \omega_1^2 \not\rightarrow (\omega, \omega, 4)^2$.

Th. 2 $\omega_1^2 \rightarrow (\omega, \omega, 3, 3)^2$.

Corollary: (Folkman) There is a finite graph G with $K_4 \not\subset G$ and $G \rightarrow (K_3)_2^2$.

H. HARBORTH: "<u>Steinhaus triangles</u>"

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1 1 1 1 1 0 1
 0 0 0 0 1 1
  0 0 0 1 0
   0 0 1 1
    0 1 0
     1 1
      0
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A Steinhaus triangle has a binary word of length n as its first row, and then under each pair of equal or opposite digits we write 0 or 1, respectively. In 1972 I have proved that for $n \equiv 0$ or 3 (mod4) there exist first rows, such that as much zeros as ones occur in the triangle. - If now for $m \geq 3$ Steinhaus triangles mod m are considered, then we have to distinguish two cases:
Under each pair either their sum (mod m), or their absolute difference (mod m) is written. - Together with M. Bartsch we here give in both cases partial results to the following questions:
1) How often can a fixed digit occur in maximum? 2) what is the maximum number of digits $\neq 0$? 3) Is it possible, that the number of zeros equals the number of non-zero digits? 4) Is it possible, that all m digits occur in the same number?

W. IMRICH (joint work with N.Seifter):

On groups and graphs with linear growth

A finitely generated infinite group has linear growth if the number $f(n)$ of group elements representable by a word of at most length n in the generators is bounded by a linear polynomial in n. It is known that such a group has a subgroup isomorphic to Z of finite index. In particular, Wilkie and van den Dries showed that the existence of a $k > 0$ with $f(k) - f(k-1) \leq k$ implies linear growth and that, setting $c = f(k) - f(k-1)$, there always is a subgroup isomorphic to Z of index $\leq \frac{c^4}{2}$. We show that the sharp bound is c.

The proof uses properties of groups and graphs with two ends.

D.M. JACKSON: Counting cycles in permutations by group characters, with an application to a topological problem

Let γ be an (integer) partition. We write $\gamma = \langle \underset{\sim}{a} \rangle$ if γ has a_i parts of size i, $i=1,2,\ldots$, where $\underset{\sim}{a} = (a_1, a_2, \ldots)$. Let b_γ denote the conjugacy class of S_N consisting of all permutations of cycle type γ. Let e_k^γ be the number of permutations in S_N which have k cycles, and which may be expressed in the form $(12\ldots N)\sigma$ for some $\sigma \in b_\gamma$ (γ a partition of N). We show that

$$z + \sum_{k,N\geq 1} \sum_{\langle \underset{\sim}{a} \rangle \vdash N} e_k^{\langle \underset{\sim}{a} \rangle} \underset{\sim}{w}^{\underset{\sim}{a}} \frac{u^N}{N!} \varphi_z z^k = z \exp\left\{ \sum_{i\geq 1} \frac{1}{i} \{(1+z)^i - z^i\} u^i w_i \right\}.$$

The special case $\langle \underset{\sim}{a} \rangle = (2^n)$, corresponding to the conjugacy class of fixed point free involutions, arises in connexion with a topological problem. The mapping φ_z is defined by $\varphi_z \binom{z}{k} = z^k$. The result is obtained by using combinatorial properties of the group algebra $\mathbb{C}S_N$ of S_N over $\mathbb{C}$, and properties of the characters of S_N.

The above equation can be given a purely combinatorial interpretation. The righthand side involves cycles whose elements can be distinguished Since $[x^m]\varphi_z z^n$ = number of ordered partitions of an n-set into m non-empty sets, the left hand side involves ordered partitions of cycles of permutations. The above equation states that these sets are of the same size. It would be of interest to obtain a bijective proof of this fact.

D. JUNGNICKEL (joint work with S.A.Vanstone): On a series of resolvable 3-designs

We show that the necessary condition $v \equiv 0 \mod 4$ for the existence of a resolvable 3-design $S_3(3,4;v)$ is also sufficient. The proof uses 1-factorizations of complete graphs. This result yields the first triple (t,k,λ) with $t \geq 3$ for which the necessary conditions for resolvable t-design are known to be sufficient. (To appear in J.Comb.Th. (A)).

