MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1986

TOPICS IN PSEUDO-DIFFERENTIAL OPERATORS

2.2. bis 8.2.1986

TAGUNGSLEITUNG:

H.O. CORDES (Berkeley) **B. GRAMSCH (Mainz)** **H. WIDOM (Santa Cruz)**

Im Mittelpunkt dieser Tagung standen neuere Entwicklungen auf dem Gebiet der Pseudodifferentialoperatoren unter Einschluß der Theorie der partiellen Differentialgleichungen und nichtlinearer Probleme. Es nahmen 50 Mathematiker teil; 35 Vorträge wurden gehalten.

Themenkreise waren u.a.:

- Nichtlineare hyperbolische Gleichungen
- Pseudodifferentialoperatoren auf beschränkten Gebieten und Mannigfaltigkeiten mit Singularitäten, asymptotische Enwicklungen
- Fréchet- und Ψ-Algebren von Pseudodifferentialoperatoren, nichtlineare Probleme für Okasches Prinzip
- Fourierintegraloperatoren, Fréchet-Liegruppen und Toeplitzoperatoren
- Operatorenkalkül, insbesondere Potenzen von Operatoren
- Indextheorie und Spurformeln
- L^p-Beschränktheit, C^*-Algebren und Analysis auf Netzen
- Fragen zur Regularität und Existenz von Lösungen
- Randwertprobleme verschiedener Art, numerische Methoden

Die freundliche Atmosphäre und viele angeregte Diskussionen trugen zum Gelingen der Tagung bei. Alle Teilnehmer bedauerten den Tod von Herrn Professor H.G. Garnir, der an dieser Konferenz hatte teilnehmen wollen.

VORTRAGSAUSZÜGE

MALCOLM R. ADAMS:

A LIE GROUP STRUCTURE FOR FOURIER INTEGRAL OPERATORS

Let M be a compact manifold and let FIO_* denote the group of invertible classical Fourier integral operators on M (modulo smoothing operators). In joint work with R.Schmid and T.Ratiu we endow this space with a Lie group structure in the following manner.

Let ΨDO_* denote the group of invertible classical pseudodifferential operators on M (modulo smoothing operators) and let $D_\theta(T^*M\backslash 0)$ be the group of θ preserving diffeomorphisms on $T^*M\backslash 0$ where θ is the canonical 1-form. We have the exact sequence of groups

$$I \to \Psi DO_* \to FIO_* \overset{\pi}{\to} D_\theta(T^*M\backslash 0) \to e$$

where π sends $A \in FIO_*$ to its canonical relation. The procedure we use is:

1) Endow ΨDO_* and $D_\theta(T^*M\backslash 0)$ with Lie group structures.

2) Construct a local section $\sigma: U \subset D_\theta(T^*M\backslash 0) \to FIO_*$ near e.

3) Identify $\pi^{-1}(U) \sim U \times \Psi DO_*$ by means of σ and give charts on FIO_* by translating this around.

4) Check overlap conditions and smoothness of group relations.

To do this, we consider $(FIO_{0,k})_*$, the group of invertible zero order FIO's modulo those of order -k-1. We give this space an ILH Lie group structure and then use this to define the Lie group structure of FIO_*.

FELIX ALI MEHMETI:

A CHARACTERIZATION OF A GENERALIZED C^∞-NOTION ON NETS

Starting point of our considerations is the mathematical description of vibrating nets (e.g. electric networks with selfinduction or systems of elastic strings). Solutions of such problems are often given in terms of

$$t \to e^{iA^\sigma t}\varphi,$$

where A: D(A) $\to$ H is a selfadjoint elliptic differential operator in some Hilbert space H of functions φ on the net (e.g. $\sigma = 1/2$ for $\ddot{u}(t)+Au(t)=0$). C^∞ solutions require special initial conditions:

$$(1)\ \varphi \in D(A^\infty) \iff (2)\ t \to e^{iA^\sigma t}\varphi \in C^\infty(\mathbb{R},H),$$

(cf. semi-group theory). In special cases (1) includes a scale of compatibility conditions imposed on $\varphi \in H$ in every node of the net, generalizing the usual C^∞-notion in the case of two branches connected by a node. (2) is shown to be equivalent to

$$t \to \Sigma_{k=0}^{\infty}\ e^{ik^\sigma t}[\varphi_k,\varphi]\varphi_k \in C^\infty(\mathbb{R},H)$$

where $\{\varphi_k\}$ is an orthonormal base of eigenfunctions of A to the eigenvalues λ_k. To this purpose, λ_k must be replaced by k in the $\{\varphi_k\}$-representation of (2) and therefore the asymptotic behaviour of the λ_k is determined.

These considerations are related to the work of G.Lumer and S.Nicaise (Mons) on diffusion problems on ramified spaces and to the research of B.Gramsch and K.G.Kalb (Mainz) on φ-algebras and abstract C^∞-notions.

