MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 8/1986

Halbgruppentheorie

23.2. bis 1.3.1986

Diese dritte Tagung über Halbgruppentheorie in Oberwolfach stand unter der Leitung von H. Jürgensen (London/Ontario), G. Lallement (Pennsylvania State University) und H. J. Weinert (Clausthal-Zellerfeld).

Von den 53 Teilnehmern kamen 11 aus der Bundesrepublik Deutschland, 25 aus 9 anderen europäischen Ländern (einschließlich der UdSSR), 15 aus Nordamerika sowie je ein Teilnehmer aus Australien und Taiwan.

Ein Schwerpunkt der 43 Vorträge waren neuere Entwicklungen auf dem Gebiet der kombinatorischen Halbgruppen sowie Anwendungen der Halbgruppentheorie auf die Theorie der formalen Sprachen und Automaten. Weitere Gegenstände waren inverse Halbgruppen, Varietäten von Halbgruppen und halbgruppentheoretische Fragen der Universellen Algebra. Viele Vorträge regten weitere Diskussionen an und führten zu entsprechenden Kontakten in kleineren Kreisen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

B. P. ALIMPIĆ:

Some congruences on a regular semigroup

Let Con S be the congruence lattice of a regular semigroup S, K and T equivalences on Con S defined by $\varrho K \xi \Leftrightarrow \ker \varrho = \ker \xi$, and $\varrho T \xi \Leftrightarrow \operatorname{tr} \varrho = \operatorname{tr} \xi$. It is known that K-classes $[\varrho_K, \varrho^K]$ and T-classes $[\varrho_T, \varrho^T]$ are intervals on Con S.

In this paper K-classes with $\operatorname{tr} \varrho^K = \omega_E$ and T-classes with $\ker \varrho^T = S$ are considered. It turns out that such a K-class consists exactly of E-unitary congruences, and such a T-class consists exactly of bands of group congruences. Similarly, K-classes for which ϱ^K is a Clifford congruence consist of E-reflexive congruences. These results generalize corresponding statements for inverse semigroups (see M. Petrich, Inverse Semigroups, Wiley, New York, 1984).

J. ALMEIDA:

On Pseudovarieties of monoids

For a pseudovariety $\underline{V}$ of semigroups (i. e., a class of finite semigroups which is closed under division and finite direct products) $\underline{M}\,\underline{V}$ represents the pseudovariety of monoids generated by all S^1 with S in $\underline{V}$. The operator $\underline{M}$ was introduced by Pin in connection with his studies of power semigroups. An analogous operator M may be defined at the level of varieties. The main lemma describes a finite basis of identities for MV where V is a variety defined by a single identity of the form $x_1 \ldots x_p y_1 \ldots y_m z_1 \ldots z_q = x_1 \ldots x_p y_1' \ldots y_{m+1}' z_1 \ldots z_q$. From it, one can derive by a direct method several known results. The same argument also yields an answer to a question posed by Pin concerning the effective characterization of $\underline{M}\,\underline{LI}$ where $\underline{LI}$ denotes the pseudovariety of all locally trivial finite semigroups.

R. BOOK:

Thue systems and word problems

We consider finitely presented monoids. Using length as a basis for the notion of "reduction", we investigate Thue systems (i. e., presentations) such that each congruence class has a unique irreducible element. Such a Thue system is called Church-Rosser.

Theorem. If a finite Thue system is Church-Rosser, then there is a linear time algorithm to solve its word problem.

Theorem. There is a polynomial-time algorithm to determine whether a finite Thue system is Church-Rosser.

We will use these facts to consider the question of whether Dehn's algorithm can be applied to finitely presented monoids.

M. DEMLOVÀ and V. KOUBEK:

Coextensions and minimal congruences

A congruence τ of a semigroup S is called minimal if τ is not identical and only the identical congruence is finer than τ. A surjective semigroup homomorphism $\varphi: T \to S$ is called a coextension of S. Our aim is to characterize all minimal congruences and construct all coextensions $\varphi: T \to S$ such that φ induces on T a minimal congruence. We say that a minimal congruce τ is outer if there exist two elements in different $\mathcal{J}$-classes which are τ-equivalent, else τ is inner. If τ is inner then there exists exactly one $\mathcal{J}$-class J of S such that for every pair (x,y) of different τ-equivalent elements we have $x, y \in J$. We characterize all outer minimal congruences and construct all coextensions inducing an outer minimal congruence. All inner minimal congruences and coextensions inducing an inner minimal congruence are described for the class of Green semigroups (a semigroup S is a Green semigroup if for every pair of elements (x,y) we have: if $y \in S^1x$ and $x \in S^1yS^1$ then $x \in S^1y$, and, if $y \in xS^1$ and $x \in S^1yS^1$ then $x \in yS^1$). In particular, we characterize all minimal coextensions $\varphi: T \to S$ of a Green semigroup S such that T is a Green semigroup. As a consequence we obtain a recurrent construction of all finite semigroups from the trivial semigroup.