D.J. KLEITMAN (joint work with R. Fellows): Radius and Diameter in Euclidean Lattices

Let $a_1,...,a_n$ be integers, with $1 \leq a_i$ for all i, and consider the integer coordinate points in n dimensional space that lie in the range $0 \leq x \leq a_i$.

We measure distance between two such points in the "Manhatten" sense, as the sum of their coordinate distances. Erdös asked: for what values of r is the set of such points within radius r of some integer coordinate origin point x, of maximum cardinality among point sets of diameter $2r$.

The answer for the n-cube (all $a_i=1$) is known, and the condition is:

$$2r < \Sigma\, [(a_i+1)/2]$$

We believe that this is necessary, when all a_i are odd.

E. KÖHLER: <u>Komplizierte Graphen</u>

Sei $\mathbb{G}$:= die Menge (von Isomorphieklassen) aller endlicher Graphen.

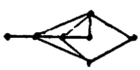
Für $\mathbb{K} \subseteq \mathbb{G}$ definiere man: $\mathbb{K} \leq \mathbb{G} :\Leftrightarrow \bigwedge_{K\in\mathbb{K}} \bigwedge_{e\in K} (K-e) \in \mathbb{K}$.

Sei nun $\mathbb{K} \leq \mathbb{G}$ fest gewählt, und man habe für $n \in \mathbb{N}$ die Menge $K_n := \{K \in \mathbb{K} \mid |K| = n\}$ schon in $r(n)$-viele Klassen $R_1^{(n)}, \ldots, R_{r(n)}^{(n)}$ eingeteilt. Dann definiere man $\varphi : K_{n+1} \rightarrow \{0,1,2,\ldots,n+1\}^{r(n)}$ vermöge $\varphi(K) := (k_1,\ldots,k_{r(n)})$ mit $k_i := |\{K-e \mid e\in K \;,\; (K-e)\in R_i^{(n)}\}|$, und für $K,K' \in K_{n+1}$ gelte $K \sim K' :\Leftrightarrow \varphi(K) = \varphi(K')$. Nun setze man fest:

$$r(n+1) := |K_{n+1}/_\sim| \quad \text{und} \quad K_{n+1}/_\sim =: \bigcup_{i=1}^{r(n+1)} R_i^{(n+1)} \;, \text{ wobei für}$$

$i < j$, $K \in R_i^{(n+1)}$ und $K^* \in R_j^{(n+1)}$ genau dann richtig sei, wenn $k_1+\ldots+k_{r(n)}$ für $\varphi(K)$ kleiner als $k_1+\ldots+k_{r(n)}$ für $\varphi(K^*)$ ist, und im Falle der Gleichheit beider Summen $\varphi(K)$ lexikographisch größer als $\varphi(K^*)$ ist.

Damit ist in jedem $\mathbb{K} \leq \mathbb{G}$ (insbesondere also auch im Falle $\mathbb{K}=\mathbb{G}$) eine "vernünftige" lineare Ordnung auf $\mathbb{K}$ definiert, und die Elemente von $R_{r(i)}^{(i)}$ heißen dann die "komplizierten" Graphen von $\mathbb{K}$.

Für $\mathbb{K}$ = Menge der Isomorphieklassen von Wäldern bestehen dann z.B. $R_{r(7)}^{(7)}$ aus dem Graphen  , und für $\mathbb{K} = \mathbb{G}$ besteht $R_{r(7)}^{(7)}$ aus dem Graphen  und seinem Komplement.

B. KORTE (joint work with L. Lovász): Convex hulls of convex geometries

Matroids behave "nicely" with linear objective functions in two respects: It is easy to optimize a linear objective function (by the greedy algorithm) over matroids and there exists an elegant characterization of the convex hull of all feasible sets of a matroid by linear inequalities.

Although greedoids can be considered as straightforward relaxations of matroids, they do not enjoy these nice linear properties in general. It is NP-hard to optimize a linear objective function over the bases of an arbitrary greedoid given by a feasibility oracle. Thus, there is no hope to obtain a nice characterization of the convex hull of the feasible sets of a greedoid.