MICHAEL BEALS:

REFLECTION OF TRANSVERSAL PROGRESSING WAVES IN NONLINEAR STRICTLY HYPERBOLIC MIXED PROBLEMS

In joint work with G.Metivier, we consider a mixed problem for a semilinear strictly hyperbolic equation. It is assumed that the boundary operators satisfy a uniform Lopatinski condition, and that in the past, the solution is "conormal" with respect to a characteristic hypersurface. If the hypersurface intersects the boundary transversally and a non-grazing hypothesis is satisfied, it is proved that the solution is conormal with respect to the reflected family of hypersurfaces. In particular, if the solution was smooth off the hypersurfaces and conormal in the past, it remains smooth off of the reflected surfaces, as in the linear case.

JOCHEN BRUNING:

L^2 INDEX THEOREMS FOR REGULAR SINGULAR OPERATORS

We propose a natural class of first order elliptic operators (called "regular singular") on noncompact Riemannian manifolds that have a finite L^2 index. The index can be computed in terms of integrals of characteristic classes, η-invariants, and dimensions of certain eigenspaces. The result can be applied to the geometric operators e.g. on manifolds with cone-like singularities (which provide the model case), asymptotically Euclidean spaces, and manifolds with cusp-like ends.

LEWIS A. COBURN:

TOEPLITZ OPERATORS ON THE SEGAL-BARGMAN SPACE

This talk describes joint work with C.A.Berger and subsequent work with C.A.Berger and K.H.Zhu. We consider Gaussian measure $d\mu$ on complex n-space and the subspace $H^2(d\mu)$ consisting of entire functions in $L^2(d\mu)$. For P the orthogonal projection from $L^2(d\mu)$ onto $H^2(d\mu)$ and f in L^∞, we define the Toeplitz operator T_f on $H^2(d\mu)$ by $T_f h = P(fh)$. We have been interested in the structure of C^*-algebras of such operators and their relation to the natural irreducible action of the Heisenberg group on $H^2(d\mu)$. Recently, Berger and I were able to characterize the maximal subalgebra Q in L^∞ for which $T_f T_g - T_{fg}$ is a compact operator for all f,g in Q. Functions in Q are completely characterized by a condition of "vanishing mean oscillation at infinity". Extension of this work to other domains is possible. Recent work in this direction has been done by Axler and Zhu.

HEINZ O. CORDES:

ON FRECHET-*-ALGEBRAS OF PSEUDODIFFERENTIAL OPERATORS

We are looking at three special types of Fréchet-*-subalgebras of ψdo's, subalgebras of $\mathcal{L}(\mathcal{H})$, $\mathcal{H} = L^2(\Omega)$, Ω a C^∞-manifold.

1) The closure $\mathcal{C}$ of an algebra of classical ψdo's (of order 0) under the Fréchet topology of all Sobolev norms.

2) For $\Omega = \mathbb{R}^n$, the algebra $\mathcal{A}$ of all $A \in \mathcal{L}(\mathcal{H})$, which are C^∞(GS) with a specific Lie group of unitary operators over $\mathcal{H}$ containing rotations, translations, dilations and gauge transforms.

3) The $*$-subalgebra $\mathcal{P}$ of $\mathcal{A}$ of all $A \in \mathcal{A}$ with $e^{iHt}Ae^{-iHt} \in C^\infty(\mathbb{R},\mathcal{A})$, where H is the Dirac Hamiltonian.

One finds that (i) all three algebras are ψ^*-subalgebras of $\mathcal{L}(\mathcal{H})$: they contain their inverses in $\mathcal{L}(\mathcal{H})$. Also (ii) $\mathcal{C}$ has a symbol map which is surjective, just as for a C^*-algebra. Finally, (iii) $\mathcal{P}$ is investigated as an algebra of observables for the Dirac equation.

ALBERT K. ERKIP:

ELLIPTIC BOUNDARY VALUE PROBLEMS IN THE HALF-SPACE

The boundary value problem: $P_u=f$ in $\mathbb{R}_+^{n+1}$, $B_ju=g_j$ on $\partial\mathbb{R}_+^{n+1}$, $j=1,\ldots,r$ is investigated. P, B_j are differential operators with smooth coefficients of polynomial growth. P is assumed to be elliptic, (P,B_j) satisfy the Lopatinski-Shapiro conditions. More restrictions have to be imposed at infinity, both on P and the system (P,B_j). The problem is then shown to be normally solvable (Fredholm) for data in appropriate weighted Sobolev spaces. The Calderon-Seeley approach is taken to reduce the problem to a system of pseudo-differential equations on the boundary. This system is analysed through the global pseudo-differential operator theory of Cordes.

DAISUKE FUJIWARA:

A REMARK ON TANIGUCHI-KUMANOGO THEOREM FOR PRODUCT OF FOURIER INTEGRAL OPERATORS

I consider many Fourier integral operators $I(\nu;\varphi_j,a_j)$, $j = 1,\ldots,k$:

$$I(\nu;\varphi_j,a_j)f(x) = \left(\frac{\nu}{2\pi t_j}\right)^{n/2} \int_{\mathbb{R}^n} a_j(x,y)\ \exp\{i\nu\varphi_j(t_j;x,y)\}\ f(y)\ dy,$$

where $\nu > 1$ and $t_j > 0$ are constants, $a_j \in B(\mathbb{R}^n \times \mathbb{R}^n)$ and

$$\varphi_j(x,y) = \frac{|x-y|^2}{2t_j} + t_j\omega_j(x,y).$$

We assume that $\kappa_m = \sup_j \Sigma_{2\leq|\alpha|+|\beta|\leq m} \sup |(\partial/\partial x)^\alpha(\partial/\partial y)^\beta\omega(x,y)| < \infty$.