J. FOUNTAIN:

Free right h-adequate semigroups

Elements a,b of a semigroup S are $\mathcal{L}^*$-related if and only if they are $\mathcal{L}$-related in some oversemigroup of S. S is right adequate if every $\mathcal{L}^*$-class of S contains an idempotent and the idempotents of S commute. These semigroups generalize inverse semigroups. If S is right adequate, then each $\mathcal{L}^*$-class contains exactly one idempotent. We denote the idempotent in the $\mathcal{L}^*$-class of a by a* and the semilattice of idempotents of S by E. For each a in S, the map $\alpha_a: E \to E$ defined by $x\alpha_a = (xa)^*$ is isotone. We say that S is right h-adequate if α_a is a homomorphism for each a.

A right h-adequate semigroup may be regarded as an algebra with two operations, multiplication and $a \mapsto a^*$. Looked at in this way the class of all right h-adequate semigroups forms a quasivariety and so free right h-adequate semigroups exist. A construction is given for such semigroups and a normal form found for their elements. Using the normal form one obtains many properties of these semigroups.

S. M. GOBERSTEIN:

Correspondences of semigroups

A correspondence of a universal algebra A is any subalgebra (perhaps empty) of A×A. The set $\mathcal{C}(A)$ of all correspondences of A is closed under operations of composition of binary relations and taking the inverse of a binary relation and is partially ordered by set-theoretic inclusion. The partially ordered involuted semigroup $(C(A),\circ,^{-1},\subseteq)$ is called a bundle of correspondences of A (the term belongs to A. G. Kurosh). We will give a survey of results about the characterization of semigroups from various classes by their bundles of correspondences and present some new results in this direction. Our main theorem states that every fundamental inverse semigroup is determined (up to isomorphism) by its bundle of correspondences in the class of all semigroups.

P. GORALCIK and V. KOUBEK:

On universality of extensions of a given semigroup

A categroy V is called universal (or binding) if every category of algebras is isomorphic to a full subcategory of V. The universality of the category of all semigroups and homomorphisms was established by Hedrlin and Lambek, and we also have a nice characterization of semigroup varieties due to Koubek and Sichler. Here we prove that the category of all semigroups containing a given semigroup S as a subsemigroup, together with all their semigroup homomorphisms, is universal if and only if the semigroup S is idempotent-free.

V. GOULD:

Axiomatisability of flat and projective S-systems

To a given class of algebraic structures C, there corresponds at least one first order language L. An immediate question is then, which of the properties of interest to us are expressible in terms of sentences of L. If this is true for a given property, then the subclass of objects having that property is said to be axiomatisable. We characterize those monoids whose flat (left) S-systems are axiomatisable.

A monoid S is (left) perfect if all flat S-systems are projective. We show that in certain cases, for example if S is regular, then the projective S-systems are axiomatisable if and only if the flat S-systems are axiomatisable and S is perfect.

E. HOTZEL:

Infinite versions of the Krohn-Rhodes theorem

The Krohn-Rhodes theorem (1962/65) states that every finite semigroup S is a homomorphic image of a subsemigroup of a finite wreath product whose components are copies of the three-element monoid with two right zeroes and of subgroups of S (or of composition factors of such subgroups). Refined versions of this theorem consider the multiplicity and the relative position of the components. We present a version of this theorem that does not need any finiteness assumption. It is based on the $\mathcal{J}$-class structure of S and uses left cancellative monoids instead of groups. A comparision with Eilenberg's holonomy theorem (1976) shows that either one or the other version may be more economical for a specific finite semigroup.

J. M. HOWIE:

Generation by nilpotents in symmetric inverse semigroups

If X is an infinite set then the nilpotent elements of the symmetric inverse semigroup $\mathcal{I}(X)$ generate the subsemigroup $K = \{\alpha \in \mathcal{I}(X) : |X \setminus \mathrm{dom}\,\alpha| = |X \setminus \mathrm{im}\,\alpha| = |X|\}$. In fact every element of K is a product of at most 3 nilpotents of index 2.

Now let I_n be the symmetric inverse semigroup on the finite set $\{1,\ldots,n\}$, let $SP_n = I_n \setminus S_n$ be the inverse semigroup of all proper subpermutations of $\{1,\ldots,n\}$ and let N be the set of nilpotents in I_n. Then $\langle N \rangle = SP_n$ if n is even; if n is odd then $\langle N \rangle = SP_n \setminus W$, where

$$W = \{\alpha \in SP_n : |\mathrm{dom}\,\alpha| \ (= |\mathrm{im}\,\alpha|) = n-1,\ \bar{\alpha} \in S_n \setminus A_n\}.$$

(Here $\bar{\alpha}$ is the unique completion of α to a permutation of $\{1,\dots,n\}$; A_n is the alternating group.)

The rank of I_n (as an inverse semigroup) is 3; the rank of $\langle N \rangle$ is n+1.

P. R. JONES:
Congruence semimodular varieties of semigroups

The universal algebraists have rather beaten to death the concept of a congruence modular variety of algebras. (Each member of such a variety must have a modular lattice of congruences). The purpose of this talk is to show that there are significant congruence semimodular varieties of semigroups. (A lattice is semimodular if $a \succ a \wedge b$ implies $a \vee b \succ b$, where $\succ$ denotes the covering relation).