However, we are able to obtain non-trivial linear characterizations of the polytope of feasible sets for certain classes of convex geometries. In one case we got a surprising result: The cone of feasible sets has an easy linear characterization, but a membership test for the associated polytope is already NP-complete.

H. LENZ (joint work with D. Jungnickel): Minimal linear spaces

A decent linear space DLS(k,v) is a finite incidence structure of v points and $b > 1$ lines, with maximal line size k, such that no lines of size 2 occur.

Problem 1: Which pairs (k,v) are possible, in particular which is the smallest possible $v=v_k$ for given k?

Problem 2: Which is the minimal line number $b=b_k$ for given k?

Result

k	2n-1	6n-2	6n-4	6	6n (n>1)
v_k	4n-1	12n-2	12n-4	15	12n+4
b_k	$2n^2-n+1$	$18n^2-7n+1$	$18n^2-15n+1$	25	$18n^2+9n-1$

Moreover, a DLS(v,k) with $b=b_k$ has $v=v_k$.
The proofs use elementary counting and factorizations of graphs.

P. LEROUX (joint work with G.Viennot): The combinatorial method of separation of variables

Classicaly, the differential equation $J' = g(J)f(t)$ is solved by separating the variables: writing $(1/g(y))dy = f(t)dt$ and integrating on both sides, the solution is given by $J = \Phi^{<-1>}(\int_0^t f(x)dx)$, where $\Phi^{<-1>}$ is the inverse function of $\Phi(y) = \int (1/g(y)dy$.

This method can be given a combinatorial interpretation and then extended to differential equations of the form $y' = \Sigma_j G_j(y)F_j(t)$. The combinatorial approach is that of "L-species", that is of types of structures that can be constructed on linearly ordered sets. The connection with analysis is via the exponential generating function

$$A(t) = \Sigma_{n\geq 0} |A[n]| t^n/n !$$

of an L-species A , where |A[n]| is the number of A-structures on any linearly ordered n-element set. It is then possible to lift differential and integral calculus, including differential equations, to the combinatorial level, and to give enplicit solutions using standard bijective methods.

H.J. PRÖMEL: Asymptotic enumeration and 0-1 lows in graph theory

It is obvious that every graph which contains a clique of size $\ell+1$ is not ℓ-colorable, and hence has chromatic number at least $\ell+1$. Also it is well known that there are $K_{\ell+1}$ - free graphs of arbitrary large chromatic number. In contrast to this we show that 'almost all' labeled $K_{\ell+1}$ - free graphs are ℓ-colorable for any $\ell \geq 2$, that is to say that if $L_\ell(n)$ denotes the number of ℓ-colorable graphs on $\{0,\ldots,n-1\}$ and $S_\ell(n)$ denotes the number of $K_{\ell+1}$ - free graphs on $\{0,\ldots,n-1\}$, then $\lim_{n\to\infty} (L_\ell(n)/S_\ell(n)) = 1$. In addition to this we derive detailed information about the structure of almost all $K_{\ell+1}$ - free graphs. We use this to prove first-order 0-1 laws for the classes $S(\ell)$ of $K_{\ell+1}$ - free graphs. This turns out to be the final step in proving that any infinite class of finite labeled undirected graphs having amalgams and being closed under induced subgraphs and isomorphisms has a first order 0-1 law. These results are from Ph.G. Kolaitis, H.J. Prömel, B.L. Rothschild, Bull.Amer. Soc. (NS)13(1985) 160-162. The corresponding results for unlabeled graphs can be obtained by using the theorem that whenever C is an infinite class of finite labeled graphs closed under subgraphs and isomorphisms and $C(n) \geq 2^{dn^2}$ for some $d>0$ and all sufficiently large n (where $C(n)$ denotes the number of elements in C on n vertices), then the average size of the automorphism groups of graphs in C is asymptotically equal to 1.

A. RECSKI: Duality of graphs and what the engineers think about it ...

There are two classical engineering applications of graphs and matroids in statics and in electric network theory.