If $T = t_1+t_2+\ldots+t_k$ is small, then there exists a phase function $\varphi(T,x,y) = \frac{|x-y|^2}{2T} + T\omega(T,x,y)$ and an amplitude function $b(x,y)$ such that

$$I(\nu;\varphi,1+b) = I(\nu;\varphi_k,1+a_k)\cdot I(\nu;\varphi_{k-1},1+a_{k-1})\cdot\ldots\cdot I(\nu;\varphi_1,1+a_1)$$

Let $\|a_j\|_m = \Sigma_{0\leq|\alpha|+|\beta|\leq m} \sup |(\partial/\partial x)^\alpha(\partial/\partial y)^\beta a_j(x,y)|$, $m = 0,1,\ldots$.

Then we can prove

<u>Theorem</u>: There exists a positive τ such that if $T \leq \tau$ then for any $m = 0,1,\ldots,$ we have

$$1 + \|b\|_m \leq \Pi_{j=1}^k (1 + C(m)\|a_j\|_{M(m)})(1 + C(m)t_j)$$

with some $C(m)$ and $M(m)$ independent of k.

DARYL GELLER:

AN ANALYTIC WEYL CALCULUS AND ANALYSIS ON THE HEISENBERG GROUP

In the Weyl calculus, consider the symbol classes $S^j=\{r(p,q)\in C^\infty(\mathbb{R}^n\times\mathbb{R}^n$: there exist $r_l(p,q)$, homogeneous of degree $j-l$ in (p,q) and smooth away from 0, such that for $|(p,q)|\geq 1$, $|\partial_p^\alpha\partial_q^\beta(r-\Sigma_{l=0}^{L-1}r_l)(p,q)| < C_{\alpha\beta L}|(p,q)|^{j-L-|\alpha|-|\beta|}\}$ and $AS^j=\{r\in S^j$: (1) r has an extension to $\mathbb{C}^{2n}$ as an entire function of exponential order 2, (2) In a sector S about $\mathbb{R}^{2n}$ in $\mathbb{C}^{2n}$, all the r_l may be extended to holomorphic functions (also denoted r_l) such that $|r_l(\zeta)|<Cr^l l!^{1/2}$ for $|\zeta|=1$, $\zeta\in S$; (3) $|(r-\Sigma_{l=0}^{L-1}r_l)(p,q)|<CR^L L!^{1/2}|(p,q)|^{j-L}$ for $|(p,q)|\geq 1\}$.

If $R=Op^W(r)$, $r\sim\Sigma r_l$, we say R is Hermite-like, if $r_0(p,q)\neq 0$ for $(p,q)\neq 0$. (If $r(p,q)=p^2+q^2$, R is the Hermite operator.) The $\{S^j\}$ compose properly under the Weyl calculus (Grossman-Loupias-Stein). If $R\in Op^W(S^j)$, then R satisfies $R:\mathcal{S}\to\mathcal{S}$, $R':\mathcal{S}'\to\mathcal{S}'$, and if R is Hermite-like, then $(f\in\mathcal{S}', Rf\in\mathcal{S}) \Rightarrow f\in\mathcal{S}$, and R has a parametrix in $Op^W(S^{-j})$.

We show that AS^j is an "analytic analogue" of S^j. In particular, if $R \in Op^W(AS^j)$, then $R:Z_2^2 \to Z_2^2$; and if R is Hermite-like, then $(f\in\mathcal{S}', Rf\in Z_2^2) \Rightarrow f\in Z_2^2$, and R has a parametrix in $Op^{w}(AS^{-j})$. Here Z_2^2 is a Gelfand-Shilov-space. The author and Peter Heller apply this theory to the construction of relative analytic parametrices, built out of homogeneous distributions, for transversally elliptic operators on the Heisenberg group.

PAUL GODIN:

ANALYTIC REGULARITY OF UNIFORMLY STABLE SHOCK FRONTS WITH ANALYTIC DATA

Majda has recently proved that large classes of conservation laws possess shock front solutions provided a condition of uniform stability is assumed. We show that under natural analyticity assumptions on the data, the shock surface will be locally analytic and the solution will also be locally analytic up to the shock surface.

BERNHARD GRAMSCH:

ON THE OKA PRINCIPLE FOR SOME CLASSES OF PSEUDO-DIFFERENTIAL OPERATORS

For special Ψ-algebras (cf. Math.Ann. 269, 27-71 (1984)) of classical pseudo-differential operators the Oka principle is discussed:

$$\pi_0(\mathscr{H}(\Omega,\Phi^{\Psi}_{m,n})) \cong \pi_0(\mathscr{C}(\Omega,\Phi^{\Psi}_{m,n})$$

for the canonical inclusion with respect to the connected components (Ω a holomorphy region, $\Phi^{\Psi}_{m,n}$ the set of Fredholm operators with kernel dimension m and cokernel dimension n in the algebra Ψ). Some methods of proof are discussed for this recent result. In the direction of K-theory the Oka principle has been established for crossed products by J.-B. Bost (C.R.Paris 1985). Results of Shubin (1970) and Gohberg and Leiterer (1973) can be proved for special classes of Fréchet algebras of pseudo-differential operators.