In fact we describe all such varieties defined by regular identities. Of relevance here are the "block-group" semigroups. groups.

All congruence semimodular varieties of inverse semigroups and of completely regular semigroups are described, as are all pseudovarieties of finite semigroups.

J. KARHUMÄKI:
The Ehrenfeucht Conjecture: consequences and generalizations

We consider possibilities of generalizing the Ehrenfeucht Conjecture for different kinds of mappings, in particular for finite substitutions and classes of finite transductions. Moreover, we deduce from the conjecture, using a decidability result of Makanin, that the equivalence problem for some classes of finite transductions is decidable as well.

U. KNAUER:

Wreath products of monoids, acts, and graphs

Let R,S be monoids and ${}_R A$, ${}_S B$ left R or S-acts, respectively. By $T = R \times S^A$ denote the wreath product of the monoids R and S by A and by ${}_T(A \times B)$ denote the wreath product of the acts A and B which then becomes a left T-act. Let X,Y be two finite undirected graphs and let M(X), M(Y) be monoids of endomorphisms (i. e., mappings preserving adjacency). The lexicographic product X[Y] is a left $M(X) \times M(Y)^X$-act. $M(X) \times M(Y)^X = M(X[Y])$ has been established for $M = \mathrm{Aut}$ (Sabidussi) and $M = \mathrm{SEnd}$ (Nummert). SEnd denotes endomorphisms which map only adjacent points onto adjacent points. For $X \in \{K_n, C_{2n+1}\}$ (i. e., complete graphs and uneven cycles) we have End X[Y] = Aut X[Y], End X[Y] = SEnd X[Y], SEnd X[Y] = Aut X[Y] iff the respective equality holds for Y.

Call an R-act A regular (inverse) if for all $a \in A$ there exists (exactly) one $e^2 = e \in R$ such that $ea = a$ and $ra = pa$ implies $re = pe$, $r,p \in R$. Regular (inverse) wreath products ${}_T(A \times B)$ of acts are characterized.

G. LALLEMENT:

Some algorithmic problems on semigroup presentations

The talk will discuss algorithmic questions related to the decidability of the word problem for semigroups presented by a single relation.

W. LEX:

Remarks on acts and the lattice of their torsion theories

Let a G-act be a triple (G,A,δ) where G is a non-empty set, A any set and δ a mapping from G×A into A. After a sketch of the torsion theory for G-acts developed in [1] the following results, amongst others, are given:

I. The torsion theories, i. e. pairs of a torsion class and a corresponding torsionfree class, of the underlying category $\mathfrak{C}$ of G-acts form a complete lattice V. - Those theories of $\mathfrak{C}$ with abstract classes - i. e. isomorphically closed classes containing the trivial acts - of irreducible acts (without non-trivial subacts) as torsion classes constitute a complete atomistic boolean sublattice of V.

II. Assuming reasonable restrictions for $\mathfrak{C}$ the non-trivial simple abelian groups G are just those groups, operating as permutation groups, where V is a pentagon.

[1] W. Lex, R. Wiegandt: Torsion Theory for Acts. Stud. Sci. Math. Hung., 16(1981), 263 - 280.

E. S. LJAPIN:

Semigroup continuations of partial groupoids

For the partial weakly assocative groupoid P we build a new partial groupoid $\bar{P}$.

For P to have a semigroup continuation it is necessary and sufficient that $\bar{P}$ should have the inner continuation answering certain properties among which the main thing is the condition of dense embedding of the minimal ideal.

These conditions are satisfied if there is a set of partial transformations of the set $\bar{P}$, which must possess certain definite properties.

S. W. MARGOLIS:

Word problems for inverse semigroups

We discuss some automata theoretic methods for solving word problems for inverse semigroups. We show that if $S = \langle X \mid u_i = v_i,\ i = 1,\dots,n\rangle$, where u_i and v_i are positive words with the same length, then the congruence classes are regular sets.

In particular, the inverse semigroup $C_n = \langle \{a_1,\dots,a_n\} \mid a_i a_j = a_j,\ 1 \leq i \neq j \leq n\rangle$ has a solvable word problem.

D. B. McALISTER:

Quasi-ideal extensions of regular semigroups

A subset Q of a regular semigroup S is called a quasi-ideal if $QSQ \subseteq Q$. This paper explores the relationship between quasi-ideal extensions S of a regular semigroup Q [i. e. Q is a quasi-ideal of S] and Rees matrix covers of Q [i. e. regular Rees matrix semigroups R such that there is a homomorphism θ of R onto Q which is one to one on each subset eRf of R, $e^2 = e$, $f^2 = f$].

R. B. McFADDEN:

Automated theorem proving applied to the theory of semigroups

An automated reasoning program may be used in many different ways to assist with research in mathematics and related fields. The program used here for research in semigroup theory is ITP (Interactive Theorem Prover), designed at Argonne National Laboratory and at Northern Illinois University. It is a general purpose program, flexible in that it may call upon one inference rule or another, with choice dependent upon a given task in a given environment. Each step is available for scruting by the user; runs may be stopped at any time, and changes may be made as the user dictates.