Electric engineers have developed a voltage-current symmetry, called duality, about 80-100 years ago. Most people (engineers and mathematicians alike) believe that this duality and the mathematical one (with orthogonal vector spaces, matroid theory etc) are essential the same, apart from differences in terminology. Masao IRI (University of Tokyo) and the present author have shown that this is not the case; hence there are two different voltage-current symmetries, bot related to the duality of planar graphs ("What does duality really mean?", Internat. Journal Circuit Theory & Applications 8 (1980) 317-324).

Civil engineers have also developed a "duality" between static and dynamic properties about 120-150 years ago. However, the exact relation between this "duality" and the mathematical one does not seem to be fully clarified.

In the present talk some observations towards this direction are presented.

I. RIVAL: Is there a diagram invariant?

There are no known examples of a nontrivial, ordertheoretical 'diagram invariant". Thus, while many familiar order-theoretical properties such as length, width, order dimension, jump number, fixed point property are preserved among all orientations of a given comparability graph, none at all of these need be preserved among all orientatious of a given covering graph (of an ordered set).

Indeed, there is some evidence to support the bold conjecture that there is no such invariant at all!

THEOREM (M. Pozet and I. Rival) Let (*) be a property about non-empty, finite, ordered sets. If (*) is a diagram invariant which is closed under order retract and direct products then either (i) (*) holds for all finite ordered sets or, (ii) (*) holds just for all connected, finite ordered sets or, (iii) (*) holds just for all finite antichains or, (iv) (*) holds just for singletons.

R.W. ROBINSON: Counting labeled degree-restricted digraphs

Counting labeled digraphs on p vertices having a specified set of in-degrees, or having all indegrees exceed a fixed lower bound, is easy. The same is true for out-degrees in place of in-degrees. However, when in-degrees and out-degrees are both restricted, counting is harder and seems to necessitate some form of inversion.

Labeled digraphs with every in- and out-degree $\geq \delta$ are counted by direct applications of inclusion-exclusion for $\delta = 1$ and 2. The resulting expressions require $O(p^4)$ and $O(p^8)$ arithmetic operations to calculate the numbers on up to p vertices. It is shown how to transform these expressions so as to accomplish the calculations in time $O(p^2)$ and $O(p^5)$, respectively.

In joint work with R.C.Read, labeled pseudodigraphs with given in- and out-degrees are counted by an intransitive extension to the classical Redfield-Read superposition theorem. A suitable linear transformation of variables performs an inversion which yields the corresponding numbers of digraphs (without loops). This method is polynomial when all degrees are $\leq d$ for fixed d; however the computational complexity increases rapidly as a function of d.

A. SCHRIJVER: Disjoint paths in planar graphs

We discuss the following theorem (conjectured by K. Mehlhorn, and proved with C.van Hoesch): Let $G=(V,E)$ be a planar graph, embedded in $\mathbb{R}^2$, let O be its outer face, let I be some fixed inner face, and let $C_1,\dots,C_h$ be curves in $\mathbb{R}^2 \setminus (I \cup O)$, each connecting two points of G on the boundary of $I \cup O$, so that for each point vor G we have: $\deg_G(r)$+(number of curves C_i having r as end point) is even. Then there exists pairwise edge-disjoint paths $P_1,\dots,P_h$ in G so that P_i is homotopic with C_i in $\mathbb{R}^2 \setminus (I \cup O)$, for $i=1,\dots,h$ if and only if for each dual path Q from $I \cup O$ to $I \cup O$ one has number of edges intersected by $Q \geq \sum_{i=1}^{h}$ (number of curves C_i necessarily intersected by Q). The proof yields a polynomial-time algorithm.

J. SPENCER (joint work with L. Lovász, K. Vesztergombi): Descrepancy, linear discrepancy, hereditary discrepancy

The discrepancy of a family $Q \subseteq Z^{\Omega}$ is a measure of how well Ω may be Red-Blue-colored so that every $A \in Q$ has nearly $|A|/2$ red points. Hereditary discrepancy is the maximum discrepancy of the restriction of Q to any $\Omega' \subset \Omega$. Linear discrepancy measures how well, given weights $p_x \in [0,1]$ for each $x \in \Omega$, Q may be Red-Blue colored so that every $A \in Q$ has nearly $\sum_{x \in Q} p_x$ red points. These notions extend naturally to matrices and simultaneous round off problems.