ALAIN GRIGIS:

ON THE ASYMPTOTIC OF GAPS IN HILL'S EQUATION

We give an estimation of the exponential decreasing of the width of the gaps in the spectrum of $-d^2/dx^2 + V(x)$ on $L^2(\mathbb{R}^n)$, when V is real, real-analytic and periodic (Hill's equation).

When V is a trigonometric polynomial satisfying some condition we have an asymptotic development. This extends previous results of Avron and Simon in Annals of Physics 1981 on the case $V(x) = \mu \cos 2x$ (Mathieu's equation). Our method uses a precise study of solutions extended to the complex plane.

GERD GRUBB:

FUNCTIONAL CALCULUS OF PSEUDO-DIFFERENTIAL BOUNDARY PROBLEMS

Pseudo-differential boundary problems arise from manipulations with differential operator problems (e.g. compositions involving inverses, reductions of matrices, reductions of singular perturbations). L. Boutet de Monvel introduced a convenient framework for such problems, of interest in itself. We define functions f(A) of elliptic Boutet de Monvel systems in the quadratic case, and, more interestingly, functions f(B) of realizations B defined from elliptic systems of the form

$$\begin{pmatrix} P_\Omega + G \\ T \end{pmatrix} : C^\infty(\bar{\Omega})^N \to \begin{matrix} C^\infty(\bar{\Omega})^N \\ \times \\ C^\infty(\partial\Omega)^M \end{matrix}$$

with P and G of order $d>0$; here B acts like $P_\Omega+G$ with domain $D(B)=\{u\in H^d(\bar{\Omega}): Tu=0\}$ in $L^2(\bar{\Omega})^N$, and Ω is a smooth bounded domain in $\mathbb{R}^n$. The lecture describes in particular the results for complex powers B^z:

1) The characterization of $D(B^\theta)$, $0<\theta<1$, in certain selfadjoint cases;

2) the detailed symbol structure of B^z for $\mathrm{Re}(z)<0$;

3) the spectral estimate $s_k(B^z) \le Ck^{-d|\mathrm{Re}(z)|/(n-1)}$ (for $\mathrm{Re}(z) < -(2d)^{-1}$);

4) the extension of the trace $\mathrm{tr}(B^z)$ to a meromorphic function of z for $\mathrm{Re}(z)<\nu_1$, where ν_1 is a certain positive number associated with the problem; as a corollary a new index formula for normal elliptic realizations.

VICTOR GUILLEMIN:

THE TRACE FORMULA FOR VECTOR BUNDLES

I describe an analogue of the standard trace formula for the wave group in which the set-up is the following: Let X be a compact Riemannian manifold, $P \to X$ a principal G-bundle, G compact, and ρ_e, e=1,2,... a ladder of irreducible representations of G. Let Δ_e be the Laplace operator on the bundle associated with ρ_e and λ_u^e, u=1,2,... its spectrum. For $d \in S(R)$, $d>0$, set $N_{d,r}(\lambda) = \Sigma_e \Sigma_{\mu_i^e < \lambda} d(\mu_i^e - re)$ with $\mu_i^e = (\lambda_i^e + e^2)^{1/2}$. I show the Fourier transform of $N_{d,r}$ is a conormal distribution whose singularities are related to a certain "minimally coupled" flow.

SÖNKE HANSEN:

AN AIRY OPERATOR CALCULUS

The standard pseudo-differential operator calculus is not adequate in the study of the Neumann operator N of a diffractive boundary problem. To overcome this limitation, Melrose developed the Airy operator calculus. Using it he proved, in particular the hypoellipticity of N. We show that N, after conjugating it with a suitable Fourier integral operator, is an elliptic pseudo-differential operator in the setting of Hörmander's calculus with the symbol spaces S(m,g). Here the metric g is defined using only the diffractive hypersurface.

LARS HÖRMANDER:

THE LIFESPAN OF CLASSICAL SOLUTIONS OF NON-LINEAR HYPERBOLIC EQUATIONS OF SECOND ORDER

F. John and S. Klainerman have proved that the Cauchy problem

(1) $\Sigma_{j,k=0}^{3} g^{jk}(u')\partial_j\partial_k u = 0$ when $x_0>0$; $u=\epsilon u_0$, $\partial_0 u=\epsilon u_1$ when $x_0=0$,

has a C^∞-solution for $x_0 \le \exp(c/\epsilon)$, some $c > 0$, if $u_j \in C_0^\infty(\mathbb{R}^3)$. Here $\Sigma_{j,k=0}^{3} g^{jk}(0)\partial_j\partial_k$ is the standard wave operator $\Box$, and $g^{jk} \in C^\infty$.