ITP has been used to generate examples of several types of semigroups, and to provide detail analyses of Green's relations on these examples. It has been used to establish connections between $\mathcal{R}$, $\mathcal{L}$ and $\mathcal{D}$, and between these equivalence relations and regularity. All the proofs generated are readily accessible, and in some cases exhibit novel features. These results are preliminary to the planned project of equipping ITP with sufficient knowledge of semigroup theory to enable it to prove theorems in more depth. A third use has been to use the defining axioms for a local semilattice to generate some facts about free such objects. The results, while elementary, are encouraging.

J. MEAKIN:

The E-unitary problem for inverse semigroup presentations

A construction is provided of a universal object in the category of X-generated E-unitary inverse semigroups with prescribed maximum group image. Some graphical techniques from combinatorial group theory are employed to study the question of when an inverse monoid presented by generators and relations is E-unitary and these techniques are applied to the study of the word problem for a class of one-relator inverse monoids.

W. D. MUNN:

Inverse semigroup algebras

For a field F and a semigroup S we denote the semigroup algebra of S over F by F[S].

Since inverse semigroups have many group-like properties, it is reasonable to conjecture that the following generalizations of well-known theorems on group algebras hold.

(i) Let F be a field and let S be an inverse semigroup in which no subgroup has an element of order p if F has prime characteristic p. Then F[S] has no non-zero nil ideals.

(ii) Let F be a field which is not algebraic over its prime subfield and let S be an inverse semigroup in which no subgroup has an element of order p if F has prime characteristic p. Then the Jacobson radical of F[S] is zero.

In 1976, A. I. Domanov estiablished (ii). Some special cases of (i) were obtained by the author in 1982 and (subject to scruting!) a full proof of (i) has recently been found. From (i) it is possible to deduce (ii) by exactly the method used for group algebras.

A. NAGY:

Subdirect irreducible WE-2 semigroups with globally idempotent core

It is a very natural thing to study subdirect products of semigroups and the subdirect irreducible semigroups. In the literature there are a lot of theorems about the exponential semigroups and it's generalizations, for example, the WE-m semigroups which make possible to describe the subdirect irreducible WE-2 semigroups with globally idempotent core.

The object of this talk is to give a short summary of the mentioned theorems and prove the main theorem which characterize the subdirect irreducible WE-2 semigroups with globally idempotent core. The theorem we shall establish is that, a semigroup is a subdirectly irreducible WE-2 semigroup with globally idempotent core if and only if it is either a subgroup of a p^{∞}-group (p is a prime) or a subdirect irreducible band. This theorem reduces the problem of finding all subdirectly irreducible WE-2 semigroups with globally idempotent core to the problem of finding all subdirect irreducible bands.

J. OKNIŃSKI:

Semigroup rings with chain conditions

Semigroup rings satisfying certain classical chain conditions are considered. In particular, we discuss necessary and sufficient conditions for a semigroup ring to be artinian, noetherian and to have the Krull dimension (in the sense of Gabriel). In the commutative case relations with the classical Krull dimension are presented. The results involve some interesting invariants of semigroups.

F. OTTO:

On groups having finite monadic Church-Rosser presentations

Thue systems are used as part of monoid-presentations to describe monoids and groups. So it is only natural to ask about the descriptive power of the various restricted versions of Thue systems. Here the ultimate goal would be to establish a close relationship between algebraic properties of monoids and combinatorial properties of the systems presenting these monoids. So far only a very few first results into this direction could be obtained.

Based on the work of Muller and Schupp on context-free groups, and more specific on the work of Haring-Smith on groups having a presentation the reduced word problem for which is a simple language, it is shown that a group G can be defined by a presentation involving a finite two-monadic Church-Rosser Thue system iff G is the free product of a finitely generated free group and finitely many finite groups. It has been conjectured that this very characterization also holds for the class of all presentations involving finite monadic Church-Rosser systems, but so far this conjecture is still open.

I. PEÁK:

Characteristic semigroups of Mealy automata

It is known, that the characteristic semigroup S(A) of an automaton A = (A,X,δ) without ouputs is very useful in the description of the structure and behaviour of such automata. To a Mealy automaton A = (A,X,Y,δ,λ) we can associate a characteristic semigroup S(A) (= S(A*)) of the projection A* = (A,X,δ) of A, but, using S(A), we can study the state behaviour of A only.

Therefore, in 1978 we have extended the concept of the characteristic semigroup to Mealy automata with the purpose, that the new characteristic semigroup S'(A) of a Mealy automaton let be able to reflect the output behaviour of Mealy automata, too.

In the next years several results were published in this area. In our lecture we give an overview on some recent results concerning the new characteristic semigroup of a Mealy automaton and on the connections between S(A) and S'(A) for an arbitrary Mealy automaton A.

D. PERRIN:

Finite monoids and automata on infinite objects

The theory of automata on infinite objects has been developped since 1960 by R. Büchi, R. McNaughton and M. Rabin. I will show how a number of concepts and results of this theory are well explained by considering finite monoids instead of automata.

G. POLLÁK:

A syntactic method in the theory of semigroup varieties

Let X be a countable alphabet, $F = F(X)$ the free semigroup and F^0 the free monoid over X.