D. STANTON: t-designs and classical association schemes

Given a finite ranked poset P and a level L_n of P, a classical t-design Y is a subset of L_n, such that $|\{y \in Y : \alpha \subset y\}|$ is independent of α, $\mathrm{rank}(\alpha) = t$. There is also a notion of a t-design for a Q-polynomial association scheme due to Delsarte. For the eight infinite families of association schemes X there are associated posets P. An equivalence of these two definitions is shown. All that is necessary is the known decomposition of the permutation representation of the automorphism group of P. These harmonics are the eigenspaces of the association scheme X.

V. STREHL: Jacobi polynomials: A rational approximation of Jacobi's generating function

A combinatorial model for the Jacobi-polynomials has been introduced by Foata / Leroux (Proc. AMS 1983) in order to give a purely combinatorial proof of Jacobi's generating function. The combinatorial operations "reduction" and "compression" lead to the notion of "order" for these Jacobi-configurations. The (exponential) generating function for Jacobi-configurations of bounded order is given, which turns out to be a rational function involving matching polynomials (indeed: variants of the Tschebycheff-polynomials). The essential step in the (inductive) proof consists in doubling matching configurations combinatorially.

J.H. VAN LINT: Neighbourhood regular graphs

A neighbourhood regular graph (or Γ-Δ-regular graph) is a graph g such that for every vertex x the induced subgraph on the neighbours $\Gamma(x)$ resp. the non-neighbours $\Delta(x)$ is regular. This idea was suggested by Seidel and in 1979 Godsil and McKay (Proc. Australian Comb. Conf. Newcastle 1979, Springer Verlag) found several properties of Γ-Δ-regular graphs and a number of non-existence results. What about examples? By excluding strongly regular graphs and graphs G for which G or $\bar{G}$ are disconnected the "non-trivial" cases are left to study. The only examples known so far were a graph on 4 vertices, one on 8, two on 28, and two on 32 vertices.

May student A. Kloks has recently found an infinite sequence of Γ-Δ-regular graphs. The construction is a follows. First, a symmetic conference matrix C of size p^2+1 is constructed via a net in the affine plane of order p. With the aid of a regular subgraph of the corresponding strongly regular graph, C is switched to a regular symmetic conference matrix $\tilde{C}$.

Let $P := \begin{pmatrix} I+\tilde{C} & I-\tilde{C} \\ I-\tilde{C} & I+\tilde{C} \end{pmatrix}$, $Q := \begin{pmatrix} I+\tilde{C} & -I+\tilde{C} \\ -I+\tilde{C} & I+\tilde{C} \end{pmatrix}$. Then

$\bar{N} := \begin{pmatrix} P & Q \\ Q & -P \end{pmatrix}$ is a Hadamard matrix with the property

that $N := \bar{N} - \begin{pmatrix} I & O \\ O & -I \end{pmatrix}$ is the $(0, \pm 1)$ - adjacency matrix

of a Γ-Δ-regular graph of order $4(p^2+1)$.

S.A. VANSTONE (joint work with D. Jungnickel): Hyperfactorizations and t-designs

A hyperfactorization of index λ and order $2n$ is a collection F of 1-factors of K_{2n} with the property that any two disjoint edges are contained in precisely λ members of F. When $\lambda = 1$ these are the 2-(2,2n) partition systems of P.Cameron. Hyperfactorizations seem difficult to construct. Apart from trivial examples there is only one infinite class known and two sporadic examples. They are of interest because they can be used to construct 5-designs.