THEOREM: Let $\Box U = 0$ in $\mathbb{R}^4$ and $U = u_0$, $\partial_0 U = u_1$ when $x_0 = 0$. Then (Friedlander) $rU(t,r\omega) = F(\omega,r-t) + O(1/r)$ $(r>0,\ \omega\in S^2)$, where

$$F(\omega,\rho) = (4\pi)^{-1}\left(\int_{\langle x,\omega\rangle=\rho} g\, dS - \partial/\partial\rho \int_{\langle x,\omega\rangle=\rho} f\, dS\right).$$

Set $G^{jkl} = \partial g^{jk}(0)/\partial\xi_l$ and $G(\omega) = \Sigma G^{jkl}\hat\omega_j\hat\omega_k\hat\omega_l$; $\hat\omega = (-1,\omega)$. Then

a) for $0 \le s < A = \max\left(\frac{1}{2} G(\omega)\ \partial^2 F(\omega,\rho)/\partial\rho^2\right)^{-1}$ the Cauchy problem

(2) $\partial V(\omega,s,\rho)/\partial s = \frac{1}{4} G(\omega)\ (\partial V(\omega,s,\rho)/\partial\rho)^2;\ V(\omega,0,\rho) = F(\omega,\rho),$

has a C^∞ solution; the second derivatives blow up as $s \to A$.

b) if $B < A$ then the Cauchy problem (1) has a C^∞ solution u_ϵ for $\epsilon\cdot\log(t) < B$ if ϵ is small enough, and locally uniformly in $S^2 \times (0,A) \times \mathbb{R}$

$$\epsilon^{-1} e^{s/\epsilon} u_\epsilon(e^{s/\epsilon}, (e^{s/\epsilon}+r)\omega) - V(\omega,s,r) = O(\epsilon\cdot\log(\epsilon)) \text{ as } \epsilon \to 0.$$

A closely related result has been proved by F.John.

CHISATO IWASAKI:

PSEUDO-DIFFERENTIAL OPERATORS ON GEVREY CLASSES

It is well-known that elliptic operators with analytic coefficients are analytic-hypoelliptic. But, in general, it is not true for degenerate operators even if they are hypoelliptic in C^∞-sense. Under the following conditions $(A_1),(A_2)$, I can prove $G^{(s)}$-hypoellipticity for an operator P if $s \ge 2$, by constructing the fundamental solution for $L = \partial/\partial t + P$.

(A_1) $p = p_m+p_{m-1}+p_{m-2},\ p_j \in S^j_{1,0}$ $(j=m,\ m-1,\ m-2)$, $p_m \ge 0$, P has coefficients belonging to G^s.

(A_2) $\exists\ c>0$ such that

$$\mathrm{Re}\ p_{m-1} + \frac{1}{2}\,\mathrm{tr}^+ A \ge c\cdot|\xi|^{m-1} \text{ on } \Sigma = \{X \in T^*\mathbb{R}^n\backslash 0;\ p_m(X) = 0\}.$$

Here, $A = iF$, F is the fundamental matrix of p_m.

For the proof of the theorem we must study symbols of multi-products of pseudo-differential operator of subclasses of $S^m_{\rho,\delta}$, which are attached to the functions of G^s-class.

NOBUHISA IWASAKI:

EXAMPLES OF EFFECTIVELY HYPERBOLIC EQUATIONS

We know that the effectively hyperbolicity is equivalent to the strong hyperbolicity for single equations. The sufficient part (P is well-posed if P is hyperbolic) is extendable to the general nonlinear equations. We shall give two examples of effectively hyperbolic equations to show how critical the problem treated is. They are the Monge-Ampère equation and the compressible Euler equation.

SHMUEL KIRO:

ON THE GLOBAL EXISTENCE OF REAL ANALYTIC SOLUTIONS OF LINEAR PDE

Let $\Omega \subset \mathbb{R}^n$ be an open set, $A(\Omega)$ the space of real analytic functions in Ω and $P = P(x,D_x)$ be a partial differential operator with analytic coefficients. We consider conditions to be imposed on Ω and P for global existence of real analytic solutions of Pu = f for every $f \in A(\Omega)$, i.e. when $PA(\Omega) = A(\Omega)$.
We give convexity type conditions for the boundary for surjectivity of P generalizing the one introduced by Kawai in the case of constant coefficients and consider to Phragmen-Lindelöf principles of Hörmander characterizing surjectivity for convex sets Ω and operators with constant coefficients.

PASCAL LAUBIN:

SECOND MICROLOCALIZATION AND OPERATORS WITH INVOLUTIVE DOUBLE CHARACTERISTICS

We introduce a class of FBI-phase functions of second type that characterize the second analytic wave front set of a distribution along an involutive or isotropic submanifold whose projection on the base space is regular. This class is large enough to solve 2-microlocal eiconal equations.
Using the WKB-method we prove a propagation theorem for the second analytic wave front set of solutions to equations with involutive double characteristics. It turns out that the second wave front set propagates along null bicharacteristics associated to the second order derivatives of the principal part of the operator.

RICHARD B. MELROSE:

RINGS OF PSEUDODIFFERENTIAL OPERATORS ASSOCIATED TO BOUNDARIES, CONES AND CUSPS

After blow-up of a singular space the Laplacian becomes an elliptic operator degenerating in various ways at the boundary components of a manifold with corners, X. The geometric structure of "true boundaries", "cones" and "cusps" can be reduced to a Lie algebra of smooth vector fields, V, consisting in these three cases of, respectively, vector fields vanishing at the boundary (V_0), tangent to the boundary (V_b) or annihilating to second order a defining function ρ, for the boundary (V_ρ). The Laplacian is a multiple of an operator in the ring generated by $C^\infty(X)$ and V. In each case there is an associated space of pseudodifferential operators distinguished by the conormal regularity of their Schwartz kernels lifted to the stretched product $X \times_V X$. This latter space is obtained from $X \times X$ by blowing up a manifold defined by V. Under suitable hypotheses, inverses of elliptic operators can be constructed symbolically in these spaces.