Definition 1. $a \lhd b$ (a is embeddable in b) iff $b = b'a\varphi b''$ for some $b', b'' \in F^0$, $\varphi \in \operatorname{End} F$.

Definition 2. If $A, B \subset F$ then $A \lhd B$ (B can be replaced by A) iff $\forall a \in A\ \exists b \in B (a \lhd b)$.

Definition 3. A set $D \subset F$ is said to be d-independent if $D = \{u_i = u_i' u_i'' u_i'''; \ u_i'' \in F,\ u_i', u_i''' \in F^0\}$ and $u_j = vu_i\varphi v'$ implies that either $u_j' = vu_i\varphi w$ or $u_j''' = wu_i\varphi v'$ $(w \in F^0)$.

$\mathbb{D}$ denotes the set of all infinite d-independent sets.

Problem: Find $\inf_{\lhd} \mathbb{D}$ (if it exists).

This is motivated by an attempt to standardize a method often used for constructing infinite independent systems of identities. As an approach, a part of $\inf_{\lhd} \mathbb{D}$ is found in the form of infima of two large subsets of $\mathbb{D}$. Also a conjecture concerning a third subset is formulated. If this later on turns out to be true, then the Problem seems hopeful.

N. R. REILLY:

The lattice of varieties of inverse semigroups

The first layers of the lattice L of varieties of inverse semigroups (consisting of varieties of groups, semilattices of groups or subdirect products of Brandt semigroups) are familiar and constitute a well behaved ideal of L. Two natural classes in which to try to extend the study of L are (i) those varieties for which every member is completely semisimple and for which H is a congruence and (ii) those varieties which are generated by simple objects such as groups, semilattices and monogenic inverse semigroups. Some questions relating to these classes will be considered: are there minimal elements outside the first class and is the second class convex?

C. REUTENAUER:

Contribution to the Burnside problem for semigroups

A semigroup S possesses the permutation property if for some $n \geq 2$, one has: $\forall\ s_1, \dots, s_n \in S$, $\exists\ \sigma \in \mathfrak{S}_n \setminus \{id\}$ such that $s_1 \dots s_n = s_{\sigma(1)} \dots s_{\sigma(n)}$. The following conditions are equivalent:

(i) S is finite.

(ii) S is finitely generated, has the permutation property, and is a torsion semigroup (i. e. each s in S generates a finite subsemigroup).

J. SAKAROVITCH:

Freeness, rationality and unambiguity

Say a monoid M is fair if

C1. the rational sets of M form a Boolean algebra,

C2. the rational sets of M are all unambiguous.

From Kleene's theorem follows that a finitely generated (f. g.) free monoid is fair. A result of M. Benois implies that a f. g. free group is fair. It is also known that a f. g. free commutative monoid is fair.

Theorem: If M and N are two fair monoids, then the free product M*N is fair.

From this one deduces that a f. g. free partially commutative monoid is fair iff it is a free product of free commutative monoids, and one can also deduce that the free group is fair.

It is an open problem to know whether one of the conditions C1 or C2 implies the other.

In the same area let me mention that G. Sénizergues very recently establishes a conjecture of mine, that can be considered as a generalization of Kleene's theorem for free groups.

Theorem: The index of a non disjunctive rational set of the free group is finite.

A. SALOMAA:

Ehrenfeucht's conjecture: Background and proof

Ehrenfeucht's conjecture has been very important in certain areas of automata and language theory during the past 15 years. The conjecture can be stated as follows. Every language $L \subseteq \Sigma^*$ possesses a finite subset $F \subseteq L$ such that, whenever two morphisms $g,h: \Sigma^* \to \Delta^*$ satisfy $g(w) = h(w) \ \forall \ w \in F$, then they also satisfy $g(w) = h(w) \ \forall \ w \in L$.

At least seven different proofs have been presented during the past year, probably the first one being due to V. Guba in Moscow, the lecture discusses the background of the problem, gives examples of some early attempts for a proof, and outlines one proof in more detail.

B. M. SCHEIN:

Semigroups and groups with small doubling

The results of this talk are obtained in collaboration with Gregory Freiman (Tel Aviv University). They represent an attempt to consider for groups and semigroups problems with a number-theoretic flavor (namely, inverse problems of the additive number theory). The direct problems of the additive number theory consider the sets of numbers (usually, integers) which are sums of a given number of summands of a given form (a typical example is the Waring problem). Here is one of the typical problems considered by us. Let A be a subset of a semigroup (or group) S. Then $A^2 = A \cdot A$ is called the doubling of A. If $|A| = n > 0$, then $1 \leq |A^2| \leq n^2$. We say that S is a semigroup with small doubling if $|A^2| \leq 2$ for every two-element subset A of S. We consider the structure of such semigroups and groups as well as analogous problems.

K. D. SCHMIDT:
Lattice-ordered partial semigroups

An intersting problem posed in Birkhoff's book is the following: "Develop a common abstraction which includes Boolean algebras (rings) and lattice ordered groups as special cases." Here we propose a solution to Birkhoff's problem which is motivated by a problem in the theory of measure and integration and which is partly inspired by the work of Dinges: We consider lattices on which a partial addition is defined such that order and addition are compatible. This concept is formalized in the notion of a lattice ordered partial semigroup (LOPS), which can be used to give a common approach to vector measures on a Boolean ring and to linear operators on a vector lattice.