G. VIENNOT (joint work with M. Desainte-Catherine): Bijections for Young tableaux enumeration

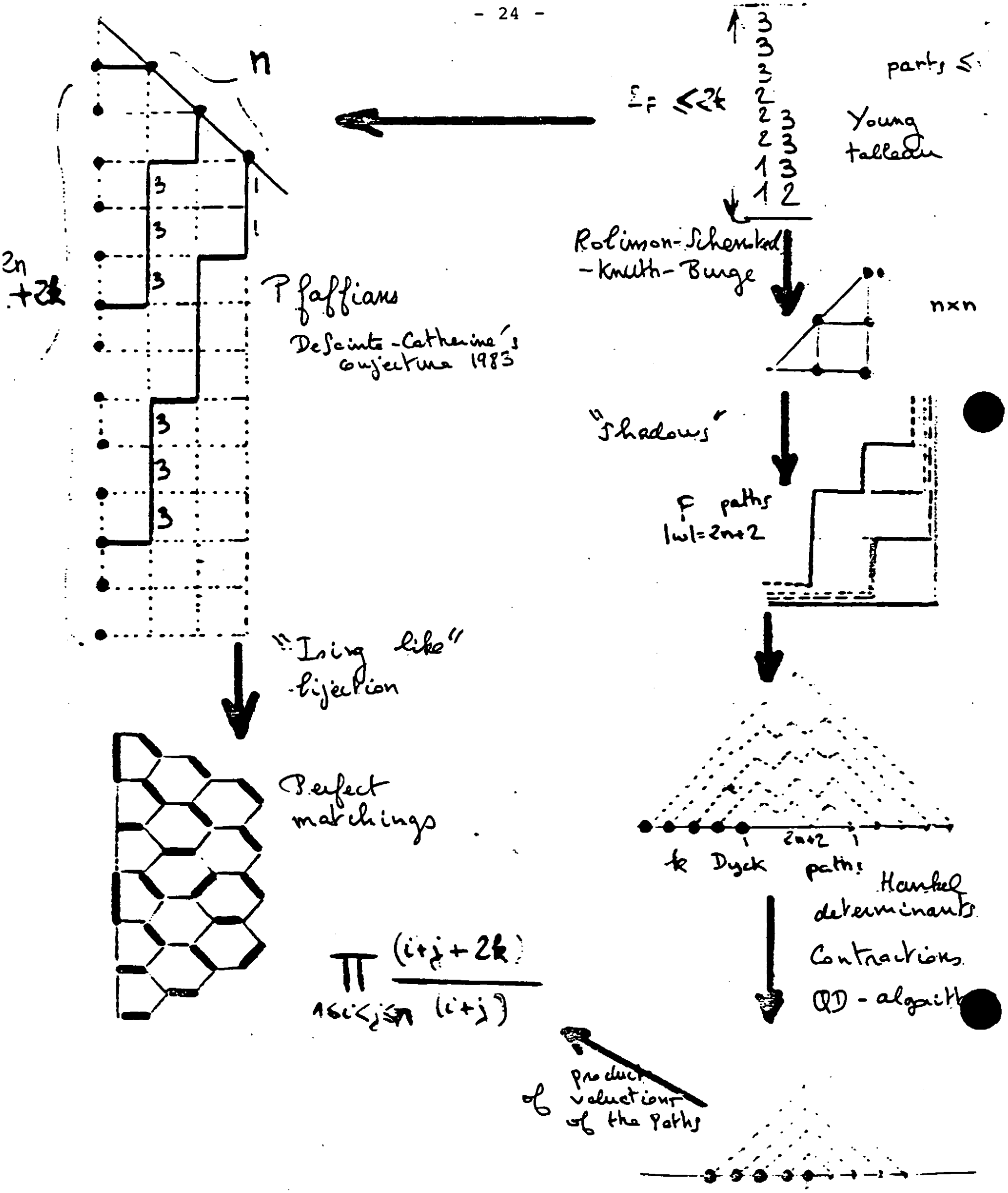
We consider the number $a_{n,k}$ of Young tableaux, strictly increasing in rows, weakly increasing in columns, with entries in $\{1,2,...,n\}$, having at most $2k$ rows, and each column has an even number of cells. We prove that

$$a_{n,k} = \prod_{1 \leq i < j \leq n} \frac{i+j+2k}{i+j} .$$

The proof is in steps:

1) Bijection between Young tableaux and certain configuration of points using Burge's version of Knuth's extension of Robinson-Schensted algorithm.
2) Bijection with configurations of Dyck paths using Viennot's geometric version of Robinson-Schensted with the "shadows".
3) Applying Gessel-Viennot methodology with determinants and non-crossing configurations of paths.
4) Applying the combinatorial theory of orthogonal polynomials and the quotient-difference algorithm (in Padé approximants theory) with weighted Dyck paths.