The general method is illustrated by recent joint work with R.Mazzev. If h is a Riemann metric on a compact manifold with boundary, X, and ρ is a defining function for ∂X then $g = h/\rho^2$ is a complete metric on the interior. It is shown that $[\Delta + |\partial\rho|_h^{-2} \varsigma(\varsigma-n)]^{-1}$, dim X = n+1, extends to be meromorphic in ς, and an element of the V_0-space of pseudo-differential operators.

HENRI MOSCOVICI:

HIGHER INDICES OF ELLIPTIC OPERATORS

The talk reports on joint work with A. Connes. Let M be a smooth closed manifold. We introduce a certain algebra A_∞ of families $\{P_\epsilon\}_{\epsilon>0}$ of smoothing pseudo-differential operators and show that any (Alexander-Spanier) cohomology class $\alpha \in H^{ev}(M,\mathbb{R})$ defines a cyclic cohomology class τ_α on A_∞. If now D: $C^\infty(E) \to C^\infty(F)$ is an elliptic operator on M, it determines a K-theory class $e_D \in K_0(A_\infty)$, which can be paired with τ_α to give the "higher index" of D:

$$\iota_\alpha(D) = \tau_\alpha(e_D,\dots,e_D).$$

The following "higher index formula" holds in this setting:

$$\iota_a(D) = (\mathrm{ch}(\sigma_D)\cup \mathrm{Td}(T_{\mathbb{C}}M)\cup\alpha)[M].$$

TOSINOBU MURAMATU:

ESTIMATES FOR THE NORM OF PSEUDO-DIFFERENTIAL OPERATORS BY MEANS OF BESOV SPACES

Theory of Besov spaces, which is developed by the speaker, is very much fit for studying boundedness of pseudo-differential operators. As a result, we have obtained sufficient conditions imposed on the symbols for operators to be bounded in L_2, which are the best ones as far as the order of regularity of symbols.

Our main result:

If $a(x,\xi) \in S^0_{\rho,\delta}B^{(\lambda,n/2)}_{\infty,1}(\mathbb{R}^n_x \times \mathbb{R}^n_\xi)$, $0 \leq \delta \leq \rho < 1$, $\lambda = \frac{1-\rho}{1-\delta}\cdot\frac{n}{2}$,
then $a(X,D)$ is L_2-bounded.

Here the superscript figures indicate the order of regularity in x and ξ, respectively, and $S^0_{\rho,\delta}$ corresponds to that of Hörmander. Our theory includes operator-valued symbols.

MICHIHIRO NAGASE:

SUFFICIENT CONDITIONS FOR PSEUDO-DIFFERENTIAL OPERATORS TO BE L^p-BOUNDED

Let $0 \leq \delta \leq \rho \leq 1$ and $\delta < 1$. It is known that pseudodifferential operators with symbol in $S^{-m}_{\rho,\delta}$ are L^p-bounded, if $m \geq m_p = (1-\rho)|1/2 - 1/p|$. The purpose of this talk is to introduce L^p-boundedness theorems, which generalize the above results for the case of non-smooth symbols. In fact we show L^p-boundedness of pseudodifferential operators for $p \geq 2$ under $[n/2]+1$ differentiability of symbols $p(x,\xi)$ in ξ and less differentiability in the space variables x.

DIDIER ROBERT:

WEYL FORMULA FOR PSEUDODIFFERENTIAL OPERATORS OF NEGATIVE ORDER

Weyl formulae for elliptic or hypoelliptic operators are now well-known. In this talk we consider cases of integral operators in $L^2(\mathbb{R}^n)$ whose eigenvalues may be positive or negative. We obtain Weyl formulae for the positive part and the negative part of the spectrum. These results generalize in some sense previous ones of Birman and Solomjak and were obtained jointly with M.Dauge.

ELMAR SCHROHE:

COMPLEX POWERS OF ELLIPTIC PSEUDODIFFERENTIAL OPERATORS

Complex powers of elliptic ΨDO's are constructed on compact and noncompact manifolds, then the analytic behaviour of their kernels, zeta and eta functions is analyzed. Starting from Seeley's classical result that the kernel $k_s(x,y)$ of A^s has an analytic extension to $\mathbb{C}$ for $x \neq y$ as a function of s and that $k_s(x,x)$ is meromorphic with only simple poles on the real axis, it is shown that for other asymptotic expansions of the symbol, $k_s(x,y)$, $x \neq y$ will still have an analytic extension to $\mathbb{C}$, but $k_s(x,x)$ in general can only be continued to a larger half-plane depending on the "quality" of the asymptotic expansion. An example is given, where $k_s(x,x)$ has a natural boundary. Similar results are proven for the zeta and eta function. For noncompact manifolds, a class of weighted symbols on $\mathbb{R}^n$ is transfered to manifolds with a suitable compatible structure, and for the corresponding ΨDO's, essentially the same results are shown to hold as in the compact case.