M. P. SCHÜTZENBERGER:
The plactic monoid

We consider: A set of variables $A = \{a < b < \dots\}$, the free algebra $\mathbb{Z}(A^*)$ generated by A, the evaluation morphism Ev from $\mathbb{Z}(A^*)$ onto the free commutative algebra on the same variables and the subring of $(\mathbb{Z}(A^*))$Ev made up of the symmetric polynomials, say Sym(A^*). We look for a quotient $A^*/_{\equiv}$ of the free monoid A^* such that the inverse image (by Ev^{-1}) of Sym(A^*)

in the algebra $\mathbb{Z}(A^*/_{\equiv})$ be a commutative algebra. It turns out that the problem admitts very few solutions. One is the plactic monoid which gives an isomorphism with the ring of representations of the linear (or symmetric) group.

H. J. SHYR:

Decomposition of languages into disjunctive outfix codes

Let X be a finite alphabet and let X* be the free monoid generated by X. Let $X^+ = X^* \setminus \{1\}$, where 1 is the empty word. A language $L \subseteq X^+$ is an outfix code if uv, $uxv \in L$ implies that $x = 1$, $x,u,v \in X^*$. Every outfix code is a prefix and suffix code and hence a code. By regular free language we mean that every regular language contained in it is finite. The family of outfix codes is a subfamily of the family of regular free languages. We say that a language L is disjunctive outfix splittable if L is a disjoint union of infinitely many disjunctive outfix codes. Shyr has shown that every right ideal of X* with infinite prefix root is a disjoint union of infinitely many regular free disjunctive prefix codes. By using the concept of ordered catenation introduced by Shyr, we investigate those languages which are disjunctive outfix splittable. In particular we show that every right ideal of X* which is not principal is disjunctive outfix splittable. This result is a generalization of the result done by Shyr.

J.-C. SPEHNER:

Every finitely generated submonoid of a free monoid has a finite Malcev's presentation

There exist finitely generated submonoids of a free monoid which are not finitely presented and have even not a finite cancellative presentation.

If Σ^* is the free monoid on the alphabet Σ and if $\varrho \subset \Sigma^* \times \Sigma^*$, let $m(\varrho)$ be the smallest congruence of Σ^* containing ϱ such that the monoid $\Sigma^*/m(\varrho)$ can be embedded in a group. If a monoid M is isomorphic to $\Sigma^*/m(\varrho)$, then (Σ,ϱ) is said to be a Malcev's presentation of M.

We shall prove here the following theorem "Every finitely generated submonoid of a free monoid has a finite Malcev's presentation and such a presentation can be effectively found".

The necessary and sufficient conditions for embedding a semigroup in a group given by A. I. Malcev and a graph for determining the relators are used to prove this theorem.

H. STRAUBING and D. THÉRIEN:

Application of categories in the theory of semigroups and automata I, II

A category can be viewed as a generalized monoid, in which the arrows (or morphisms) correspond to elements of the monoid, and only certain products of elements are defined. (From this point of view, a monoid is a category having only a single object.) We discuss the extension of a number of ideas from semigroup theory to the domain of categories (a theory recently developed in detail by B. Tilson), as well as a number of applications to problems involving recognizable languages. These include a partial solution to the problem of effectively determining if a given star-free language has dot-depth 2, and the characterization of varieties of languages closed under unambiguous products.

M. B. SZENDREI:

Certain relatively free *-orthodox semigroups

For any variety $\mathcal{U}$ of *-bands, one can consider the class $\mathcal{OU}$ of all *-orthodox semigroups which have a band of idempotents belonging to $\mathcal{U}$. The class $\mathcal{OU}$ is easily seen to be a variety of *-semigroups. In particular, if $\mathcal{U}$ equals $\mathcal{B}$, the variety of all *-bands, $\mathcal{NB}$, that of normal *-bands and $\mathcal{S}$, that of semilattices, then $\mathcal{OU}$ is just the variety of all *-orthodox semigroups, that of all generalized inverse *-semigroups and that of all inverse semigroups, respectively. The free objects in $\mathcal{OB}$, $\mathcal{ONB}$ and $\mathcal{OS}$ were described by the author (1985) and by Scheiblich (1982 and 1973), respectively. For any variety $\mathcal{U}$ of *-bands, we present here a solution of the word problem of the free objects in $\mathcal{OU}$. This result enables us to represent the free object of $\mathcal{OU}$ on a set X as a well-determined subsemigroup of a semidirect product of the free object of $\mathcal{U}$ on the set $\{\{g,gx\}: x \in X, g \in G_X\}$ by the free group G_X on X. In the case of free inverse semigroups this interpretation is closely related to their P-representation.