The numbers $a_{r,k}$ also enumerate perfect matchings of graphs formed with pentagons and hexagons - in relation with Pfaffians and Ising, model -. This solves Myriam Desainte-Catherine's conjecture (Ste. Croix-aux-Mines 1983).


n
2n +2k
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Young tableau
Robinson-Schensted-Knuth-Burge
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"Shadows"
F paths
|w|=2n+2
"Ising like" bijection
Perfect matchings
$\prod_{1\le i<j\le n} \frac{(i+j+2k)}{(i+j)}$
k Dyck paths
Hankel determinants
Contractions
QD - algorith
product of valuations of the paths

B. VOIGT: On discrepancies of 0·1 sequences

Let λ be a probability and let $\alpha = (\alpha(0), \alpha(1), \ldots)$ be an infinite 0-1 sequence. We put $\lambda = \lambda(1)$, $(1-\lambda) = \lambda(0)$ and for longer (finite) sequences $(a_0, \ldots, a_{s-1})$ we put $\lambda(a_0, \ldots, a_{s-1}) = \pi \lambda(a_i)$, i.e., we take the product probability. The s-discrepancy of α is defined by

$$D^s(\lambda,\alpha) := \underset{\vec{a}\in\{0,1\}^s}{\mathrm{MAX}} \ \underset{N}{\mathrm{Limsup}} \ N \left| \frac{\#\ \vec{a}\text{-subsequ. in } \alpha \rceil N}{\binom{N}{s}} - \lambda(\vec{a}) \right| .$$

In a recent article, Kirschenhofer and Tichy (J.Number Theory 1985) showed that $\underset{\alpha}{\mathrm{INF}} \ \underset{\lambda}{\mathrm{SUP}} \ D(\lambda\alpha) = \frac{s}{2}$. They also put the question to determine $D(\lambda) = \underset{\alpha}{\mathrm{INF}} \ D(\lambda\alpha)$ explicitly.

We can show that

Result: For irrational $\lambda > 1/2$ it follows that

$$D^s(\lambda) = \binom{s}{2}(\lambda^{s-1} - \lambda^s) + \frac{s}{2}\lambda^{s-1} .$$

R. WILLE: A construction method for finite distributive lattices

A complete tolerance relation Θ on a complete lattice L yields a decomposition of L understandable as an atlas of subintervals which themselves form a complete lattice L/Θ .

For a finite distributive lattice L_o with a ranked ordered set of join-irreducible elements, a specific sequence of tolerance relations Θ_{i-1} of L_{i-1} with $L_i := L_{i-1}/\Theta_{i-1}$ $(1 \leq i \leq n)$ gives rise to a construction method for L_o : this starts with L_n and iteratively builds up $L_{n-1}, L_{n-2}, \ldots, L_1, L_o$ according to the ranking of the join-irreducible elements of L_o . The method can be demonstrated by constructing the free distributive lattices over n generators $(n \leq 5)$. For counting elements the following formula is useful (Θ tolerance relation of a finite lattice L and $S \subseteq L$):

$$|S| = \sum_{I_1 \leq I_2 \text{ in } L/\Theta} \mu_{L/\Theta}(I_1, I_2) \cdot |S \cap I_1 \cap I_2| \ .$$

G.M. ZIEGLER: <u>Branchings in rooted graphs and the diameter of greedoids</u>

Greedoids are finite accessible set systems satisfying the matroid exchange axiom. We study the complexity of exchange rules on the bases of 2-connected greedoids, as measured by the diameter of the basis graph. We show that this diameter (depending on the rank r) is bounded by 2^r-1 for 2-connected greedoids, by $r^2-r+1 \, (r > 0)$ for 2-connected branching greedoids, which are the collection of rooted trees in a 2-connected graph ("branchings").
This particularly answers a question by L. Lovâsz on the maximal number of exchanges of leaves required to transform a spanning tree of a 2-connected rooted graph or digraph into a given second one.

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