BERT-WOLFGANG SCHULZE:

PSEUDO-DIFFERENTIAL OPERATORS AND MELLIN EXPANSIONS IN SPACES WITH CONORMAL ASYMPTOTICS

There is given a complete Mellin and symbolic pseudo-differential symbolic calculus for classical ΨDO's (pseudo-differential operators) without the transmission property. This class is reached by reducing elliptic mixed boundary problems to the boundary (e.g. the Zaremba problem) and basically all interesting mixed problems for differential operators lead to the class of ΨDO's with violated transmission condition with respect to the jump Γ of the data (Γ being a submanifold of the boundary of codimension 1). The "regularity" near Γ is a conormal singularity in terms of the distance to Γ

and the type can be explained by Mellin expansions of the pseudo-differential actions. A similar approach with operator-valued symbols applies for operators on manifolds with singularities, e.g. conical ones or edges. It is a basic question to understand the branching of complex exponents in the conormal asymptotics with varying parameter $\in \Gamma$. This follows in the context of C^∞-functions of analytic functionals that are pointwise discrete and of finite order. The results are part of a book on the asymptotics for mixed boundary problems jointly with S.Rempel.

JOHANNES SJÖSTRAND:

SEMICLASSICAL RESONANCES

In [1] (see also [2]) we developped with B.Helffer a microlocal approach to resonances for (in particular) the semiclassical Schrödinger operator $P = -h^2\Delta + V(x)$ on $\mathbb{R}^n$, when $h > 0$ is very small, and we gave rather precise results about resonances generated by tunneling from a potential well in an island. In this talk we announced complete asymptotic descriptions of the resonances as $h \to 0$ in two cases when the classical dynamics are particularly simple:

I. The union K^0 of trapped classical trajectories in $\xi^2+V(x) = 0$ is reduced to a point $(x_0,0)$ and $V''(x_0)$ is non-degenerate.

II. (Joint work with C.Gerard). K^0 is a simple closed trajectory of hyperbolic type.

In case I we gave examples of double resonances which are also second order poles of the (holomorphically extended) resolvent.

References:

1. Resonances en limite semiclassique. To appear as a supplement to Bull.S.M.F..

2. Lecture notes from the NATO symposium, Il Castelvecchio 1985. Reidel Publ. Co.,to appear.

HOUSHANG H. SOHRAB:

A CLASS OF PSEUDO-DIFFERENTIAL C^*-ALGEBRAS

Following an approach introduced by Prof. H.O.Cordes, a comparison algebra $\underline{A}$ of $L^2(\mathbb{R}^n)$-bounded pseudo-differential operators is constructed, using the Schrödinger operator $H=D^2+q(x)$, with the negative Laplacian D^2, and a smooth potential $q(x)\geq 1$, for all x in $\mathbb{R}^n$, such that $q(x) \to \infty$, as $|x| \to \infty$, and that $|\mathrm{grad}(q(x))|=o(q(x))$ at infinity. $\underline{A}$ is generated by $C(\underline{\mathbb{R}}^n)$, $\underline{K}$ and the bounded operators $q^{-1/2}Hq^{1/2}$, $D_1H^{-1/2},\ldots,D_nH^{-1/2}$. Here, H is the selfadjoint closure of our Schrödinger operator on $C_0^\infty(\mathbb{R}^n)$, $D_j=-i\partial/\partial x_j$, $\underline{K}$ = compact ideal of the algebra of bounded operators of L^2, and $\underline{\mathbb{R}}^n$ is a suitable compactification of $\mathbb{R}^n$. We show that $\underline{A}/\underline{K} \simeq C(M)$, describe as much as possible the Gelfand space M, and indicate applications to normal solvability of related classes of partial differential operators.

FRANK-OLME SPECK:

DIFFRACTION PROBLEMS WITH IMPEDANCE CONDITIONS

The two- or three-dimensional electromagnetic diffraction problem for a half-plane impedance sheet belongs to a class of elliptic transmission problems of mixed type. Sobolev spaces of order 1 and ± 1/2 are naturally involved according to the energy norm and the trace theorem, respectively. This operator theoretic approach presents the equivalence to Wiener-Hopf equations and their solutions in the sense of a well-posed problem with respect to the spaces under consideration. Slightly different impedance numbers for the two banks of the screen lead to a perturbation problem. All results yield direct a priori estimates for the solution.

GUNTHER UHLMANN:

ON AN INVERSE BOUNDARY VALUE PROBLEM

In this talk we will outline a proof (joint work with J.Sylvester) that one can determine uniquely smooth conductivities from boundary measurements in all dimensions, $n \geq 2$. More precisely: Let Ω be a smooth bounded domain in $\mathbb{R}^n$, let γ be a smooth strictly positive function in $\bar{\Omega}$. We consider the operator defined by

(1) $$L_\gamma \mu = \mathrm{div}(\gamma \nabla \mu) = \gamma \Delta \mu + \nabla\gamma \cdot \nabla \mu.$$