G. THIERRIN:

Languages induced by certain homomorphisms of a free monoid

Let X be a nonempty set and X* be the free monoid on X. To every equivalence relation ϱ on X* one may associate the family of all languages over X which are saturated by ϱ. Some interesting families of languages may be obtained in this way. A language belonging to such a family has the property of being closed under certain transformations of the words which reflect the identities valid in the corresponding homomorphic image. By varying these transformations, we obtain further languages by the requirement that they be closed under all such transformations.

W. THOMAS:

Concatenation games

We deal with connections between formal language theory and mathematical logic as they appear in the characterization of star-free languages (resp. the dot-depth hierarchy) in terms of first-order logic (resp. the quantifier alternation hierarchy for formulas in prenex normal form). It is shown that in a slightly more general context the correspondence between concatenation and existential quantification fails: There is a regular language L_0 such that the star-free closure of L_0 is distinct from the "first-order closure" of L_0 (i. e. the class of languages that are first-order definable in terms of atomic formulas "segment from x to y belongs to L_0"). The proof involves a concatenation game derived from the Ehrenfeucht-Fraïssé game of mathematical logic. Modifications of the game are mentioned which have been (or might be) useful for proving other hierarchy results in language theory, concerning, e. g., the dot-depth hierarchy, its refinements, or the generalized star-height problem.

P. G. TROTTER:

Injectives in varieties of completely regular semigroups

Let V be a variety of completely regular semigroups. Define $I \in V$ to be a V-injective if and only if for each $S,T \in V$ such that S is a subsemigroup of T, any homomorphism from S into I extends to a homomorphism from T into I. In this talk the semigroups that are V-injectives will be described.

H. J. WEINERT:

Generalized semigroup semirings and semialgebras

At first let S be an associative ring and $A = {}_S A$ a free left S-module which is also a (not necessary associative) ring. Then

- in a classical meaning - A is called an algebra over S iff (1) $\alpha(ab) = (\alpha a)b = a(\alpha b)$ holds for all $\alpha \in S$, $a,b \in A$. Let U be a basis of ${}_S A$ and use the unique presentation (2) $a = \Sigma\alpha_u u$ (for all $u \in U$, nearly all $\alpha_u \in S$ are zero) for each $a \in S$. Then the multiplication on A is uniquely determined by the products uv for all $u,v \in U$. Conversely, the latter provides an algebra A over S. Unfortunately, (1) forces that S is commutative apart from some pathological exceptions. In particular, polynomial rings $S[x]$, $S[x_1,x_2,...]$, rings of matrices over S, or (0-restricted) semigroups rings are algebras over S only if S is commutative - a completely useless restriction depending on (1).

A better concept, called a generalized algebra over a ring S which avoids these disadvantages, was considered by H. Zassenhaus (1937), G. Pickert (1947) and the author (1965). The topic of the talk is to present both concepts and to generalize the latter again in two directions: a) The ring S can be replaced by an additively commutative semiring. b) The semimodule ${}_S A$ over the semiring S may consist of infinite sums of the form (2). The resulting concept of generalized semialgebras includes among others polynomial semirings, semirings of formal power series, semirings of matrices as well as those semirings which are used in the theory of formal languages.

Berichterstatter: U. Hebisch

Tagungsteilnehmer

Branka Alimpic
Faculty of Science
Studentski trg 16/IV
Beograd 11 000

Jugoslawien

Prof. Dr. J. Almeida
Centro de Matemåtica
Universidade do Minho

4700 Braga

Portugal

Dr. G. Betsch
Im Furtbrunnen 17

7031 Weil im Schönbuch

Prof. Dr. R. V. Book
Mathematics
UCSB
Santa Barbara, Ca 93106

USA

Prof. Dr. B. Bosbach
Fachbereich Mathematik
GHS Kassel

3500 Kassel

Prof. Dr. M. Demlovå
Dep. of Math.
Fac. Electr. Engni.
Technical University of Prague
Suchbatarova 2

16627 Praha 6
CSSR

Prof. Dr. P. Dubreil
Residence Fremiet
Domaine des Reaux

F-91840 Soisy-sur-Ecole

Frankreich

Prof. Dr. John Fountain
Dept. of Mathematics
University of York
Heslington
York Y01 5DD

Großbritannien

Prof. Dr. M. L. Galvao
Dept. Matemåtica
Fac. Ciencias/Univ. Lisboa
Rua Ernesto de vasconcelos

1700 Lisboa

Portugal

Prof. Dr. S. M. Goberstein
Department of Mathematics
California State University
Chico, CA 95929

USA

Prof. Dr. G. Gomes
Dept. Matemåtica
Fac. Ciencias/Univ. Lisboa
Rua Ernesto de vasconcelos

1700 Lisboa

Portugal

Dr. P. Goralcik
MFFKU
Sokolovskå 83

18600 PRAHA 8

CSSR

Dr. V. Gould
School of Mathematics
University Walk
Bristol BS8 1TW

GB

Prof. Dr. K. Keimel
Fachbereich Mathematik
TH Darmstadt

6100 Darmstadt

Dr. Udo Hebisch
Institut für Mathematik
TU Clausthal
Erzstraße 1

3392 Clausthal-Zellerfeld

Prof. Dr. Ulrich Knauer
Fachbereich Mathematik
Universität Oldenburg
Carl-von-Ossietzky-Straße