Let $\varphi \in C^\infty(\partial\Omega)$. We solve the Dirichlet problem

(2) $$L_\gamma \mu = 0 \quad \text{in } \Omega,$$
$$\mu|_{\partial\Omega} = \varphi$$

and we consider the associated Dirichlet integral $Q_\gamma(\varphi) = \int_\Omega \gamma|\nabla u|^2$. The question proposed by A.Calderon is whether the map

(3) $$\gamma \to Q_\gamma \quad \text{is injective.}$$

We prove that this is indeed the case. The uniqueness of the map (3) is equivalent to the uniqueness of the Neumann map

(4) $$\varphi = u|_{\partial\Omega} \to \left(\gamma \frac{\partial u}{\partial \nu}\right)\Big|_{\partial\Omega} \quad \text{with } u \text{ solution of (2).}$$

Thus for every potential φ on the boundary we measure the induced current on the boundary. The question is whether these measurements, made only at the boundary, can determine uniquely the conductivity γ in the interior.

WOLFGANG WENDLAND:

ON THE NUMERICAL SOLUTION OF CERTAIN PSEUDODIFFERENTIAL EQUATIONS

This is a brief survey on asymptotic error estimates for the so called boundary element methods in the form of Galerkin and collocation methods. The "direct method" is based on the formulation of pseudodifferential equations on the boundary in terms of the boundary conditions and the Calderon projector for regular elliptic boundary value problems.

The unknown part of the Cauchy data is approximated by boundary elements of a regular $S_k^{d,m}$ system in the sense of Babushka and Aziz. For Galerkins's method we find optimal convergence provided the boundary ψdos satisfy a Garding inequality on the boundary. The latter holds for one choice of boundary ψdos if the original boundary value problem possesses a strongly elliptic variational formulation for a suitable transmission comparison problem. The optimal convergence result extends to compact and small perturbations, special regular transformations and it provides a suitable localization principle. This implies that optimal convergence of the boundary element method takes place if and only if the boundary ψdos are strongly elliptic (in a modified version). For two-dimensional boundary value problems where the ψdos live on simple closed curves these results have been extended to naive collocation with "smoothest" splines. Again, above mentioned strong ellipticity is sufficient and necessary for optimal order convergence. The survey concerns results by D.N.Arnold, M.Costabel, G.C.Hsiao, S.Prössdorf, J. Nedelec, J. Saranen, G.Schmidt, E.Stephan, and the lecturer.

HAROLD WIDOM:

ASYMPTOTIC EXPANSIONS FOR PSEUDODIFFERENTIAL OPERATORS ON BOUNDED DOMAINS

We derive first a formal "asymptotic" expansion for the trace of $f(P_\Omega A P_\Omega)$ where A is a ψdo of negative order on $\mathbb{R}^n$ (with scalar or matrix symbol), P_Ω is the projection from $L^2(\mathbb{R}^n)$ to $L^2(\Omega)$ (Ω being a bounded region in $\mathbb{R}^n$ with smooth boundary) and f is a suitable analytic or C^∞ function. In case the symbol of A is of the form $\sigma(x,\epsilon\xi)$ where σ is a fixed symbol and $\epsilon\to 0$, the formal expansion gives a correct asymptotic expansion of the form $\Sigma_{k=0}^{\infty} a_k \epsilon^{k-n}$, where the a_k are given by explicit integral formulas. In the case $f(\lambda) = e^{-t/\lambda}$ and A is a positive elliptic operator satisfying certain conditions, the formal expansion is, to n+1 terms, a correct asymptotic expansion as $t \to 0+$, and the trace is of the form $\Sigma_{k=0}^{n} a_k t^{(k-n)/r} + O(t^\delta)$ with $\delta > 0$.

MASAO YAMAZAKI:

PROPAGATION OF SINGULARITIES OF SOLUTIONS TO NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Here we generalize Bony's results for non-linear equations in two points. First we consider quasi-homogeneous wave front sets and their location along the same line as R.Lascar. Secondly we relax the conditions on regularity originally posed on the solution.

Given a nonlinear equation, we can calculate real numbers ρ, σ and $\mu(s)$ (defined for a number $s>\rho$) and show that the microlocal regularity theorem on noncharacteristic points is valid for solutions in $H^{M,s}_{loc}$ $(s>\rho)$ and the propagation theorem is valid for solutions in $H^{M,s}_{loc}$ $(s>\rho)$ modulo terms belonging to $H^{M,t+m}_{loc}$ [resp. $H^{M,t+m-1}$], where $H^{M,s}$ denotes the anisotropic Sobolev space.

Some conditions imposed on ρ and σ are merely to define the nonlinear terms, but others are more essential, and we see that the conditions cannot be relaxed by giving counter-examples for the microlocal regularity theorem on noncharacteristic points.

STEVEN ZELDITCH:

SELBERG TRACE FORMULAE, ΨDO'S AND EQUIDISTRIBUTION OF CLOSED GEODESICS

Selberg's trace formula for compact hyperbolic surfaces will be generalized to operators of the form $Op(\varphi_m)f(\Delta)$ where φ_m is an automorphic form of some weight m (φ_m is to be an eigenfunction of the Casimir operator). The geometric side of the formula involves only geodesic integrals of φ_m. Letting f and φ_m vary, one expects to recapture Bowen's equidistribution theory of closed geodesics. One also gets an equidistribution theorem for eigenfunctions of the Laplacian.

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