2900 Oldenburg

Dr. E. Hotzel
GMD

5205 St. Augustin 1

Dr. V. Koubek
MFFUK
Malostranskê nâm. 25
11200 Praha

CSSR

Prof. Dr. J. M. Howie
Mathematical Inst.
North Haugh
St. Andrews, Fife KY16 9SS

Schottland

Prof. Dr. G. Lallement
Department of Mathematics
Pennsylvania State University
University Park PA 16802

USA

Prof. Dr. P. R. Jones
Department of Mathematics
Statistics and Computer Science
Marquette University
Milwaukee WI 53233

USA

Prof. Dr. L. C. A. van Leeuwen
Dep. of Math.
University of Groningen

GP 800 Groningen

Niederlande

Prof. Dr. Helmut Jürgensen
University of Western Ontario
Department of Computer Science
Middlesex College
London, Canada, N6A 5B7

Kanada

Prof. Dr. Wilfried Lex
Institut für Informatik
Technische Universität Clausthal
Erzstraße 1

3392 Clausthal

Prof. J. Karhumäki
University of Turku
Department of Mathematics
SF 20500 T u r k u 50

Finland

Prof. Dr. E. S. Ljapin
196066 Leningrad

M-66 Moskovskij pr. d. 208, kv. 16

UdSSR

Prof. Dr. S. W. Margolis
Department of Mathematics
University of Nebraska

Lincoln, Nebraska 68588

USA

Prof. Dr. D. B. Mc Alister
Department of Mathematical Sciences
Northern Illinois University
DeKalb, IL 60115

USA

Prof. Dr. R. B. McFadden
Dept.of Mathematical Sciences
Northern Illinois University
De Kalb, Il 60115

USA

Prof. Dr. J. C. Meakin
Department of Mathematics
University of Nebraska

Lincoln, Nebraska 68588

USA

Prof. Dr. W. D. Munn
Department of Mathematics
University of Glasgow
Glasgow G12 8QW

Schottland

Prof. Dr. A. Nagy
Technical University of Budapest
Transport Engineering Faculty
Department of Mathematics

Budapest, Muegyetem rkp. 9. H. êp.

Ungarn

Prof. Dr. J. Okniński
Institute of Mathematics
University of Warsaw

00-901 Warsaw, PKIN

Polen

Prof. Dr. Friedrich Otto
Fachbereich Informatik
Universität Kaiserslautern
Postfach 30 49

6750 Kaiserslautern

Prof. Dr. I. Peák
K. Marx University of Economics
Inst. of Math. and Computer Science
1092 Budapest

Dimitrov tér. 8

Ungarn

Prof. Dr. D. Perrin
Université Paris 7
2 Place Jussien, LITP

75251 Paris Cedex 05

Frankreich

Prof. G. Pollak
Mathematical Institute
Reáltanoda u. 13 - 15

H-1053 Budapest

Ungarn

Prof. Dr. N. R. Reilly
Dept. of Mathematics
Simon Fraser University
Burnaby
British Columbia
Canada V5A 1S6

Kanada

Prof. Dr. C. Reutenauer
Maths-Info
UQAM
CP 8888 succ A
Montreal (Quebec)
Canada H3C 3P8

Kanada

Prof. Dr. J. Sakarovitch
Université Paris 6
LITP
4 place Iussien

75230 Paris CEDEX 05

Frankreich

Prof. Dr. A. Salomaa
University of Turku
Department of Mathematics
SF 20500 T u r k u 50

Finland

Prof. Dr. M. B. Szendrei
Jósef Attila University
Bolyai Institute
Aradi vertanuk tere 1
6720 Szeged

Ungarn

Prof. Dr. Boris M. Schein
Department of Mathematics
University of Arkansas
Fayetteville, AR 72701

USA

Prof. D. Thérien
School of Computer Science
McGill University
Montreal, Quebec H3A 2K6

Kanada

Prof. Dr. K. D. Schmidt
Seminar für Statistik
Universität Mannheim
A 5

6800 Mannheim

Prof. Dr. G. Thierrin
Department of Mathematics
University of Western Ontario
London, Ontario, N6A 5B7

Kanada

Prof. Dr. M. P. Schützenberger
Université Paris 7
2 Place Jussien, LITP

75251 Paris Cedex 05

Frankreich

Prof. Dr. W. Thomas
Lehrstuhl für Informatik II
TH Aachen
Büchel 29-31

5100 Aachen

Prof. Dr. H. J. Shyr
Department of Applied Mathematics
National Chung-Hsing University

Taichung

Taiwan 400

Prof. Dr. P. G. Trotter
Department of Mathematics
University of Tasmania
Hobart
Tasmania

Australien

Prof. Dr. J.-C. Spehner
Université de Haute Alsace
I. S. E. A.
4, sur des Frères Lumière

68093 Mulhouse Cédex

Frankreich

Prof. Dr. H. J. Weinert
Institut für Mathematik
TUC
Erzstraße 1

3392 Clausthal-Zellerfeld

Prof. H. Straubing
Department of Computer Science
Boston College
Chestnut Hill, Massachusetts 02167

USA